Computer algebra independent integration tests

1_Algebraic_functions/1.2_Trinomial_products/1.2.2Quartic/1.2.2.3(d+ex^2)^m(a+bx^2+c

Nasser M. Abbasi

December 15, 2018 Compiled on December 15, 2018 at 2:29am

Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

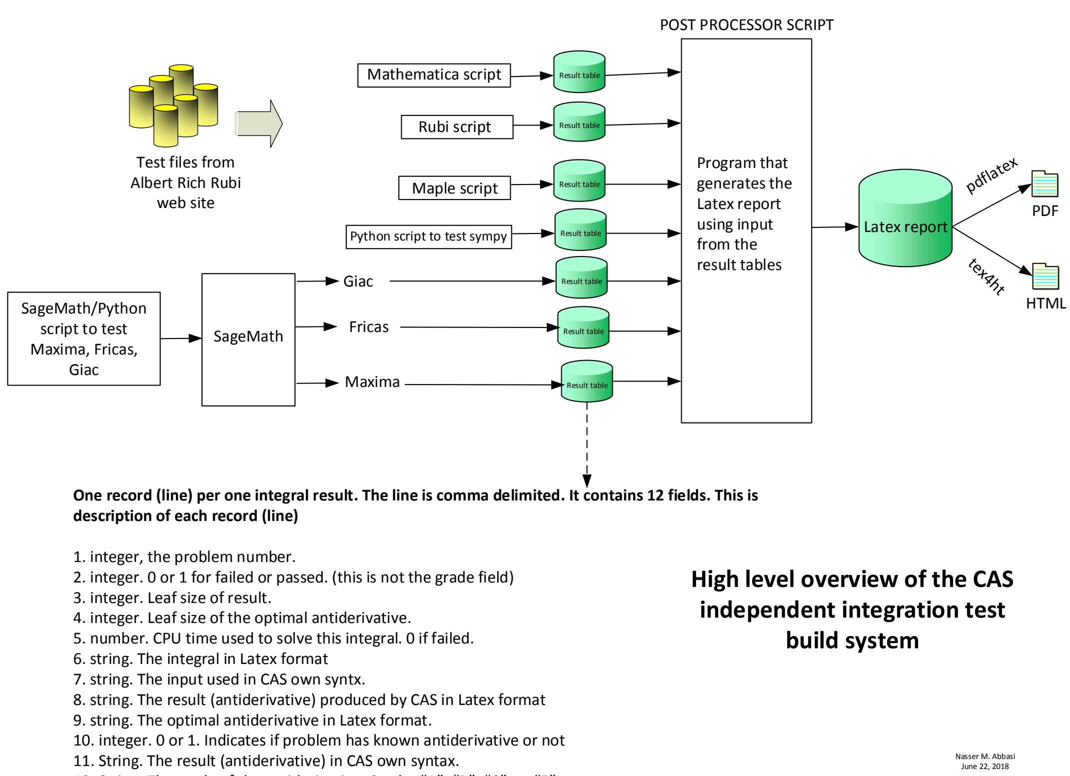
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.76 (408)	% 0.24 (1)
Rubi in Sympy	% 82.15 (336)	% 17.85 (73)
Mathematica	% 98.04 (401)	% 1.96 (8)
Maple	% 96.58 (395)	% 3.42 (14)
Maxima	% 15.4 (63)	% 84.6 (346)
Fricas	% 49.39 (202)	% 50.61 (207)
Sympy	% 45.23 (185)	% 54.77 (224)
Giac	% 36.67 (150)	% 63.33 (259)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

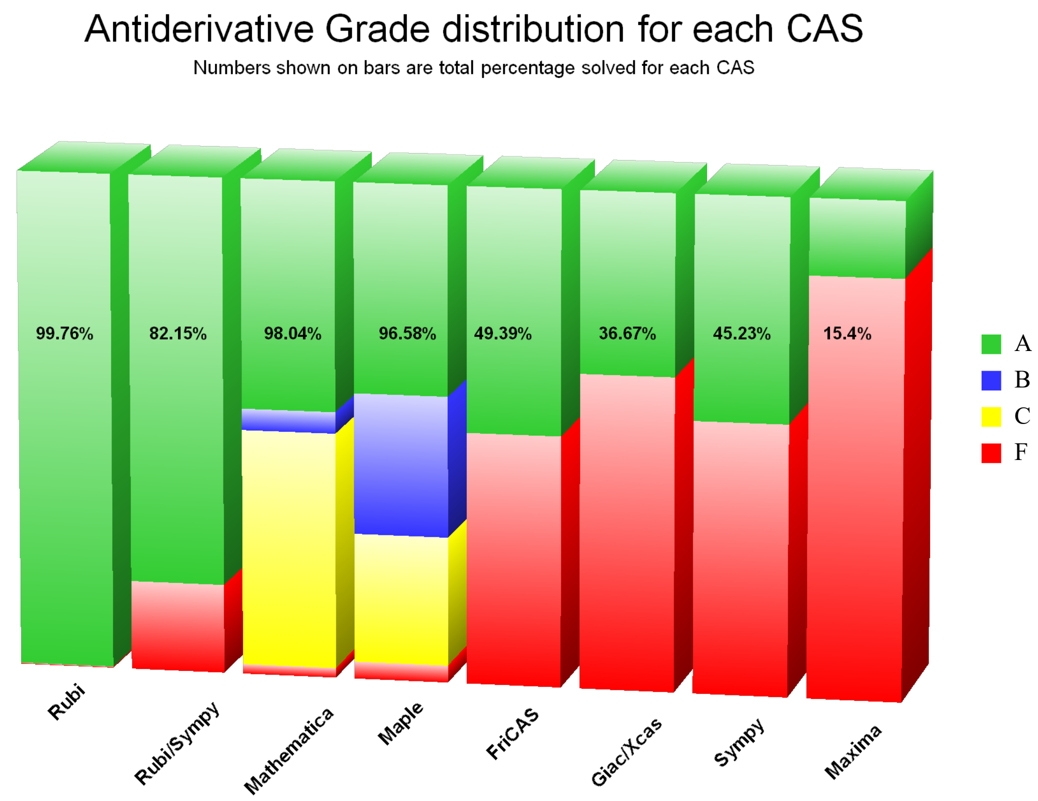
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

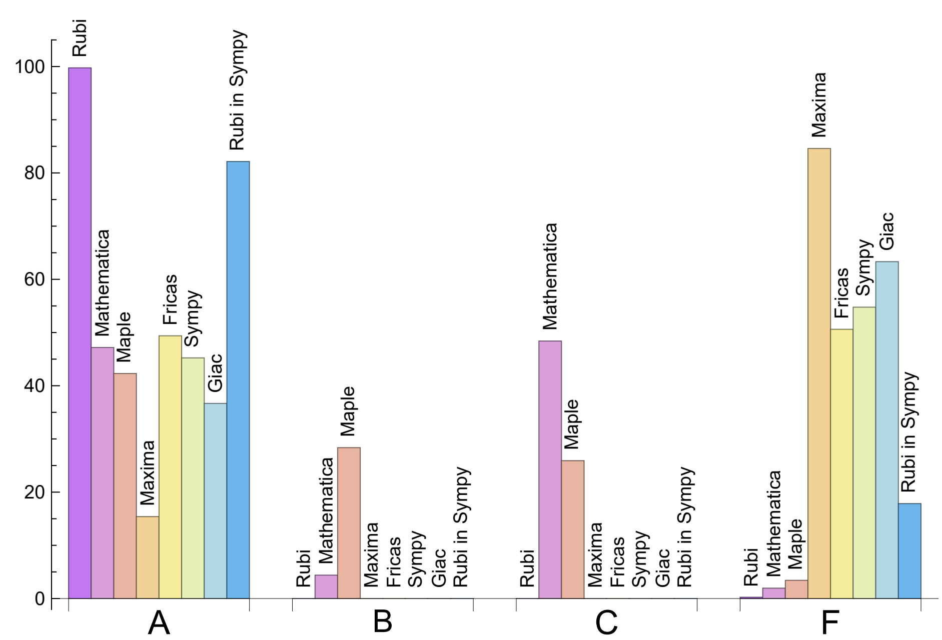
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.76	0.	0.	0.24
Rubi in Sympy	82.15	0.	0.	17.85
Mathematica	47.19	4.4	48.41	1.96
Maple	42.3	28.36	25.92	3.42
Maxima	15.4	0.	0.	84.6
Fricas	49.39	0.	0.	50.61
Sympy	45.23	0.	0.	54.77
Giac	36.67	0.	0.	63.33

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.41	151.88	1.	109.5	1.
Rubi in Sympy	36.97	133.32	1.03	104.	0.93
Mathematica	0.46	173.15	1.32	112.	1.
Maple	0.03	367.44	2.28	165.	1.34
Maxima	0.73	75.76	1.36	45.	1.2
Fricas	2.02	570.91	2.64	1.	0.03
Sympy	8.49	150.24	1.62	78.	1.09
Giac	0.43	125.03	1.34	58.5	1.19

1.8 list of integrals that has no closed form antiderivative

{179, 403, 408, 409}

1.9 list of integrals not solved by each system

Not solved by Rubi {178}

Not solved by Rubi in Sympy {1, 2, 3, 4, 5, 6, 7, 8, 128, 129, 130, 131, 136, 137, 138, 139, 140, 145, 147, 157, 178, 190, 191, 192, 217, 218, 219, 234, 235, 241, 246, 247, 248, 249, 250, 251, 256, 257, 258, 259, 260, 267, 268, 269, 273, 274, 278, 279, 294, 296, 303, 308, 309, 310, 317, 319, 327, 335, 342, 351, 358, 359, 366, 367, 374, 382, 383, 389, 393, 404, 405, 408, 409}

Not solved by Mathematica {7, 8, 178, 184, 185, 190, 191, 192}

Not solved by Maple {178, 180, 181, 182, 183, 184, 185, 190, 191, 192, 404, 405, 406, 407}

Not solved by Maxima {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 53, 54, 56, 57, 58, 61, 62, 63, 67, 68, 70, 71, 72, 75, 76, 77, 78, 79, 83, 86, 87, 88, 89, 90, 91, 95, 98, 99, 105, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 132, 133, 134, 135, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 252, 253, 254, 255, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407}

Not solved by Fricas {1, 2, 3, 4, 5, 6, 7, 8, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 110, 112, 113, 119, 120, 121, 122, 123, 124, 125, 126, 127, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 267, 272, 273, 278, 279, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407}

Not solved by Sympy {1, 2, 3, 4, 7, 8, 30, 32, 110, 111, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 150, 151, 156, 157, 161, 162, 166, 167, 169, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 184, 185, 186, 187, 190, 191, 192, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 263, 264, 267, 268, 272, 273, 274, 275, 278, 279, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409}

Not solved by Giac {1, 2, 3, 4, 5, 6, 7, 8, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 53, 54, 56, 57, 58, 67, 68, 70, 71, 72, 76, 88, 108, 109, 110, 111, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 197, 198, 199, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 272, 274, 275, 276, 277, 278, 279, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {1, 7, 162, 384, 385, 388}

Mathematica {3, 4, 5, 6, 22, 24, 31, 106, 107, 121, 122, 123, 124, 126, 404, 405, 406, 407}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	275	251	0	0	0	0	0
normalized size	1	1.	0.6	0.55	0.	0.	0.	0.	0.
time (sec)	N/A	1.034	1.668	0.022	0.	0.	0.	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	719	247	0	0	0	0	0
normalized size	1	1.	3.3	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.604	2.008	0.039	0.	0.	0.	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	685	327	0	0	0	0	0
normalized size	1	1.	10.54	5.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.226	2.566	0.236	0.	0.	0.	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	1137	311	0	0	0	0	0
normalized size	1	1.	18.05	4.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.228	6.281	0.232	0.	0.	0.	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	623	336	0	0	0	0	0
normalized size	1	1.	8.65	4.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.197	2.403	0.225	0.	0.	11.305	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	1198	337	0	0	0	0	0
normalized size	1	1.	17.11	4.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.198	6.222	0.217	0.	0.	11.224	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	561	561	0	437	0	0	0	0	0
normalized size	1	1.	0.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	1.4	0.836	0.034	0.	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	527	527	0	439	0	0	0	0	0
normalized size	1	1.	0.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	1.856	0.866	0.057	0.	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	183	260	0	1035	109	325	230
normalized size	1	1.	0.74	1.05	0.	4.19	0.44	1.32	0.93
time (sec)	N/A	0.326	0.123	0.009	0.	0.326	1.914	0.307	61.89

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	184	260	0	1035	110	325	230
normalized size	1	1.	0.74	1.05	0.	4.19	0.45	1.32	0.93
time (sec)	N/A	0.28	0.087	0.005	0.	0.317	2.743	0.293	62.351

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	95	122	0	1019	110	347	76
normalized size	1	1.	1.1	1.42	0.	11.85	1.28	4.03	0.88
time (sec)	N/A	0.101	0.047	0.006	0.	0.332	1.998	0.291	16.019

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	95	122	0	1019	110	347	76
normalized size	1	1.	1.1	1.42	0.	11.85	1.28	4.03	0.88
time (sec)	N/A	0.089	0.035	0.003	0.	0.318	2.812	0.286	16.802

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	33	122	53	38	41	70	32
normalized size	1	1.	0.82	3.05	1.32	0.95	1.02	1.75	0.8
time (sec)	N/A	0.042	0.019	0.009	0.848	0.288	0.236	0.274	5.347

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	44	82	53	55	49	54	46
normalized size	1	1.	0.86	1.61	1.04	1.08	0.96	1.06	0.9
time (sec)	N/A	0.048	0.021	0.003	0.836	0.285	0.194	0.276	12.344

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	32	13	34	42	32	39	15
normalized size	1	1.	2.	0.81	2.12	2.62	2.	2.44	0.94
time (sec)	N/A	0.011	0.023	0.002	0.812	0.301	0.166	0.27	2.62

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	16	15	16	15
normalized size	1	1.	1.	0.81	1.	1.	0.94	1.	0.94
time (sec)	N/A	0.009	0.008	0.004	0.862	0.28	0.178	0.268	2.582

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	254	0	1	138	0	73
normalized size	1	1.	0.8	3.39	0.	0.01	1.84	0.	0.97
time (sec)	N/A	0.085	0.043	0.006	0.	0.313	1.928	0.	13.192

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	91	254	0	1	131	0	100
normalized size	1	1.	0.86	2.4	0.	0.01	1.24	0.	0.94
time (sec)	N/A	0.088	0.042	0.006	0.	0.33	1.942	0.	37.836

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	290	0	1	87	300	73
normalized size	1	1.	0.8	3.87	0.	0.01	1.16	4.	0.97
time (sec)	N/A	0.097	0.051	0.013	0.	0.289	0.464	0.278	12.574

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	75	290	0	1	80	300	83
normalized size	1	1.	0.83	3.22	0.	0.01	0.89	3.33	0.92
time (sec)	N/A	0.085	0.036	0.004	0.	0.291	1.284	0.277	27.973

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	18	23	23	22	26	14
normalized size	1	1.	1.92	1.38	1.77	1.77	1.69	2.	1.08
time (sec)	N/A	0.018	0.009	0.007	0.861	0.278	0.418	0.269	4.45

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	27	100	0	0	70	0	15
normalized size	1	1.	1.69	6.25	0.	0.	4.38	0.	0.94
time (sec)	N/A	0.059	0.047	0.017	0.	0.	3.62	0.	12.047

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	46	99	0	0	70	0	34
normalized size	1	1.	1.31	2.83	0.	0.	2.	0.	0.97
time (sec)	N/A	0.112	0.05	0.008	0.	0.	4.038	0.	26.392

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	54	107	0	0	61	0	39
normalized size	1	1.	1.26	2.49	0.	0.	1.42	0.	0.91
time (sec)	N/A	0.083	0.047	0.017	0.	0.	3.644	0.	18.211

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	73	108	0	0	60	0	82
normalized size	1	1.	0.82	1.21	0.	0.	0.67	0.	0.92
time (sec)	N/A	0.151	0.05	0.008	0.	0.	3.942	0.	32.269

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	52	120	0	0	66	0	76
normalized size	1	1.	0.58	1.35	0.	0.	0.74	0.	0.85
time (sec)	N/A	0.052	0.064	0.019	0.	0.	3.863	0.	7.226

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	51	120	0	0	66	0	134
normalized size	1	1.	0.34	0.79	0.	0.	0.43	0.	0.88
time (sec)	N/A	0.1	0.051	0.006	0.	0.	3.397	0.	13.228

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	79	122	0	0	70	0	80
normalized size	1	1.	0.88	1.36	0.	0.	0.78	0.	0.89
time (sec)	N/A	0.053	0.07	0.016	0.	0.	4.017	0.	8.116

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	78	122	0	0	71	0	139
normalized size	1	1.	0.5	0.78	0.	0.	0.46	0.	0.89
time (sec)	N/A	0.104	0.056	0.005	0.	0.	3.598	0.	14.539

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	15	0	0	0	0	8
normalized size	1	1.	1.	1.5	0.	0.	0.	0.	0.8
time (sec)	N/A	0.034	0.037	0.035	0.	0.	0.	0.	9.08

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	31	118	0	0	71	0	8
normalized size	1	1.	3.1	11.8	0.	0.	7.1	0.	0.8
time (sec)	N/A	0.056	0.049	0.019	0.	0.	3.698	0.	13.462

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	24	28	0	0	0	0	20
normalized size	1	1.	1.04	1.22	0.	0.	0.	0.	0.87
time (sec)	N/A	0.094	0.035	0.016	0.	0.	0.	0.	21.18

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	52	117	0	0	71	0	20
normalized size	1	1.	2.26	5.09	0.	0.	3.09	0.	0.87
time (sec)	N/A	0.113	0.055	0.01	0.	0.	4.142	0.	27.207

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	181	71	0	1	122	1	73
normalized size	1	1.	2.21	0.87	0.	0.01	1.49	0.01	0.89
time (sec)	N/A	0.188	0.183	0.037	0.	0.287	1.32	0.454	17.748

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	181	71	0	1	122	1	73
normalized size	1	1.	2.21	0.87	0.	0.01	1.49	0.01	0.89
time (sec)	N/A	0.204	0.184	0.037	0.	0.283	1.37	0.453	17.776

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	189	75	0	1	110	1	75
normalized size	1	1.	2.42	0.96	0.	0.01	1.41	0.01	0.96
time (sec)	N/A	0.174	0.181	0.033	0.	0.293	1.487	0.439	18.13

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	189	75	0	1	121	1	75
normalized size	1	1.	2.2	0.87	0.	0.01	1.41	0.01	0.87
time (sec)	N/A	0.195	0.18	0.031	0.	0.283	1.354	0.439	18.198

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	182	88	0	1	121	1	66
normalized size	1	1.	2.33	1.13	0.	0.01	1.55	0.01	0.85
time (sec)	N/A	0.104	0.227	0.022	0.	0.29	2.15	0.453	31.47

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	182	69	0	1	110	1	66
normalized size	1	1.	2.33	0.88	0.	0.01	1.41	0.01	0.85
time (sec)	N/A	0.089	0.239	0.022	0.	0.29	2.275	0.454	31.567

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	190	61	0	1	112	1	66
normalized size	1	1.	2.71	0.87	0.	0.01	1.6	0.01	0.94
time (sec)	N/A	0.083	0.235	0.023	0.	0.278	2.234	0.44	32.386

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	190	61	0	1	112	1	66
normalized size	1	1.	2.71	0.87	0.	0.01	1.6	0.01	0.94
time (sec)	N/A	0.092	0.231	0.022	0.	0.291	2.205	0.441	32.234

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	250	582	0	1	158	1	110
normalized size	1	1.	1.87	4.34	0.	0.01	1.18	0.01	0.82
time (sec)	N/A	0.228	0.271	0.086	0.	0.276	2.82	0.703	46.312

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	248	582	0	1	160	1	121
normalized size	1	1.	1.91	4.48	0.	0.01	1.23	0.01	0.93
time (sec)	N/A	0.309	0.21	0.03	0.	0.292	2.791	0.703	32.108

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	248	582	0	1	160	1	121
normalized size	1	1.	1.91	4.48	0.	0.01	1.23	0.01	0.93
time (sec)	N/A	0.259	0.071	0.003	0.	0.286	2.841	0.706	39.126

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	34	34	26	34	26
normalized size	1	1.	1.	0.9	1.17	1.17	0.9	1.17	0.9
time (sec)	N/A	0.051	0.03	0.01	0.733	0.28	1.874	0.278	24.184

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	138	52	0	1	117	69	58
normalized size	1	1.	2.3	0.87	0.	0.02	1.95	1.15	0.97
time (sec)	N/A	0.129	0.347	0.012	0.	0.296	1.225	0.275	17.344

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	126	277	0	1	95	1	49
normalized size	1	1.	2.03	4.47	0.	0.02	1.53	0.02	0.79
time (sec)	N/A	0.108	0.098	0.051	0.	0.279	0.792	0.325	12.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	134	277	0	1	83	1	49
normalized size	1	1.	2.03	4.2	0.	0.02	1.26	0.02	0.74
time (sec)	N/A	0.109	0.096	0.034	0.	0.294	0.834	0.318	12.583

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	83	136	0	35	42	53	70
normalized size	1	1.	1.84	3.02	0.	0.78	0.93	1.18	1.56
time (sec)	N/A	0.115	0.123	0.06	0.	0.282	0.257	0.276	9.451

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	12	15	26	22	15	10
normalized size	1	1.	1.13	0.8	1.	1.73	1.47	1.	0.67
time (sec)	N/A	0.024	0.011	0.013	0.805	0.284	0.215	0.271	7.809

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	15	15	14	15	14
normalized size	1	1.	1.	0.86	1.07	1.07	1.	1.07	1.
time (sec)	N/A	0.015	0.005	0.003	0.803	0.281	0.183	0.27	4.611

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	97	34	45	38	44	45	42
normalized size	1	1.	2.55	0.89	1.18	1.	1.16	1.18	1.11
time (sec)	N/A	0.071	0.313	0.01	0.828	0.281	0.251	0.27	8.454

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	99	40	0	32	42	0	49
normalized size	1	1.	2.06	0.83	0.	0.67	0.88	0.	1.02
time (sec)	N/A	0.08	0.164	0.036	0.	0.28	0.245	0.	9.498

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	97	40	0	38	44	0	49
normalized size	1	1.	2.11	0.87	0.	0.83	0.96	0.	1.07
time (sec)	N/A	0.079	0.384	0.034	0.	0.271	0.248	0.	8.562

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	18	23	20	14	62	15
normalized size	1	1.	0.81	0.86	1.1	0.95	0.67	2.95	0.71
time (sec)	N/A	0.029	0.01	0.007	0.808	0.283	0.19	0.27	5.036

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	101	40	0	35	42	0	49
normalized size	1	1.	2.2	0.87	0.	0.76	0.91	0.	1.07
time (sec)	N/A	0.078	0.476	0.036	0.	0.272	0.251	0.	8.742

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	99	40	0	28	29	0	46
normalized size	1	1.	2.25	0.91	0.	0.64	0.66	0.	1.05
time (sec)	N/A	0.068	0.171	0.039	0.	0.29	0.232	0.	8.727

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	14	20	0	20	12	0	19
normalized size	1	1.	0.61	0.87	0.	0.87	0.52	0.	0.83
time (sec)	N/A	0.05	0.011	0.035	0.	0.303	0.212	0.	8.347

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	12	11	16	16	8	16	8
normalized size	1	1.	1.09	1.	1.45	1.45	0.73	1.45	0.73
time (sec)	N/A	0.01	0.008	0.009	0.732	0.292	0.157	0.27	6.049

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	30	39	34	26	45	29
normalized size	1	1.	0.74	0.77	1.	0.87	0.67	1.15	0.74
time (sec)	N/A	0.041	0.01	0.013	0.738	0.282	0.206	0.272	8.978

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	82	0	65	46	104	48
normalized size	1	1.	0.95	1.86	0.	1.48	1.05	2.36	1.09
time (sec)	N/A	0.072	0.021	0.048	0.	0.277	0.208	0.308	8.804

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	127	279	0	1	94	1	53
normalized size	1	1.	1.92	4.23	0.	0.02	1.42	0.02	0.8
time (sec)	N/A	0.063	0.123	0.022	0.	0.291	0.783	0.327	16.135

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	84	136	0	34	39	53	68
normalized size	1	1.	1.83	2.96	0.	0.74	0.85	1.15	1.48
time (sec)	N/A	0.074	0.122	0.019	0.	0.275	0.247	0.278	9.422

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	12	10	12	23	14	12	7
normalized size	1	1.	1.33	1.11	1.33	2.56	1.56	1.33	0.78
time (sec)	N/A	0.023	0.011	0.009	0.841	0.315	0.215	0.269	7.907

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	15	15	7	15	8
normalized size	1	1.	1.	1.	1.36	1.36	0.64	1.36	0.73
time (sec)	N/A	0.01	0.007	0.008	0.732	0.275	0.154	0.271	6.092

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	34	34	26	34	26
normalized size	1	1.	1.	0.9	1.17	1.17	0.9	1.17	0.9
time (sec)	N/A	0.033	0.01	0.005	0.745	0.282	0.192	0.271	11.41

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	62	46	0	46
normalized size	1	1.	0.84	0.78	0.	1.24	0.92	0.	0.92
time (sec)	N/A	0.048	0.02	0.014	0.	0.28	0.2	0.	13.546

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	59	46	0	46
normalized size	1	1.	0.84	0.78	0.	1.18	0.92	0.	0.92
time (sec)	N/A	0.046	0.022	0.015	0.	0.278	0.201	0.	13.886

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	36	36	22	46	26
normalized size	1	1.	1.	0.9	1.16	1.16	0.71	1.48	0.84
time (sec)	N/A	0.029	0.007	0.005	0.741	0.28	0.192	0.275	9.239

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	62	46	0	46
normalized size	1	1.	0.84	0.78	0.	1.24	0.92	0.	0.92
time (sec)	N/A	0.048	0.024	0.015	0.	0.271	0.196	0.	13.831

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	62	46	0	46
normalized size	1	1.	0.84	0.78	0.	1.24	0.92	0.	0.92
time (sec)	N/A	0.051	0.031	0.015	0.	0.282	0.205	0.	15.012

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	62	46	0	46
normalized size	1	1.	0.84	0.78	0.	1.24	0.92	0.	0.92
time (sec)	N/A	0.05	0.026	0.015	0.	0.302	0.202	0.	13.878

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	32	12	35	42	32	39	14
normalized size	1	1.	2.29	0.86	2.5	3.	2.29	2.79	1.
time (sec)	N/A	0.014	0.011	0.003	0.826	0.28	0.169	0.272	4.577

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	31	30	39	36	29	45	29
normalized size	1	1.	0.79	0.77	1.	0.92	0.74	1.15	0.74
time (sec)	N/A	0.04	0.01	0.011	0.747	0.281	0.2	0.271	9.088

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	42	82	0	62	46	104	53
normalized size	1	1.	0.88	1.71	0.	1.29	0.96	2.17	1.1
time (sec)	N/A	0.086	0.031	0.018	0.	0.276	0.207	0.309	8.778

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	124	277	0	1	88	0	49
normalized size	1	1.	2.	4.47	0.	0.02	1.42	0.	0.79
time (sec)	N/A	0.104	0.094	0.049	0.	0.29	0.735	0.	10.965

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	83	136	0	35	41	35	87
normalized size	1	1.	1.69	2.78	0.	0.71	0.84	0.71	1.78
time (sec)	N/A	0.155	0.231	0.058	0.	0.265	0.236	0.277	8.991

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	81	110	0	35	41	35	66
normalized size	1	1.	1.88	2.56	0.	0.81	0.95	0.81	1.53
time (sec)	N/A	0.089	0.109	0.048	0.	0.293	0.239	0.279	7.88

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	83	104	0	35	41	35	87
normalized size	1	1.	1.69	2.12	0.	0.71	0.84	0.71	1.78
time (sec)	N/A	0.123	0.165	0.042	0.	0.281	0.237	0.273	8.948

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	3	2	3	2
normalized size	1	1.	1.	1.5	1.5	1.5	1.	1.5	1.
time (sec)	N/A	0.005	0.004	0.002	0.836	0.283	0.152	0.269	2.769

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	99	34	45	35	41	35	42
normalized size	1	1.	2.61	0.89	1.18	0.92	1.08	0.92	1.11
time (sec)	N/A	0.06	0.321	0.007	0.835	0.286	0.226	0.273	7.142

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	88	53	32	39	53	32
normalized size	1	1.	0.86	2.51	1.51	0.91	1.11	1.51	0.91
time (sec)	N/A	0.042	0.021	0.006	0.82	0.274	0.213	0.268	4.604

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	12	20	0	9	7	41	19
normalized size	1	1.	0.52	0.87	0.	0.39	0.3	1.78	0.83
time (sec)	N/A	0.036	0.011	0.018	0.	0.284	0.192	0.274	7.654

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	10	16	14	14	7	15	5
normalized size	1	1.	0.91	1.45	1.27	1.27	0.64	1.36	0.45
time (sec)	N/A	0.007	0.006	0.008	0.737	0.26	0.145	0.268	5.049

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	29	22	28	28	19	58	53
normalized size	1	1.	0.45	0.34	0.43	0.43	0.29	0.89	0.82
time (sec)	N/A	0.069	0.009	0.007	0.765	0.283	0.19	0.272	8.295

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	70	0	53	39	53	44
normalized size	1	1.	0.93	1.63	0.	1.23	0.91	1.23	1.02
time (sec)	N/A	0.064	0.019	0.043	0.	0.307	0.193	0.281	7.972

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	82	0	57	39	53	51
normalized size	1	1.	0.87	1.78	0.	1.24	0.85	1.15	1.11
time (sec)	N/A	0.076	0.019	0.045	0.	0.288	0.199	0.279	7.775

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	125	279	0	1	87	0	46
normalized size	1	1.	2.02	4.5	0.	0.02	1.4	0.	0.74
time (sec)	N/A	0.056	0.125	0.024	0.	0.293	0.736	0.	13.903

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	87	136	0	38	42	35	88
normalized size	1	1.	1.74	2.72	0.	0.76	0.84	0.7	1.76
time (sec)	N/A	0.129	0.217	0.017	0.	0.274	0.255	0.278	9.367

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	82	111	0	38	42	35	68
normalized size	1	1.	1.86	2.52	0.	0.86	0.95	0.8	1.55
time (sec)	N/A	0.066	0.126	0.017	0.	0.283	0.242	0.276	8.187

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	10	104	0	18	10	35	88
normalized size	1	1.	0.26	2.67	0.	0.46	0.26	0.9	2.26
time (sec)	N/A	0.092	0.01	0.017	0.	0.264	0.202	0.274	9.291

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	12	5	9	5
normalized size	1	1.	1.	1.11	1.33	1.33	0.56	1.	0.56
time (sec)	N/A	0.007	0.007	0.008	0.737	0.294	0.156	0.27	5.425

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	28	19	47	19
normalized size	1	1.	1.	0.88	1.12	1.12	0.76	1.88	0.76
time (sec)	N/A	0.026	0.008	0.005	0.751	0.275	0.188	0.272	9.755

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	62	46	50	39	46	39
normalized size	1	1.	0.87	1.35	1.	1.09	0.85	1.	0.85
time (sec)	N/A	0.037	0.017	0.003	0.812	0.279	0.183	0.273	9.397

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	35	0	57	39	53	39
normalized size	1	1.	0.87	0.76	0.	1.24	0.85	1.15	0.85
time (sec)	N/A	0.04	0.02	0.014	0.	0.29	0.197	0.27	12.818

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	19	3	18	18	12	20	2
normalized size	1	1.	9.5	1.5	9.	9.	6.	10.	1.
time (sec)	N/A	0.006	0.003	0.001	0.735	0.28	0.166	0.27	3.885

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	34	74	57	39	53	46
normalized size	1	1.	1.05	0.89	1.95	1.5	1.03	1.39	1.21
time (sec)	N/A	0.064	0.023	0.005	0.829	0.288	0.205	0.27	7.905

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	40	70	0	57	39	53	49
normalized size	1	1.	0.85	1.49	0.	1.21	0.83	1.13	1.04
time (sec)	N/A	0.079	0.031	0.016	0.	0.269	0.218	0.278	8.376

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	82	0	57	39	53	53
normalized size	1	1.	0.87	1.78	0.	1.24	0.85	1.15	1.15
time (sec)	N/A	0.075	0.024	0.018	0.	0.285	0.202	0.279	8.046

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	99	34	45	38	46	45	42
normalized size	1	1.	2.3	0.79	1.05	0.88	1.07	1.05	0.98
time (sec)	N/A	0.069	0.167	0.01	0.841	0.281	0.267	0.27	8.384

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	99	34	45	38	46	45	44
normalized size	1	1.	2.3	0.79	1.05	0.88	1.07	1.05	1.02
time (sec)	N/A	0.061	0.036	0.004	0.846	0.282	0.275	0.27	8.432

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	28	31	46	22	34	14
normalized size	1	1.	1.29	1.33	1.48	2.19	1.05	1.62	0.67
time (sec)	N/A	0.016	0.017	0.013	0.754	0.283	0.194	0.268	5.502

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	53	26	51	78	53	59	24
normalized size	1	1.	1.89	0.93	1.82	2.79	1.89	2.11	0.86
time (sec)	N/A	0.031	0.03	0.01	0.847	0.286	1.856	0.27	8.619

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	72	56	69	85	474	81	32
normalized size	1	1.	2.	1.56	1.92	2.36	13.17	2.25	0.89
time (sec)	N/A	0.074	0.067	0.009	0.847	0.294	3.33	0.27	9.379

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	104	0	196	46	55	90
normalized size	1	1.	0.99	1.41	0.	2.65	0.62	0.74	1.22
time (sec)	N/A	0.132	0.159	0.03	0.	0.29	0.501	0.269	9.244

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	97	114	93	99	740	93	80
normalized size	1	1.	1.17	1.37	1.12	1.19	8.92	1.12	0.96
time (sec)	N/A	0.125	0.209	0.007	0.861	0.295	2.957	0.271	19.396

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	147	168	142	261	876	147	109
normalized size	1	1.	1.24	1.41	1.19	2.19	7.36	1.24	0.92
time (sec)	N/A	0.188	0.412	0.016	0.842	0.298	4.766	0.272	30.952

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	111	710	0	7285	122	0	212
normalized size	1	1.	0.47	3.03	0.	31.13	0.52	0.	0.91
time (sec)	N/A	0.477	0.195	0.103	0.	0.789	3.21	0.	33.067

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	165	1506	0	8505	167	0	275
normalized size	1	1.	0.52	4.77	0.	26.91	0.53	0.	0.87
time (sec)	N/A	0.764	0.354	0.316	0.	0.998	4.924	0.	49.467

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-2)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	53	199	0	0	0	0	240
normalized size	1	1.	0.33	1.24	0.	0.	0.	0.	1.5
time (sec)	N/A	0.342	0.069	0.089	0.	0.	0.	0.	29.842

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	53	199	0	595	0	0	240
normalized size	1	1.	0.31	1.16	0.	3.46	0.	0.	1.4
time (sec)	N/A	0.34	0.055	0.083	0.	0.339	0.	0.	28.853

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	137	285	0	0	332	1	134
normalized size	1	1.	0.86	1.78	0.	0.	2.08	0.01	0.84
time (sec)	N/A	0.267	0.145	0.029	0.	0.	6.294	0.317	34.6

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	136	283	0	0	330	1	134
normalized size	1	1.	0.85	1.77	0.	0.	2.06	0.01	0.84
time (sec)	N/A	0.23	0.094	0.026	0.	0.	6.344	0.317	34.322

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	115	92	0	926	27	0	128
normalized size	1	1.	1.01	0.81	0.	8.12	0.24	0.	1.12
time (sec)	N/A	0.158	0.311	0.037	0.	0.315	1.35	0.	36.916

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F(-2)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	115	96	0	350	0	0	134
normalized size	1	1.	0.94	0.79	0.	2.87	0.	0.	1.1
time (sec)	N/A	0.165	0.303	0.05	0.	0.313	0.	0.	39.65

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	115	89	170	1	143	124	117
normalized size	1	1.	0.93	0.72	1.37	0.01	1.15	1.	0.94
time (sec)	N/A	0.162	0.226	0.021	0.817	0.324	1.413	0.282	32.44

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	130	190	0	9914	172	0	148
normalized size	1	1.	0.96	1.4	0.	72.9	1.26	0.	1.09
time (sec)	N/A	0.23	0.265	0.026	0.	2.504	4.173	0.	41.742

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F(-2)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	138	198	0	5455	0	0	168
normalized size	1	1.	0.86	1.24	0.	34.09	0.	0.	1.05
time (sec)	N/A	0.248	0.242	0.041	0.	50.272	0.	0.	43.284

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F(-2)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	247	404	0	0	0	0	386
normalized size	1	1.	0.6	0.98	0.	0.	0.	0.	0.93
time (sec)	N/A	0.892	0.317	0.074	0.	0.	0.	0.	106.045

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F(-2)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	163	320	0	0	0	0	240
normalized size	1	1.	0.7	1.37	0.	0.	0.	0.	1.03
time (sec)	N/A	0.392	0.309	0.07	0.	0.	0.	0.	88.361

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	103	200	0	0	0	0	114
normalized size	1	1.	1.07	2.08	0.	0.	0.	0.	1.19
time (sec)	N/A	0.295	0.141	0.125	0.	0.	0.	0.	32.458

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	19	113	0	0	0	0	27
normalized size	1	1.	0.76	4.52	0.	0.	0.	0.	1.08
time (sec)	N/A	0.099	0.043	0.019	0.	0.	0.	0.	18.821

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	103	204	0	0	0	0	114
normalized size	1	1.	1.07	2.12	0.	0.	0.	0.	1.19
time (sec)	N/A	0.45	0.183	0.121	0.	0.	0.	0.	32.935

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	107	204	0	0	0	0	109
normalized size	1	1.	1.16	2.22	0.	0.	0.	0.	1.18
time (sec)	N/A	0.309	0.155	0.115	0.	0.	0.	0.	32.298

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	35	95	0	0	0	0	27
normalized size	1	1.	1.3	3.52	0.	0.	0.	0.	1.
time (sec)	N/A	0.132	0.055	0.013	0.	0.	0.	0.	22.694

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	107	204	0	0	0	0	109
normalized size	1	1.	1.16	2.22	0.	0.	0.	0.	1.18
time (sec)	N/A	0.451	0.197	0.117	0.	0.	0.	0.	33.633

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	187	515	0	0	0	0	269
normalized size	1	1.	0.63	1.74	0.	0.	0.	0.	0.91
time (sec)	N/A	0.27	0.367	0.062	0.	0.	0.	0.	35.838

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	97	127	1	110	127	0
normalized size	1	1.	1.	0.92	1.2	0.01	1.04	1.2	0.
time (sec)	N/A	0.163	0.031	0.002	0.7	0.275	0.135	0.268	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	72	96	1	78	96	0
normalized size	1	1.	1.	0.91	1.22	0.01	0.99	1.22	0.
time (sec)	N/A	0.112	0.025	0.002	0.733	0.257	0.115	0.269	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	49	65	1	56	68	0
normalized size	1	1.	1.	0.88	1.16	0.02	1.	1.21	0.
time (sec)	N/A	0.069	0.017	0.002	0.745	0.265	0.101	0.269	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	35	1	29	38	0
normalized size	1	1.	1.	0.84	1.09	0.03	0.91	1.19	0.
time (sec)	N/A	0.033	0.003	0.001	0.727	0.261	0.075	0.269	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	57	0	1	104	59	49
normalized size	1	1.	1.	1.04	0.	0.02	1.89	1.07	0.89
time (sec)	N/A	0.076	0.057	0.008	0.	0.296	1.663	0.268	17.054

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	78	82	0	1	138	84	65
normalized size	1	1.	1.05	1.11	0.	0.01	1.86	1.14	0.88
time (sec)	N/A	0.113	0.083	0.012	0.	0.291	2.291	0.27	23.838

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	92	99	0	1	219	104	85
normalized size	1	1.	0.99	1.06	0.	0.01	2.35	1.12	0.91
time (sec)	N/A	0.143	0.099	0.012	0.	0.29	3.117	0.271	24.203

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	113	122	0	1	204	135	112
normalized size	1	1.	0.9	0.97	0.	0.01	1.62	1.07	0.89
time (sec)	N/A	0.222	0.14	0.013	0.	0.288	4.033	0.271	28.812

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	133	130	174	1	144	173	0
normalized size	1	1.	1.	0.98	1.31	0.01	1.08	1.3	0.
time (sec)	N/A	0.222	0.04	0.002	0.737	0.255	0.166	0.269	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	90	120	1	104	123	0
normalized size	1	1.	1.	0.93	1.24	0.01	1.07	1.27	0.
time (sec)	N/A	0.143	0.03	0.001	0.74	0.253	0.138	0.268	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	68	1	60	72	0
normalized size	1	1.	1.	0.85	1.13	0.02	1.	1.2	0.
time (sec)	N/A	0.065	0.005	0.001	0.732	0.257	0.108	0.269	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	1	22	28	0
normalized size	1	1.	1.	0.88	1.12	0.04	0.88	1.12	0.
time (sec)	N/A	0.019	0.001	0.001	0.749	0.259	0.078	0.27	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	97	136	0	1	235	142	0
normalized size	1	1.	0.9	1.26	0.	0.01	2.18	1.31	0.
time (sec)	N/A	0.148	0.127	0.005	0.	0.319	2.196	0.269	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	134	170	0	1	314	173	146
normalized size	1	1.	1.02	1.3	0.	0.01	2.4	1.32	1.11
time (sec)	N/A	0.3	0.19	0.013	0.	0.29	3.763	0.27	79.125

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	154	211	0	1	257	196	177
normalized size	1	1.	0.99	1.36	0.	0.01	1.66	1.26	1.14
time (sec)	N/A	0.399	0.187	0.015	0.	0.286	6.493	0.274	85.923

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	174	262	0	1	292	225	207
normalized size	1	1.	0.93	1.4	0.	0.01	1.56	1.2	1.11
time (sec)	N/A	0.499	0.252	0.016	0.	0.297	10.523	0.271	82.609

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	200	231	0	1	335	267	255
normalized size	1	1.	0.88	1.02	0.	0.	1.48	1.18	1.13
time (sec)	N/A	0.572	0.33	0.016	0.	0.282	17.519	0.276	92.556

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	444	741	0	3885	500	672	0
normalized size	1	1.	1.02	1.7	0.	8.89	1.14	1.54	0.
time (sec)	N/A	0.876	0.729	0.014	0.	4.591	12.388	0.279	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	360	572	0	2880	350	547	345
normalized size	1	1.	0.97	1.55	0.	7.78	0.95	1.48	0.93
time (sec)	N/A	0.909	0.519	0.006	0.	1.22	7.947	0.28	88.348

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	269	412	0	1998	238	454	0
normalized size	1	1.	0.91	1.39	0.	6.73	0.8	1.53	0.
time (sec)	N/A	0.531	0.486	0.005	0.	0.461	5.086	0.28	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	183	260	0	1035	109	331	230
normalized size	1	1.	0.74	1.05	0.	4.19	0.44	1.34	0.93
time (sec)	N/A	0.287	0.113	0.004	0.	0.285	1.91	0.279	59.68

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	134	128	0	142	20	242	172
normalized size	1	1.	0.72	0.69	0.	0.77	0.11	1.31	0.93
time (sec)	N/A	0.194	0.034	0.003	0.	0.299	0.35	0.273	46.257

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	234	363	0	1	0	458	304
normalized size	1	1.	0.7	1.08	0.	0.	0.	1.36	0.9
time (sec)	N/A	0.518	0.307	0.01	0.	0.916	0.	0.285	91.131

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	453	362	650	0	1	0	698	425
normalized size	1	1.	0.8	1.43	0.	0.	0.	1.54	0.94
time (sec)	N/A	0.729	1.085	0.015	0.	19.115	0.	0.284	133.508

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	371	624	0	2857	352	574	321
normalized size	1	1.	1.02	1.72	0.	7.87	0.97	1.58	0.88
time (sec)	N/A	0.733	0.488	0.014	0.	0.404	12.645	0.282	98.998

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	295	464	0	2155	275	497	318
normalized size	1	1.	0.85	1.33	0.	6.17	0.79	1.42	0.91
time (sec)	N/A	0.584	0.291	0.012	0.	0.502	7.22	0.282	94.833

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	267	303	0	1179	136	369	255
normalized size	1	1.	0.97	1.1	0.	4.29	0.49	1.34	0.93
time (sec)	N/A	0.389	0.606	0.007	0.	0.29	3.824	0.278	75.165

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	183	143	0	213	39	262	190
normalized size	1	1.	0.91	0.71	0.	1.05	0.19	1.3	0.94
time (sec)	N/A	0.24	0.259	0.006	0.	0.293	1.875	0.271	57.185

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	689	689	429	873	0	1	0	814	636
normalized size	1	1.	0.62	1.27	0.	0.	0.	1.18	0.92
time (sec)	N/A	1.146	0.535	0.021	0.	22.185	0.	0.288	174.876

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	864	864	540	1169	0	1	0	1	0
normalized size	1	1.	0.62	1.35	0.	0.	0.	0.	0.
time (sec)	N/A	1.685	1.16	0.026	0.	174.494	0.	0.286	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	235	388	0	0	173	0	299
normalized size	1	1.	0.72	1.19	0.	0.	0.53	0.	0.92
time (sec)	N/A	0.545	0.557	0.017	0.	0.	7.89	0.	69.767

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	195	266	0	0	124	0	243
normalized size	1	1.	0.74	1.01	0.	0.	0.47	0.	0.92
time (sec)	N/A	0.302	0.391	0.01	0.	0.	5.69	0.	40.981

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	131	169	0	0	78	0	204
normalized size	1	1.	0.58	0.75	0.	0.	0.35	0.	0.9
time (sec)	N/A	0.175	0.111	0.006	0.	0.	3.69	0.	20.276

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	95	107	0	0	0	0	275
normalized size	1	1.	0.29	0.33	0.	0.	0.	0.	0.85
time (sec)	N/A	0.409	0.083	0.041	0.	0.	0.	0.	21.588

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	570	713	522	556	0	0	0	0	619
normalized size	1	1.25	0.92	0.98	0.	0.	0.	0.	1.09
time (sec)	N/A	0.919	1.001	0.032	0.	0.	0.	0.	82.345

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	232	360	0	0	180	0	194
normalized size	1	1.	1.09	1.69	0.	0.	0.85	0.	0.91
time (sec)	N/A	0.58	0.811	0.028	0.	0.	8.706	0.	81.845

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	192	246	0	0	129	0	150
normalized size	1	1.	1.19	1.52	0.	0.	0.8	0.	0.93
time (sec)	N/A	0.339	0.439	0.01	0.	0.	6.256	0.	56.921

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	127	154	0	0	82	0	112
normalized size	1	1.	1.02	1.24	0.	0.	0.66	0.	0.9
time (sec)	N/A	0.22	0.127	0.006	0.	0.	4.113	0.	36.283

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	91	97	0	0	0	0	63
normalized size	1	1.	1.26	1.35	0.	0.	0.	0.	0.88
time (sec)	N/A	0.121	0.087	0.028	0.	0.	0.	0.	15.386

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	508	523	0	0	0	0	265
normalized size	1	1.	1.7	1.75	0.	0.	0.	0.	0.89
time (sec)	N/A	0.518	1.561	0.033	0.	0.	0.	0.	69.386

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	128	160	0	0	73	0	112
normalized size	1	1.	1.02	1.27	0.	0.	0.58	0.	0.89
time (sec)	N/A	0.214	0.131	0.011	0.	0.	4.143	0.	35.894

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	92	99	0	0	0	0	63
normalized size	1	1.	1.26	1.36	0.	0.	0.	0.	0.86
time (sec)	N/A	0.12	0.07	0.023	0.	0.	0.	0.	15.597

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	78	158	0	0	70	0	48
normalized size	1	1.	1.44	2.93	0.	0.	1.3	0.	0.89
time (sec)	N/A	0.136	0.098	0.067	0.	0.	4.29	0.	25.598

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	142	165	0	0	76	0	41
normalized size	1	1.	2.73	3.17	0.	0.	1.46	0.	0.79
time (sec)	N/A	0.134	0.149	0.042	0.	0.	4.397	0.	18.476

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	134	175	0	0	83	0	209
normalized size	1	1.	0.57	0.74	0.	0.	0.35	0.	0.89
time (sec)	N/A	0.176	0.129	0.014	0.	0.	4.015	0.	21.548

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	98	110	0	0	0	0	284
normalized size	1	1.	0.29	0.33	0.	0.	0.	0.	0.85
time (sec)	N/A	0.398	0.087	0.022	0.	0.	0.	0.	22.485

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	43	79	0	0	0	0	42
normalized size	1	1.	1.08	1.98	0.	0.	0.	0.	1.05
time (sec)	N/A	0.175	0.063	0.072	0.	0.	0.	0.	23.705

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	50	86	0	0	0	0	253
normalized size	1	1.	0.18	0.3	0.	0.	0.	0.	0.89
time (sec)	N/A	0.348	0.063	0.141	0.	0.	0.	0.	13.349

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	59	78	0	0	0	0	41
normalized size	1	1.	1.48	1.95	0.	0.	0.	0.	1.02
time (sec)	N/A	0.06	0.068	0.033	0.	0.	0.	0.	8.25

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	65	86	0	0	0	0	253
normalized size	1	1.	0.23	0.3	0.	0.	0.	0.	0.88
time (sec)	N/A	0.29	0.063	0.033	0.	0.	0.	0.	18.695

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.064	0.118	0.	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.081	0.117	0.	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	196	136	0	0	0	0	0	168
normalized size	1	0.96	0.67	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.406	0.093	0.073	0.	0.	0.	0.	33.991

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	142	106	0	0	0	0	0	124
normalized size	1	0.95	0.71	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	0.27	0.055	0.062	0.	0.	0.	0.	25.496

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	75	0	0	0	0	0	76
normalized size	1	1.	0.78	0.	0.	0.	0.	0.	0.79
time (sec)	N/A	0.112	0.03	0.044	0.	0.	0.	0.	14.842

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	34	0	34
normalized size	1	1.	1.	0.	0.	0.	0.77	0.	0.77
time (sec)	N/A	0.023	0.01	0.034	0.	0.	38.595	0.	3.987

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	0	0	0	97
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.79
time (sec)	N/A	0.281	0.045	0.066	0.	0.	0.	0.	57.12

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	0	0	0	0	0	0	155
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.462	0.063	0.081	0.	0.	0.	0.	110.37

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	103	86	75	0	0	0	0	70
normalized size	1	0.95	0.8	0.69	0.	0.	0.	0.	0.65
time (sec)	N/A	0.205	0.033	0.424	0.	0.	0.	0.	14.286

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	79	65	56	0	0	0	0	53
normalized size	1	0.92	0.76	0.65	0.	0.	0.	0.	0.62
time (sec)	N/A	0.148	0.021	0.069	0.	0.	0.	0.	12.351

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	0	0	61	0	32
normalized size	1	1.	1.	0.88	0.	0.	1.45	0.	0.76
time (sec)	N/A	0.048	0.016	0.042	0.	0.	155.555	0.	6.668

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	0	0	29	0	14
normalized size	1	1.	1.	0.94	0.	0.	1.61	0.	0.78
time (sec)	N/A	0.01	0.007	0.03	0.	0.	32.457	0.	1.277

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	0.043	0.064	0.	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.198	0.048	0.081	0.	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.27	0.085	0.107	0.	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	42	0	1	75	194	49
normalized size	1	1.	1.	0.82	0.	0.02	1.47	3.8	0.96
time (sec)	N/A	0.072	0.038	0.005	0.	0.298	1.739	0.281	16.606

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	0	1	58	166	36
normalized size	1	1.	1.	0.82	0.	0.03	1.53	4.37	0.95
time (sec)	N/A	0.061	0.027	0.003	0.	0.272	1.517	0.28	15.406

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	0	1	34	159	26
normalized size	1	1.	1.	0.76	0.	0.03	1.17	5.48	0.9
time (sec)	N/A	0.041	0.014	0.003	0.	0.278	1.315	0.281	11.722

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	1	46	157	22
normalized size	1	1.	1.	0.67	0.	0.04	1.92	6.54	0.92
time (sec)	N/A	0.023	0.007	0.002	0.	0.271	0.336	0.279	5.608

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	65	55	0	1	226	0	63
normalized size	1	1.	0.9	0.76	0.	0.01	3.14	0.	0.88
time (sec)	N/A	0.141	0.059	0.014	0.	0.29	2.371	0.	35.094

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	76	73	0	1	255	0	82
normalized size	1	1.	0.85	0.82	0.	0.01	2.87	0.	0.92
time (sec)	N/A	0.212	0.1	0.015	0.	0.295	3.239	0.	58.147

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	61	1442	0	1	0	0	56
normalized size	1	1.	0.98	23.26	0.	0.02	0.	0.	0.9
time (sec)	N/A	0.121	0.042	0.066	0.	0.307	0.	0.	21.582

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	986	0	1	0	177	36
normalized size	1	1.	1.	25.95	0.	0.03	0.	4.66	0.95
time (sec)	N/A	0.076	0.025	0.034	0.	0.296	0.	0.327	14.474

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	57	441	0	1	0	0	54
normalized size	1	1.	0.93	7.23	0.	0.02	0.	0.	0.89
time (sec)	N/A	0.114	0.052	0.03	0.	0.293	0.	0.	21.391

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	68	911	0	1	0	0	73
normalized size	1	1.	0.85	11.39	0.	0.01	0.	0.	0.91
time (sec)	N/A	0.207	0.155	0.033	0.	0.328	0.	0.	42.707

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	98	132	0	1	0	0	121
normalized size	1	1.	0.64	0.86	0.	0.01	0.	0.	0.79
time (sec)	N/A	0.158	0.228	0.079	0.	0.306	0.	0.	28.015

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	86	109	0	1	0	0	88
normalized size	1	1.	0.78	0.99	0.	0.01	0.	0.	0.8
time (sec)	N/A	0.102	0.074	0.026	0.	0.302	0.	0.	15.963

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	50	69	0	1	0	0	56
normalized size	1	1.	0.77	1.06	0.	0.02	0.	0.	0.86
time (sec)	N/A	0.065	0.037	0.025	0.	0.279	0.	0.	9.677

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	248	0	1	0	0	70
normalized size	1	1.	1.	3.18	0.	0.01	0.	0.	0.9
time (sec)	N/A	0.103	0.05	0.058	0.	0.281	0.	0.	15.485

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	111	504	0	1	0	0	104
normalized size	1	1.	0.89	4.03	0.	0.01	0.	0.	0.83
time (sec)	N/A	0.147	0.095	0.089	0.	0.29	0.	0.	22.453

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	123	729	0	1	0	0	136
normalized size	1	1.	0.73	4.34	0.	0.01	0.	0.	0.81
time (sec)	N/A	0.247	0.124	0.072	0.	0.279	0.	0.	43.888

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	123	105	0	1	0	0	121
normalized size	1	1.	0.81	0.69	0.	0.01	0.	0.	0.8
time (sec)	N/A	0.151	0.298	0.022	0.	0.283	0.	0.	28.73

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	110	85	0	1	0	0	88
normalized size	1	1.	1.01	0.78	0.	0.01	0.	0.	0.81
time (sec)	N/A	0.102	0.153	0.017	0.	0.287	0.	0.	17.362

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	67	54	0	1	0	0	56
normalized size	1	1.	1.05	0.84	0.	0.02	0.	0.	0.88
time (sec)	N/A	0.069	0.028	0.015	0.	0.277	0.	0.	12.73

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	266	0	1	0	0	70
normalized size	1	1.	1.	3.45	0.	0.01	0.	0.	0.91
time (sec)	N/A	0.101	0.047	0.074	0.	0.28	0.	0.	21.525

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	110	526	0	1	0	0	104
normalized size	1	1.	0.89	4.24	0.	0.01	0.	0.	0.84
time (sec)	N/A	0.149	0.093	0.062	0.	0.282	0.	0.	29.13

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	122	757	0	1	0	0	136
normalized size	1	1.	0.73	4.53	0.	0.01	0.	0.	0.81
time (sec)	N/A	0.248	0.124	0.067	0.	0.285	0.	0.	52.73

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	38	25	3	99	0	0	27
normalized size	1	1.	1.27	0.83	0.1	3.3	0.	0.	0.9
time (sec)	N/A	0.029	0.014	0.014	0.85	0.289	0.	0.	5.606

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	40	34	33	19	88	0	0	36
normalized size	1	1.67	1.42	1.38	0.79	3.67	0.	0.	1.5
time (sec)	N/A	0.033	0.013	0.011	0.876	0.271	0.	0.	4.926

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	72	71	59	26	185	0	0	0
normalized size	1	0.99	0.97	0.81	0.36	2.53	0.	0.	0.
time (sec)	N/A	0.204	0.021	0.003	0.881	0.287	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	121	226	0	1	343	0	0
normalized size	1	1.	1.	1.87	0.	0.01	2.83	0.	0.
time (sec)	N/A	0.276	0.123	0.014	0.	0.281	3.153	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	84	142	0	1	275	0	0
normalized size	1	1.	0.98	1.65	0.	0.01	3.2	0.	0.
time (sec)	N/A	0.183	0.073	0.004	0.	0.288	2.572	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	63	79	0	1	212	0	54
normalized size	1	1.	0.98	1.23	0.	0.02	3.31	0.	0.84
time (sec)	N/A	0.122	0.093	0.003	0.	0.277	1.952	0.	27.18

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	33	0	1	124	0	42
normalized size	1	1.	0.98	0.67	0.	0.02	2.53	0.	0.86
time (sec)	N/A	0.052	0.02	0.001	0.	0.268	0.565	0.	15.381

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	133	155	0	1	2664	0	117
normalized size	1	1.	0.98	1.14	0.	0.01	19.59	0.	0.86
time (sec)	N/A	0.4	0.388	0.024	0.	0.42	60.368	0.	87.44

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	177	319	0	1	0	0	170
normalized size	1	1.	0.95	1.71	0.	0.01	0.	0.	0.91
time (sec)	N/A	0.61	0.778	0.017	0.	1.322	0.	0.	135.329

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	134	7043	0	1	0	0	124
normalized size	1	1.	0.96	50.67	0.	0.01	0.	0.	0.89
time (sec)	N/A	0.552	0.393	0.069	0.	0.783	0.	0.	102.221

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	103	4332	0	1	0	0	94
normalized size	1	1.	0.95	40.11	0.	0.01	0.	0.	0.87
time (sec)	N/A	0.258	0.093	0.031	0.	0.385	0.	0.	66.529

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	73	2264	0	1	0	0	66
normalized size	1	1.	0.96	29.79	0.	0.01	0.	0.	0.87
time (sec)	N/A	0.159	0.062	0.031	0.	0.367	0.	0.	50.859

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	775	0	1	0	0	90
normalized size	1	1.	1.	7.31	0.	0.01	0.	0.	0.85
time (sec)	N/A	0.273	0.254	0.029	0.	0.431	0.	0.	63.211

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	134	1647	0	1	0	0	131
normalized size	1	1.	0.9	11.05	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.604	0.553	0.032	0.	0.704	0.	0.	118.803

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	169	263	0	0	0	0	172
normalized size	1	1.	0.92	1.44	0.	0.	0.	0.	0.94
time (sec)	N/A	0.18	0.373	0.226	0.	0.	0.	0.	37.518

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	162	248	0	0	0	0	156
normalized size	1	1.	0.99	1.51	0.	0.	0.	0.	0.95
time (sec)	N/A	0.14	0.233	0.011	0.	0.	0.	0.	28.837

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	168	233	0	0	0	0	138
normalized size	1	1.	1.16	1.61	0.	0.	0.	0.	0.95
time (sec)	N/A	0.104	0.327	0.008	0.	0.	0.	0.	18.432

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	118	293	0	0	0	0	320
normalized size	1	1.	0.86	2.14	0.	0.	0.	0.	2.34
time (sec)	N/A	0.181	0.101	0.091	0.	0.	0.	0.	91.864

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	164	224	0	0	0	0	541
normalized size	1	1.	3.35	4.57	0.	0.	0.	0.	11.04
time (sec)	N/A	0.035	0.536	0.025	0.	0.	0.	0.	140.011

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	176	333	0	0	0	0	0
normalized size	1	1.	1.89	3.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.748	0.349	0.033	0.	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	240	438	0	0	0	0	0
normalized size	1	1.	1.45	2.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.934	0.472	0.034	0.	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	157	233	0	0	0	0	150
normalized size	1	1.	0.99	1.47	0.	0.	0.	0.	0.94
time (sec)	N/A	0.148	0.219	0.037	0.	0.	0.	0.	30.579

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	143	218	0	0	0	0	128
normalized size	1	1.	1.04	1.59	0.	0.	0.	0.	0.93
time (sec)	N/A	0.11	0.194	0.01	0.	0.	0.	0.	20.989

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	94	205	0	0	0	0	105
normalized size	1	1.	0.82	1.78	0.	0.	0.	0.	0.91
time (sec)	N/A	0.065	0.09	0.007	0.	0.	0.	0.	11.774

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	73	104	0	0	0	0	236
normalized size	1	1.	1.06	1.51	0.	0.	0.	0.	3.42
time (sec)	N/A	0.11	0.038	0.022	0.	0.	0.	0.	66.113

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	226	397	0	0	0	0	328
normalized size	1	1.	1.92	3.36	0.	0.	0.	0.	2.78
time (sec)	N/A	0.198	0.743	0.03	0.	0.	0.	0.	80.059

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	235	418	0	0	0	0	0
normalized size	1	1.	1.65	2.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.433	0.413	0.031	0.	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	136	268	0	0	0	0	133
normalized size	1	1.	0.94	1.86	0.	0.	0.	0.	0.92
time (sec)	N/A	0.114	0.273	0.044	0.	0.	0.	0.	22.227

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	158	268	0	0	0	0	90
normalized size	1	1.	1.61	2.73	0.	0.	0.	0.	0.92
time (sec)	N/A	0.073	0.294	0.011	0.	0.	0.	0.	16.245

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	160	247	0	0	0	0	85
normalized size	1	1.	1.67	2.57	0.	0.	0.	0.	0.89
time (sec)	N/A	0.059	0.301	0.01	0.	0.	0.	0.	11.466

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	204	398	0	0	0	0	348
normalized size	1	1.	1.23	2.4	0.	0.	0.	0.	2.1
time (sec)	N/A	0.223	0.292	0.026	0.	0.	0.	0.	130.828

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	168	419	0	0	0	0	0
normalized size	1	1.	1.51	3.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.435	0.575	0.034	0.	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	192	439	0	0	0	0	0
normalized size	1	1.	1.01	2.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.912	0.38	0.035	0.	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	135	136	182	1	156	192	0
normalized size	1	1.	1.	1.01	1.35	0.01	1.16	1.42	0.
time (sec)	N/A	0.255	0.071	0.002	0.739	0.249	0.154	0.267	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	104	103	138	1	112	146	0
normalized size	1	1.	1.01	1.	1.34	0.01	1.09	1.42	0.
time (sec)	N/A	0.197	0.049	0.001	0.762	0.244	0.142	0.266	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	93	1	78	103	0
normalized size	1	1.	1.	0.96	1.27	0.01	1.07	1.41	0.
time (sec)	N/A	0.129	0.036	0.002	0.744	0.257	0.154	0.269	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	49	1	39	58	0
normalized size	1	1.	1.	0.88	1.17	0.02	0.93	1.38	0.
time (sec)	N/A	0.057	0.014	0.001	0.737	0.273	0.117	0.266	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	84	0	1	117	76	58
normalized size	1	1.	0.98	1.27	0.	0.02	1.77	1.15	0.88
time (sec)	N/A	0.096	0.105	0.003	0.	0.286	2.165	0.268	19.989

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	88	118	0	1	153	101	78
normalized size	1	1.	1.06	1.42	0.	0.01	1.84	1.22	0.94
time (sec)	N/A	0.158	0.098	0.011	0.	0.274	3.763	0.271	33.067

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	110	131	0	1	196	136	110
normalized size	1	1.	0.96	1.14	0.	0.01	1.7	1.18	0.96
time (sec)	N/A	0.205	0.183	0.012	0.	0.277	6.592	0.273	34.014

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	142	158	0	1	241	181	144
normalized size	1	1.	0.95	1.05	0.	0.01	1.61	1.21	0.96
time (sec)	N/A	0.308	0.25	0.013	0.	0.281	11.494	0.27	38.86

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	223	219	294	1	272	344	0
normalized size	1	1.	1.	0.98	1.32	0.	1.22	1.54	0.
time (sec)	N/A	0.424	0.177	0.002	0.715	0.245	0.238	0.269	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	156	155	198	1	192	244	0
normalized size	1	1.	1.01	1.	1.28	0.01	1.24	1.57	0.
time (sec)	N/A	0.281	0.1	0.001	0.737	0.242	0.191	0.267	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	91	122	1	107	143	0
normalized size	1	1.	1.	0.95	1.27	0.01	1.11	1.49	0.
time (sec)	N/A	0.137	0.043	0.001	0.743	0.238	0.152	0.269	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	61	1	48	58	0
normalized size	1	1.	1.	0.86	1.24	0.02	0.98	1.18	0.
time (sec)	N/A	0.048	0.009	0.001	0.73	0.238	0.123	0.269	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	144	267	0	1	366	250	0
normalized size	1	1.	1.01	1.87	0.	0.01	2.56	1.75	0.
time (sec)	N/A	0.297	0.126	0.005	0.	0.275	4.837	0.267	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	183	320	0	1	479	279	212
normalized size	1	1.	1.1	1.93	0.	0.01	2.89	1.68	1.28
time (sec)	N/A	0.535	0.186	0.015	0.	0.285	12.483	0.265	155.121

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	217	402	0	1	396	329	279
normalized size	1	1.	1.08	2.	0.	0.	1.97	1.64	1.39
time (sec)	N/A	0.827	0.213	0.016	0.	0.296	53.154	0.268	170.48

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	267	506	0	1	0	400	332
normalized size	1	1.	1.07	2.02	0.	0.	0.	1.6	1.33
time (sec)	N/A	0.951	0.281	0.018	0.	0.299	0.	0.266	162.457

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	345	412	0	1	0	491	415
normalized size	1	1.	1.09	1.3	0.	0.	0.	1.55	1.31
time (sec)	N/A	1.239	0.441	0.016	0.	0.281	0.	0.268	173.066

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	88	118	0	1	153	101	78
normalized size	1	1.	1.06	1.42	0.	0.01	1.84	1.22	0.94
time (sec)	N/A	0.154	0.102	0.	0.	0.276	3.695	0.266	31.155

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	88	118	0	1	153	101	78
normalized size	1	1.	1.06	1.42	0.	0.01	1.84	1.22	0.94
time (sec)	N/A	0.121	0.029	0.007	0.	0.266	3.715	0.264	35.492

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	570	1888	0	0	0	1	0
normalized size	1	1.	1.24	4.11	0.	0.	0.	0.	0.
time (sec)	N/A	4.615	1.671	0.061	0.	0.	0.	2.237	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	402	1211	0	12938	0	1	0
normalized size	1	1.	1.27	3.83	0.	40.94	0.	0.	0.
time (sec)	N/A	2.274	1.188	0.046	0.	20.587	0.	1.742	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	269	695	0	6332	920	1	0
normalized size	1	1.	1.13	2.92	0.	26.61	3.87	0.	0.
time (sec)	N/A	1.238	0.632	0.034	0.	1.923	123.774	1.244	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	172	328	0	2059	314	1	185
normalized size	1	1.	0.99	1.89	0.	11.83	1.8	0.01	1.06
time (sec)	N/A	0.425	0.269	0.024	0.	0.311	16.168	0.809	32.053

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	129	116	0	828	87	1	138
normalized size	1	1.	0.86	0.77	0.	5.52	0.58	0.01	0.92
time (sec)	N/A	0.209	0.161	0.002	0.	0.266	3.037	0.369	19.613

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	274	480	0	0	0	0	255
normalized size	1	1.	1.08	1.89	0.	0.	0.	0.	1.
time (sec)	N/A	1.192	0.517	0.027	0.	0.	0.	0.	93.174

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	354	1141	0	0	0	1	0
normalized size	1	1.	0.83	2.66	0.	0.	0.	0.	0.
time (sec)	N/A	2.813	1.658	0.037	0.	0.	0.	17.026	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	563	540	8504	0	16358	0	0	0
normalized size	1	1.	0.96	15.1	0.	29.06	0.	0.	0.
time (sec)	N/A	7.333	3.571	0.204	0.	44.489	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	415	5421	0	9906	0	0	415
normalized size	1	1.	1.08	14.04	0.	25.66	0.	0.	1.08
time (sec)	N/A	4.469	2.547	0.18	0.	5.846	0.	0.	151.364

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	310	1761	0	6174	1180	0	279
normalized size	1	1.	1.06	6.01	0.	21.07	4.03	0.	0.95
time (sec)	N/A	1.444	1.508	0.1	0.	0.995	143.205	0.	168.52

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	243	733	0	3117	394	0	230
normalized size	1	1.	0.96	2.91	0.	12.37	1.56	0.	0.91
time (sec)	N/A	0.897	0.818	0.003	0.	0.321	15.299	0.	61.76

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	660	660	708	10749	0	0	0	0	0
normalized size	1	1.	1.07	16.29	0.	0.	0.	0.	0.
time (sec)	N/A	6.516	6.283	0.153	0.	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1087	1088	1235	14860	0	0	0	0	0
normalized size	1	1.	1.14	13.67	0.	0.	0.	0.	0.
time (sec)	N/A	24.445	8.01	0.193	0.	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	177	283	0	1	505	243	211
normalized size	1	1.	0.82	1.32	0.	0.	2.35	1.13	0.98
time (sec)	N/A	0.303	0.383	0.014	0.	0.494	166.516	0.287	27.214

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	146	229	0	1	413	196	170
normalized size	1	1.	0.83	1.31	0.	0.01	2.36	1.12	0.97
time (sec)	N/A	0.245	0.197	0.014	0.	0.365	85.921	0.269	23.384

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	112	175	0	1	272	143	124
normalized size	1	1.	0.85	1.33	0.	0.01	2.06	1.08	0.94
time (sec)	N/A	0.211	0.127	0.012	0.	0.328	33.455	0.266	19.855

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	85	122	0	1	230	107	90
normalized size	1	1.	0.88	1.26	0.	0.01	2.37	1.1	0.93
time (sec)	N/A	0.132	0.093	0.011	0.	0.295	19.5	0.269	16.893

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	87	112	0	1	134	108	85
normalized size	1	1.	0.98	1.26	0.	0.01	1.51	1.21	0.96
time (sec)	N/A	0.152	0.153	0.011	0.	0.299	22.174	0.27	19.591

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	91	124	0	1	450	119	97
normalized size	1	1.	0.9	1.23	0.	0.01	4.46	1.18	0.96
time (sec)	N/A	0.149	0.244	0.011	0.	0.306	55.07	0.272	28.782

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	67	66	234	126	639	101	119
normalized size	1	1.	0.78	0.77	2.72	1.47	7.43	1.17	1.38
time (sec)	N/A	0.193	0.085	0.006	0.764	0.308	160.57	0.271	28.717

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	101	100	306	184	0	153	155
normalized size	1	1.	0.8	0.79	2.43	1.46	0.	1.21	1.23
time (sec)	N/A	0.268	0.11	0.007	0.757	0.403	0.	0.27	31.585

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	164	132	136	379	239	0	200	192
normalized size	1	0.99	0.8	0.82	2.3	1.45	0.	1.21	1.16
time (sec)	N/A	0.39	0.149	0.008	0.747	0.556	0.	0.271	35.196

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	167	172	452	302	0	255	240
normalized size	1	1.	0.8	0.82	2.15	1.44	0.	1.21	1.14
time (sec)	N/A	0.438	0.157	0.009	0.739	0.845	0.	0.271	39.315

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	119	172	0	0	0	0	182
normalized size	1	1.	0.62	0.89	0.	0.	0.	0.	0.94
time (sec)	N/A	0.208	0.094	0.029	0.	0.	0.	0.	36.092

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	114	155	0	0	0	0	162
normalized size	1	1.	0.68	0.92	0.	0.	0.	0.	0.96
time (sec)	N/A	0.162	0.076	0.011	0.	0.	0.	0.	26.832

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	109	137	0	0	0	0	139
normalized size	1	1.	0.73	0.92	0.	0.	0.	0.	0.93
time (sec)	N/A	0.115	0.067	0.009	0.	0.	0.	0.	17.932

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	102	121	0	0	0	0	128
normalized size	1	1.	0.72	0.86	0.	0.	0.	0.	0.91
time (sec)	N/A	0.099	0.06	0.005	0.	0.	0.	0.	15.321

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	232	90	138	0	0	0	0	0
normalized size	1	1.3	0.51	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.332	0.088	0.036	0.	0.	0.	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	208	162	0	0	0	0	173
normalized size	1	1.	1.	0.78	0.	0.	0.	0.	0.83
time (sec)	N/A	0.336	0.21	0.027	0.	0.	0.	0.	52.747

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	174	186	0	0	0	0	0
normalized size	1	1.	0.73	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	1.158	0.3	0.029	0.	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	129	206	0	0	0	0	209
normalized size	1	1.	0.59	0.94	0.	0.	0.	0.	0.95
time (sec)	N/A	0.242	0.087	0.026	0.	0.	0.	0.	42.27

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	124	189	0	0	0	0	189
normalized size	1	1.	0.63	0.95	0.	0.	0.	0.	0.95
time (sec)	N/A	0.196	0.083	0.011	0.	0.	0.	0.	33.077

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	119	172	0	0	0	0	163
normalized size	1	1.	0.66	0.96	0.	0.	0.	0.	0.91
time (sec)	N/A	0.151	0.075	0.008	0.	0.	0.	0.	23.305

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	114	155	0	0	0	0	156
normalized size	1	1.	0.66	0.9	0.	0.	0.	0.	0.91
time (sec)	N/A	0.135	0.067	0.005	0.	0.	0.	0.	21.119

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	148	170	0	0	0	0	214
normalized size	1	1.	0.71	0.82	0.	0.	0.	0.	1.03
time (sec)	N/A	0.489	0.133	0.021	0.	0.	0.	0.	63.717

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	333	213	177	0	0	0	0	258
normalized size	1	1.5	0.96	0.8	0.	0.	0.	0.	1.16
time (sec)	N/A	0.868	0.214	0.028	0.	0.	0.	0.	131.853

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	288	174	186	0	0	0	0	0
normalized size	1	1.25	0.75	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	1.267	0.32	0.029	0.	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	106	138	0	0	0	0	151
normalized size	1	1.	0.68	0.88	0.	0.	0.	0.	0.96
time (sec)	N/A	0.17	0.105	0.025	0.	0.	0.	0.	31.075

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	104	121	0	0	0	0	131
normalized size	1	1.	0.73	0.85	0.	0.	0.	0.	0.92
time (sec)	N/A	0.129	0.091	0.01	0.	0.	0.	0.	21.396

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	69	106	0	0	0	0	116
normalized size	1	1.	0.57	0.88	0.	0.	0.	0.	0.96
time (sec)	N/A	0.08	0.06	0.007	0.	0.	0.	0.	12.291

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	0	0	0	0	42
normalized size	1	1.	1.04	0.96	0.	0.	0.	0.	0.88
time (sec)	N/A	0.017	0.025	0.004	0.	0.	0.	0.	2.695

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	55	47	0	0	0	0	0
normalized size	1	1.	0.52	0.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.211	0.037	0.018	0.	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	208	162	0	0	0	0	0
normalized size	1	1.	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.334	0.218	0.026	0.	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	186	186	0	0	0	0	0
normalized size	1	1.	0.78	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.527	0.317	0.028	0.	0.	0.	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	109	274	0	0	0	0	177
normalized size	1	1.	0.58	1.45	0.	0.	0.	0.	0.94
time (sec)	N/A	0.216	0.098	0.048	0.	0.	0.	0.	23.565

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	104	234	0	0	0	0	158
normalized size	1	1.	0.61	1.38	0.	0.	0.	0.	0.93
time (sec)	N/A	0.174	0.088	0.014	0.	0.	0.	0.	16.669

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	99	196	0	0	0	0	139
normalized size	1	1.	0.66	1.32	0.	0.	0.	0.	0.93
time (sec)	N/A	0.132	0.105	0.011	0.	0.	0.	0.	11.242

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	99	173	0	0	0	0	139
normalized size	1	1.	0.66	1.16	0.	0.	0.	0.	0.93
time (sec)	N/A	0.13	0.081	0.01	0.	0.	0.	0.	11.23

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	97	150	0	0	0	0	133
normalized size	1	1.	0.67	1.03	0.	0.	0.	0.	0.92
time (sec)	N/A	0.107	0.071	0.008	0.	0.	0.	0.	10.256

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	99	129	0	0	0	0	138
normalized size	1	1.	0.66	0.87	0.	0.	0.	0.	0.93
time (sec)	N/A	0.101	0.061	0.006	0.	0.	0.	0.	15.647

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	207	138	161	0	0	0	0	0
normalized size	1	1.2	0.8	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.367	0.131	0.022	0.	0.	0.	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	208	185	0	0	0	0	330
normalized size	1	1.	0.89	0.79	0.	0.	0.	0.	1.4
time (sec)	N/A	0.806	0.225	0.03	0.	0.	0.	0.	129.892

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	159	209	0	0	0	0	0
normalized size	1	1.	0.6	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	1.459	0.553	0.031	0.	0.	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	112	193	0	0	0	0	112
normalized size	1	1.	0.97	1.66	0.	0.	0.	0.	0.97
time (sec)	N/A	0.263	0.125	0.035	0.	0.	0.	0.	54.171

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	107	176	0	0	0	0	94
normalized size	1	1.	1.13	1.85	0.	0.	0.	0.	0.99
time (sec)	N/A	0.224	0.104	0.011	0.	0.	0.	0.	42.098

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	102	159	0	0	0	0	75
normalized size	1	1.	1.38	2.15	0.	0.	0.	0.	1.01
time (sec)	N/A	0.19	0.101	0.011	0.	0.	0.	0.	33.03

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	94	141	0	0	0	0	51
normalized size	1	1.	2.04	3.07	0.	0.	0.	0.	1.11
time (sec)	N/A	0.153	0.103	0.008	0.	0.	0.	0.	24.257

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	90	125	0	0	0	0	42
normalized size	1	1.	2.05	2.84	0.	0.	0.	0.	0.95
time (sec)	N/A	0.131	0.086	0.004	0.	0.	0.	0.	19.953

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	51	141	0	0	0	0	51
normalized size	1	1.	1.11	3.07	0.	0.	0.	0.	1.11
time (sec)	N/A	0.265	0.085	0.02	0.	0.	0.	0.	39.126

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	196	165	0	0	0	0	178
normalized size	1	1.	2.65	2.23	0.	0.	0.	0.	2.41
time (sec)	N/A	0.268	0.267	0.027	0.	0.	0.	0.	37.347

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	244	189	0	0	0	0	0
normalized size	1	1.	2.39	1.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.891	0.468	0.029	0.	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	122	227	0	0	0	0	138
normalized size	1	1.	0.86	1.6	0.	0.	0.	0.	0.97
time (sec)	N/A	0.289	0.118	0.032	0.	0.	0.	0.	60.592

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	117	210	0	0	0	0	119
normalized size	1	1.	0.97	1.74	0.	0.	0.	0.	0.98
time (sec)	N/A	0.253	0.114	0.011	0.	0.	0.	0.	48.281

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	112	193	0	0	0	0	99
normalized size	1	1.	1.12	1.93	0.	0.	0.	0.	0.99
time (sec)	N/A	0.22	0.112	0.011	0.	0.	0.	0.	38.98

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	107	176	0	0	0	0	76
normalized size	1	1.	1.32	2.17	0.	0.	0.	0.	0.94
time (sec)	N/A	0.178	0.101	0.008	0.	0.	0.	0.	29.261

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	102	159	0	0	0	0	70
normalized size	1	1.	1.38	2.15	0.	0.	0.	0.	0.95
time (sec)	N/A	0.161	0.089	0.005	0.	0.	0.	0.	25.794

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	130	173	0	0	0	0	172
normalized size	1	1.	1.81	2.4	0.	0.	0.	0.	2.39
time (sec)	N/A	0.441	0.178	0.021	0.	0.	0.	0.	82.179

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	201	180	0	0	0	0	241
normalized size	1	1.	2.16	1.94	0.	0.	0.	0.	2.59
time (sec)	N/A	0.722	0.292	0.028	0.	0.	0.	0.	114.061

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	244	189	0	0	0	0	0
normalized size	1	1.	2.39	1.85	0.	0.	0.	0.	0.
time (sec)	N/A	1.106	0.355	0.03	0.	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	97	142	0	0	0	0	65
normalized size	1	1.	1.49	2.18	0.	0.	0.	0.	1.
time (sec)	N/A	0.198	0.124	0.024	0.	0.	0.	0.	39.036

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	92	125	0	0	0	0	48
normalized size	1	1.	2.	2.72	0.	0.	0.	0.	1.04
time (sec)	N/A	0.165	0.109	0.01	0.	0.	0.	0.	29.01

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	34	110	0	0	0	0	29
normalized size	1	1.	1.36	4.4	0.	0.	0.	0.	1.16
time (sec)	N/A	0.128	0.058	0.007	0.	0.	0.	0.	19.73

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	19	47	0	0	0	0	12
normalized size	1	1.	1.9	4.7	0.	0.	0.	0.	1.2
time (sec)	N/A	0.031	0.028	0.005	0.	0.	0.	0.	6.491

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	24	48	0	0	0	0	17
normalized size	1	1.	1.41	2.82	0.	0.	0.	0.	1.
time (sec)	N/A	0.109	0.044	0.018	0.	0.	0.	0.	17.747

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	196	165	0	0	0	0	73
normalized size	1	1.	2.65	2.23	0.	0.	0.	0.	0.99
time (sec)	N/A	0.263	0.317	0.026	0.	0.	0.	0.	38.583

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	108	189	0	0	0	0	0
normalized size	1	1.	1.06	1.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.437	0.426	0.028	0.	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	97	280	0	0	0	0	88
normalized size	1	1.	1.04	3.01	0.	0.	0.	0.	0.95
time (sec)	N/A	0.232	0.126	0.048	0.	0.	0.	0.	53.086

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	92	240	0	0	0	0	71
normalized size	1	1.	1.24	3.24	0.	0.	0.	0.	0.96
time (sec)	N/A	0.203	0.116	0.012	0.	0.	0.	0.	39.636

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	79	202	0	0	0	0	54
normalized size	1	1.	1.44	3.67	0.	0.	0.	0.	0.98
time (sec)	N/A	0.168	0.126	0.01	0.	0.	0.	0.	29.598

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	79	179	0	0	0	0	54
normalized size	1	1.	1.44	3.25	0.	0.	0.	0.	0.98
time (sec)	N/A	0.169	0.124	0.01	0.	0.	0.	0.	29.549

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	79	156	0	0	0	0	54
normalized size	1	1.	1.44	2.84	0.	0.	0.	0.	0.98
time (sec)	N/A	0.155	0.116	0.008	0.	0.	0.	0.	24.093

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	79	133	0	0	0	0	49
normalized size	1	1.	1.44	2.42	0.	0.	0.	0.	0.89
time (sec)	N/A	0.137	0.108	0.006	0.	0.	0.	0.	21.014

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	101	164	0	0	0	0	73
normalized size	1	1.	1.4	2.28	0.	0.	0.	0.	1.01
time (sec)	N/A	0.297	0.213	0.023	0.	0.	0.	0.	81.754

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	196	188	0	0	0	0	299
normalized size	1	1.	1.96	1.88	0.	0.	0.	0.	2.99
time (sec)	N/A	0.662	0.293	0.03	0.	0.	0.	0.	144.434

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	244	212	0	0	0	0	0
normalized size	1	1.	1.91	1.66	0.	0.	0.	0.	0.
time (sec)	N/A	1.271	0.38	0.033	0.	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	354	292	0	0	0	0	241
normalized size	1	1.	1.46	1.21	0.	0.	0.	0.	1.
time (sec)	N/A	0.28	0.989	0.208	0.	0.	0.	0.	56.684

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	349	275	0	0	0	0	219
normalized size	1	1.	1.58	1.24	0.	0.	0.	0.	0.99
time (sec)	N/A	0.233	0.91	0.013	0.	0.	0.	0.	44.381

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	343	258	0	0	0	0	199
normalized size	1	1.	1.73	1.3	0.	0.	0.	0.	1.01
time (sec)	N/A	0.186	1.037	0.011	0.	0.	0.	0.	35.309

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	338	240	0	0	0	0	177
normalized size	1	1.	1.91	1.36	0.	0.	0.	0.	1.
time (sec)	N/A	0.132	0.776	0.008	0.	0.	0.	0.	26.613

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	331	224	0	0	0	0	168
normalized size	1	1.	1.96	1.33	0.	0.	0.	0.	0.99
time (sec)	N/A	0.117	0.717	0.005	0.	0.	0.	0.	30.96

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	283	386	0	0	0	0	253
normalized size	1	1.	0.88	1.2	0.	0.	0.	0.	0.79
time (sec)	N/A	0.234	0.328	0.08	0.	0.	0.	0.	33.496

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	481	410	0	0	0	0	0
normalized size	1	1.	1.69	1.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.23	1.242	0.032	0.	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	308	434	0	0	0	0	0
normalized size	1	1.	0.99	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.85	0.884	0.034	0.	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	364	326	0	0	0	0	269
normalized size	1	1.	1.36	1.22	0.	0.	0.	0.	1.
time (sec)	N/A	0.32	1.191	0.049	0.	0.	0.	0.	63.075

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	358	309	0	0	0	0	248
normalized size	1	1.	1.45	1.25	0.	0.	0.	0.	1.
time (sec)	N/A	0.273	0.993	0.01	0.	0.	0.	0.	50.961

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	354	292	0	0	0	0	226
normalized size	1	1.	1.57	1.29	0.	0.	0.	0.	1.
time (sec)	N/A	0.223	0.912	0.013	0.	0.	0.	0.	41.674

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	349	275	0	0	0	0	202
normalized size	1	1.	1.69	1.33	0.	0.	0.	0.	0.98
time (sec)	N/A	0.17	1.04	0.01	0.	0.	0.	0.	31.782

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	343	258	0	0	0	0	194
normalized size	1	1.	1.73	1.3	0.	0.	0.	0.	0.98
time (sec)	N/A	0.158	0.922	0.005	0.	0.	0.	0.	36.53

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	477	418	0	0	0	0	301
normalized size	1	1.	1.68	1.47	0.	0.	0.	0.	1.06
time (sec)	N/A	0.403	1.77	0.026	0.	0.	0.	0.	88.183

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	372	309	425	0	0	0	0	0
normalized size	1	1.22	1.01	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.711	0.727	0.034	0.	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	440	440	309	434	0	0	0	0	0
normalized size	1	1.	0.7	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	1.059	0.885	0.034	0.	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	337	241	0	0	0	0	189
normalized size	1	1.	1.8	1.29	0.	0.	0.	0.	1.01
time (sec)	N/A	0.189	0.844	0.038	0.	0.	0.	0.	39.207

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	331	224	0	0	0	0	172
normalized size	1	1.	1.95	1.32	0.	0.	0.	0.	1.01
time (sec)	N/A	0.147	0.757	0.01	0.	0.	0.	0.	29.766

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	214	209	0	0	0	0	153
normalized size	1	1.	1.42	1.38	0.	0.	0.	0.	1.01
time (sec)	N/A	0.096	0.264	0.007	0.	0.	0.	0.	20.671

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	142	85	0	0	0	0	63
normalized size	1	1.	2.22	1.33	0.	0.	0.	0.	0.98
time (sec)	N/A	0.021	0.096	0.004	0.	0.	0.	0.	9.965

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	159	107	0	0	0	0	165
normalized size	1	1.	0.95	0.64	0.	0.	0.	0.	0.98
time (sec)	N/A	0.085	0.112	0.023	0.	0.	0.	0.	17.027

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	481	410	0	0	0	0	280
normalized size	1	1.	1.68	1.43	0.	0.	0.	0.	0.98
time (sec)	N/A	0.233	1.224	0.031	0.	0.	0.	0.	32.984

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	308	434	0	0	0	0	0
normalized size	1	1.	0.98	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.438	1.307	0.031	0.	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	339	379	0	0	0	0	216
normalized size	1	1.	1.55	1.73	0.	0.	0.	0.	0.99
time (sec)	N/A	0.239	0.903	0.067	0.	0.	0.	0.	55.224

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	333	339	0	0	0	0	197
normalized size	1	1.	1.66	1.7	0.	0.	0.	0.	0.98
time (sec)	N/A	0.193	0.798	0.014	0.	0.	0.	0.	41.721

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	328	301	0	0	0	0	178
normalized size	1	1.	1.81	1.66	0.	0.	0.	0.	0.98
time (sec)	N/A	0.149	0.691	0.01	0.	0.	0.	0.	31.376

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	329	278	0	0	0	0	178
normalized size	1	1.	1.82	1.54	0.	0.	0.	0.	0.98
time (sec)	N/A	0.146	0.704	0.01	0.	0.	0.	0.	31.684

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	329	255	0	0	0	0	178
normalized size	1	1.	1.82	1.41	0.	0.	0.	0.	0.98
time (sec)	N/A	0.135	0.8	0.008	0.	0.	0.	0.	26.078

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	328	232	0	0	0	0	177
normalized size	1	1.	1.81	1.28	0.	0.	0.	0.	0.98
time (sec)	N/A	0.12	0.67	0.006	0.	0.	0.	0.	30.652

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	483	409	0	0	0	0	279
normalized size	1	1.	1.7	1.44	0.	0.	0.	0.	0.98
time (sec)	N/A	0.272	0.956	0.028	0.	0.	0.	0.	78.02

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	311	433	0	0	0	0	0
normalized size	1	1.	1.	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.625	0.782	0.035	0.	0.	0.	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	320	457	0	0	0	0	0
normalized size	1	1.	0.94	1.34	0.	0.	0.	0.	0.
time (sec)	N/A	1.214	1.015	0.037	0.	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	467	467	584	1186	0	0	0	0	444
normalized size	1	1.	1.25	2.54	0.	0.	0.	0.	0.95
time (sec)	N/A	0.851	5.24	0.023	0.	0.	0.	0.	105.783

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	488	756	0	0	0	0	328
normalized size	1	1.	1.37	2.12	0.	0.	0.	0.	0.92
time (sec)	N/A	0.461	2.989	0.012	0.	0.	0.	0.	54.248

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	302	362	0	0	0	0	257
normalized size	1	1.	1.07	1.28	0.	0.	0.	0.	0.91
time (sec)	N/A	0.23	0.431	0.006	0.	0.	0.	0.	30.159

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	214	200	0	0	0	0	332
normalized size	1	1.	0.55	0.52	0.	0.	0.	0.	0.86
time (sec)	N/A	0.5	0.228	0.038	0.	0.	0.	0.	29.312

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	703	889	1069	1279	0	0	0	0	784
normalized size	1	1.26	1.52	1.82	0.	0.	0.	0.	1.12
time (sec)	N/A	1.455	3.092	0.039	0.	0.	0.	0.	143.104

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	596	1195	0	0	0	0	0
normalized size	1	1.	1.08	2.16	0.	0.	0.	0.	0.
time (sec)	N/A	2.619	4.444	0.022	0.	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	454	503	761	0	0	0	0	405
normalized size	1	1.	1.11	1.68	0.	0.	0.	0.	0.89
time (sec)	N/A	1.812	2.443	0.012	0.	0.	0.	0.	142.436

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	293	364	0	0	0	0	340
normalized size	1	1.	0.76	0.95	0.	0.	0.	0.	0.88
time (sec)	N/A	0.96	0.447	0.007	0.	0.	0.	0.	103.691

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	205	201	0	0	0	0	173
normalized size	1	1.	1.04	1.02	0.	0.	0.	0.	0.88
time (sec)	N/A	0.523	0.222	0.038	0.	0.	0.	0.	61.454

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	718	718	1341	1293	0	0	0	0	0
normalized size	1	1.	1.87	1.8	0.	0.	0.	0.	0.
time (sec)	N/A	1.812	6.492	0.039	0.	0.	0.	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	304	355	0	0	0	0	420
normalized size	1	1.	0.63	0.74	0.	0.	0.	0.	0.88
time (sec)	N/A	1.379	0.477	0.013	0.	0.	0.	0.	142.476

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	216	198	0	0	0	0	411
normalized size	1	1.	1.06	0.97	0.	0.	0.	0.	2.01
time (sec)	N/A	0.579	0.226	0.036	0.	0.	0.	0.	153.612

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	295	357	0	0	0	0	260
normalized size	1	1.	1.01	1.22	0.	0.	0.	0.	0.89
time (sec)	N/A	0.235	0.449	0.037	0.	0.	0.	0.	33.864

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	207	199	0	0	0	0	338
normalized size	1	1.	0.52	0.5	0.	0.	0.	0.	0.85
time (sec)	N/A	0.486	0.218	0.035	0.	0.	0.	0.	31.717

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	154	380	0	0	0	0	211
normalized size	1	1.	0.67	1.66	0.	0.	0.	0.	0.92
time (sec)	N/A	0.294	0.304	0.014	0.	0.	0.	0.	54.623

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	127	235	0	0	0	0	148
normalized size	1	1.	0.76	1.4	0.	0.	0.	0.	0.88
time (sec)	N/A	0.174	0.201	0.014	0.	0.	0.	0.	27.162

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	73	108	0	0	0	0	116
normalized size	1	1.	0.6	0.89	0.	0.	0.	0.	0.95
time (sec)	N/A	0.083	0.082	0.008	0.	0.	0.	0.	13.87

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	59	55	0	0	0	0	102
normalized size	1	1.	0.48	0.44	0.	0.	0.	0.	0.82
time (sec)	N/A	0.254	0.059	0.024	0.	0.	0.	0.	35.01

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	399	175	443	0	0	0	0	332
normalized size	1	1.26	0.55	1.4	0.	0.	0.	0.	1.05
time (sec)	N/A	0.492	0.76	0.037	0.	0.	0.	0.	77.532

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.114	0.109	0.	0.	0.	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	498	498	1871	0	0	0	0	0	0
normalized size	1	1.	3.76	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.599	18.326	0.077	0.	0.	0.	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	358	345	1410	0	0	0	0	0	0
normalized size	1	0.96	3.94	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.77	6.058	0.059	0.	0.	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	706	0	0	0	0	0	233
normalized size	1	1.	2.58	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.512	1.259	0.034	0.	0.	0.	0.	67.942

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	487	0	0	0	0	0	112
normalized size	1	1.	3.66	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.138	0.782	0.02	0.	0.	0.	0.	30.396

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.059	0.057	0.	0.	0.	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.089	0.077	0.	0.	0.	0.	0.

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [155] had the largest ratio of [0.7778]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	7	1.	24	0.292
2	A	9	9	1.	26	0.346
3	A	2	2	1.	40	0.05

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
4	A	2	2	1.	40	0.05
5	A	2	2	1.	46	0.043
6	A	2	2	1.	46	0.043
7	A	7	7	1.	29	0.241
8	A	9	9	1.	31	0.29
9	A	9	6	1.	17	0.353
10	A	9	6	1.	18	0.333
11	A	3	3	1.	18	0.167
12	A	3	3	1.	19	0.158
13	A	5	3	1.	17	0.176
14	A	3	2	1.	17	0.118
15	A	2	2	1.	17	0.118
16	A	2	2	1.	17	0.118
17	A	5	3	1.	27	0.111
18	A	3	2	1.	28	0.071
19	A	5	3	1.	21	0.143
20	A	3	2	1.	22	0.091
21	A	3	3	1.	15	0.2
22	A	2	2	1.	22	0.091
23	A	5	5	1.	23	0.217
24	A	3	3	1.	21	0.143
25	A	6	6	1.	22	0.273
26	A	1	1	1.	22	0.045
27	A	3	3	1.	21	0.143
28	A	1	1	1.	23	0.043
29	A	3	3	1.	22	0.136
30	A	1	1	1.	28	0.036
31	A	2	2	1.	24	0.083
32	A	4	4	1.	28	0.143
33	A	5	5	1.	25	0.2
34	A	5	3	1.	26	0.115
35	A	5	3	1.	26	0.115
36	A	5	3	1.	27	0.111
37	A	5	3	1.	27	0.111
38	A	3	2	1.	27	0.074
39	A	3	2	1.	27	0.074
40	A	3	2	1.	28	0.071
41	A	3	2	1.	28	0.071
42	A	3	2	1.	30	0.067
43	A	5	3	1.	29	0.103
44	A	6	4	1.	29	0.138
45	A	3	2	1.	32	0.062
46	A	5	3	1.	31	0.097
47	A	5	3	1.	22	0.136
48	A	5	3	1.	23	0.13
49	A	3	2	1.	22	0.091
50	A	3	2	1.	22	0.091

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
51	A	3	3	1.	22	0.136
52	A	5	3	1.	22	0.136
53	A	5	3	1.	22	0.136
54	A	5	3	1.	20	0.15
55	A	5	3	1.	17	0.176
56	A	5	3	1.	22	0.136
57	A	5	3	1.	22	0.136
58	A	5	3	1.	22	0.136
59	A	2	2	1.	22	0.091
60	A	7	3	1.	22	0.136
61	A	5	3	1.	22	0.136
62	A	3	2	1.	22	0.091
63	A	3	2	1.	22	0.091
64	A	3	2	1.	22	0.091
65	A	2	2	1.	22	0.091
66	A	3	2	1.	22	0.091
67	A	3	2	1.	22	0.091
68	A	3	2	1.	20	0.1
69	A	3	2	1.	17	0.118
70	A	3	2	1.	22	0.091
71	A	3	2	1.	22	0.091
72	A	3	2	1.	22	0.091
73	A	3	3	1.	22	0.136
74	A	7	3	1.	22	0.136
75	A	5	3	1.	22	0.136
76	A	5	3	1.	18	0.167
77	A	3	2	1.	18	0.111
78	A	3	2	1.	18	0.111
79	A	3	2	1.	18	0.111
80	A	2	2	1.	18	0.111
81	A	5	3	1.	16	0.188
82	A	5	3	1.	13	0.231
83	A	5	3	1.	18	0.167
84	A	2	2	1.	18	0.111
85	A	7	3	1.	18	0.167
86	A	5	3	1.	18	0.167
87	A	5	3	1.	18	0.167
88	A	3	2	1.	20	0.1
89	A	3	2	1.	20	0.1
90	A	3	2	1.	20	0.1
91	A	3	2	1.	20	0.1
92	A	2	2	1.	20	0.1
93	A	3	2	1.	18	0.111
94	A	3	2	1.	15	0.133
95	A	3	2	1.	20	0.1
96	A	3	3	1.	20	0.15
97	A	5	3	1.	20	0.15

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
98	A	5	3	1.	20	0.15
99	A	5	3	1.	20	0.15
100	A	5	3	1.	23	0.13
101	A	5	3	1.	22	0.136
102	A	3	3	1.	20	0.15
103	A	3	2	1.	22	0.091
104	A	3	2	1.	22	0.091
105	A	3	2	1.	18	0.111
106	A	9	5	1.	18	0.278
107	A	10	6	1.	18	0.333
108	A	9	5	1.	18	0.278
109	A	10	6	1.	18	0.333
110	A	9	5	1.	29	0.172
111	A	9	5	1.	26	0.192
112	A	9	5	1.	24	0.208
113	A	9	5	1.	22	0.227
114	A	9	5	1.	25	0.2
115	A	9	5	1.	31	0.161
116	A	9	5	1.	32	0.156
117	A	9	5	1.	23	0.217
118	A	9	5	1.	25	0.2
119	A	9	5	1.	29	0.172
120	A	9	5	1.	32	0.156
121	A	4	4	1.	22	0.182
122	A	5	5	1.	24	0.208
123	A	4	4	1.	24	0.167
124	A	4	4	1.	24	0.167
125	A	4	4	1.	24	0.167
126	A	4	4	1.	24	0.167
127	A	3	3	1.	39	0.077
128	A	2	1	1.	17	0.059
129	A	2	1	1.	17	0.059
130	A	2	1	1.	17	0.059
131	A	2	1	1.	15	0.067
132	A	3	2	1.	17	0.118
133	A	3	3	1.	17	0.176
134	A	3	3	1.	17	0.176
135	A	4	4	1.	17	0.235
136	A	2	1	1.	19	0.053
137	A	2	1	1.	19	0.053
138	A	2	1	1.	17	0.059
139	A	2	1	1.	9	0.111
140	A	3	2	1.	19	0.105
141	A	4	3	1.	19	0.158
142	A	5	3	1.	19	0.158
143	A	5	3	1.	19	0.158
144	A	5	3	1.	19	0.158

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
145	A	11	7	1.	19	0.368
146	A	11	7	1.	19	0.368
147	A	11	7	1.	19	0.368
148	A	9	6	1.	17	0.353
149	A	9	6	1.	9	0.667
150	A	12	8	1.	19	0.421
151	A	14	9	1.	19	0.474
152	A	11	8	1.	19	0.421
153	A	11	8	1.	19	0.421
154	A	10	7	1.	17	0.412
155	A	10	7	1.	9	0.778
156	A	22	9	1.	19	0.474
157	A	24	10	1.	19	0.526
158	A	5	5	1.	21	0.238
159	A	4	4	1.	21	0.19
160	A	3	3	1.	19	0.158
161	A	2	2	1.	21	0.095
162	A	6	5	1.25	21	0.238
163	A	8	8	1.	22	0.364
164	A	7	7	1.	22	0.318
165	A	6	6	1.	20	0.3
166	A	2	2	1.	22	0.091
167	A	9	9	1.	22	0.409
168	A	6	6	1.	21	0.286
169	A	2	2	1.	23	0.087
170	A	3	3	1.	29	0.103
171	A	3	3	1.	29	0.103
172	A	3	3	1.	22	0.136
173	A	2	2	1.	24	0.083
174	A	2	2	1.	21	0.095
175	A	2	2	1.	21	0.095
176	A	1	1	1.	22	0.045
177	A	2	2	1.	21	0.095
178	F	0	0	N/A	0	N/A
179	A	0	0	0.	0	0.
180	A	9	6	0.96	19	0.316
181	A	7	6	0.95	19	0.316
182	A	6	5	1.	17	0.294
183	A	2	2	1.	9	0.222
184	A	6	5	1.	19	0.263
185	A	8	5	1.	19	0.263
186	A	6	4	0.95	19	0.21
187	A	5	4	0.92	19	0.21
188	A	4	3	1.	17	0.176
189	A	1	1	1.	9	0.111
190	A	4	3	1.	19	0.158
191	A	5	3	1.	19	0.158

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
192	A	6	3	1.	19	0.158
193	A	4	3	1.	24	0.125
194	A	4	3	1.	24	0.125
195	A	3	3	1.	24	0.125
196	A	2	2	1.	22	0.091
197	A	5	5	1.	24	0.208
198	A	6	6	1.	24	0.25
199	A	6	6	1.	26	0.231
200	A	3	3	1.	26	0.115
201	A	4	4	1.	26	0.154
202	A	6	6	1.	26	0.231
203	A	5	5	1.	28	0.179
204	A	4	4	1.	28	0.143
205	A	3	3	1.	28	0.107
206	A	3	3	1.	28	0.107
207	A	4	4	1.	28	0.143
208	A	6	6	1.	28	0.214
209	A	5	5	1.	29	0.172
210	A	4	4	1.	29	0.138
211	A	3	3	1.	29	0.103
212	A	3	3	1.	29	0.103
213	A	4	4	1.	29	0.138
214	A	6	6	1.	29	0.207
215	A	2	2	1.	19	0.105
216	A	3	3	1.67	19	0.158
217	A	7	5	0.99	31	0.161
218	A	4	3	1.	39	0.077
219	A	4	3	1.	39	0.077
220	A	3	3	1.	39	0.077
221	A	2	2	1.	37	0.054
222	A	5	5	1.	39	0.128
223	A	6	6	1.	39	0.154
224	A	7	7	1.	41	0.171
225	A	6	6	1.	41	0.146
226	A	3	3	1.	41	0.073
227	A	4	4	1.	41	0.098
228	A	6	6	1.	41	0.146
229	A	6	6	1.	20	0.3
230	A	5	5	1.	20	0.25
231	A	4	4	1.	18	0.222
232	A	8	7	1.	20	0.35
233	A	1	1	1.	20	0.05
234	A	25	12	1.	20	0.6
235	A	28	13	1.	20	0.65
236	A	5	5	1.	20	0.25
237	A	4	4	1.	20	0.2
238	A	3	3	1.	18	0.167

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
239	A	4	4	1.	20	0.2
240	A	8	7	1.	20	0.35
241	A	11	10	1.	20	0.5
242	A	4	4	1.	20	0.2
243	A	2	2	1.	20	0.1
244	A	2	2	1.	18	0.111
245	A	9	8	1.	20	0.4
246	A	16	9	1.	20	0.45
247	A	25	13	1.	20	0.65
248	A	2	1	1.	22	0.045
249	A	2	1	1.	22	0.045
250	A	2	1	1.	22	0.045
251	A	2	1	1.	20	0.05
252	A	3	2	1.	22	0.091
253	A	3	3	1.	22	0.136
254	A	3	3	1.	22	0.136
255	A	4	4	1.	22	0.182
256	A	2	1	1.	24	0.042
257	A	2	1	1.	24	0.042
258	A	2	1	1.	22	0.045
259	A	2	1	1.	14	0.071
260	A	3	2	1.	24	0.083
261	A	4	3	1.	24	0.125
262	A	5	3	1.	24	0.125
263	A	5	3	1.	24	0.125
264	A	5	3	1.	24	0.125
265	A	3	3	1.	22	0.136
266	A	3	3	1.	23	0.13
267	A	5	3	1.	24	0.125
268	A	5	3	1.	24	0.125
269	A	5	3	1.	24	0.125
270	A	3	2	1.	22	0.091
271	A	3	2	1.	14	0.143
272	A	6	3	1.	24	0.125
273	A	8	4	1.	24	0.167
274	A	4	3	1.	24	0.125
275	A	4	3	1.	24	0.125
276	A	4	3	1.	22	0.136
277	A	4	3	1.	14	0.214
278	A	10	4	1.	24	0.167
279	A	12	5	1.	24	0.208
280	A	7	5	1.	24	0.208
281	A	6	5	1.	24	0.208
282	A	5	5	1.	24	0.208
283	A	4	4	1.	24	0.167
284	A	4	4	1.	24	0.167
285	A	4	4	1.	24	0.167

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
286	A	4	4	1.	24	0.167
287	A	5	5	1.	24	0.208
288	A	6	5	0.99	24	0.208
289	A	7	5	1.	24	0.208
290	A	6	6	1.	24	0.25
291	A	5	5	1.	24	0.208
292	A	4	4	1.	22	0.182
293	A	4	4	1.	14	0.286
294	A	8	7	1.3	24	0.292
295	A	8	7	1.	24	0.292
296	A	24	11	1.	24	0.458
297	A	7	6	1.	24	0.25
298	A	6	5	1.	24	0.208
299	A	5	4	1.	22	0.182
300	A	5	5	1.	14	0.357
301	A	13	8	1.	24	0.333
302	A	20	9	1.5	24	0.375
303	A	26	11	1.25	24	0.458
304	A	5	5	1.	24	0.208
305	A	4	4	1.	24	0.167
306	A	3	3	1.	22	0.136
307	A	1	1	1.	14	0.071
308	A	4	4	1.	24	0.167
309	A	8	7	1.	24	0.292
310	A	10	9	1.	24	0.375
311	A	6	5	1.	24	0.208
312	A	5	5	1.	24	0.208
313	A	4	4	1.	24	0.167
314	A	4	4	1.	24	0.167
315	A	4	4	1.	22	0.182
316	A	4	4	1.	14	0.286
317	A	9	8	1.2	24	0.333
318	A	18	9	1.	24	0.375
319	A	28	12	1.	24	0.5
320	A	8	7	1.	24	0.292
321	A	7	7	1.	24	0.292
322	A	6	6	1.	24	0.25
323	A	5	5	1.	22	0.227
324	A	5	5	1.	14	0.357
325	A	7	7	1.	24	0.292
326	A	7	7	1.	24	0.292
327	A	20	11	1.	24	0.458
328	A	9	7	1.	24	0.292
329	A	8	7	1.	24	0.292
330	A	7	6	1.	24	0.25
331	A	6	5	1.	22	0.227
332	A	6	6	1.	14	0.429

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
333	A	13	8	1.	24	0.333
334	A	20	12	1.	24	0.5
335	A	26	14	1.	24	0.583
336	A	6	6	1.	24	0.25
337	A	5	5	1.	24	0.208
338	A	4	4	1.	22	0.182
339	A	2	2	1.	14	0.143
340	A	2	2	1.	24	0.083
341	A	7	7	1.	24	0.292
342	A	9	9	1.	24	0.375
343	A	7	6	1.	24	0.25
344	A	6	6	1.	24	0.25
345	A	5	5	1.	24	0.208
346	A	5	5	1.	24	0.208
347	A	5	5	1.	22	0.227
348	A	5	5	1.	14	0.357
349	A	8	8	1.	24	0.333
350	A	16	9	1.	24	0.375
351	A	25	12	1.	24	0.5
352	A	7	6	1.	24	0.25
353	A	6	6	1.	24	0.25
354	A	5	5	1.	24	0.208
355	A	4	4	1.	22	0.182
356	A	4	4	1.	14	0.286
357	A	6	5	1.	24	0.208
358	A	6	5	1.	24	0.208
359	A	18	9	1.	24	0.375
360	A	8	6	1.	24	0.25
361	A	7	6	1.	24	0.25
362	A	6	5	1.	24	0.208
363	A	5	4	1.	22	0.182
364	A	5	5	1.	14	0.357
365	A	11	6	1.	24	0.25
366	A	18	8	1.22	24	0.333
367	A	22	10	1.	24	0.417
368	A	5	5	1.	24	0.208
369	A	4	4	1.	24	0.167
370	A	3	3	1.	22	0.136
371	A	1	1	1.	14	0.071
372	A	2	2	1.	24	0.083
373	A	6	5	1.	24	0.208
374	A	8	7	1.	24	0.292
375	A	6	5	1.	24	0.208
376	A	5	5	1.	24	0.208
377	A	4	4	1.	24	0.167
378	A	4	4	1.	24	0.167
379	A	4	4	1.	22	0.182

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
380	A	4	4	1.	14	0.286
381	A	7	6	1.	24	0.25
382	A	14	7	1.	24	0.292
383	A	22	10	1.	24	0.417
384	A	5	5	1.	26	0.192
385	A	4	4	1.	26	0.154
386	A	3	3	1.	24	0.125
387	A	2	2	1.	26	0.077
388	A	6	5	1.26	26	0.192
389	A	6	6	1.	27	0.222
390	A	5	5	1.	27	0.185
391	A	4	4	1.	25	0.16
392	A	2	2	1.	27	0.074
393	A	7	7	1.	27	0.259
394	A	5	5	1.	26	0.192
395	A	2	2	1.	28	0.071
396	A	3	3	1.	27	0.111
397	A	2	2	1.	29	0.069
398	A	5	5	1.	24	0.208
399	A	4	4	1.	24	0.167
400	A	3	3	1.	22	0.136
401	A	4	4	1.	24	0.167
402	A	8	7	1.26	24	0.292
403	A	0	0	0.	0	0.
404	A	8	7	1.	24	0.292
405	A	7	6	0.96	24	0.25
406	A	6	5	1.	22	0.227
407	A	2	2	1.	14	0.143
408	A	0	0	0.	0	0.
409	A	0	0	0.	0	0.

3 Listing of integrals

3.1 $\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=458

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{a}eg+\sqrt{c}df\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)} - \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(ef-dg)\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)} + \frac{(ef-dg)\tan^{-1}\left(\frac{x\sqrt{-\frac{ae^4+cd^4}{d^2e^2}}}{\sqrt{a+cx^4}}\right)}{2de\sqrt{-\frac{ae^4+cd^4}{d^2e^2}}} - \frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

```
[Out] ((e*f - d*g)*ArcTan[(Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*x)/Sqrt[a
 + c*x^4]])/(2*d*e*Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]) - ((e*f -
d*g)*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*
x^4])])/(2*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[c]*d*f + Sqrt[a]*e*g)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4
)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sqrt[c]*d^2 -
Sqrt[a]*e^2)*(e*f - d*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)
/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2
)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(4*a^(1/4)*c^(1/4)*d*e*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a +
c*x^4])
```

Rubi [A] time = 1.0339, antiderivative size = 458, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{a}eg+\sqrt{c}df\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)} - \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(ef-dg)\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)} + \frac{(ef-dg)\tan^{-1}\left(\frac{x\sqrt{-\frac{ae^4+cd^4}{d^2e^2}}}{\sqrt{a+cx^4}}\right)}{2de\sqrt{-\frac{ae^4+cd^4}{d^2e^2}}} - \frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(f + g*x)/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] ((e*f - d*g)*ArcTan[(Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*x)/Sqrt[a
 + c*x^4]])/(2*d*e*Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]) - ((e*f -
d*g)*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*
x^4])])/(2*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[c]*d*f + Sqrt[a]*e*g)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4
)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sqrt[c]*d^2 -
Sqrt[a]*e^2)*(e*f - d*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)
/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2
```

```
)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(4*a^(1/4)*c^(1/4)*d*e*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a +
c*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x+f)/(e*x+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.66753, size = 275, normalized size = 0.6

$$\frac{\frac{(dg-ef)\left(\sqrt[4]{c}de\sqrt{a+cx^4}\left(\log\left(\sqrt{a+cx^4}\sqrt{ae^4+cd^4}+ae^2+cd^2x^2\right)-\log\left(e^2x^2-d^2\right)\right)+2\sqrt[4]{-1}\sqrt[4]{a}\sqrt{\frac{cx^4}{a}+1}\sqrt{ae^4+cd^4}\left(\frac{i\sqrt{a}e^2}{\sqrt{c}d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)\right)}{\sqrt[4]{c}d\sqrt{ae^4+cd^4}}-\frac{2ig\sqrt{\frac{cx^4}{a}}}{}}{2e\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(f + g*x)/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] (((-2*I)*g*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c
])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]] + ((-(e*f) + d*g)*
(2*(-1)^(1/4)*a^(1/4)*Sqrt[c*d^4 + a*e^4]*Sqrt[1 + (c*x^4)/a]*Ell
ipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)
*x)/a^(1/4)], -1] + c^(1/4)*d*e*Sqrt[a + c*x^4]*(-Log[-d^2 + e^2*
x^2] + Log[a*e^2 + c*d^2*x^2 + Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4
]])))/(c^(1/4)*d*Sqrt[c*d^4 + a*e^4]))/(2*e*Sqrt[a + c*x^4])
```

Maple [C] time = 0.022, size = 251, normalized size = 0.6

$$\frac{g}{e}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

$$+\frac{-dg+ef}{e^2}\left(-\frac{1}{2}Artanh\left(\frac{1}{2}\left(2\frac{cd^2x^2}{e^2}+2a\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}+a}}\frac{1}{\sqrt{cx^4+a}}\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}+a}}+\frac{e}{d}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticPi\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/e*g/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(
1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(
1/2)*c^(1/2))^(1/2),I)+(-d*g+e*f)/e^2*(-1/2/(c*d^4/e^4+a)^(1/2)*a
rctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(c*d^4/e^4+a)^(1/2)/(c*x^4+a)^(1
/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/d*e*(1-I/a^(1/2)*c^(1/2)*x^2)^(1
/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*
(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/2)*
c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx+f}{\sqrt{cx^4+a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x + f)/(sqrt(c*x^4 + a)*(e*x + d)),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x + f)/(sqrt(c*x^4 + a)*(e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{f+gx}{\sqrt{a+cx^4}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x+f)/(e*x+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)/(sqrt(a + c*x**4)*(d + e*x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx+f}{\sqrt{cx^4+a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x + f)/(sqrt(c*x^4 + a)*(e*x + d)),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

3.2 $\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=218

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(ef-dg)\left(\frac{\sqrt{a}e^2}{\sqrt{c}d^2};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}de\sqrt{cx^4-a}}+\frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}+\frac{\sqrt[4]{a}g\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}e\sqrt{cx^4-a}}$$

```
[Out] ((e*f - d*g)*ArcTanh[(a*e^2 - c*d^2*x^2)/(Sqrt[c*d^4 - a*e^4]*Sqr
t[-a + c*x^4])])/(2*Sqrt[c*d^4 - a*e^4]) + (a^(1/4)*g*Sqrt[1 - (c
*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*e*S
qrt[-a + c*x^4]) + (a^(1/4)*(e*f - d*g)*Sqrt[1 - (c*x^4)/a]*Ellip
ticPi[(Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[(c^(1/4)*x)/a^(1/4)], -
1])/(c^(1/4)*d*e*Sqrt[-a + c*x^4])
```

Rubi [A] time = 0.60376, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(ef-dg)\left(\frac{\sqrt{a}e^2}{\sqrt{c}d^2};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}de\sqrt{cx^4-a}}+\frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}+\frac{\sqrt[4]{a}g\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}e\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(f + g*x)/((d + e*x)*Sqrt[-a + c*x^4]),x]
```

```
[Out] ((e*f - d*g)*ArcTanh[(a*e^2 - c*d^2*x^2)/(Sqrt[c*d^4 - a*e^4]*Sqr
t[-a + c*x^4])])/(2*Sqrt[c*d^4 - a*e^4]) + (a^(1/4)*g*Sqrt[1 - (c
*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*e*S
qrt[-a + c*x^4]) + (a^(1/4)*(e*f - d*g)*Sqrt[1 - (c*x^4)/a]*Ellip
ticPi[(Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[(c^(1/4)*x)/a^(1/4)], -
1])/(c^(1/4)*d*e*Sqrt[-a + c*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x+f)/(e*x+d)/(c*x**4-a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.00761, size = 719, normalized size = 3.3

$$\frac{if\sqrt{-\frac{(1-i)\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[4]{c}x+i\sqrt[4]{a}}}\sqrt{\frac{(1+i)\left(\sqrt[4]{a}+i\sqrt[4]{c}x\right)\left(\sqrt[4]{a}+\sqrt[4]{c}x\right)}{\left(\sqrt[4]{a}-i\sqrt[4]{c}x\right)^2}}\left(\sqrt[4]{a}-i\sqrt[4]{c}x\right)^2\left(\left(\sqrt[4]{a}e-\sqrt[4]{c}d\right)F\left(\sin^{-1}\left(\sqrt{\frac{(1+i)\left(\sqrt[4]{c}x+\sqrt[4]{a}\right)}{2\sqrt[4]{c}x+2i\sqrt[4]{a}}}\right)\middle|2\right)-(1-i)\sqrt[4]{a}e\left(\frac{(1-i)\left(\sqrt[4]{c}d-i\sqrt[4]{a}e\right)}{\sqrt[4]{c}d-\sqrt[4]{a}e};\sin^{-1}\right.\right.}{\sqrt[4]{a}\left(\sqrt[4]{a}e-\sqrt[4]{c}d\right)\left(\sqrt[4]{a}e+i\sqrt[4]{c}d\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(f + g*x)/((d + e*x)*Sqrt[-a + c*x^4]),x]
```

```
[Out] (((-I)*g*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/S
qrt[a])]*x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*e) + (I*f*(a^(1/4) -
I*c^(1/4)*x)^2*Sqrt[((-1 + I)*(a^(1/4) - c^(1/4)*x))/(I*a^(1/4) +
 c^(1/4)*x)]*Sqrt[((1 + I)*(a^(1/4) + I*c^(1/4)*x)*(a^(1/4) + c^(
1/4)*x))/(a^(1/4) - I*c^(1/4)*x)^2]*((-(c^(1/4)*d) + a^(1/4)*e)*E
llipticF[ArcSin[Sqrt[((1 + I)*(a^(1/4) + c^(1/4)*x))/((2*I)*a^(1/
4) + 2*c^(1/4)*x)]], 2] - (1 - I)*a^(1/4)*e*EllipticPi[((1 - I)*(
c^(1/4)*d - I*a^(1/4)*e))/(c^(1/4)*d - a^(1/4)*e), ArcSin[Sqrt[((
1 + I)*(a^(1/4) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2]
))/(a^(1/4)*(-(c^(1/4)*d) + a^(1/4)*e)*(I*c^(1/4)*d + a^(1/4)*e))
 + (d*g*(a^(1/4) - I*c^(1/4)*x)^2*Sqrt[((-1 + I)*(a^(1/4) - c^(1/
4)*x))/(I*a^(1/4) + c^(1/4)*x)]*Sqrt[((1 + I)*(a^(1/4) + I*c^(1/4
)*x)*(a^(1/4) + c^(1/4)*x))/(a^(1/4) - I*c^(1/4)*x)^2]*(I*(c^(1/4
)*d - a^(1/4)*e)*EllipticF[ArcSin[Sqrt[((1 + I)*(a^(1/4) + c^(1/4
)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2] + (1 + I)*a^(1/4)*e*Ell
ipticPi[((1 - I)*(c^(1/4)*d - I*a^(1/4)*e))/(c^(1/4)*d - a^(1/4)*
e), ArcSin[Sqrt[((1 + I)*(a^(1/4) + c^(1/4)*x))/((2*I)*a^(1/4) +
2*c^(1/4)*x)]], 2]))/(a^(1/4)*e*(-(c^(1/4)*d) + a^(1/4)*e)*(I*c^(
1/4)*d + a^(1/4)*e)))/Sqrt[-a + c*x^4]
```

Maple [A] time = 0.039, size = 247, normalized size = 1.1

$$\frac{g}{e}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4-a}}$$
$$+\frac{-dg+ef}{e^2}\left(-\frac{1}{2}Artanh\left(\frac{1}{2}\left(2\frac{cd^2x^2}{e^2}-2a\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}-a}}\frac{1}{\sqrt{cx^4-a}}\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}-a}}+\frac{e}{d}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticPi\left(x\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x)
```

```
[Out] 1/e*g/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*
(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a
^(1/2)*c^(1/2))^(1/2),I)+(-d*g+e*f)/e^2*(-1/2/(c*d^4/e^4-a)^(1/2)
*arctanh(1/2*(2*c*x^2*d^2/e^2-2*a)/(c*d^4/e^4-a)^(1/2)/(c*x^4-a)^
(1/2))+1/(-1/a^(1/2)*c^(1/2))^(1/2)/d*e*(1+1/a^(1/2)*c^(1/2)*x^2)
^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticPi
(x*(-1/a^(1/2)*c^(1/2))^(1/2),-e^2*a^(1/2)/d^2/c^(1/2),(1/a^(1/2)
*c^(1/2))^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx+f}{\sqrt{cx^4-a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x + f)/(sqrt(c*x^4 - a)*(e*x + d)),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 - a)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x + f)/(sqrt(c*x^4 - a)*(e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{f + gx}{\sqrt{-a + cx^4}\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4-a)**(1/2),x)
```

```
[Out] Integral((f + g*x)/(sqrt(-a + c*x**4)*(d + e*x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx + f}{\sqrt{cx^4 - a}(ex + d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x + f)/(sqrt(c*x^4 - a)*(e*x + d)),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 - a)*(e*x + d)), x)
```

3.3 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\tanh^{-1}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}x^2-4}}\right)$$

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2
*Sqrt[3])]*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3
```

Rubi [A] time = 0.226246, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\tanh^{-1}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2
*Sqrt[3])]*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*3**(1/2)*x**2)**(1/2),
```

```
[Out] Timed out
```

Mathematica [C] time = 2.56647, size = 685, normalized size = 10.54

$$\frac{\left(x+\sqrt{3}-1\right)^2\sqrt{-x^3+\left(\sqrt{3}-1\right)x^2-2\left(2+\sqrt{3}\right)x+2\left(1+\sqrt{3}\right)}\sqrt{\frac{-\frac{4}{x+\sqrt{3}-1}+\sqrt{3}+1}{3+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}}}\left(2\sqrt{6}\sqrt{\frac{x^2+2\sqrt{3}+4}{\left(x+\sqrt{3}-1\right)^2}}\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-i\left(\cdots\right.}\right.}{\left(\sqrt{2\cdots\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] ((-1 + Sqrt[3] + x)^2*Sqrt[2*(1 + Sqrt[3]) - 2*(2 + Sqrt[3])*x +
(-1 + Sqrt[3])*x^2 - x^3]*Sqrt[(1 + Sqrt[3] - 4/(-1 + Sqrt[3] + x
```

```
))/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])]*((I*(-1 + Sqrt[3] + I
*Sqrt[2*(2 + Sqrt[3])]) + (2*((2*I)*Sqrt[3] - Sqrt[2*(2 + Sqrt[3]
)] + Sqrt[6*(2 + Sqrt[3])]))/(-1 + Sqrt[3] + x))*Sqrt[Sqrt[2*(2 +
 Sqrt[3])] + I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]*EllipticF[Ar
cSin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3
] + x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(2 + Sqrt[
3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])] + 2*Sqrt[6]*Sqrt[(
4 + 2*Sqrt[3] + x^2)/(-1 + Sqrt[3] + x)^2]*Sqrt[Sqrt[2*(2 + Sqrt[
3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]*EllipticPi[(2*Sqrt
[2*(2 + Sqrt[3])])/(Sqrt[2*(2 + Sqrt[3])] + I*(3 + Sqrt[3])), Arc
Sin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3]
 + x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(2 + Sqrt[3
])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])]))/((Sqrt[2*(2 + Sqr
t[3])] + I*(3 + Sqrt[3]))*Sqrt[1 + Sqrt[3] - (2 + Sqrt[3])*x + ((
-1 + Sqrt[3])*x^2)/2 - x^3/2]*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]*Sqrt
[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))])
```

Maple [C] time = 0.236, size = 327, normalized size = 5.

$$\frac{EllipticF\left(x\left(\frac{i}{2}\sqrt{3}-\frac{i}{2}\right),i\sqrt{1+4\sqrt{3}\left(1+1/2\sqrt{3}\right)}\right)}{\frac{i}{2}\sqrt{3}-\frac{i}{2}}\sqrt{1-\left(\frac{\sqrt{3}}{2}-1\right)x^2}\sqrt{1-\left(1+\frac{\sqrt{3}}{2}\right)x^2}\frac{1}{\sqrt{-4+x^4+4x^2\sqrt{3}}}$$
$$-2\sqrt{3}\left(-1/2\frac{1}{\sqrt{\left(-1-\sqrt{3}\right)^4+4\sqrt{3}\left(-1-\sqrt{3}\right)^2-4}}Artanh\left(1/2\frac{4\sqrt{3}\left(-1-\sqrt{3}\right)^2-8+4x^2\sqrt{3}+2x^2\left(-1-\sqrt{3}\right)^2}{\sqrt{\left(-1-\sqrt{3}\right)^4+4\sqrt{3}\left(-1-\sqrt{3}\right)^2-4}\sqrt{-4+x^4+4x^2\sqrt{3}}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*x^2*3^(1/2))^(1/2),x)
```

```
[Out] 1/(1/2*I*3^(1/2)-1/2*I)*(1-(1/2*3^(1/2)-1)*x^2)^(1/2)*(1-(1+1/2*3
^(1/2))*x^2)^(1/2)/(-4+x^4+4*x^2*3^(1/2))^(1/2)*EllipticF(x*(1/2*
I*3^(1/2)-1/2*I),I*(1+4*3^(1/2)*(1+1/2*3^(1/2)))^(1/2))-2*3^(1/2)
*(-1/2/((-1-3^(1/2))^4+4*3^(1/2)*(-1-3^(1/2))^2-4)^(1/2)*arctanh(
1/2*(4*3^(1/2)*(-1-3^(1/2))^2-8+4*x^2*3^(1/2)+2*x^2*(-1-3^(1/2))^
2)/((-1-3^(1/2))^4+4*3^(1/2)*(-1-3^(1/2))^2-4)^(1/2)/(-4+x^4+4*x^
2*3^(1/2))^(1/2))-1/(1/2*3^(1/2)-1)^(1/2)/(-1-3^(1/2))*(1-(1/2*3^
(1/2)-1)*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4+4*x^2*3
^(1/2))^(1/2)*EllipticPi((1/2*3^(1/2)-1)^(1/2)*x,1/(1/2*3^(1/2)-1
)/(-1-3^(1/2))^2,(1+1/2*3^(1/2))^(1/2)/(1/2*3^(1/2)-1)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1)),x, a
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + s
qrt(3) + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1)),x, a
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\left(x+1+\sqrt{3}\right)\sqrt{x^4+4\sqrt{3}x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))*sqrt(x**4 + 4*sqrt(
3)*x**2 - 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1)),x, a
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + s
qrt(3) + 1)), x)
```

3.4 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx$

Optimal. Leaf size=63

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}x^2-4}}\right)$$

```
[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*S
qrt[3])]*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4])])/3
```

Rubi [A] time = 0.228188, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*S
qrt[3])]*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4])])/3
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*3**(1/2)*x**2)**(1/2),
```

```
[Out] Timed out
```

Mathematica [C] time = 6.28109, size = 1137, normalized size = 18.05

$$-\frac{\left(x-\sqrt{3}-1\right)^2\sqrt{\frac{\sqrt{3}-1+\frac{4}{x-\sqrt{3}-1}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}}\sqrt{\left(x-\sqrt{3}+1\right)^3+\left(-2+4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)^2+\left(20-8\sqrt{3}\right)\left(x-\sqrt{3}+1\right)+16\sqrt{3}-24}\left(\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] -(((-1 - Sqrt[3] + x)^2*Sqrt[(-1 + Sqrt[3] + 4/(-1 - Sqrt[3] + x)
)/(-3 + Sqrt[3] - I*Sqrt[4 - 2*Sqrt[3]])]*Sqrt[-24 + 16*Sqrt[3] +
```

```
(20 - 8*Sqrt[3])*(1 - Sqrt[3] + x) + (-2 + 4*Sqrt[3])*(1 - Sqrt[
3] + x)^2 + (1 - Sqrt[3] + x)^3]*((I*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I
*(1 + Sqrt[3]) + (8*I)/(-1 - Sqrt[3] + x)] + I*Sqrt[3]*Sqrt[Sqrt[
4 - 2*Sqrt[3]] + I*(1 + Sqrt[3]) + (8*I)/(-1 - Sqrt[3] + x)] + Sq
rt[-2*I + (2*I)*Sqrt[3] - 2*Sqrt[12 - 6*Sqrt[3]] + 4*Sqrt[4 - 2*S
qrt[3]] - ((16*I)*(-2 + Sqrt[3]))/(-1 - Sqrt[3] + x)] + (2*((2*I)
*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 + Sqrt[3]) + (8*I)/(-1 -
 Sqrt[3] + x)] + Sqrt[6]*Sqrt[-I + I*Sqrt[3] - Sqrt[12 - 6*Sqrt[3
]] + 2*Sqrt[4 - 2*Sqrt[3]] - ((8*I)*(-2 + Sqrt[3]))/(-1 - Sqrt[3]
 + x)] + Sqrt[-2*I + (2*I)*Sqrt[3] - 2*Sqrt[12 - 6*Sqrt[3]] + 4*S
qrt[4 - 2*Sqrt[3]] - ((16*I)*(-2 + Sqrt[3]))/(-1 - Sqrt[3] + x)])
)/(-1 - Sqrt[3] + x))*EllipticF[ArcSin[Sqrt[Sqrt[4 - 2*Sqrt[3]] -
 I*(1 + Sqrt[3]) - (8*I)/(-1 - Sqrt[3] + x)]/(2^(3/4)*(2 - Sqrt[3
])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3
+ Sqrt[3]))] + 2*Sqrt[6]*Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3
]) - (8*I)/(-1 - Sqrt[3] + x)]*Sqrt[1 + 8/(-1 - Sqrt[3] + x)^2 +
(2*(1 + Sqrt[3]))/(-1 - Sqrt[3] + x)]*EllipticPi[(2*Sqrt[4 - 2*Sq
rt[3]])/(Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3])), ArcSin[Sqrt[Sqr
t[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3]) - (8*I)/(-1 - Sqrt[3] + x)]/(2
^(3/4)*(2 - Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2
*Sqrt[3]] + I*(-3 + Sqrt[3]))]))/((Sqrt[4 - 2*Sqrt[3]] - I*(-3 +
Sqrt[3]))*Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3]) - (8*I)/(-1
- Sqrt[3] + x)]*Sqrt[8*(1 + Sqrt[3]) + 4*(3 + Sqrt[3])*(-1 - Sqrt
[3] + x) + 2*(1 + Sqrt[3])*(-1 - Sqrt[3] + x)^2 + (-1 - Sqrt[3] +
 x)^3/2]*Sqrt[48 - 32*Sqrt[3] - 64*(1 - Sqrt[3] + x) + 32*Sqrt[3]
*(1 - Sqrt[3] + x) + 24*(1 - Sqrt[3] + x)^2 - 16*Sqrt[3]*(1 - Sqr
t[3] + x)^2 - 4*(1 - Sqrt[3] + x)^3 + 4*Sqrt[3]*(1 - Sqrt[3] + x)
^3 + (1 - Sqrt[3] + x)^4]))
```

Maple [C] time = 0.232, size = 311, normalized size = 4.9

$$\frac{EllipticF\left(x\left(\frac{i}{2}+\frac{i}{2}\sqrt{3}\right),i\sqrt{1-4\sqrt{3}\left(-1/2\sqrt{3}+1\right)}\right)}{\frac{i}{2}+\frac{i}{2}\sqrt{3}}\sqrt{1-\left(-1-\frac{\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{\sqrt{3}}{2}+1\right)x^2}\frac{1}{\sqrt{-4+x^4-4x^2\sqrt{3}}}+2\sqrt{3}\left(-1/2\frac{1}{\sqrt{\left(\sqrt{3}-1\right)^4-4\sqrt{3}\left(\sqrt{3}-1\right)^2-4}}Artanh\left(1/2\frac{-4\sqrt{3}\left(\sqrt{3}-1\right)^2-8-4x^2\sqrt{3}+2x^2\left(\sqrt{3}-1\right)^2}{\sqrt{\left(\sqrt{3}-1\right)^4-4\sqrt{3}\left(\sqrt{3}-1\right)^2-4}\sqrt{-4+x^4-4x^2\sqrt{3}}}\right)-\frac{\sqrt{1}}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*x^2*3^(1/2))^(1/2),x)
```

```
[Out] 1/(1/2*I+1/2*I*3^(1/2))*(1-(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(-1/2*3
^(1/2)+1)*x^2)^(1/2)/(-4+x^4-4*x^2*3^(1/2))^(1/2)*EllipticF(x*(1/
2*I+1/2*I*3^(1/2)),I*(1-4*3^(1/2)*(-1/2*3^(1/2)+1))^(1/2))+2*3^(1
/2)*(-1/2/((3^(1/2)-1)^4-4*3^(1/2)*(3^(1/2)-1)^2-4)^(1/2)*arctanh
(1/2*(-4*3^(1/2)*(3^(1/2)-1)^2-8-4*x^2*3^(1/2)+2*x^2*(3^(1/2)-1)^
2)/((3^(1/2)-1)^4-4*3^(1/2)*(3^(1/2)-1)^2-4)^(1/2)/(-4+x^4-4*x^2*
3^(1/2))^(1/2))-1/(-1-1/2*3^(1/2))^(1/2)/(3^(1/2)-1)*(1-(-1-1/2*3
^(1/2))*x^2)^(1/2)*(1-(-1/2*3^(1/2)+1)*x^2)^(1/2)/(-4+x^4-4*x^2*3
^(1/2))^(1/2)*EllipticPi((-1-1/2*3^(1/2))^(1/2)*x,1/(-1-1/2*3^(1/
2))/(3^(1/2)-1)^2,(-1/2*3^(1/2)+1)^(1/2)/(-1-1/2*3^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1)),x, a
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - s
qrt(3) + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1)),x, a
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1+\sqrt{3}}{\left(x-\sqrt{3}+1\right)\sqrt{x^4-4\sqrt{3}x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)*sqrt(x**4 - 4*sqrt(
3)*x**2 - 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\,\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1)),x, a
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - s
qrt(3) + 1)), x)
```

3.5 $\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\tanh^{-1}\left(\frac{\left(2x-\sqrt{3}+1\right)^2}{2\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{4x^4+4\sqrt{3}x^2-1}}\right)$$

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + 2*x)^2/(2*Sqrt[3*(-3
 + 2*Sqrt[3])]*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^4])])/3
```

Rubi [A] time = 0.196658, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\tanh^{-1}\left(\frac{\left(2x-\sqrt{3}+1\right)^2}{2\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{4x^4+4\sqrt{3}x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + 2*x)/((1 + Sqrt[3] + 2*x)*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + 2*x)^2/(2*Sqrt[3*(-3
 + 2*Sqrt[3])]*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^4])])/3
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+2*x-3**(1/2))/(1+2*x+3**(1/2))/(-1+4*x**4+4*3**(1/2)*x**2)**
```

```
[Out] Timed out
```

Mathematica [C] time = 2.40256, size = 623, normalized size = 8.65

$$\frac{\left(2x+\sqrt{3}-1\right)^2\sqrt{\frac{-\frac{4}{2x+\sqrt{3}-1}+\sqrt{3}+1}{3+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}}}\left(4\sqrt{3}\sqrt{\frac{2x^2+\sqrt{3}+2}{\left(2x+\sqrt{3}-1\right)^2}}\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-i\left(\frac{8}{2x+\sqrt{3}-1}-\sqrt{3}+1\right)}\left(\frac{2\sqrt{2\left(2+\sqrt{3}\right)}}{\sqrt{2\left(2+\sqrt{3}\right)}+i\left(3+\sqrt{3}\right)};\sin^{-1}\left(\sqrt{\sqrt{2(\ldots}\right.\right.\right.}{\left(\sqrt{2\ldots}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + 2*x)/((1 + Sqrt[3] + 2*x)*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^4
```

```
[Out] ((-1 + Sqrt[3] + 2*x)^2*Sqrt[(1 + Sqrt[3] - 4/(-1 + Sqrt[3] + 2*x
))/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])]*((I*(-1 + Sqrt[3] + I
```

```
*Sqrt[2*(2 + Sqrt[3])]) + (2*((2*I)*Sqrt[3] - Sqrt[2*(2 + Sqrt[3]
)] + Sqrt[6*(2 + Sqrt[3])]))/(-1 + Sqrt[3] + 2*x))*Sqrt[Sqrt[2*(2
 + Sqrt[3])] + I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + 2*x))]*Elliptic
F[ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sq
rt[3] + 2*x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(2 +
 Sqrt[3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])] + 4*Sqrt[3]*
Sqrt[(2 + Sqrt[3] + 2*x^2)/(-1 + Sqrt[3] + 2*x)^2]*Sqrt[Sqrt[2*(2
 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + 2*x))]*Elliptic
Pi[(2*Sqrt[2*(2 + Sqrt[3])])/(Sqrt[2*(2 + Sqrt[3])] + I*(3 + Sqrt
[3])), ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1
 + Sqrt[3] + 2*x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2
*(2 + Sqrt[3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])]))/((Sqr
t[2*(2 + Sqrt[3])] + I*(3 + Sqrt[3]))*Sqrt[-2 + 8*Sqrt[3]*x^2 + 8
*x^4]*Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[
3] + 2*x))])
```

Maple [C] time = 0.225, size = 336, normalized size = 4.7

$$\frac{EllipticF\left(x\left(i\sqrt{3}-i\right), i\sqrt{1+\sqrt{3}\left(2\sqrt{3}+4\right)}\right)}{i\sqrt{3}-i}\sqrt{1-\left(2\sqrt{3}-4\right)x^2}\sqrt{1-\left(2\sqrt{3}+4\right)x^2}\frac{1}{\sqrt{-1+4x^4+4x^2\sqrt{3}}}$$
$$-2\sqrt{3}\left(-1/4\frac{1}{\sqrt{4\left(-1/2-1/2\sqrt{3}\right)^4+4\sqrt{3}\left(-1/2-1/2\sqrt{3}\right)^2-1}}Artanh\left(1/2\frac{4\sqrt{3}\left(-1/2-1/2\sqrt{3}\right)^2-2+4x^2\sqrt{3}}{\sqrt{4\left(-1/2-1/2\sqrt{3}\right)^4+4\sqrt{3}\left(-1/2-1/2\right.}}\right.\right.$$
$$-1/2\frac{\sqrt{1-\left(2\sqrt{3}-4\right)x^2}\sqrt{1-\left(2\sqrt{3}+4\right)x^2}}{\sqrt{2\sqrt{3}-4}\left(-1/2-1/2\sqrt{3}\right)\sqrt{-1+4x^4+4x^2\sqrt{3}}}EllipticPi\left(\sqrt{2\sqrt{3}-4}x, \frac{1}{\left(2\sqrt{3}-4\right)\left(-1/2-1/2\sqrt{3}\right)^2}, \frac{\sqrt{2\sqrt{3}+4}}{\sqrt{2\sqrt{3}-4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*x^2*3^(1/2))^(1/2),x)
```

```
[Out] 1/(I*3^(1/2)-I)*(1-(2*3^(1/2)-4)*x^2)^(1/2)*(1-(2*3^(1/2)+4)*x^2)
^(1/2)/(-1+4*x^4+4*x^2*3^(1/2))^(1/2)*EllipticF(x*(I*3^(1/2)-I),I
*(1+3^(1/2)*(2*3^(1/2)+4))^(1/2))-2*3^(1/2)*(-1/4/(4*(-1/2-1/2*3^
(1/2))^4+4*3^(1/2)*(-1/2-1/2*3^(1/2))^2-1)^(1/2)*arctanh(1/2*(4*3
^(1/2)*(-1/2-1/2*3^(1/2))^2-2+4*x^2*3^(1/2)+8*x^2*(-1/2-1/2*3^(1/
2))^2)/(4*(-1/2-1/2*3^(1/2))^4+4*3^(1/2)*(-1/2-1/2*3^(1/2))^2-1)^
(1/2)/(-1+4*x^4+4*x^2*3^(1/2))^(1/2))-1/2/(2*3^(1/2)-4)^(1/2)/(-1
/2-1/2*3^(1/2))*(1-(2*3^(1/2)-4)*x^2)^(1/2)*(1-(2*3^(1/2)+4)*x^2)
^(1/2)/(-1+4*x^4+4*x^2*3^(1/2))^(1/2)*EllipticPi((2*3^(1/2)-4)^(1
/2)*x,1/(2*3^(1/2)-4)/(-1/2-1/2*3^(1/2))^2,(2*3^(1/2)+4)^(1/2)/(2
*3^(1/2)-4)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x-\sqrt{3}+1}{\sqrt{4x^4+4\sqrt{3}x^2-1}\left(2x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2*x + sqrt(3) + 1)
```

```
[Out] integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2
*x + sqrt(3) + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2*x + sqrt(3) + 1)
```

```
[Out] Timed out
```

Sympy [A] time = 11.3049, size = 0, normalized size = 0.

NaN

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x-3**(1/2))/(1+2*x+3**(1/2))/(-1+4*x**4+4*3**(1/2)*x**2)**(1/2)
```

```
[Out] nan
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x - \sqrt{3} + 1}{\sqrt{4\,x^4 + 4\,\sqrt{3}x^2 - 1}\left(2\,x + \sqrt{3} + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2*x + sqrt(3) + 1)
```

```
[Out] integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2
*x + sqrt(3) + 1)), x)
```

3.6 $\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(2x+\sqrt{3}+1\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{4x^4-4\sqrt{3}x^2-1}}\right)$$

```
[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + 2*x)^2/(2*Sqrt[3*(3 +
 2*Sqrt[3])]*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^4])])/3
```

Rubi [A] time = 0.198258, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(2x+\sqrt{3}+1\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{4x^4-4\sqrt{3}x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + 2*x)/((1 - Sqrt[3] + 2*x)*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^4]),x]
```

```
[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + 2*x)^2/(2*Sqrt[3*(3 +
 2*Sqrt[3])]*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^4])])/3
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+2*x+3**(1/2))/(1+2*x-3**(1/2))/(-1+4*x**4-4*3**(1/2)*x**2)**
```

```
[Out] Timed out
```

Mathematica [C] time = 6.2223, size = 1198, normalized size = 17.11

$$-\frac{\left(2x-\sqrt{3}-1\right)^2\sqrt{\frac{\sqrt{3}-1+\frac{4}{2x-\sqrt{3}-1}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}}\sqrt{\left(2x-\sqrt{3}+1\right)^3+\left(-2+4\sqrt{3}\right)\left(2x-\sqrt{3}+1\right)^2+\left(20-8\sqrt{3}\right)\left(2x-\sqrt{3}+1\right)+16\sqrt{3}-}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + 2*x)/((1 - Sqrt[3] + 2*x)*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^4
```

```
[Out] -((-1 - Sqrt[3] + 2*x)^2*Sqrt[(-1 + Sqrt[3] + 4/(-1 - Sqrt[3] + 2
*x))/(-3 + Sqrt[3] - I*Sqrt[4 - 2*Sqrt[3]])]*Sqrt[-24 + 16*Sqrt[3
```

```
] + (20 - 8*Sqrt[3])*(1 - Sqrt[3] + 2*x) + (-2 + 4*Sqrt[3])*(1 -
Sqrt[3] + 2*x)^2 + (1 - Sqrt[3] + 2*x)^3]*((I*Sqrt[Sqrt[4 - 2*Sqr
t[3]] + I*(1 + Sqrt[3]) + (8*I)/(-1 - Sqrt[3] + 2*x)] + I*Sqrt[3]
*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 + Sqrt[3]) + (8*I)/(-1 - Sqrt[3]
 + 2*x)] + Sqrt[-2*I + (2*I)*Sqrt[3] - 2*Sqrt[12 - 6*Sqrt[3]] + 4
*Sqrt[4 - 2*Sqrt[3]] - ((16*I)*(-2 + Sqrt[3]))/(-1 - Sqrt[3] + 2*
x)] + (2*((2*I)*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 + Sqrt[3]
) + (8*I)/(-1 - Sqrt[3] + 2*x)] + Sqrt[6]*Sqrt[-I + I*Sqrt[3] - S
qrt[12 - 6*Sqrt[3]] + 2*Sqrt[4 - 2*Sqrt[3]] - ((8*I)*(-2 + Sqrt[3
]))/(-1 - Sqrt[3] + 2*x)] + Sqrt[-2*I + (2*I)*Sqrt[3] - 2*Sqrt[12
 - 6*Sqrt[3]] + 4*Sqrt[4 - 2*Sqrt[3]] - ((16*I)*(-2 + Sqrt[3]))/(
-1 - Sqrt[3] + 2*x)]))/(-1 - Sqrt[3] + 2*x))*EllipticF[ArcSin[Sqr
t[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3]) - (8*I)/(-1 - Sqrt[3] + 2
*x)]/(2^(3/4)*(2 - Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqr
t[4 - 2*Sqrt[3]] + I*(-3 + Sqrt[3]))] + 2*Sqrt[6]*Sqrt[Sqrt[4 - 2
*Sqrt[3]] - I*(1 + Sqrt[3]) - (8*I)/(-1 - Sqrt[3] + 2*x)]*Sqrt[1
+ 8/(-1 - Sqrt[3] + 2*x)^2 + (2*(1 + Sqrt[3]))/(-1 - Sqrt[3] + 2*
x)]*EllipticPi[(2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] - I*(
-3 + Sqrt[3])), ArcSin[Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3])
 - (8*I)/(-1 - Sqrt[3] + 2*x)]/(2^(3/4)*(2 - Sqrt[3])^(1/4))], (2
*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3 + Sqrt[3]))]))
/(2*(Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3]))*Sqrt[Sqrt[4 - 2*Sqrt
[3]] - I*(1 + Sqrt[3]) - (8*I)/(-1 - Sqrt[3] + 2*x)]*Sqrt[8*(1 +
Sqrt[3]) + 4*(3 + Sqrt[3])*(-1 - Sqrt[3] + 2*x) + 2*(1 + Sqrt[3])
*(-1 - Sqrt[3] + 2*x)^2 + (-1 - Sqrt[3] + 2*x)^3/2]*Sqrt[12 - 8*S
qrt[3] - 16*(1 - Sqrt[3] + 2*x) + 8*Sqrt[3]*(1 - Sqrt[3] + 2*x) +
 6*(1 - Sqrt[3] + 2*x)^2 - 4*Sqrt[3]*(1 - Sqrt[3] + 2*x)^2 - (1 -
 Sqrt[3] + 2*x)^3 + Sqrt[3]*(1 - Sqrt[3] + 2*x)^3 + (1 - Sqrt[3]
+ 2*x)^4/4])
```

Maple [C] time = 0.217, size = 337, normalized size = 4.8

$$\frac{\mathit{EllipticF}\left(x\left(i+i\sqrt{3}\right),i\sqrt{1-\sqrt{3}\left(-2\sqrt{3}+4\right)}\right)}{i+i\sqrt{3}}\sqrt{1-\left(-2\sqrt{3}-4\right)x^2}\sqrt{1-\left(-2\sqrt{3}+4\right)x^2}\frac{1}{\sqrt{-1+4x^4-4x^2\sqrt{3}}}$$

$$+2\sqrt{3}\left(-1/4\frac{1}{\sqrt{4\left(1/2\sqrt{3}-1/2\right)^4-4\sqrt{3}\left(1/2\sqrt{3}-1/2\right)^2-1}}\mathit{Artanh}\left(1/2\frac{-4\sqrt{3}\left(1/2\sqrt{3}-1/2\right)^2-2-4x^2\sqrt{3}+8x}{\sqrt{4\left(1/2\sqrt{3}-1/2\right)^4-4\sqrt{3}\left(1/2\sqrt{3}-1/2\right)^2-}}\right.\right.$$

$$-1/2\frac{\sqrt{1-\left(-2\sqrt{3}-4\right)x^2}\sqrt{1-\left(-2\sqrt{3}+4\right)x^2}}{\sqrt{-2\sqrt{3}-4}\left(1/2\sqrt{3}-1/2\right)\sqrt{-1+4x^4-4x^2\sqrt{3}}}\mathit{EllipticPi}\left(\sqrt{-2\sqrt{3}-4}x,\frac{1}{\left(-2\sqrt{3}-4\right)\left(1/2\sqrt{3}-1/2\right)^2},\frac{\sqrt{-2\sqrt{3}+}}{\sqrt{-2\sqrt{3}-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*x^2*3^(1/2))^(1/2),x)
```

```
[Out] 1/(I+I*3^(1/2))*(1-(-2*3^(1/2)-4)*x^2)^(1/2)*(1-(-2*3^(1/2)+4)*x^
2)^(1/2)/(-1+4*x^4-4*x^2*3^(1/2))^(1/2)*EllipticF(x*(I+I*3^(1/2))
,I*(1-3^(1/2)*(-2*3^(1/2)+4))^(1/2))+2*3^(1/2)*(-1/4/(4*(1/2*3^(1
/2)-1/2)^4-4*3^(1/2)*(1/2*3^(1/2)-1/2)^2-1)^(1/2)*arctanh(1/2*(-4
*3^(1/2)*(1/2*3^(1/2)-1/2)^2-2-4*x^2*3^(1/2)+8*x^2*(1/2*3^(1/2)-1
/2)^2)/(4*(1/2*3^(1/2)-1/2)^4-4*3^(1/2)*(1/2*3^(1/2)-1/2)^2-1)^(1
/2)/(-1+4*x^4-4*x^2*3^(1/2))^(1/2))-1/2/(-2*3^(1/2)-4)^(1/2)/(1/2
*3^(1/2)-1/2)*(1-(-2*3^(1/2)-4)*x^2)^(1/2)*(1-(-2*3^(1/2)+4)*x^2)
^(1/2)/(-1+4*x^4-4*x^2*3^(1/2))^(1/2)*EllipticPi((-2*3^(1/2)-4)^(
1/2)*x,1/(-2*3^(1/2)-4)/(1/2*3^(1/2)-1/2)^2,(-2*3^(1/2)+4)^(1/2)/
(-2*3^(1/2)-4)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x + \sqrt{3} + 1}{\sqrt{4x^4 - 4\sqrt{3}x^2 - 1}\left(2x - \sqrt{3} + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2*x - sqrt(3) + 1)
```

```
[Out] integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2
*x - sqrt(3) + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2*x - sqrt(3) + 1)
```

```
[Out] Timed out
```

Sympy [A] time = 11.2237, size = 0, normalized size = 0.

NaN

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x+3**(1/2))/(1+2*x-3**(1/2))/(-1+4*x**4-4*3**(1/2)*x**2)**(1/2)
```

```
[Out] nan
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x + \sqrt{3} + 1}{\sqrt{4x^4 - 4\sqrt{3}x^2 - 1}\left(2x - \sqrt{3} + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2*x - sqrt(3) + 1)
```

```
[Out] integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2
*x - sqrt(3) + 1)), x)
```

3.7 $\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=561

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}eg+\sqrt{c}df\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)} - \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(ef-dg)\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)} + \frac{(ef-dg)\tan^{-1}\left(\frac{x\sqrt{-\frac{ae^4+cd^4}{d^2e^2}-b}}{\sqrt{a+bx^2+cx^4}}\right)}{2de\sqrt{-\frac{ae^4+cd^4}{d^2e^2}-b}} - \frac{(ef-dg)\tanh^{-1}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}$$

```
[Out] ((e*f - d*g)*ArcTan[(Sqrt[-b - (c*d^4 + a*e^4)/(d^2*e^2)]*x)/Sqrt
[a + b*x^2 + c*x^4]])/(2*d*e*Sqrt[-b - (c*d^4 + a*e^4)/(d^2*e^2)]
) - ((e*f - d*g)*ArcTanh[(b*d^2 + 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2
)/(2*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]*Sqrt[a + b*x^2 + c*x^4])])/(
2*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]) + ((Sqrt[c]*d*f + Sqrt[a]*e*g)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt
[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*
Sqrt[a + b*x^2 + c*x^4]) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(e*f - d*
g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]
*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*(Sqrt[c]*d^2 + Sqrt[a]*e^2)
*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 1.39989, antiderivative size = 561, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}eg+\sqrt{c}df\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)} - \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(ef-dg)\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)} + \frac{(ef-dg)\tan^{-1}\left(\frac{x\sqrt{-\frac{ae^4+cd^4}{d^2e^2}-b}}{\sqrt{a+bx^2+cx^4}}\right)}{2de\sqrt{-\frac{ae^4+cd^4}{d^2e^2}-b}} - \frac{(ef-dg)\tanh^{-1}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(f + g*x)/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]),x]
```



```
[Out] ((e*f - d*g)*ArcTan[(Sqrt[-b - (c*d^4 + a*e^4)/(d^2*e^2)]*x)/Sqrt
[a + b*x^2 + c*x^4]])/(2*d*e*Sqrt[-b - (c*d^4 + a*e^4)/(d^2*e^2)]
) - ((e*f - d*g)*ArcTanh[(b*d^2 + 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2
)/(2*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]*Sqrt[a + b*x^2 + c*x^4])])/(
2*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]) + ((Sqrt[c]*d*f + Sqrt[a]*e*g)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt
[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*
Sqrt[a + b*x^2 + c*x^4]) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(e*f - d*
g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]
```

```
*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*(Sqrt[c]*d^2 + Sqrt[a]*e^2)
*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x+f)/(e*x+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.836456, size = 0, normalized size = 0.

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(f + g*x)/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] Integrate[(f + g*x)/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]), x]
```

Maple [A] time = 0.034, size = 437, normalized size = 0.8

$$\frac{g\sqrt{2}}{4e}\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\frac{b\left(b\right.}{}}\right.$$

$$+\frac{-dg+ef}{e^2}\left(-\frac{1}{2}Artanh\left(\frac{1}{2}\left(2\frac{cx^2d^2}{e^2}+bx^2+\frac{bd^2}{e^2}+2a\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}+\frac{bd^2}{e^2}+a}}\frac{1}{\sqrt{cx^4+bx^2+a}}\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}+\frac{bd^2}{e^2}+a}}+\frac{\sqrt{2}e}{d}\sqrt{1-\frac{x^2}{2a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4/e*g*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+
(-d*g+e*f)/e^2*(-1/2/(c*d^4/e^4+b*d^2/e^2+a)^(1/2)*arctanh(1/2*(2
*c*x^2*d^2/e^2+b*x^2+b*d^2/e^2+2*a)/(c*d^4/e^4+b*d^2/e^2+a)^(1/2)
/(c*x^4+b*x^2+a)^(1/2))+2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
/d*e*(1-1/2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(1+1/2*(b+(-4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),2/(-b+(-4*a*c+b^2)^(1/
2))*a/d^2*e^2,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx + f}{\sqrt{cx^4 + bx^2 + a}(ex + d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{f + gx}{(d + ex)\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)/((d + e*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.8 $\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=527

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}(ef-dg)\left(-\frac{\left(b-\sqrt{b^2+4ac}\right)e^2}{2cd^2};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2+4ac}-b}}\right)|\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}de\sqrt{-a+bx^2+cx^4}}+\frac{g\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}-\frac{(ef-dg)\tanh^{-1}\left(\frac{-2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{-a+bx^2+cx^4}\sqrt{-ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{-ae^4+bd^2e^2+cd^4}}$$

```
[Out] -((e*f - d*g)*ArcTanh[(b*d^2 - 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(
2*Sqrt[c*d^4 + b*d^2*e^2 - a*e^4]*Sqrt[-a + b*x^2 + c*x^4])])/(2*
Sqrt[c*d^4 + b*d^2*e^2 - a*e^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*g
*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))*EllipticF[ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (-2*Sqrt[b^2 + 4*a*c])
/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*e*Sqrt[(1 + (2*c*x^2)
/(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]))
]*Sqrt[-a + b*x^2 + c*x^4]) + (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*(e*f
- d*g)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[-((b - Sqrt[b^2 + 4*a*c]
)*e^2)/(2*c*d^2), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 +
 4*a*c]]], (b - Sqrt[b^2 + 4*a*c])/(b + Sqrt[b^2 + 4*a*c])])/(Sqr
t[2]*Sqrt[c]*d*e*Sqrt[-a + b*x^2 + c*x^4])
```

Rubi [A] time = 1.85558, antiderivative size = 527, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}(ef-dg)\left(-\frac{\left(b-\sqrt{b^2+4ac}\right)e^2}{2cd^2};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2+4ac}-b}}\right)|\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}de\sqrt{-a+bx^2+cx^4}}+\frac{g\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}-\frac{(ef-dg)\tanh^{-1}\left(\frac{-2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{-a+bx^2+cx^4}\sqrt{-ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{-ae^4+bd^2e^2+cd^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(f + g*x)/((d + e*x)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] -((e*f - d*g)*ArcTanh[(b*d^2 - 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(
2*Sqrt[c*d^4 + b*d^2*e^2 - a*e^4]*Sqrt[-a + b*x^2 + c*x^4])])/(2*
Sqrt[c*d^4 + b*d^2*e^2 - a*e^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*g
*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))*EllipticF[ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (-2*Sqrt[b^2 + 4*a*c])
/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*e*Sqrt[(1 + (2*c*x^2)
/(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]))
]*Sqrt[-a + b*x^2 + c*x^4]) + (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*(e*f
- d*g)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[-((b - Sqrt[b^2 + 4*a*c]
)*e^2)/(2*c*d^2), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 +
```

```
 4*a*c]]], (b - Sqrt[b^2 + 4*a*c])/(b + Sqrt[b^2 + 4*a*c])])/(Sqr
t[2]*Sqrt[c]*d*e*Sqrt[-a + b*x^2 + c*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x+f)/(e*x+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.865718, size = 0, normalized size = 0.

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(f + g*x)/((d + e*x)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Integrate[(f + g*x)/((d + e*x)*Sqrt[-a + b*x^2 + c*x^4]), x]
```

Maple [A] time = 0.057, size = 439, normalized size = 0.8

$$\frac{g}{2e}\sqrt{4+2\frac{\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\sqrt{4-2\frac{\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}EllipticF\left(\frac{x}{2}\sqrt{-2\frac{-b+\sqrt{4ac+b^2}}{a}},\frac{1}{2}\sqrt{-4-2\frac{b\left(b+\sqrt{4ac+b\right.}}{ac}}\right.$$

$$+\frac{-dg+ef}{e^2}\left(-\frac{1}{2}Artanh\left(\frac{1}{2}\left(2\frac{cx^2d^2}{e^2}+bx^2+\frac{bd^2}{e^2}-2a\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}+\frac{bd^2}{e^2}-a}}\frac{1}{\sqrt{cx^4+bx^2-a}}\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}+\frac{bd^2}{e^2}-a}}+\frac{e}{d}\sqrt{1+\frac{x^2}{2a}\left(-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/2/e*g/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^
4+b*x^2-a)^(1/2)*EllipticF(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1
/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))+(-d*g+e*f)/e^2*
(-1/2/(c*d^4/e^4+b*d^2/e^2-a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+
b*x^2+b*d^2/e^2-2*a)/(c*d^4/e^4+b*d^2/e^2-a)^(1/2)/(c*x^4+b*x^2-a
)^(1/2))+1/(-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)/d*e*(1+1/2*(-b+(
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(1-1/2*(b+(4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)/(c*x^4+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(4*a*c+b^2)^(1
/2))/a)^(1/2)*x,-2/(-b+(4*a*c+b^2)^(1/2))*a/d^2*e^2,1/2*2^(1/2)*(
(b+(4*a*c+b^2)^(1/2))/a)^(1/2)/(-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1
/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx+f}{\sqrt{cx^4+bx^2-a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 - a)*(e*x + d)),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 - a)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 - a)*(e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{f + gx}{(d + ex)\sqrt{-a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral((f + g*x)/((d + e*x)*sqrt(-a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 - a)*(e*x + d)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.9 $\int \frac{c+dx^2}{a+bx^4} \, dx$

Optimal. Leaf size=247

$$-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

```
[Out] -((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
)/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((
Sqrt[b]*c - Sqrt[a]*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]
*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sq
rt[2]*a^(3/4)*b^(3/4))
```

Rubi [A] time = 0.326324, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$

$$-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(a + b*x^4),x]
```

```
[Out] -((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
)/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((
Sqrt[b]*c - Sqrt[a]*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]
*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sq
rt[2]*a^(3/4)*b^(3/4))
```

Rubi in Sympy [A] time = 61.89, size = 230, normalized size = 0.93

$$\frac{\sqrt{2}\left(\sqrt{a}d-\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}d-\sqrt{b}c\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}d+\sqrt{b}c\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}d+\sqrt{b}c\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(b*x**4+a),x)
```

```
[Out] sqrt(2)*(sqrt(a)*d - sqrt(b)*c)*log(-sqrt(2)*a**(1/4)*b**(3/4)*x
+ sqrt(a)*sqrt(b) + b*x**2)/(8*a**(3/4)*b**(3/4)) - sqrt(2)*(sqrt
```

```
(a)*d - sqrt(b)*c)*log(sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt
(b) + b*x**2)/(8*a**(3/4)*b**(3/4)) - sqrt(2)*(sqrt(a)*d + sqrt(b
)*c)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(3/4)*b**(3/4))
+ sqrt(2)*(sqrt(a)*d + sqrt(b)*c)*atan(1 + sqrt(2)*b**(1/4)*x/a**
(1/4))/(4*a**(3/4)*b**(3/4))
```

Mathematica [A] time = 0.123167, size = 183, normalized size = 0.74

$$-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\right)-2\left(\sqrt{a}d+\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+}{4\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)] + 2*(Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(
1/4)] - (Sqrt[b]*c - Sqrt[a]*d)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*x + Sqrt[b]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
+ Sqrt[b]*x^2]))/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

Maple [A] time = 0.009, size = 260, normalized size = 1.1

$$\frac{c\sqrt{2}}{8\,a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{c\sqrt{2}}{4\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{c\sqrt{2}}{4\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{d\sqrt{2}}{8\,b}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{d\sqrt{2}}{4\,b}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{d\sqrt{2}}{4\,b}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(b*x^4+a),x)
```

```
[Out] 1/8*c*(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(
1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4*c*(a/b)^(1/4)/
a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4*c*(a/b)^(1/4)/a*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/8*d/b/(a/b)^(1/4)*2^(1/2)*
ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(
1/2)+(a/b)^(1/2)))+1/4*d/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
b)^(1/4)*x+1)+1/4*d/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.326442, size = 1035, normalized size = 4.19

$$-\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x\right.$$
$$\left.+\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right)$$
$$+\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x\right.$$
$$\left.-\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right)$$
$$+\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x\right.$$
$$\left.+\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3+a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right)$$
$$-\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x\right.$$
$$\left.-\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3+a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^
3)) + 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(
-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2
*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a
^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b
*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(b^2*c^4 - a^
2*d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(
a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 -
2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt((
a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)
/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 -
2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 + a^2*b*c*d^2)*sq
rt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*
c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2
*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^
3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*
b^2*c^3 + a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 +
 a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b)))
```

Sympy [A] time = 1.91401, size = 109, normalized size = 0.44

$$\operatorname{RootSum}\left(256t^4a^3b^3 + 64t^2a^2b^2cd + a^2d^4 + 2abc^2d^2 + b^2c^4, \left(t \mapsto t\log\left(x + \frac{64t^3a^3b^2d + 12ta^2bcd^2 - 4tab^2c^3}{a^2d^4 - b^2c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3 + 64*_t**2*a**2*b**2*c*d + a**2*d**4
+ 2*a*b*c**2*d**2 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a*
*3*b**2*d + 12*_t*a**2*b*c*d**2 - 4*_t*a*b**2*c**3)/(a**2*d**4 -
b**2*c**4))))
```

GIAC/XCAS [A] time = 0.307205, size = 325, normalized size = 1.32

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}d\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}d\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*arctan(1/2*sq
rt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt
(2)*((a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*
b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4
) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^
3)^(3/4)*d)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

3.10 $\int \frac{c-dx^2}{a+bx^4}\,dx$

Optimal. Leaf size=247

$$-\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

```
[Out] -((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
)/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((
Sqrt[b]*c + Sqrt[a]*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]
*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sq
rt[2]*a^(3/4)*b^(3/4))
```

Rubi [A] time = 0.280385, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c - d*x^2)/(a + b*x^4),x]
```

```
[Out] -((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
)/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((
Sqrt[b]*c + Sqrt[a]*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]
*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sq
rt[2]*a^(3/4)*b^(3/4))
```

Rubi in Sympy [A] time = 62.351, size = 230, normalized size = 0.93

$$\frac{\sqrt{2}\left(\sqrt{a}d-\sqrt{b}c\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}d-\sqrt{b}c\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}d+\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}d+\sqrt{b}c\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)/(b*x**4+a),x)
```

```
[Out] sqrt(2)*(sqrt(a)*d - sqrt(b)*c)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1
/4))/(4*a**(3/4)*b**(3/4)) - sqrt(2)*(sqrt(a)*d - sqrt(b)*c)*atan
```

```
(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(3/4)*b**(3/4)) - sqrt(2)
*(sqrt(a)*d + sqrt(b)*c)*log(-sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(
a)*sqrt(b) + b*x**2)/(8*a**(3/4)*b**(3/4)) + sqrt(2)*(sqrt(a)*d +
 sqrt(b)*c)*log(sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b
*x**2)/(8*a**(3/4)*b**(3/4))
```

Mathematica [A] time = 0.087005, size = 184, normalized size = 0.74

$$-\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\right)+\left(2\sqrt{a}d-2\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c - d*x^2)/(a + b*x^4),x]
```

```
[Out] ((-2*Sqrt[b]*c + 2*Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1
/4)] + 2*(Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a
^(1/4)] - (Sqrt[b]*c + Sqrt[a]*d)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2]))/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

Maple [A] time = 0.005, size = 260, normalized size = 1.1

$$\frac{c\sqrt{2}}{8\,a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{c\sqrt{2}}{4\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{c\sqrt{2}}{4\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)-\frac{d\sqrt{2}}{8\,b}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{d\sqrt{2}}{4\,b}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{d\sqrt{2}}{4\,b}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)/(b*x^4+a),x)
```

```
[Out] 1/8*c*(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(
1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4*c*(a/b)^(1/4)/
a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4*c*(a/b)^(1/4)/a*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)-1/8*d/b/(a/b)^(1/4)*2^(1/2)*
ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(
1/2)+(a/b)^(1/2)))-1/4*d/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
b)^(1/4)*x+1)-1/4*d/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(d*x^2 - c)/(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.316516, size = 1035, normalized size = 4.19

$$-\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x+\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right)+\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x-\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right)+\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x+\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right)-\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x-\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(d*x^2 - c)/(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3
)) + 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-
(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*
b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3
*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^
2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d
^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3
*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*
b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt(-(a*b
*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a
*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a
*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 + a^2*b*c*d^2)*sqrt(
-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*
d)/(a*b))) - 1/4*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*
d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3
*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*b
^2*c^3 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 +
 a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b)))
```

Sympy [A] time = 2.74251, size = 110, normalized size = 0.45

$$-\operatorname{RootSum}\left(256t^4a^3b^3 - 64t^2a^2b^2cd + a^2d^4 + 2abc^2d^2 + b^2c^4, \left(t \mapsto t\log\left(x + \frac{64t^3a^3b^2d - 12ta^2bcd^2 + 4tab^2c^3}{a^2d^4 - b^2c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2+c)/(b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**3 - 64*_t**2*a**2*b**2*c*d + a**2*d**4
 + 2*a*b*c**2*d**2 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a
**3*b**2*d - 12*_t*a**2*b*c*d**2 + 4*_t*a*b**2*c**3)/(a**2*d**4 -
 b**2*c**4))))
```

GIAC/XCAS [A] time = 0.29328, size = 325, normalized size = 1.32

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{3}{4}}d\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{3}{4}}d\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(d*x^2 - c)/(b*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*arctan(1/2*sq
rt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt
(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*
b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4
) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c + (a*b^
3)^(3/4)*d)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

3.11 $\int \frac{c+dx^2}{a-bx^4}\,dx$

Optimal. Leaf size=86

$$\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

```
[Out] ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*
b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*d)*ArcTanh[(b^(1/4)*x)/a^(1/4)])
/(2*a^(3/4)*b^(3/4))
```

Rubi [A] time = 0.100966, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*
b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*d)*ArcTanh[(b^(1/4)*x)/a^(1/4)])
/(2*a^(3/4)*b^(3/4))
```

Rubi in Sympy [A] time = 16.0188, size = 76, normalized size = 0.88

$$-\frac{\left(\sqrt{a}d-\sqrt{b}c\right)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(-b*x**4+a),x)
```

```
[Out] -(sqrt(a)*d - sqrt(b)*c)*atan(b**(1/4)*x/a**(1/4))/(2*a**(3/4)*b*
*(3/4)) + (sqrt(a)*d + sqrt(b)*c)*atanh(b**(1/4)*x/a**(1/4))/(2*a
**(3/4)*b**(3/4))
```

Mathematica [A] time = 0.0469434, size = 95, normalized size = 1.1

$$\frac{2\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(\sqrt{a}d+\sqrt{b}c\right)\left(\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)-\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\right)}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(a - b*x^4),x]
```

```
[Out] (2*(Sqrt[b]*c - Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]
*c + Sqrt[a]*d)*(Log[a^(1/4) - b^(1/4)*x] - Log[a^(1/4) + b^(1/4)
*x]))/(4*a^(3/4)*b^(3/4))
```

Maple [B] time = 0.006, size = 122, normalized size = 1.4

$$\frac{c}{4a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{c}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)-\frac{d}{2b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{d}{4b}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(-b*x^4+a),x)
```

```
[Out] 1/4*c*(a/b)^(1/4)/a*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/2*c*(a/
b)^(1/4)/a*arctan(x/(a/b)^(1/4))-1/2*d/b/(a/b)^(1/4)*arctan(x/(a/
b)^(1/4))+1/4*d/b/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(d*x^2 + c)/(b*x^4 - a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.331886, size = 1019, normalized size = 11.85

$$\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x+\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right)-\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x-\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right)-\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x+\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right)+\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x-\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(d*x^2 + c)/(b*x^4 - a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))
 + 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b^
2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c
*d^2)*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3
)) + 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2
 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x
 - (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))
 - a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d
^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*sqrt((
b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log
(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^
2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 + a^2*b*c*d^2)*sqrt(-(a*b*sqr
t((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b)))
 + 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b
^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt
((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 + a^2
*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^
3*b^3)) - 2*c*d)/(a*b)))
```

Sympy [A] time = 1.99784, size = 110, normalized size = 1.28

$$-\operatorname{RootSum}\left(256t^4a^3b^3-64t^2a^2b^2cd-a^2d^4+2abc^2d^2-b^2c^4,\left(t\mapsto t\log\left(x+\frac{-64t^3a^3b^2d+12ta^2bcd^2+4tab^2c^3}{a^2d^4-b^2c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**3 - 64*_t**2*a**2*b**2*c*d - a**2*d**4
 + 2*a*b*c**2*d**2 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*
a**3*b**2*d + 12*_t*a**2*b*c*d**2 + 4*_t*a*b**2*c**3)/(a**2*d**4
- b**2*c**4))))
```

GIAC/XCAS [A] time = 0.290926, size = 347, normalized size = 4.03

$$\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2c+\left(-ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}+\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2c+\left(-ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}+\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{3}{4}}d\right)\ln\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8ab^3}-\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{3}{4}}d\right)\ln\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(d*x^2 + c)/(b*x^4 - a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4)*d)*arctan(1/2*
sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/4*
sqrt(2)*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4)*d)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/8*sqrt
(2)*((-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*d)*ln(x^2 + sqrt(2)*x*
(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*((-a*b^3)^(1/4)*
b^2*c - (-a*b^3)^(3/4)*d)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(
-a/b))/(a*b^3)
```

3.12 $\int \frac{c-dx^2}{a-bx^4}\,dx$

Optimal. Leaf size=86

$$\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

```
[Out] ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*
b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*d)*ArcTanh[(b^(1/4)*x)/a^(1/4)])
/(2*a^(3/4)*b^(3/4))
```

Rubi [A] time = 0.088682, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c - d*x^2)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*
b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*d)*ArcTanh[(b^(1/4)*x)/a^(1/4)])
/(2*a^(3/4)*b^(3/4))
```

Rubi in Sympy [A] time = 16.8022, size = 76, normalized size = 0.88

$$-\frac{\left(\sqrt{a}d-\sqrt{b}c\right)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)/(-b*x**4+a),x)
```

```
[Out] -(sqrt(a)*d - sqrt(b)*c)*atanh(b**(1/4)*x/a**(1/4))/(2*a**(3/4)*b
**(3/4)) + (sqrt(a)*d + sqrt(b)*c)*atan(b**(1/4)*x/a**(1/4))/(2*a
**(3/4)*b**(3/4))
```

Mathematica [A] time = 0.035346, size = 95, normalized size = 1.1

$$\frac{2\left(\sqrt{a}d+\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}c-\sqrt{a}d\right)\left(\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)-\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\right)}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c - d*x^2)/(a - b*x^4),x]
```

```
[Out] (2*(Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]
*c - Sqrt[a]*d)*(Log[a^(1/4) - b^(1/4)*x] - Log[a^(1/4) + b^(1/4)
*x]))/(4*a^(3/4)*b^(3/4))
```

Maple [B] time = 0.003, size = 122, normalized size = 1.4

$$\frac{c}{4a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{c}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$
$$+\frac{d}{2b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{d}{4b}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)/(-b*x^4+a),x)
```

```
[Out] 1/4*c*(a/b)^(1/4)/a*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/2*c*(a/
b)^(1/4)/a*arctan(x/(a/b)^(1/4))+1/2*d/b/(a/b)^(1/4)*arctan(x/(a/
b)^(1/4))-1/4*d/b/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 - c)/(b*x^4 - a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.318288, size = 1019, normalized size = 11.85

$$\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x+\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right)-\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x-\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right)-\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x+\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3+a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right)+\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x-\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3+a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 - c)/(b*x^4 - a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)
) + 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b
^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*
c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b
^3)) + 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*
d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4
)*x - (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^
3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c
^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqr
t((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*
log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 + a^2*b*c*d^2)*sqrt((a*b*s
qrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b)
)) + 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*
b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqr
t((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 + a^
2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^
3*b^3)) - 2*c*d)/(a*b)))
```

Sympy [A] time = 2.81242, size = 110, normalized size = 1.28

$$\mathrm{RootSum}\left(256t^4a^3b^3+64t^2a^2b^2cd-a^2d^4+2abc^2d^2-b^2c^4,\left(t\mapsto t\log\left(x+\frac{-64t^3a^3b^2d-12ta^2bcd^2-4tab^2c^3}{a^2d^4-b^2c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2+c)/(-b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3 + 64*_t**2*a**2*b**2*c*d - a**2*d**4
+ 2*a*b*c**2*d**2 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a
**3*b**2*d - 12*_t*a**2*b*c*d**2 - 4*_t*a*b**2*c**3)/(a**2*d**4 -
 b**2*c**4))))
```

GIAC/XCAS [A] time = 0.286423, size = 347, normalized size = 4.03

$$\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}+\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}+\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2c+\left(-ab^3\right)^{\frac{3}{4}}d\right)\ln\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8ab^3}-\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2c+\left(-ab^3\right)^{\frac{3}{4}}d\right)\ln\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 - c)/(b*x^4 - a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*d)*arctan(1/2*
sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/4*
sqrt(2)*((-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*d)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/8*sqrt
(2)*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4)*d)*ln(x^2 + sqrt(2)*x*
(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*((-a*b^3)^(1/4)*
b^2*c + (-a*b^3)^(3/4)*d)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(
-a/b))/(a*b^3)
```

3.13 $\int \frac{2+3x^2}{4+9x^4} dx$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(1-\sqrt{3}x\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[1 - Sqrt[3]*x]/(2*Sqrt[3]) + ArcTan[1 + Sqrt[3]*x]/(2*Sqr
t[3])
```

Rubi [A] time = 0.0422042, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{\tan^{-1}\left(\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(1-\sqrt{3}x\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(4 + 9*x^4),x]

[Out] -ArcTan[1 - Sqrt[3]*x]/(2*Sqrt[3]) + ArcTan[1 + Sqrt[3]*x]/(2*Sqr
t[3])
```

Rubi in Sympy [A] time = 5.34746, size = 32, normalized size = 0.8

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}x-1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((3*x**2+2)/(9*x**4+4),x)

[Out] sqrt(3)*atan(sqrt(3)*x - 1)/6 + sqrt(3)*atan(sqrt(3)*x + 1)/6
```

Mathematica [A] time = 0.0192236, size = 33, normalized size = 0.82

$$\frac{\tan^{-1}\left(\sqrt{3}x+1\right)-\tan^{-1}\left(1-\sqrt{3}x\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(4 + 9*x^4),x]

[Out] (-ArcTan[1 - Sqrt[3]*x] + ArcTan[1 + Sqrt[3]*x])/(2*Sqrt[3])
```

Maple [B] time = 0.009, size = 122, normalized size = 3.1

$$\frac{\sqrt{6}\,\sqrt{2}}{12}\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}-1\right)+\frac{\sqrt{6}\,\sqrt{2}}{48}\ln\left(1\left(x^2+\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}\right)\left(x^2-\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}\right)^{-1}\right)+\frac{\sqrt{6}\,\sqrt{2}}{12}\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}+1\right)+\frac{\sqrt{6}\,\sqrt{2}}{48}\ln\left(1\left(x^2-\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}\right)\left(x^2+\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((3*x^2+2)/(9*x^4+4),x)
```

```
[Out] 1/12*6^(1/2)*2^(1/2)*arctan(1/2*6^(1/2)*x*2^(1/2)-1)+1/48*6^(1/2)
*2^(1/2)*ln((x^2+1/3*6^(1/2)*x*2^(1/2)+2/3)/(x^2-1/3*6^(1/2)*x*2^
(1/2)+2/3))+1/12*6^(1/2)*2^(1/2)*arctan(1/2*6^(1/2)*x*2^(1/2)+1)+
1/48*6^(1/2)*2^(1/2)*ln((x^2-1/3*6^(1/2)*x*2^(1/2)+2/3)/(x^2+1/3*
6^(1/2)*x*2^(1/2)+2/3))
```

Maxima [A] time = 0.848098, size = 53, normalized size = 1.32

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(3x+\sqrt{3}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(3x-\sqrt{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((3*x^2 + 2)/(9*x^4 + 4),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(3*x + sqrt(3))) + 1/6*sqrt(3)*arc
tan(1/3*sqrt(3)*(3*x - sqrt(3)))
```

Fricas [A] time = 0.287986, size = 38, normalized size = 0.95

$$\frac{1}{6}\sqrt{3}\left(\arctan\left(\frac{1}{4}\sqrt{3}\left(3x^3+2x\right)\right)+\arctan\left(\frac{1}{2}\sqrt{3}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((3*x^2 + 2)/(9*x^4 + 4),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(arctan(1/4*sqrt(3)*(3*x^3 + 2*x)) + arctan(1/2*sqrt(
3)*x))
```

Sympy [A] time = 0.235763, size = 41, normalized size = 1.02

$$\frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{\sqrt{3}x}{2}\right)+2\operatorname{atan}\left(\frac{3\sqrt{3}x^3}{4}+\frac{\sqrt{3}x}{2}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((3*x**2+2)/(9*x**4+4),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/2) + 2*atan(3*sqrt(3)*x**3/4 + sqrt(3)*
x/2))/12
```

GIAC/XCAS [A] time = 0.274163, size = 70, normalized size = 1.75

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{9}{8}\sqrt{2}\left(\frac{4}{9}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{9}{8}\sqrt{2}\left(\frac{4}{9}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(9*x^4 + 4),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(9/8*sqrt(2)*(4/9)^(3/4)*(2*x + sqrt(2)*(4/9)^(
1/4))) + 1/6*sqrt(3)*arctan(9/8*sqrt(2)*(4/9)^(3/4)*(2*x - sqrt(2
)*(4/9)^(1/4)))
```

3.14 $\int \frac{2-3x^2}{4+9x^4}\,dx$

Optimal. Leaf size=51

$$\frac{\log\left(3x^2+2\sqrt{3}x+2\right)}{4\sqrt{3}}-\frac{\log\left(3x^2-2\sqrt{3}x+2\right)}{4\sqrt{3}}$$

```
[Out] -Log[2 - 2*Sqrt[3]*x + 3*x^2]/(4*Sqrt[3]) + Log[2 + 2*Sqrt[3]*x +
 3*x^2]/(4*Sqrt[3])
```

Rubi [A] time = 0.0483462, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{\log\left(3x^2+2\sqrt{3}x+2\right)}{4\sqrt{3}}-\frac{\log\left(3x^2-2\sqrt{3}x+2\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - 3*x^2)/(4 + 9*x^4),x]
```

```
[Out] -Log[2 - 2*Sqrt[3]*x + 3*x^2]/(4*Sqrt[3]) + Log[2 + 2*Sqrt[3]*x +
 3*x^2]/(4*Sqrt[3])
```

Rubi in Sympy [A] time = 12.3444, size = 46, normalized size = 0.9

$$-\frac{\sqrt{3}\log\left(3x^2-2\sqrt{3}x+2\right)}{12}+\frac{\sqrt{3}\log\left(3x^2+2\sqrt{3}x+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-3*x**2+2)/(9*x**4+4),x)
```

```
[Out] -sqrt(3)*log(3*x**2 - 2*sqrt(3)*x + 2)/12 + sqrt(3)*log(3*x**2 +
2*sqrt(3)*x + 2)/12
```

Mathematica [A] time = 0.020653, size = 44, normalized size = 0.86

$$\frac{\log\left(3x^2+2\sqrt{3}x+2\right)-\log\left(-3x^2+2\sqrt{3}x-2\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - 3*x^2)/(4 + 9*x^4),x]
```

```
[Out] (-Log[-2 + 2*Sqrt[3]*x - 3*x^2] + Log[2 + 2*Sqrt[3]*x + 3*x^2])/(
4*Sqrt[3])
```

Maple [B] time = 0.003, size = 82, normalized size = 1.6

$$\frac{\sqrt{6}\sqrt{2}}{48}\ln\left(1\left(x^2+\frac{\sqrt{6}x\sqrt{2}}{3}+\frac{2}{3}\right)\left(x^2-\frac{\sqrt{6}x\sqrt{2}}{3}+\frac{2}{3}\right)^{-1}\right)-\frac{\sqrt{6}\sqrt{2}}{48}\ln\left(1\left(x^2-\frac{\sqrt{6}x\sqrt{2}}{3}+\frac{2}{3}\right)\left(x^2+\frac{\sqrt{6}x\sqrt{2}}{3}+\frac{2}{3}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3*x^2+2)/(9*x^4+4),x)
```

```
[Out] 1/48*6^(1/2)*2^(1/2)*ln((x^2+1/3*6^(1/2)*x*2^(1/2)+2/3)/(x^2-1/3*
6^(1/2)*x*2^(1/2)+2/3))-1/48*6^(1/2)*2^(1/2)*ln((x^2-1/3*6^(1/2)*
x*2^(1/2)+2/3)/(x^2+1/3*6^(1/2)*x*2^(1/2)+2/3))
```

Maxima [A] time = 0.836451, size = 53, normalized size = 1.04

$$\frac{1}{12}\sqrt{3}\log\left(3x^2+2\sqrt{3}x+2\right)-\frac{1}{12}\sqrt{3}\log\left(3x^2-2\sqrt{3}x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(3*x^2 - 2)/(9*x^4 + 4),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*log(3*x^2 + 2*sqrt(3)*x + 2) - 1/12*sqrt(3)*log(3*x^
2 - 2*sqrt(3)*x + 2)
```

Fricas [A] time = 0.28456, size = 55, normalized size = 1.08

$$\frac{1}{12}\sqrt{3}\log\left(\frac{36x^3+\sqrt{3}\left(9x^4+24x^2+4\right)+24x}{9x^4+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(3*x^2 - 2)/(9*x^4 + 4),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log((36*x^3 + sqrt(3)*(9*x^4 + 24*x^2 + 4) + 24*x)/(
9*x^4 + 4))
```

Sympy [A] time = 0.194143, size = 49, normalized size = 0.96

$$-\frac{\sqrt{3}\log\left(x^2-\frac{2\sqrt{3}x}{3}+\frac{2}{3}\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\frac{2\sqrt{3}x}{3}+\frac{2}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*x**2+2)/(9*x**4+4),x)
```

```
[Out] -sqrt(3)*log(x**2 - 2*sqrt(3)*x/3 + 2/3)/12 + sqrt(3)*log(x**2 +
2*sqrt(3)*x/3 + 2/3)/12
```

GIAC/XCAS [A] time = 0.276163, size = 54, normalized size = 1.06

$$\frac{1}{12}\sqrt{3}\ln\left(x^2 + \sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}} x + \frac{2}{3}\right) - \frac{1}{12}\sqrt{3}\ln\left(x^2 - \sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}} x + \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(3*x^2 - 2)/(9*x^4 + 4),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*ln(x^2 + sqrt(2)*(4/9)^(1/4)*x + 2/3) - 1/12*sqrt(3)
*ln(x^2 - sqrt(2)*(4/9)^(1/4)*x + 2/3)
```

3.15 $\int \frac{2+3x^2}{4-9x^4}\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

```
[Out] ArcTanh[Sqrt[3/2]*x]/Sqrt[6]
```

Rubi [A] time = 0.0110161, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{\tanh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(4 - 9*x^4),x]

[Out] ArcTanh[Sqrt[3/2]*x]/Sqrt[6]
```

Rubi in Sympy [A] time = 2.62034, size = 15, normalized size = 0.94

$$\frac{\sqrt{6}\,\text{atanh}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/(-9*x**4+4),x)

[Out] sqrt(6)*atanh(sqrt(6)*x/2)/6
```

Mathematica [A] time = 0.0230237, size = 32, normalized size = 2.

$$\frac{\log\left(3x+\sqrt{6}\right)-\log\left(\sqrt{6}-3x\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/(4 - 9*x^4),x]

[Out] (-Log[Sqrt[6] - 3*x] + Log[Sqrt[6] + 3*x])/(2*Sqrt[6])
```

Maple [A] time = 0.002, size = 13, normalized size = 0.8

$$\frac{\sqrt{6}}{6}\,Artanh\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/(-9*x^4+4),x)
```

```
[Out] 1/6*arctanh(1/2*x*6^(1/2))*6^(1/2)
```

Maxima [A] time = 0.811624, size = 34, normalized size = 2.12

$$-\frac{1}{12}\sqrt{6}\log\left(\frac{3x-\sqrt{6}}{3x+\sqrt{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(3*x^2 + 2)/(9*x^4 - 4),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(6)*log((3*x - sqrt(6))/(3*x + sqrt(6)))
```

Fricas [A] time = 0.301299, size = 42, normalized size = 2.62

$$\frac{1}{12}\sqrt{6}\log\left(\frac{\sqrt{6}\left(3x^2+2\right)+12x}{3x^2-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(3*x^2 + 2)/(9*x^4 - 4),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*log((sqrt(6)*(3*x^2 + 2) + 12*x)/(3*x^2 - 2))
```

Sympy [A] time = 0.166318, size = 32, normalized size = 2.

$$-\frac{\sqrt{6}\log\left(x-\frac{\sqrt{6}}{3}\right)}{12}+\frac{\sqrt{6}\log\left(x+\frac{\sqrt{6}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/(-9*x**4+4),x)
```

```
[Out] -sqrt(6)*log(x - sqrt(6)/3)/12 + sqrt(6)*log(x + sqrt(6)/3)/12
```

GIAC/XCAS [A] time = 0.269926, size = 39, normalized size = 2.44

$$\frac{1}{12}\sqrt{6}\ln\left(\left|x+\frac{1}{3}\sqrt{6}\right|\right)-\frac{1}{12}\sqrt{6}\ln\left(\left|x-\frac{1}{3}\sqrt{6}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(3*x^2 + 2)/(9*x^4 - 4),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*ln(abs(x + 1/3*sqrt(6))) - 1/12*sqrt(6)*ln(abs(x - 1
/3*sqrt(6)))
```

3.16 $\int \frac{2-3x^2}{4-9x^4}\,dx$

Optimal. Leaf size=16

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

```
[Out] ArcTan[Sqrt[3/2]*x]/Sqrt[6]
```

Rubi [A] time = 0.00853843, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - 3*x^2)/(4 - 9*x^4),x]

[Out] ArcTan[Sqrt[3/2]*x]/Sqrt[6]
```

Rubi in Sympy [A] time = 2.58155, size = 15, normalized size = 0.94

$$\frac{\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-3*x**2+2)/(-9*x**4+4),x)

[Out] sqrt(6)*atan(sqrt(6)*x/2)/6
```

Mathematica [A] time = 0.00814293, size = 16, normalized size = 1.

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - 3*x^2)/(4 - 9*x^4),x]

[Out] ArcTan[Sqrt[3/2]*x]/Sqrt[6]
```

Maple [A] time = 0.004, size = 13, normalized size = 0.8

$$\frac{\sqrt{6}}{6}\arctan\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3*x^2+2)/(-9*x^4+4),x)
```

```
[Out] 1/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Maxima [A] time = 0.862497, size = 16, normalized size = 1.

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 - 2)/(9*x^4 - 4),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

Fricas [A] time = 0.279951, size = 16, normalized size = 1.

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 - 2)/(9*x^4 - 4),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

Sympy [A] time = 0.177656, size = 15, normalized size = 0.94

$$\frac{\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*x**2+2)/(-9*x**4+4),x)
```

```
[Out] sqrt(6)*atan(sqrt(6)*x/2)/6
```

GIAC/XCAS [A] time = 0.267811, size = 16, normalized size = 1.

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 - 2)/(9*x^4 - 4),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

3.17 $\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

```
[Out] -((b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1
/4))) + (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2
]*a^(1/4))
```

Rubi [A] time = 0.085071, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a]*Sqrt[b] + b*x^2)/(a + b*x^4),x]
```

```
[Out] -((b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1
/4))) + (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2
]*a^(1/4))
```

Rubi in Sympy [A] time = 13.192, size = 73, normalized size = 0.97

$$-\frac{\sqrt{2}\sqrt[4]{b}\,\text{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}+\frac{\sqrt{2}\sqrt[4]{b}\,\text{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a**(1/2)*b**(1/2))/(b*x**4+a),x)
```

```
[Out] -sqrt(2)*b**(1/4)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(2*a**(1/
4)) + sqrt(2)*b**(1/4)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(2*a
**(1/4))
```

Mathematica [A] time = 0.042832, size = 60, normalized size = 0.8

$$\frac{\sqrt[4]{b}\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)-\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right)}{\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a]*Sqrt[b] + b*x^2)/(a + b*x^4),x]
```

```
[Out] (b^(1/4)*(-ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + ArcTan[1 + (
Sqrt[2]*b^(1/4)*x)/a^(1/4)]))/(Sqrt[2]*a^(1/4))
```

Maple [B] time = 0.006, size = 254, normalized size = 3.4

$$\frac{\sqrt{2}}{8}\sqrt{b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt{a}}$$
$$+\frac{\sqrt{2}}{4}\sqrt{b}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt{a}}+\frac{\sqrt{2}}{4}\sqrt{b}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt{a}}$$
$$+\frac{\sqrt{2}}{8}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{\sqrt{2}}{4}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{\sqrt{2}}{4}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x)
```

```
[Out] 1/8/a^(1/2)*b^(1/2)*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(
1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4/a^
(1/2)*b^(1/2)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)
+1/4/a^(1/2)*b^(1/2)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x-1)+1/8/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/
b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4/(a/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4/(a/b)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + sqrt(a)*sqrt(b))/(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.313349, size = 1, normalized size = 0.01

$$\left[\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{bx^4-4\sqrt{a}\sqrt{b}x^2+4\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^3-ax\right)\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}+a}{bx^4+a}\right),\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{b}x}{\sqrt{a}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}\right)\right.$$
$$\left.+\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\frac{\sqrt{\frac{1}{2}}\left(bx^3+\sqrt{a}\sqrt{b}x\right)}{a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + sqrt(a)*sqrt(b))/(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*log((b*x^4 - 4*sqrt(a)*sqrt
(b)*x^2 + 4*sqrt(1/2)*(sqrt(a)*sqrt(b)*x^3 - a*x)*sqrt(-sqrt(b)/s
qrt(a)) + a)/(b*x^4 + a)), sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*arctan
(sqrt(1/2)*sqrt(b)*x/(sqrt(a)*sqrt(sqrt(b)/sqrt(a)))) + sqrt(1/2)
*sqrt(sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*(b*x^3 + sqrt(a)*sqrt(b)*
x)/(a*sqrt(sqrt(b)/sqrt(a))))]
```

Sympy [A] time = 1.92794, size = 138, normalized size = 1.84

$$-\frac{\sqrt{2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(-\frac{\sqrt{2}\sqrt{a}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}-\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{\sqrt{2}\sqrt{a}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}-\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a**(1/2)*b**(1/2))/(b*x**4+a),x)
```

```
[Out] -sqrt(2)*sqrt(-sqrt(b)/sqrt(a))*log(-sqrt(2)*sqrt(a)*x*sqrt(-sqrt
(b)/sqrt(a))/sqrt(b) - sqrt(a)/sqrt(b) + x**2)/4 + sqrt(2)*sqrt(-
sqrt(b)/sqrt(a))*log(sqrt(2)*sqrt(a)*x*sqrt(-sqrt(b)/sqrt(a))/sqr
t(b) - sqrt(a)/sqrt(b) + x**2)/4
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + sqrt(a)*sqrt(b))/(b*x^4 + a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.18 $\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx$

Optimal. Leaf size=106

$$\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}$$

```
[Out] -(b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])
/(2*Sqrt[2]*a^(1/4)) + (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2])/(2*Sqrt[2]*a^(1/4))
```

Rubi [A] time = 0.0875173, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a]*Sqrt[b] - b*x^2)/(a + b*x^4),x]
```

```
[Out] -(b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])
/(2*Sqrt[2]*a^(1/4)) + (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2])/(2*Sqrt[2]*a^(1/4))
```

Rubi in Sympy [A] time = 37.8359, size = 100, normalized size = 0.94

$$-\frac{\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{4\sqrt[4]{a}}+\frac{\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{4\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-b*x**2+a**(1/2)*b**(1/2))/(b*x**4+a),x)
```

```
[Out] -sqrt(2)*b**(1/4)*log(-sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt
(b) + b*x**2)/(4*a**(1/4)) + sqrt(2)*b**(1/4)*log(sqrt(2)*a**(1/4
)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(4*a**(1/4))
```

Mathematica [A] time = 0.0422349, size = 91, normalized size = 0.86

$$\frac{\sqrt[4]{b}\left(\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x-\sqrt{a}-\sqrt{b}x^2\right)\right)}{2\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a]*Sqrt[b] - b*x^2)/(a + b*x^4),x]
```

```
[Out] (b^(1/4)*(-Log[-Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x - Sqrt[b]*x^2
] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(2*S
qrt[2]*a^(1/4))
```

Maple [B] time = 0.006, size = 254, normalized size = 2.4

$$\frac{\sqrt{2}}{8}\sqrt{b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt{a}}+\frac{\sqrt{2}}{4}\sqrt{b}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt{a}}+\frac{\sqrt{2}}{4}\sqrt{b}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt{a}}-\frac{\sqrt{2}}{8}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{\sqrt{2}}{4}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{\sqrt{2}}{4}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x)
```

```
[Out] 1/8/a^(1/2)*b^(1/2)*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(
1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4/a^
(1/2)*b^(1/2)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)
+1/4/a^(1/2)*b^(1/2)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x-1)-1/8/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/
b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))-1/4/(a/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)-1/4/(a/b)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*x^2 - sqrt(a)*sqrt(b))/(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.329648, size = 1, normalized size = 0.01

$$\left[\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{bx^4+4\sqrt{a}\sqrt{b}x^2+4\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^3+ax\right)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}+a}{bx^4+a}\right),\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{b}x}{\sqrt{a}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}\right)+\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(-\frac{\sqrt{\frac{1}{2}}\left(bx^3-\sqrt{a}\sqrt{b}x\right)}{a\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*x^2 - sqrt(a)*sqrt(b))/(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*log((b*x^4 + 4*sqrt(a)*sqrt(
b)*x^2 + 4*sqrt(1/2)*(sqrt(a)*sqrt(b)*x^3 + a*x)*sqrt(sqrt(b)/sqr
t(a)) + a)/(b*x^4 + a)), sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*arctan(
sqrt(1/2)*sqrt(b)*x/(sqrt(a)*sqrt(-sqrt(b)/sqrt(a)))) + sqrt(1/2)
*sqrt(-sqrt(b)/sqrt(a))*arctan(-sqrt(1/2)*(b*x^3 - sqrt(a)*sqrt(b
)*x)/(a*sqrt(-sqrt(b)/sqrt(a))))]
```

Sympy [A] time = 1.94154, size = 131, normalized size = 1.24

$$-\frac{\sqrt{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(-\frac{\sqrt{2}\sqrt{a}x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}+\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{\sqrt{2}\sqrt{a}x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}+\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a**(1/2)*b**(1/2))/(b*x**4+a),x)
```

```
[Out] -sqrt(2)*sqrt(sqrt(b)/sqrt(a))*log(-sqrt(2)*sqrt(a)*x*sqrt(sqrt(b
)/sqrt(a))/sqrt(b) + sqrt(a)/sqrt(b) + x**2)/4 + sqrt(2)*sqrt(sqr
t(b)/sqrt(a))*log(sqrt(2)*sqrt(a)*x*sqrt(sqrt(b)/sqrt(a))/sqrt(b)
 + sqrt(a)/sqrt(b) + x**2)/4
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 - sqrt(a)*sqrt(b))/(b*x^4 + a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.19 $\int \frac{d+ex^2}{d^2+e^2x^4}\,dx$

Optimal. Leaf size=75

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e
])) + ArcTan[1 + (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sq
rt[e])
```

Rubi [A] time = 0.0971491, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e
])) + ArcTan[1 + (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sq
rt[e])
```

Rubi in Sympy [A] time = 12.5742, size = 73, normalized size = 0.97

$$-\frac{\sqrt{2}\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{e}}+\frac{\sqrt{2}\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(e**2*x**4+d**2),x)
```

```
[Out] -sqrt(2)*atan(1 - sqrt(2)*sqrt(e)*x/sqrt(d))/(2*sqrt(d)*sqrt(e))
+ sqrt(2)*atan(1 + sqrt(2)*sqrt(e)*x/sqrt(d))/(2*sqrt(d)*sqrt(e))
```

Mathematica [A] time = 0.0510194, size = 60, normalized size = 0.8

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}+1\right)-\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] (-ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]] + ArcTan[1 + (Sqrt[2]*S
qrt[e]*x)/Sqrt[d]])/(Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Maple [B] time = 0.013, size = 290, normalized size = 3.9

$$\frac{\sqrt{2}}{8d}\sqrt[4]{\frac{d^2}{e^2}}\ln\left(1\left(x^2+\sqrt[4]{\frac{d^2}{e^2}}x\sqrt{2}+\sqrt{\frac{d^2}{e^2}}\right)\left(x^2-\sqrt[4]{\frac{d^2}{e^2}}x\sqrt{2}+\sqrt{\frac{d^2}{e^2}}\right)^{-1}\right) + \frac{\sqrt{2}}{4d}\sqrt[4]{\frac{d^2}{e^2}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{d^2}{e^2}}}+1\right) + \frac{\sqrt{2}}{4d}\sqrt[4]{\frac{d^2}{e^2}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{d^2}{e^2}}}-1\right) + \frac{\sqrt{2}}{8e}\ln\left(1\left(x^2-\sqrt[4]{\frac{d^2}{e^2}}x\sqrt{2}+\sqrt{\frac{d^2}{e^2}}\right)\left(x^2+\sqrt[4]{\frac{d^2}{e^2}}x\sqrt{2}+\sqrt{\frac{d^2}{e^2}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{d^2}{e^2}}} + \frac{\sqrt{2}}{4e}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{d^2}{e^2}}}+1\right)\frac{1}{\sqrt[4]{\frac{d^2}{e^2}}} + \frac{\sqrt{2}}{4e}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{d^2}{e^2}}}-1\right)\frac{1}{\sqrt[4]{\frac{d^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(e^2*x^4+d^2),x)
```

```
[Out] 1/8/d*(d^2/e^2)^(1/4)*2^(1/2)*ln((x^2+(d^2/e^2)^(1/4)*x*2^(1/2)+(
d^2/e^2)^(1/2))/(x^2-(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(1/2)))+
1/4/d*(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x+1)
+1/4/d*(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x-1
)+1/8/e/(d^2/e^2)^(1/4)*2^(1/2)*ln((x^2-(d^2/e^2)^(1/4)*x*2^(1/2)
+(d^2/e^2)^(1/2))/(x^2+(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(1/2))
)+1/4/e/(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x+
1)+1/4/e/(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x
-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(e^2*x^4 + d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.289028, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{2}\log\left(\frac{4\,de^2x^3-4\,d^2ex+\sqrt{2}\left(e^2x^4-4\,dex^2+d^2\right)\sqrt{-de}}{e^2x^4+d^2}\right)}{4\,\sqrt{-de}}, \frac{\sqrt{2}\left(\arctan\left(\frac{\sqrt{2}\sqrt{de}x}{2\,d}\right)+\arctan\left(\frac{\sqrt{2}\left(e^2x^3+dex\right)}{2\,\sqrt{de}d}\right)\right)}{2\,\sqrt{de}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(e^2*x^4 + d^2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*log((4*d*e^2*x^3 - 4*d^2*e*x + sqrt(2)*(e^2*x^4 - 4*
d*e*x^2 + d^2)*sqrt(-d*e))/(e^2*x^4 + d^2))/sqrt(-d*e), 1/2*sqrt(
2)*(arctan(1/2*sqrt(2)*sqrt(d*e)*x/d) + arctan(1/2*sqrt(2)*(e^2*x
^3 + d*e*x)/(sqrt(d*e)*d)))/sqrt(d*e)]
```

Sympy [A] time = 0.464425, size = 87, normalized size = 1.16

$$-\frac{\sqrt{2}\sqrt{-\frac{1}{de}}\log\left(-\sqrt{2}dx\sqrt{-\frac{1}{de}}-\frac{d}{e}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{-\frac{1}{de}}\log\left(\sqrt{2}dx\sqrt{-\frac{1}{de}}-\frac{d}{e}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(e**2*x**4+d**2),x)
```

```
[Out] -sqrt(2)*sqrt(-1/(d*e))*log(-sqrt(2)*d*x*sqrt(-1/(d*e)) - d/e + x
**2)/4 + sqrt(2)*sqrt(-1/(d*e))*log(sqrt(2)*d*x*sqrt(-1/(d*e)) -
d/e + x**2)/4
```

GIAC/XCAS [A] time = 0.27754, size = 300, normalized size = 4.

$$\frac{\sqrt{2}\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}+\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}+2x\right)e^{\frac{1}{2}}}{2\left(d^2\right)^{\frac{1}{4}}}\right)e^{(-6)}}{4d^2}+\frac{\sqrt{2}\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}+\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}-2x\right)e^{\frac{1}{2}}}{2\left(d^2\right)^{\frac{1}{4}}}\right)e^{(-6)}}{4d^2}+\frac{\sqrt{2}\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}-\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)e^{(-6)}\ln\left(\sqrt{2}\left(d^2\right)^{\frac{1}{4}}xe^{\left(-\frac{1}{2}\right)}+x^2+\sqrt{d^2}e^{(-1)}\right)}{8d^2}-\frac{\sqrt{2}\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}-\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)e^{(-6)}\ln\left(-\sqrt{2}\left(d^2\right)^{\frac{1}{4}}xe^{\left(-\frac{1}{2}\right)}+x^2+\sqrt{d^2}e^{(-1)}\right)}{8d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(e^2*x^4 + d^2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*arcta
n(1/2*sqrt(2)*(sqrt(2)*(d^2)^(1/4)*e^(-1/2) + 2*x)*e^(1/2)/(d^2)^
(1/4))*e^(-6)/d^2 + 1/4*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) + (d^2)^(
3/4)*e^(11/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*(d^2)^(1/4)*e^(-1/2)
- 2*x)*e^(1/2)/(d^2)^(1/4))*e^(-6)/d^2 + 1/8*sqrt(2)*((d^2)^(1/4)
*d*e^(11/2) - (d^2)^(3/4)*e^(11/2))*e^(-6)*ln(sqrt(2)*(d^2)^(1/4)
*x*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/d^2 - 1/8*sqrt(2)*((d^2)^(1
/4)*d*e^(11/2) - (d^2)^(3/4)*e^(11/2))*e^(-6)*ln(-sqrt(2)*(d^2)^(
1/4)*x*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/d^2
```

3.20 $\int \frac{d-ex^2}{d^2+e^2x^4}\,dx$

Optimal. Leaf size=90

$$\frac{\log\left(\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\log\left(-\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

```
[Out] -Log[d - Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(2*Sqrt[2]*Sqrt[d]*Sq
rt[e]) + Log[d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(2*Sqrt[2]*Sq
rt[d]*Sqrt[e])
```

Rubi [A] time = 0.0852422, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\log\left(\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\log\left(-\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d - e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] -Log[d - Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(2*Sqrt[2]*Sqrt[d]*Sq
rt[e]) + Log[d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(2*Sqrt[2]*Sq
rt[d]*Sqrt[e])
```

Rubi in Sympy [A] time = 27.973, size = 83, normalized size = 0.92

$$-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2\right)}{4\sqrt{d}\sqrt{e}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2\right)}{4\sqrt{d}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-e*x**2+d)/(e**2*x**4+d**2),x)
```

```
[Out] -sqrt(2)*log(-sqrt(2)*sqrt(d)*sqrt(e)*x + d + e*x**2)/(4*sqrt(d)*
sqrt(e)) + sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(e)*x + d + e*x**2)/(4
*sqrt(d)*sqrt(e))
```

Mathematica [A] time = 0.0356861, size = 75, normalized size = 0.83

$$\frac{\log\left(\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2\right)-\log\left(\sqrt{2}\sqrt{d}\sqrt{e}x-d-ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d - e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] (-Log[-d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x - e*x^2] + Log[d + Sqrt[2]*S
qrt[d]*Sqrt[e]*x + e*x^2])/(2*Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Maple [B] time = 0.004, size = 290, normalized size = 3.2

$$\frac{\sqrt{2}}{8\,d}\sqrt[4]{\frac{d^{2}}{e^{2}}}\ln\left(1\left(x^{2}+\sqrt[4]{\frac{d^{2}}{e^{2}}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}\right)\left(x^{2}-\sqrt[4]{\frac{d^{2}}{e^{2}}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}}{4\,d}\sqrt[4]{\frac{d^{2}}{e^{2}}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{d^{2}}{e^{2}}}}+1\right)+\frac{\sqrt{2}}{4\,d}\sqrt[4]{\frac{d^{2}}{e^{2}}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{d^{2}}{e^{2}}}}-1\right)$$
$$-\frac{\sqrt{2}}{8\,e}\ln\left(1\left(x^{2}-\sqrt[4]{\frac{d^{2}}{e^{2}}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}\right)\left(x^{2}+\sqrt[4]{\frac{d^{2}}{e^{2}}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{d^{2}}{e^{2}}}}$$
$$-\frac{\sqrt{2}}{4\,e}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{d^{2}}{e^{2}}}}+1\right)\frac{1}{\sqrt[4]{\frac{d^{2}}{e^{2}}}}-\frac{\sqrt{2}}{4\,e}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{d^{2}}{e^{2}}}}-1\right)\frac{1}{\sqrt[4]{\frac{d^{2}}{e^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-e*x^2+d)/(e^2*x^4+d^2),x)
```

```
[Out] 1/8/d*(d^2/e^2)^(1/4)*2^(1/2)*ln((x^2+(d^2/e^2)^(1/4)*x*2^(1/2)+(
d^2/e^2)^(1/2))/(x^2-(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(1/2)))+
1/4/d*(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x+1)
+1/4/d*(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x-1
)-1/8/e/(d^2/e^2)^(1/4)*2^(1/2)*ln((x^2-(d^2/e^2)^(1/4)*x*2^(1/2)
+(d^2/e^2)^(1/2))/(x^2+(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(1/2))
)-1/4/e/(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x+
1)-1/4/e/(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x
-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 - d)/(e^2*x^4 + d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.291147, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{2}\log\left(\frac{4\,de^{2}x^{3}+4\,d^{2}ex+\sqrt{2}\left(e^{2}x^{4}+4\,dex^{2}+d^{2}\right)\sqrt{de}}{e^{2}x^{4}+d^{2}}\right)}{4\,\sqrt{de}},\ \frac{\sqrt{2}\left(\arctan\left(\frac{\sqrt{2}\sqrt{-de}x}{2\,d}\right)+\arctan\left(\frac{\sqrt{2}\left(e^{2}x^{3}-dex\right)}{2\,\sqrt{-de}d}\right)\right)}{2\,\sqrt{-de}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 - d)/(e^2*x^4 + d^2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*log((4*d*e^2*x^3 + 4*d^2*e*x + sqrt(2)*(e^2*x^4 + 4*
d*e*x^2 + d^2)*sqrt(d*e))/(e^2*x^4 + d^2))/sqrt(d*e), 1/2*sqrt(2)
*(arctan(1/2*sqrt(2)*sqrt(-d*e)*x/d) + arctan(1/2*sqrt(2)*(e^2*x^
3 - d*e*x)/(sqrt(-d*e)*d)))/sqrt(-d*e)]
```

Sympy [A] time = 1.28393, size = 80, normalized size = 0.89

$$-\frac{\sqrt{2}\sqrt{\frac{1}{de}}\log\left(-\sqrt{2}dx\sqrt{\frac{1}{de}}+\frac{d}{e}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{\frac{1}{de}}\log\left(\sqrt{2}dx\sqrt{\frac{1}{de}}+\frac{d}{e}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-e*x**2+d)/(e**2*x**4+d**2),x)
```

```
[Out] -sqrt(2)*sqrt(1/(d*e))*log(-sqrt(2)*d*x*sqrt(1/(d*e)) + d/e + x**
2)/4 + sqrt(2)*sqrt(1/(d*e))*log(sqrt(2)*d*x*sqrt(1/(d*e)) + d/e
+ x**2)/4
```

GIAC/XCAS [A] time = 0.276823, size = 300, normalized size = 3.33

$$\frac{\sqrt{2}\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}-\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}+2x\right)e^{\frac{1}{2}}}{2\left(d^2\right)^{\frac{1}{4}}}\right)e^{(-6)}}{4d^2}+\frac{\sqrt{2}\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}-\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}-2x\right)e^{\frac{1}{2}}}{2\left(d^2\right)^{\frac{1}{4}}}\right)e^{(-6)}}{4d^2}+\frac{\sqrt{2}\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}+\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)e^{(-6)}\ln\left(\sqrt{2}\left(d^2\right)^{\frac{1}{4}}xe^{\left(-\frac{1}{2}\right)}+x^2+\sqrt{d^2}e^{(-1)}\right)}{8d^2}-\frac{\sqrt{2}\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}+\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)e^{(-6)}\ln\left(-\sqrt{2}\left(d^2\right)^{\frac{1}{4}}xe^{\left(-\frac{1}{2}\right)}+x^2+\sqrt{d^2}e^{(-1)}\right)}{8d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 - d)/(e^2*x^4 + d^2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) - (d^2)^(3/4)*e^(11/2))*arcta
n(1/2*sqrt(2)*(sqrt(2)*(d^2)^(1/4)*e^(-1/2) + 2*x)*e^(1/2)/(d^2)^
(1/4))*e^(-6)/d^2 + 1/4*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) - (d^2)^(
3/4)*e^(11/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*(d^2)^(1/4)*e^(-1/2)
- 2*x)*e^(1/2)/(d^2)^(1/4))*e^(-6)/d^2 + 1/8*sqrt(2)*((d^2)^(1/4)
*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*e^(-6)*ln(sqrt(2)*(d^2)^(1/4)
*x*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/d^2 - 1/8*sqrt(2)*((d^2)^(1
/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*e^(-6)*ln(-sqrt(2)*(d^2)^(
1/4)*x*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/d^2
```

3.21 $\int \frac{5+2x^2}{-1+x^4}\,dx$

Optimal. Leaf size=13

$$-\frac{3}{2}\tan^{-1}(x)-\frac{7}{2}\tanh^{-1}(x)$$

```
[Out] (-3*ArcTan[x])/2 - (7*ArcTanh[x])/2
```

Rubi [A] time = 0.0178989, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{3}{2}\tan^{-1}(x)-\frac{7}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + 2*x^2)/(-1 + x^4),x]

[Out] (-3*ArcTan[x])/2 - (7*ArcTanh[x])/2
```

Rubi in Sympy [A] time = 4.45043, size = 14, normalized size = 1.08

$$-\frac{3\operatorname{atan}(x)}{2}-\frac{7\operatorname{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+5)/(x**4-1),x)

[Out] -3*atan(x)/2 - 7*atanh(x)/2
```

Mathematica [A] time = 0.00909264, size = 25, normalized size = 1.92

$$\frac{7}{4}\log(1-x)-\frac{7}{4}\log(x+1)-\frac{3}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 + 2*x^2)/(-1 + x^4),x]

[Out] (-3*ArcTan[x])/2 + (7*Log[1 - x])/4 - (7*Log[1 + x])/4
```

Maple [A] time = 0.007, size = 18, normalized size = 1.4

$$\frac{7\ln(-1+x)}{4}-\frac{7\ln(1+x)}{4}-\frac{3\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+5)/(x^4-1),x)
```

```
[Out] 7/4*ln(-1+x)-7/4*ln(1+x)-3/2*arctan(x)
```

Maxima [A] time = 0.860507, size = 23, normalized size = 1.77

$$-\frac{3}{2}\arctan(x)-\frac{7}{4}\log(x+1)+\frac{7}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 5)/(x^4 - 1),x, algorithm="maxima")
```

```
[Out] -3/2*arctan(x) - 7/4*log(x + 1) + 7/4*log(x - 1)
```

Fricas [A] time = 0.277699, size = 23, normalized size = 1.77

$$-\frac{3}{2}\arctan(x)-\frac{7}{4}\log(x+1)+\frac{7}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 5)/(x^4 - 1),x, algorithm="fricas")
```

```
[Out] -3/2*arctan(x) - 7/4*log(x + 1) + 7/4*log(x - 1)
```

Sympy [A] time = 0.418141, size = 22, normalized size = 1.69

$$\frac{7\log(x-1)}{4}-\frac{7\log(x+1)}{4}-\frac{3\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+5)/(x**4-1),x)
```

```
[Out] 7*log(x - 1)/4 - 7*log(x + 1)/4 - 3*atan(x)/2
```

GIAC/XCAS [A] time = 0.269146, size = 26, normalized size = 2.

$$-\frac{3}{2}\arctan(x)-\frac{7}{4}\ln(|x+1|)+\frac{7}{4}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 5)/(x^4 - 1),x, algorithm="giac")
```

```
[Out] -3/2*arctan(x) - 7/4*ln(abs(x + 1)) + 7/4*ln(abs(x - 1))
```

3.22 $\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx$

Optimal. Leaf size=16

$$\frac{E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

```
[Out] EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]
```

Rubi [A] time = 0.0592676, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + b*x^2)/Sqrt[1 - b^2*x^4],x]

[Out] EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]
```

Rubi in Sympy [A] time = 12.0469, size = 15, normalized size = 0.94

$$\frac{E\left(\operatorname{asin}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+1)/(-b**2*x**4+1)**(1/2),x)

[Out] elliptic_e(asin(sqrt(b)*x), -1)/sqrt(b)
```

Mathematica [C] time = 0.0470417, size = 27, normalized size = 1.69

$$-\frac{iE\left(i\sinh^{-1}\left(\sqrt{-b}x\right)\middle|-1\right)}{\sqrt{-b}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + b*x^2)/Sqrt[1 - b^2*x^4],x]

[Out] ((-I)*EllipticE[I*ArcSinh[Sqrt[-b]*x], -1])/Sqrt[-b]
```

Maple [B] time = 0.017, size = 100, normalized size = 6.3

$$1\sqrt{-bx^2+1}\sqrt{bx^2+1}\mathit{EllipticF}\left(x\sqrt{b},i\right)\frac{1}{\sqrt{b}}\frac{1}{\sqrt{-b^2x^4+1}}$$
$$-1\sqrt{-bx^2+1}\sqrt{bx^2+1}\left(\mathit{EllipticF}\left(x\sqrt{b},i\right)-\mathit{EllipticE}\left(x\sqrt{b},i\right)\right)\frac{1}{\sqrt{b}}\frac{1}{\sqrt{-b^2x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+1)/(-b^2*x^4+1)^(1/2),x)
```

```
[Out] 1/b^(1/2)*(-b*x^2+1)^(1/2)*(b*x^2+1)^(1/2)/(-b^2*x^4+1)^(1/2)*Ell
ipticF(x*b^(1/2),I)-1/b^(1/2)*(-b*x^2+1)^(1/2)*(b*x^2+1)^(1/2)/(-
b^2*x^4+1)^(1/2)*(EllipticF(x*b^(1/2),I)-EllipticE(x*b^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+1}{\sqrt{-b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2+1}{\sqrt{-b^2x^4+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)
```

Sympy [A] time = 3.62017, size = 70, normalized size = 4.38

$$\frac{bx^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|b^2x^4e^{2i\pi}\right)}{4\,\Gamma\left(\frac{7}{4}\right)}+\frac{x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|b^2x^4e^{2i\pi}\right)}{4\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+1)/(-b**2*x**4+1)**(1/2),x)
```

```
[Out] b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2
*I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b
**2*x**4*exp_polar(2*I*pi))/(4*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+1}{\sqrt{-b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1),x, algorithm="giac")

[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)
```

3.23 $\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx$

Optimal. Leaf size=35

$$\frac{2F\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}-\frac{E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

```
[Out] -(EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]) + (2*EllipticF[ArcSin
[Sqrt[b]*x], -1])/Sqrt[b]
```

Rubi [A] time = 0.112191, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$\frac{2F\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}-\frac{E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - b*x^2)/Sqrt[1 - b^2*x^4],x]
```

```
[Out] -(EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]) + (2*EllipticF[ArcSin
[Sqrt[b]*x], -1])/Sqrt[b]
```

Rubi in Sympy [A] time = 26.392, size = 34, normalized size = 0.97

$$-\frac{E\left(\operatorname{asin}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}+\frac{2F\left(\operatorname{asin}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+1)/(-b**2*x**4+1)**(1/2),x)
```

```
[Out] -elliptic_e(asin(sqrt(b)*x), -1)/sqrt(b) + 2*elliptic_f(asin(sqrt
(b)*x), -1)/sqrt(b)
```

Mathematica [C] time = 0.0503164, size = 46, normalized size = 1.31

$$\frac{i\left(E\left(i\sinh^{-1}\left(\sqrt{-b}x\right)\middle|-1\right)-2F\left(i\sinh^{-1}\left(\sqrt{-b}x\right)\middle|-1\right)\right)}{\sqrt{-b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - b*x^2)/Sqrt[1 - b^2*x^4],x]
```

```
[Out] (I*(EllipticE[I*ArcSinh[Sqrt[-b]*x], -1] - 2*EllipticF[I*ArcSinh[
Sqrt[-b]*x], -1]))/Sqrt[-b]
```

Maple [B] time = 0.008, size = 99, normalized size = 2.8

$$1\sqrt{-bx^2+1}\sqrt{bx^2+1}\left(EllipticF\left(x\sqrt{b},i\right)-EllipticE\left(x\sqrt{b},i\right)\right)\frac{1}{\sqrt{b}}\frac{1}{\sqrt{-b^2x^4+1}}+1\sqrt{-bx^2+1}\sqrt{bx^2+1}EllipticF\left(x\sqrt{b},i\right)\frac{1}{\sqrt{b}}\frac{1}{\sqrt{-b^2x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x)
```

```
[Out] 1/b^(1/2)*(-b*x^2+1)^(1/2)*(b*x^2+1)^(1/2)/(-b^2*x^4+1)^(1/2)*(El
lipticF(x*b^(1/2),I)-EllipticE(x*b^(1/2),I))+1/b^(1/2)*(-b*x^2+1)
^(1/2)*(b*x^2+1)^(1/2)/(-b^2*x^4+1)^(1/2)*EllipticF(x*b^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{bx^2-1}{\sqrt{-b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(-b^2*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{bx^2-1}{\sqrt{-b^2x^4+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^2 - 1)/sqrt(-b^2*x^4 + 1), x)
```

Sympy [A] time = 4.03753, size = 70, normalized size = 2.

$$-\frac{bx^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\,\Gamma\left(\frac{7}{4}\right)}+\frac{x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+1)/(-b**2*x**4+1)**(1/2),x)
```

```
[Out] -b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(
2*I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,),
b**2*x**4*exp_polar(2*I*pi))/(4*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx^2 - 1}{\sqrt{-b^2x^4 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 + 1), x)
```

3.24 $\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{1-b^2x^4}E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}$$

```
[Out] (Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqr
t[-1 + b^2*x^4])
```

Rubi [A] time = 0.0832769, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\sqrt{1-b^2x^4}E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] (Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqr
t[-1 + b^2*x^4])
```

Rubi in Sympy [A] time = 18.2114, size = 39, normalized size = 0.91

$$\frac{\sqrt{-b^2x^4+1}E\left(\operatorname{asin}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+1)/(b**2*x**4-1)**(1/2),x)
```

```
[Out] sqrt(-b**2*x**4 + 1)*elliptic_e(asin(sqrt(b)*x), -1)/(sqrt(b)*sqr
t(b**2*x**4 - 1))
```

Mathematica [C] time = 0.0471207, size = 54, normalized size = 1.26

$$-\frac{i\sqrt{1-b^2x^4}E\left(i\sinh^{-1}\left(\sqrt{-b}x\right)\middle|-1\right)}{\sqrt{-b}\sqrt{b^2x^4-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - b^2*x^4]*EllipticE[I*ArcSinh[Sqrt[-b]*x], -1])/(Sq
rt[-b]*Sqrt[-1 + b^2*x^4])
```

Maple [B] time = 0.017, size = 107, normalized size = 2.5

$$1\sqrt{bx^2+1}\sqrt{-bx^2+1}EllipticF\left(x\sqrt{-b},i\right)\frac{1}{\sqrt{-b}}\frac{1}{\sqrt{b^2x^4-1}} + 1\sqrt{bx^2+1}\sqrt{-bx^2+1}\left(EllipticF\left(x\sqrt{-b},i\right)-EllipticE\left(x\sqrt{-b},i\right)\right)\frac{1}{\sqrt{-b}}\frac{1}{\sqrt{b^2x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((b*x^2+1)/(b^2*x^4-1)^(1/2),x)
```

```
[Out] 1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-b*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*E
llipticF(x*(-b)^(1/2),I)+1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-b*x^2+1)^
(1/2)/(b^2*x^4-1)^(1/2)*(EllipticF(x*(-b)^(1/2),I)-EllipticE(x*(-
b)^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+1}{\sqrt{b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2+1}{\sqrt{b^2x^4-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + 1)/sqrt(b^2*x^4 - 1), x)
```

Sympy [A] time = 3.64356, size = 61, normalized size = 1.42

$$-\frac{ibx^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4\right)}{4\,\Gamma\left(\frac{7}{4}\right)}-\frac{ix\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4\right)}{4\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x**2+1)/(b**2*x**4-1)**(1/2),x)
```

```
[Out] -I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4)/(4*gamm
a(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4)/(4*
gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + 1}{\sqrt{b^2x^4 - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1),x, algorithm="giac")

[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1), x)
```

3.25 $\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx$

Optimal. Leaf size=89

$$\frac{2\sqrt{1-b^2x^4}F\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}-\frac{\sqrt{1-b^2x^4}E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}$$

```
[Out] -((Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*S
qrt[-1 + b^2*x^4])) + (2*Sqrt[1 - b^2*x^4]*EllipticF[ArcSin[Sqrt[
b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b^2*x^4])
```

Rubi [A] time = 0.150733, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{2\sqrt{1-b^2x^4}F\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}-\frac{\sqrt{1-b^2x^4}E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] -((Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*S
qrt[-1 + b^2*x^4])) + (2*Sqrt[1 - b^2*x^4]*EllipticF[ArcSin[Sqrt[
b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b^2*x^4])
```

Rubi in Sympy [A] time = 32.2685, size = 82, normalized size = 0.92

$$-\frac{\sqrt{-b^2x^4+1}E\left(\operatorname{asin}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}+\frac{2\sqrt{-b^2x^4+1}F\left(\operatorname{asin}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+1)/(b**2*x**4-1)**(1/2),x)
```

```
[Out] -sqrt(-b**2*x**4 + 1)*elliptic_e(asin(sqrt(b)*x), -1)/(sqrt(b)*sq
rt(b**2*x**4 - 1)) + 2*sqrt(-b**2*x**4 + 1)*elliptic_f(asin(sqrt(
b)*x), -1)/(sqrt(b)*sqrt(b**2*x**4 - 1))
```

Mathematica [C] time = 0.0499081, size = 73, normalized size = 0.82

$$\frac{i\sqrt{1-b^2x^4}\left(E\left(i\sinh^{-1}\left(\sqrt{-b}x\right)\middle|-1\right)-2F\left(i\sinh^{-1}\left(\sqrt{-b}x\right)\middle|-1\right)\right)}{\sqrt{-b}\sqrt{b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] (I*Sqrt[1 - b^2*x^4]*(EllipticE[I*ArcSinh[Sqrt[-b]*x], -1] - 2*El
lipticF[I*ArcSinh[Sqrt[-b]*x], -1]))/(Sqrt[-b]*Sqrt[-1 + b^2*x^4]
)
```

Maple [A] time = 0.008, size = 108, normalized size = 1.2

$$-1\sqrt{bx^2+1}\sqrt{-bx^2+1}\left(EllipticF\left(x\sqrt{-b},i\right)-EllipticE\left(x\sqrt{-b},i\right)\right)\frac{1}{\sqrt{-b}}\frac{1}{\sqrt{b^2x^4-1}}+1\sqrt{bx^2+1}\sqrt{-bx^2+1}EllipticF\left(x\sqrt{-b},i\right)\frac{1}{\sqrt{-b}}\frac{1}{\sqrt{b^2x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+1)/(b^2*x^4-1)^(1/2),x)
```

```
[Out] -1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-b*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*
(EllipticF(x*(-b)^(1/2),I)-EllipticE(x*(-b)^(1/2),I))+1/(-b)^(1/2
)*(b*x^2+1)^(1/2)*(-b*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*EllipticF(x*
(-b)^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{bx^2-1}{\sqrt{b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 - 1),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(b^2*x^4 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{bx^2-1}{\sqrt{b^2x^4-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 - 1),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^2 - 1)/sqrt(b^2*x^4 - 1), x)
```

Sympy [A] time = 3.94169, size = 60, normalized size = 0.67

$$\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+1)/(b**2*x**4-1)**(1/2),x)
```

```
[Out] I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4)/(4*gamma
(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4)/(4*g
amma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx^2-1}{\sqrt{b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 - 1),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 - 1), x)
```

3.26 $\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx$

Optimal. Leaf size=89

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)\mid\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\frac{x\sqrt{b^2x^4+1}}{bx^2+1}$$

```
[Out] -((x*Sqrt[1 + b^2*x^4])/(1 + b*x^2)) + ((1 + b*x^2)*Sqrt[(1 + b^2
*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b
]*Sqrt[1 + b^2*x^4])
```

Rubi [A] time = 0.0517291, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)\mid\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\frac{x\sqrt{b^2x^4+1}}{bx^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] -((x*Sqrt[1 + b^2*x^4])/(1 + b*x^2)) + ((1 + b*x^2)*Sqrt[(1 + b^2
*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b
]*Sqrt[1 + b^2*x^4])
```

Rubi in Sympy [A] time = 7.22608, size = 76, normalized size = 0.85

$$-\frac{x\sqrt{b^2x^4+1}}{bx^2+1}+\frac{\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,(bx^2+1)\,E\left(2\operatorname{atan}\left(\sqrt{b}x\right)\middle|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-b*x**2+1)/(b**2*x**4+1)**(1/2),x)
```

```
[Out] -x*sqrt(b**2*x**4 + 1)/(b*x**2 + 1) + sqrt((b**2*x**4 + 1)/(b*x**
2 + 1)**2)*(b*x**2 + 1)*elliptic_e(2*atan(sqrt(b)*x), 1/2)/(sqrt(
b)*sqrt(b**2*x**4 + 1))
```

Mathematica [C] time = 0.0644465, size = 52, normalized size = 0.58

$$-\frac{E\left(i\sinh^{-1}\left(\sqrt{ib}x\right)\middle|-1\right)-(1-i)F\left(i\sinh^{-1}\left(\sqrt{ib}x\right)\middle|-1\right)}{\sqrt{ib}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] -((EllipticE[I*ArcSinh[Sqrt[I*b]*x], -1] - (1 - I)*EllipticF[I*Ar
cSinh[Sqrt[I*b]*x], -1])/Sqrt[I*b])
```

Maple [C] time = 0.019, size = 120, normalized size = 1.4

$$-i\sqrt{1-ibx^2}\sqrt{1+ibx^2}\left(EllipticF\left(x\sqrt{ib},i\right)-EllipticE\left(x\sqrt{ib},i\right)\right)\frac{1}{\sqrt{ib}}\frac{1}{\sqrt{b^2x^4+1}}+1\sqrt{1-ibx^2}\sqrt{1+ibx^2}EllipticF\left(x\sqrt{ib},i\right)\frac{1}{\sqrt{ib}}\frac{1}{\sqrt{b^2x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+1)/(b^2*x^4+1)^(1/2),x)
```

```
[Out] -I/(I*b)^(1/2)*(1-I*b*x^2)^(1/2)*(1+I*b*x^2)^(1/2)/(b^2*x^4+1)^(1
/2)*(EllipticF(x*(I*b)^(1/2),I)-EllipticE(x*(I*b)^(1/2),I))+1/(I*
b)^(1/2)*(1-I*b*x^2)^(1/2)*(1+I*b*x^2)^(1/2)/(b^2*x^4+1)^(1/2)*El
lipticF(x*(I*b)^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx^2-1}{\sqrt{b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{bx^2-1}{\sqrt{b^2x^4+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)
```

Sympy [A] time = 3.86314, size = 66, normalized size = 0.74

$$-\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+1)/(b**2*x**4+1)**(1/2),x)
```

```
[Out] -b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(
I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*
*2*x**4*exp_polar(I*pi))/(4*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx^2 - 1}{\sqrt{b^2x^4 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)
```

3.27 $\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx$

Optimal. Leaf size=152

$$\frac{x\sqrt{b^2x^4+1}}{bx^2+1}+\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}F\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}$$

```
[Out] (x*Sqrt[1 + b^2*x^4])/(1 + b*x^2) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^
4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*S
qrt[1 + b^2*x^4]) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2
]*EllipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4]
)
```

Rubi [A] time = 0.0995589, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{x\sqrt{b^2x^4+1}}{bx^2+1}+\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}F\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] (x*Sqrt[1 + b^2*x^4])/(1 + b*x^2) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^
4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*S
qrt[1 + b^2*x^4]) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2
]*EllipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4]
)
```

Rubi in Sympy [A] time = 13.2283, size = 134, normalized size = 0.88

$$\frac{x\sqrt{b^2x^4+1}}{bx^2+1}-\frac{\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,(bx^2+1)\,E\left(2\,\mathrm{atan}\left(\sqrt{b}x\right)\middle|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}+\frac{\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,(bx^2+1)\,F\left(2\,\mathrm{atan}\left(\sqrt{b}x\right)\middle|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+1)/(b**2*x**4+1)**(1/2),x)
```

```
[Out] x*sqrt(b**2*x**4 + 1)/(b*x**2 + 1) - sqrt((b**2*x**4 + 1)/(b*x**2
 + 1)**2)*(b*x**2 + 1)*elliptic_e(2*atan(sqrt(b)*x), 1/2)/(sqrt(b
)*sqrt(b**2*x**4 + 1)) + sqrt((b**2*x**4 + 1)/(b*x**2 + 1)**2)*(b
*x**2 + 1)*elliptic_f(2*atan(sqrt(b)*x), 1/2)/(sqrt(b)*sqrt(b**2*
x**4 + 1))
```

Mathematica [C] time = 0.0512056, size = 51, normalized size = 0.34

$$\frac{E\left(i\sinh^{-1}\left(\sqrt{ib}x\right)\middle|-1\right)-(1+i)F\left(i\sinh^{-1}\left(\sqrt{ib}x\right)\middle|-1\right)}{\sqrt{ib}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] (EllipticE[I*ArcSinh[Sqrt[I*b]*x], -1] - (1 + I)*EllipticF[I*ArcS
inh[Sqrt[I*b]*x], -1])/Sqrt[I*b]
```

Maple [C] time = 0.006, size = 120, normalized size = 0.8

$$1\sqrt{1-ibx^2}\sqrt{1+ibx^2}\mathit{EllipticF}\left(x\sqrt{ib},i\right)\frac{1}{\sqrt{ib}}\frac{1}{\sqrt{b^2x^4+1}}$$
$$+i\sqrt{1-ibx^2}\sqrt{1+ibx^2}\left(\mathit{EllipticF}\left(x\sqrt{ib},i\right)-\mathit{EllipticE}\left(x\sqrt{ib},i\right)\right)\frac{1}{\sqrt{ib}}\frac{1}{\sqrt{b^2x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+1)/(b^2*x^4+1)^(1/2),x)
```

```
[Out] 1/(I*b)^(1/2)*(1-I*b*x^2)^(1/2)*(1+I*b*x^2)^(1/2)/(b^2*x^4+1)^(1/
2)*EllipticF(x*(I*b)^(1/2),I)+I/(I*b)^(1/2)*(1-I*b*x^2)^(1/2)*(1+
I*b*x^2)^(1/2)/(b^2*x^4+1)^(1/2)*(EllipticF(x*(I*b)^(1/2),I)-Elli
pticE(x*(I*b)^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+1}{\sqrt{b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2+1}{\sqrt{b^2x^4+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)
```

Sympy [A] time = 3.39688, size = 66, normalized size = 0.43

$$\frac{bx^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\,\Gamma\left(\frac{7}{4}\right)}+\frac{x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+1)/(b**2*x**4+1)**(1/2),x)
```

```
[Out] b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I
*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**
2*x**4*exp_polar(I*pi))/(4*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + 1}{\sqrt{b^2x^4 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)
```

3.28 $\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}+\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)\mid\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}$$

```
[Out] (x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2) + ((1 + b*x^2)*Sqrt[(1 + b^2*x
^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*
Sqrt[-1 - b^2*x^4])
```

Rubi [A] time = 0.0533591, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}+\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)\mid\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

```
[Out] (x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2) + ((1 + b*x^2)*Sqrt[(1 + b^2*x
^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*
Sqrt[-1 - b^2*x^4])
```

Rubi in Sympy [A] time = 8.11608, size = 80, normalized size = 0.89

$$\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}+\frac{\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\left(bx^2+1\right)E\left(2\,\mathrm{atan}\left(\sqrt{b}x\right)\middle|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+1)/(-b**2*x**4-1)**(1/2),x)
```

```
[Out] x*sqrt(-b**2*x**4 - 1)/(b*x**2 + 1) + sqrt((b**2*x**4 + 1)/(b*x**
2 + 1)**2)*(b*x**2 + 1)*elliptic_e(2*atan(sqrt(b)*x), 1/2)/(sqrt(
b)*sqrt(-b**2*x**4 - 1))
```

Mathematica [C] time = 0.0698462, size = 79, normalized size = 0.88

$$-\frac{\sqrt{b^2x^4+1}\left(E\left(i\sinh^{-1}\left(\sqrt{ib}x\right)\middle|-1\right)-(1-i)F\left(i\sinh^{-1}\left(\sqrt{ib}x\right)\middle|-1\right)\right)}{\sqrt{ib}\sqrt{-b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

```
[Out] -((Sqrt[1 + b^2*x^4]*(EllipticE[I*ArcSinh[Sqrt[I*b]*x], -1] - (1
- I)*EllipticF[I*ArcSinh[Sqrt[I*b]*x], -1]))/(Sqrt[I*b]*Sqrt[-1 -
 b^2*x^4]))
```

Maple [C] time = 0.016, size = 122, normalized size = 1.4

$$i\sqrt{1+ibx^2}\sqrt{1-ibx^2}\left(\mathit{EllipticF}\left(x\sqrt{-ib},i\right)-\mathit{EllipticE}\left(x\sqrt{-ib},i\right)\right)\frac{1}{\sqrt{-ib}}\frac{1}{\sqrt{-b^2x^4-1}}+1\sqrt{1+ibx^2}\sqrt{1-ibx^2}\mathit{EllipticF}\left(x\sqrt{-ib},i\right)\frac{1}{\sqrt{-ib}}\frac{1}{\sqrt{-b^2x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x)
```

```
[Out] I/(-I*b)^(1/2)*(1+I*b*x^2)^(1/2)*(1-I*b*x^2)^(1/2)/(-b^2*x^4-1)^(
1/2)*(EllipticF(x*(-I*b)^(1/2),I)-EllipticE(x*(-I*b)^(1/2),I))+1/
(-I*b)^(1/2)*(1+I*b*x^2)^(1/2)*(1-I*b*x^2)^(1/2)/(-b^2*x^4-1)^(1/
2)*EllipticF(x*(-I*b)^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{bx^2-1}{\sqrt{-b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 - 1),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(-b^2*x^4 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{bx\mathrm{integral}\left(-\frac{\sqrt{-b^2x^4-1}(bx^2-1)}{b^3x^6+bx^2},x\right)+\sqrt{-b^2x^4-1}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 - 1),x, algorithm="fricas")
```

```
[Out] (b*x*integral(-sqrt(-b^2*x^4 - 1)*(b*x^2 - 1)/(b^3*x^6 + b*x^2),
x) + sqrt(-b^2*x^4 - 1))/(b*x)
```

Sympy [A] time = 4.01715, size = 70, normalized size = 0.78

$$\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+1)/(-b**2*x**4-1)**(1/2),x)
```

```
[Out] I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar
(I*pi))/(4*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,),
```

```
b**2*x**4*exp_polar(I*pi))/(4*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx^2-1}{\sqrt{-b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 - 1),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 - 1), x)
```

3.29 $\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx$

Optimal. Leaf size=156

$$-\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}+\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}F\left(2\tan^{-1}\left(\sqrt{b}x\right)\mid\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}-\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)\mid\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}$$

```
[Out] -((x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2)) - ((1 + b*x^2)*Sqrt[(1 + b^
2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[
b]*Sqrt[-1 - b^2*x^4]) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x
^2)^2]*EllipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^
2*x^4])
```

Rubi [A] time = 0.104292, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}+\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}F\left(2\tan^{-1}\left(\sqrt{b}x\right)\mid\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}-\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)\mid\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

```
[Out] -((x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2)) - ((1 + b*x^2)*Sqrt[(1 + b^
2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[
b]*Sqrt[-1 - b^2*x^4]) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x
^2)^2]*EllipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^
2*x^4])
```

Rubi in Sympy [A] time = 14.5387, size = 139, normalized size = 0.89

$$-\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}-\frac{\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,(bx^2+1)\,E\left(2\operatorname{atan}\left(\sqrt{b}x\right)\middle|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}+\frac{\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,(bx^2+1)\,F\left(2\operatorname{atan}\left(\sqrt{b}x\right)\middle|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+1)/(-b**2*x**4-1)**(1/2),x)
```

```
[Out] -x*sqrt(-b**2*x**4 - 1)/(b*x**2 + 1) - sqrt((b**2*x**4 + 1)/(b*x*
*2 + 1)**2)*(b*x**2 + 1)*elliptic_e(2*atan(sqrt(b)*x), 1/2)/(sqrt
(b)*sqrt(-b**2*x**4 - 1)) + sqrt((b**2*x**4 + 1)/(b*x**2 + 1)**2)
*(b*x**2 + 1)*elliptic_f(2*atan(sqrt(b)*x), 1/2)/(sqrt(b)*sqrt(-b
**2*x**4 - 1))
```

Mathematica [C] time = 0.0556502, size = 78, normalized size = 0.5

$$\frac{\sqrt{b^2x^4+1}\left(E\left(i\sinh^{-1}\left(\sqrt{ib}x\right)\middle|-1\right)-(1+i)F\left(i\sinh^{-1}\left(\sqrt{ib}x\right)\middle|-1\right)\right)}{\sqrt{ib}\sqrt{-b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

```
[Out] (Sqrt[1 + b^2*x^4]*(EllipticE[I*ArcSinh[Sqrt[I*b]*x], -1] - (1 +
I)*EllipticF[I*ArcSinh[Sqrt[I*b]*x], -1]))/(Sqrt[I*b]*Sqrt[-1 - b
^2*x^4])
```

Maple [C] time = 0.005, size = 122, normalized size = 0.8

$$1\sqrt{1+ibx^2}\sqrt{1-ibx^2}\,EllipticF\left(x\sqrt{-ib},i\right)\frac{1}{\sqrt{-ib}}\frac{1}{\sqrt{-b^2x^4-1}} - i\sqrt{1+ibx^2}\sqrt{1-ibx^2}\left(EllipticF\left(x\sqrt{-ib},i\right)-EllipticE\left(x\sqrt{-ib},i\right)\right)\frac{1}{\sqrt{-ib}}\frac{1}{\sqrt{-b^2x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+1)/(-b^2*x^4-1)^(1/2),x)
```

```
[Out] 1/(-I*b)^(1/2)*(1+I*b*x^2)^(1/2)*(1-I*b*x^2)^(1/2)/(-b^2*x^4-1)^(
1/2)*EllipticF(x*(-I*b)^(1/2),I)-I/(-I*b)^(1/2)*(1+I*b*x^2)^(1/2)
*(1-I*b*x^2)^(1/2)/(-b^2*x^4-1)^(1/2)*(EllipticF(x*(-I*b)^(1/2),I
)-EllipticE(x*(-I*b)^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+1}{\sqrt{-b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{bx\,\mathrm{integral}\left(-\frac{\sqrt{-b^2x^4-1}(bx^2+1)}{b^3x^6+bx^2},x\right)-\sqrt{-b^2x^4-1}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1),x, algorithm="fricas")
```

```
[Out] (b*x*integral(-sqrt(-b^2*x^4 - 1)*(b*x^2 + 1)/(b^3*x^6 + b*x^2),
x) - sqrt(-b^2*x^4 - 1))/(b*x)
```

Sympy [A] time = 3.59782, size = 71, normalized size = 0.46

$$-\frac{ibx^3\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\,\Gamma\left(\frac{7}{4}\right)}-\frac{ix\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+1)/(-b**2*x**4-1)**(1/2),x)
```

```
[Out] -I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_pola
r(I*pi))/(4*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,)
, b**2*x**4*exp_polar(I*pi))/(4*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + 1}{\sqrt{-b^2x^4 - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1), x)
```

3.30 $\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

```
[Out] EllipticE[ArcSin[c*x], -1]/c
```

Rubi [A] time = 0.0342907, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + c^2*x^2]/Sqrt[1 - c^2*x^2],x]

[Out] EllipticE[ArcSin[c*x], -1]/c
```

Rubi in Sympy [A] time = 9.07964, size = 8, normalized size = 0.8

$$\frac{E\left(\operatorname{asin}(cx)\middle|-1\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c**2*x**2+1)**(1/2)/(-c**2*x**2+1)**(1/2),x)

[Out] elliptic_e(asin(c*x), -1)/c
```

Mathematica [A] time = 0.0366889, size = 10, normalized size = 1.

$$\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + c^2*x^2]/Sqrt[1 - c^2*x^2],x]

[Out] EllipticE[ArcSin[c*x], -1]/c
```

Maple [C] time = 0.035, size = 15, normalized size = 1.5

$$\frac{EllipticE\left(x\,csgn(c)\,c, i\right) csgn(c)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] EllipticE(x*csgn(c)*c,I)*csgn(c)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c^2*x^2 + 1)/sqrt(-c^2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{\sqrt{-c^2x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c^2*x^2 + 1)/sqrt(-c^2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/sqrt(-c^2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c^2*x^2 + 1)/sqrt(-c^2*x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/sqrt(-c^2*x^2 + 1), x)
```

3.31 $\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=10

$$\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

```
[Out] EllipticE[ArcSin[c*x], -1]/c
```

Rubi [A] time = 0.0561727, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + c^2*x^2)/Sqrt[1 - c^4*x^4],x]

[Out] EllipticE[ArcSin[c*x], -1]/c
```

Rubi in Sympy [A] time = 13.4616, size = 8, normalized size = 0.8

$$\frac{E\left(\operatorname{asin}\left(cx\right)\middle|-1\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c**2*x**2+1)/(-c**4*x**4+1)**(1/2),x)

[Out] elliptic_e(asin(c*x), -1)/c
```

Mathematica [C] time = 0.049367, size = 31, normalized size = 3.1

$$-\frac{iE\left(i\sinh^{-1}\left(\sqrt{-c^2}x\right)\middle|-1\right)}{\sqrt{-c^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + c^2*x^2)/Sqrt[1 - c^4*x^4],x]

[Out] ((-I)*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -1])/Sqrt[-c^2]
```

Maple [B] time = 0.019, size = 118, normalized size = 11.8

$$1\sqrt{-c^2x^2+1}\sqrt{c^2x^2+1}EllipticF\left(x\sqrt{c^2},i\right)\frac{1}{\sqrt{c^2}}\frac{1}{\sqrt{-c^4x^4+1}}$$
$$-1\sqrt{-c^2x^2+1}\sqrt{c^2x^2+1}\left(EllipticF\left(x\sqrt{c^2},i\right)-EllipticE\left(x\sqrt{c^2},i\right)\right)\frac{1}{\sqrt{c^2}}\frac{1}{\sqrt{-c^4x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] 1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^4*x^4+1)^(
1/2)*EllipticF(x*(c^2)^(1/2),I)-1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*
(c^2*x^2+1)^(1/2)/(-c^4*x^4+1)^(1/2)*(EllipticF(x*(c^2)^(1/2),I)-
EllipticE(x*(c^2)^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c^2x^2 + 1}{\sqrt{-c^4x^4 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c^2*x^2 + 1)/sqrt(-c^4*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)/sqrt(-c^4*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{c^2x^2 + 1}{\sqrt{-c^4x^4 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c^2*x^2 + 1)/sqrt(-c^4*x^4 + 1),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)/sqrt(-c^4*x^4 + 1), x)
```

Sympy [A] time = 3.69821, size = 71, normalized size = 7.1

$$\frac{c^2x^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|c^4x^4e^{2i\pi}\right)}{4\,\Gamma\left(\frac{7}{4}\right)} + \frac{x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|c^4x^4e^{2i\pi}\right)}{4\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c**2*x**2+1)/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] c**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c**4*x**4*exp_pola
r(2*I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,)
, c**4*x**4*exp_polar(2*I*pi))/(4*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c^2x^2 + 1}{\sqrt{-c^4x^4 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c^2*x^2 + 1)/sqrt(-c^4*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)/sqrt(-c^4*x^4 + 1), x)
```

3.32 $\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx$

Optimal. Leaf size=23

$$\frac{2F\left(\sin^{-1}(cx)\middle|-1\right)}{c}-\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/
c
```

Rubi [A] time = 0.0944334, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{2F\left(\sin^{-1}(cx)\middle|-1\right)}{c}-\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/
c
```

Rubi in Sympy [A] time = 21.1797, size = 20, normalized size = 0.87

$$-\frac{E\left(\operatorname{asin}(cx)\middle|-1\right)}{c}+\frac{2F\left(\operatorname{asin}(cx)\middle|-1\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-c**2*x**2+1)**(1/2)/(c**2*x**2+1)**(1/2),x)
```

```
[Out] -elliptic_e(asin(c*x), -1)/c + 2*elliptic_f(asin(c*x), -1)/c
```

Mathematica [A] time = 0.0348893, size = 24, normalized size = 1.04

$$\frac{E\left(\sin^{-1}\left(\sqrt{-c^2}x\right)\middle|-1\right)}{\sqrt{-c^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[-c^2]*x], -1]/Sqrt[-c^2]
```

Maple [C] time = 0.016, size = 28, normalized size = 1.2

$$\frac{(2\,EllipticF(x\,csgn(c)\,c,i)-EllipticE(x\,csgn(c)\,c,i))\,csgn(c)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x)

[Out] (2*EllipticF(x*csgn(c)*c,I)-EllipticE(x*csgn(c)*c,I))*csgn(c)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1),x, algorithm="maxima")

[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1),x, algorithm="fricas")

[Out] integral(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(cx-1)(cx+1)}}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-c**2*x**2+1)**(1/2)/(c**2*x**2+1)**(1/2),x)

[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/sqrt(c**2*x**2 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1),x, algorithm="giac")

[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

3.33 $\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=23

$$\frac{2F\left(\sin^{-1}(cx)\middle|-1\right)}{c}-\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/
c
```

Rubi [A] time = 0.112811, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2F\left(\sin^{-1}(cx)\middle|-1\right)}{c}-\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/
c
```

Rubi in Sympy [A] time = 27.2071, size = 20, normalized size = 0.87

$$-\frac{E\left(\operatorname{asin}(cx)\middle|-1\right)}{c}+\frac{2F\left(\operatorname{asin}(cx)\middle|-1\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-c**2*x**2+1)/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] -elliptic_e(asin(c*x), -1)/c + 2*elliptic_f(asin(c*x), -1)/c
```

Mathematica [C] time = 0.055079, size = 52, normalized size = 2.26

$$\frac{i\left(E\left(i\sinh^{-1}\left(\sqrt{-c^2}x\right)\middle|-1\right)-2F\left(i\sinh^{-1}\left(\sqrt{-c^2}x\right)\middle|-1\right)\right)}{\sqrt{-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

```
[Out] (I*(EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -1] - 2*EllipticF[I*ArcSin
h[Sqrt[-c^2]*x], -1]))/Sqrt[-c^2]
```

Maple [B] time = 0.01, size = 117, normalized size = 5.1

$$1\sqrt{-c^2x^2+1}\sqrt{c^2x^2+1}\,EllipticF\left(x\sqrt{c^2},i\right)\frac{1}{\sqrt{c^2}}\frac{1}{\sqrt{-c^4x^4+1}}$$
$$+1\sqrt{-c^2x^2+1}\sqrt{c^2x^2+1}\left(EllipticF\left(x\sqrt{c^2},i\right)-EllipticE\left(x\sqrt{c^2},i\right)\right)\frac{1}{\sqrt{c^2}}\frac{1}{\sqrt{-c^4x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] 1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^4*x^4+1)^(
1/2)*EllipticF(x*(c^2)^(1/2),I)+1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*
(c^2*x^2+1)^(1/2)/(-c^4*x^4+1)^(1/2)*(EllipticF(x*(c^2)^(1/2),I)-
EllipticE(x*(c^2)^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{c^2x^2 - 1}{\sqrt{-c^4x^4 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(c^2*x^2 - 1)/sqrt(-c^4*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((c^2*x^2 - 1)/sqrt(-c^4*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{c^2x^2 - 1}{\sqrt{-c^4x^4 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(c^2*x^2 - 1)/sqrt(-c^4*x^4 + 1),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*x^2 - 1)/sqrt(-c^4*x^4 + 1), x)
```

Sympy [A] time = 4.14197, size = 71, normalized size = 3.09

$$-\frac{c^2x^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|c^4x^4e^{2i\pi}\right)}{4\,\Gamma\left(\frac{7}{4}\right)} + \frac{x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|c^4x^4e^{2i\pi}\right)}{4\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((-c**2*x**2+1)/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] -c**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c**4*x**4*exp_pol
ar(2*I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,
), c**4*x**4*exp_polar(2*I*pi))/(4*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{c^2x^2 - 1}{\sqrt{-c^4x^4 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c^2*x^2 - 1)/sqrt(-c^4*x^4 + 1),x, algorithm="giac")

[Out] integrate(-(c^2*x^2 - 1)/sqrt(-c^4*x^4 + 1), x)
```

3.34 $\int \frac{d+ex^2}{d^2+bx^2+e^2x^4} dx$

Optimal. Leaf size=82

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-b}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}-\frac{\tan^{-1}\left(\frac{\sqrt{2de-b}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

```
[Out] -(ArcTan[(Sqrt[-b + 2*d*e] - 2*e*x)/Sqrt[b + 2*d*e]]/Sqrt[b + 2*d
*e]) + ArcTan[(Sqrt[-b + 2*d*e] + 2*e*x)/Sqrt[b + 2*d*e]]/Sqrt[b
+ 2*d*e]
```

Rubi [A] time = 0.187703, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-b}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}-\frac{\tan^{-1}\left(\frac{\sqrt{2de-b}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[-b + 2*d*e] - 2*e*x)/Sqrt[b + 2*d*e]]/Sqrt[b + 2*d
*e]) + ArcTan[(Sqrt[-b + 2*d*e] + 2*e*x)/Sqrt[b + 2*d*e]]/Sqrt[b
+ 2*d*e]
```

Rubi in Sympy [A] time = 17.7482, size = 73, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\frac{2ex-\sqrt{-b+2de}}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}+\frac{\operatorname{atan}\left(\frac{2ex+\sqrt{-b+2de}}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(e**2*x**4+b*x**2+d**2),x)
```

```
[Out] atan((2*e*x - sqrt(-b + 2*d*e))/sqrt(b + 2*d*e))/sqrt(b + 2*d*e)
+ atan((2*e*x + sqrt(-b + 2*d*e))/sqrt(b + 2*d*e))/sqrt(b + 2*d*e
)
```

Mathematica [B] time = 0.182985, size = 181, normalized size = 2.21

$$\frac{\frac{\left(\sqrt{b^2-4d^2e^2}-b+2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}+\frac{\left(\sqrt{b^2-4d^2e^2}+b-2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{b^2-4d^2e^2}+b}}\right)}{\sqrt{\sqrt{b^2-4d^2e^2}+b}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] (((-b + 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[
b - Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[b - Sqrt[b^2 - 4*d^2*e^2]] + ((
b - 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b +
Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2
```

```
]*Sqrt[b^2 - 4*d^2*e^2])
```

Maple [A] time = 0.037, size = 71, normalized size = 0.9

$$-1\arctan\left(1\left(-2\,ex+\sqrt{2\,de-b}\right)\frac{1}{\sqrt{2\,de+b}}\right)\frac{1}{\sqrt{2\,de+b}}+1\arctan\left(1\left(2\,ex+\sqrt{2\,de-b}\right)\frac{1}{\sqrt{2\,de+b}}\right)\frac{1}{\sqrt{2\,de+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x)
```

```
[Out] -arctan((-2*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)
+arctan((2*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ex^2+d}{e^2x^4+bx^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(e^2*x^4 + b*x^2 + d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(e^2*x^4 + b*x^2 + d^2), x)
```

Fricas [A] time = 0.28728, size = 1, normalized size = 0.01

$$\left[\frac{\log\left(\frac{2\left(2\,de^2+be\right)x^3-2\left(2\,d^2e+bd\right)x+\left(e^2x^4-(4\,de+b)x^2+d^2\right)\sqrt{-2\,de-b}}{e^2x^4+bx^2+d^2}\right)}{2\sqrt{-2\,de-b}},\ \frac{\arctan\left(\frac{ex}{\sqrt{2\,de+b}}\right)+\arctan\left(\frac{e^2x^3+(de+b)x}{\sqrt{2\,de+b}d}\right)}{\sqrt{2\,de+b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(e^2*x^4 + b*x^2 + d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*(2*d*e^2 + b*e)*x^3 - 2*(2*d^2*e + b*d)*x + (e^2*x^4
- (4*d*e + b)*x^2 + d^2)*sqrt(-2*d*e - b))/(e^2*x^4 + b*x^2 + d^2
))/sqrt(-2*d*e - b), (arctan(e*x/sqrt(2*d*e + b)) + arctan((e^2*x
^3 + (d*e + b)*x)/(sqrt(2*d*e + b)*d)))/sqrt(2*d*e + b)]
```

Sympy [A] time = 1.31954, size = 122, normalized size = 1.49

$$-\frac{\sqrt{-\frac{1}{b+2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{-\frac{1}{b+2de}}-2de\sqrt{-\frac{1}{b+2de}}\right)}{e}\right)}{2}+\frac{\sqrt{-\frac{1}{b+2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(b\sqrt{-\frac{1}{b+2de}}+2de\sqrt{-\frac{1}{b+2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(e**2*x**4+b*x**2+d**2),x)
```

```
[Out] -sqrt(-1/(b + 2*d*e))*log(-d/e + x**2 + x*(-b*sqrt(-1/(b + 2*d*e)
) - 2*d*e*sqrt(-1/(b + 2*d*e)))/e)/2 + sqrt(-1/(b + 2*d*e))*log(-
d/e + x**2 + x*(b*sqrt(-1/(b + 2*d*e)) + 2*d*e*sqrt(-1/(b + 2*d*e
)))/e)/2
```

GIAC/XCAS [A] time = 0.453956, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(e^2*x^4 + b*x^2 + d^2),x, algorithm="giac")

[Out] Done
```

3.35 $\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx$

Optimal. Leaf size=82

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}-\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

```
[Out] -(ArcTan[(Sqrt[2*d*e - f] - 2*e*x)/Sqrt[2*d*e + f]]/Sqrt[2*d*e +
f]) + ArcTan[(Sqrt[2*d*e - f] + 2*e*x)/Sqrt[2*d*e + f]]/Sqrt[2*d*
e + f]
```

Rubi [A] time = 0.204049, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}-\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[2*d*e - f] - 2*e*x)/Sqrt[2*d*e + f]]/Sqrt[2*d*e +
f]) + ArcTan[(Sqrt[2*d*e - f] + 2*e*x)/Sqrt[2*d*e + f]]/Sqrt[2*d*
e + f]
```

Rubi in Sympy [A] time = 17.7761, size = 73, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\frac{2ex-\sqrt{2de-f}}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}+\frac{\operatorname{atan}\left(\frac{2ex+\sqrt{2de-f}}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(e**2*x**4+f*x**2+d**2),x)
```

```
[Out] atan((2*e*x - sqrt(2*d*e - f))/sqrt(2*d*e + f))/sqrt(2*d*e + f) +
 atan((2*e*x + sqrt(2*d*e - f))/sqrt(2*d*e + f))/sqrt(2*d*e + f)
```

Mathematica [B] time = 0.183775, size = 181, normalized size = 2.21

$$\frac{\frac{\left(\sqrt{f^2-4d^2e^2}+2de-f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{f-\sqrt{f^2-4d^2e^2}}}\right)}{\sqrt{f-\sqrt{f^2-4d^2e^2}}}+\frac{\left(\sqrt{f^2-4d^2e^2}-2de+f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{f^2-4d^2e^2}+f}}\right)}{\sqrt{\sqrt{f^2-4d^2e^2}+f}}}{\sqrt{2}\sqrt{f^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] (((2*d*e - f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[
f - Sqrt[-4*d^2*e^2 + f^2]]])/Sqrt[f - Sqrt[-4*d^2*e^2 + f^2]] +
```

```
((-2*d*e + f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[
f + Sqrt[-4*d^2*e^2 + f^2]]])/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]])/(
Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

Maple [A] time = 0.037, size = 71, normalized size = 0.9

$$-1\arctan\left(1\left(-2\,ex+\sqrt{2\,de-f}\right)\frac{1}{\sqrt{2\,de+f}}\right)\frac{1}{\sqrt{2\,de+f}}+1\arctan\left(1\left(2\,ex+\sqrt{2\,de-f}\right)\frac{1}{\sqrt{2\,de+f}}\right)\frac{1}{\sqrt{2\,de+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x)
```

```
[Out] -arctan((-2*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)
+arctan((2*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{e^2x^4+fx^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(e^2*x^4 + f*x^2 + d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(e^2*x^4 + f*x^2 + d^2), x)
```

Fricas [A] time = 0.283345, size = 1, normalized size = 0.01

$$\left[\frac{\log\left(\frac{2\left(2\,de^2+ef\right)x^3-2\left(2\,d^2e+df\right)x+\left(e^2x^4-(4\,de+f)x^2+d^2\right)\sqrt{-2\,de-f}}{e^2x^4+fx^2+d^2}\right)}{2\sqrt{-2\,de-f}},\ \frac{\arctan\left(\frac{ex}{\sqrt{2\,de+f}}\right)+\arctan\left(\frac{e^2x^3+(de+f)x}{\sqrt{2\,de+f}\,d}\right)}{\sqrt{2\,de+f}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(e^2*x^4 + f*x^2 + d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*(2*d*e^2 + e*f)*x^3 - 2*(2*d^2*e + d*f)*x + (e^2*x^4
- (4*d*e + f)*x^2 + d^2)*sqrt(-2*d*e - f))/(e^2*x^4 + f*x^2 + d^2
))/sqrt(-2*d*e - f), (arctan(e*x/sqrt(2*d*e + f)) + arctan((e^2*x
^3 + (d*e + f)*x)/(sqrt(2*d*e + f)*d)))/sqrt(2*d*e + f)]
```

Sympy [A] time = 1.37044, size = 122, normalized size = 1.49

$$-\frac{\sqrt{-\frac{1}{2de+f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{-\frac{1}{2de+f}}-f\sqrt{-\frac{1}{2de+f}}\right)}{e}\right)}{2}+\frac{\sqrt{-\frac{1}{2de+f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{-\frac{1}{2de+f}}+f\sqrt{-\frac{1}{2de+f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(e**2*x**4+f*x**2+d**2),x)
```

```
[Out] -sqrt(-1/(2*d*e + f))*log(-d/e + x**2 + x*(-2*d*e*sqrt(-1/(2*d*e
+ f)) - f*sqrt(-1/(2*d*e + f)))/e)/2 + sqrt(-1/(2*d*e + f))*log(-
d/e + x**2 + x*(2*d*e*sqrt(-1/(2*d*e + f)) + f*sqrt(-1/(2*d*e + f
)))/e)/2
```

GIAC/XCAS [A] time = 0.452962, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(e^2*x^4 + f*x^2 + d^2),x, algorithm="giac")
```

```
[Out] Done
```

3.36 $\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx$

Optimal. Leaf size=78

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

```
[Out] ArcTanh[(Sqrt[b + 2*d*e] - 2*e*x)/Sqrt[b - 2*d*e]]/Sqrt[b - 2*d*e
] - ArcTanh[(Sqrt[b + 2*d*e] + 2*e*x)/Sqrt[b - 2*d*e]]/Sqrt[b - 2
*d*e]
```

Rubi [A] time = 0.174408, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[b + 2*d*e] - 2*e*x)/Sqrt[b - 2*d*e]]/Sqrt[b - 2*d*e
] - ArcTanh[(Sqrt[b + 2*d*e] + 2*e*x)/Sqrt[b - 2*d*e]]/Sqrt[b - 2
*d*e]
```

Rubi in Sympy [A] time = 18.1295, size = 75, normalized size = 0.96

$$-\frac{\operatorname{atanh}\left(\frac{2ex-\sqrt{b+2de}}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}-\frac{\operatorname{atanh}\left(\frac{2ex+\sqrt{b+2de}}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(e**2*x**4-b*x**2+d**2),x)
```

```
[Out] -atanh((2*e*x - sqrt(b + 2*d*e))/sqrt(b - 2*d*e))/sqrt(b - 2*d*e)
 - atanh((2*e*x + sqrt(b + 2*d*e))/sqrt(b - 2*d*e))/sqrt(b - 2*d*
e)
```

Mathematica [B] time = 0.181215, size = 189, normalized size = 2.42

$$\frac{\frac{\left(\sqrt{b^2-4d^2e^2}+b+2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{-\sqrt{b^2-4d^2e^2}-b}}\right)}{\sqrt{-\sqrt{b^2-4d^2e^2}-b}}+\frac{\left(\sqrt{b^2-4d^2e^2}-b-2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{b^2-4d^2e^2}-b}}\right)}{\sqrt{\sqrt{b^2-4d^2e^2}-b}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] (((b + 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-
b - Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[-b - Sqrt[b^2 - 4*d^2*e^2]] + (
(-b - 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b
 + Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]])/(Sq
```

```
rt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

Maple [A] time = 0.033, size = 75, normalized size = 1.

$$1\arctan\left(1\left(2ex+\sqrt{2de+b}\right)\frac{1}{\sqrt{2de-b}}\right)\frac{1}{\sqrt{2de-b}}-1\arctan\left(1\left(-2ex+\sqrt{2de+b}\right)\frac{1}{\sqrt{2de-b}}\right)\frac{1}{\sqrt{2de-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x)
```

```
[Out] 1/(2*d*e-b)^(1/2)*arctan((2*e*x+(2*d*e+b)^(1/2))/(2*d*e-b)^(1/2))
-1/(2*d*e-b)^(1/2)*arctan((-2*e*x+(2*d*e+b)^(1/2))/(2*d*e-b)^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{e^2x^4-bx^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(e^2*x^4 - b*x^2 + d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(e^2*x^4 - b*x^2 + d^2), x)
```

Fricas [A] time = 0.293069, size = 1, normalized size = 0.01

$$\left[\frac{\log\left(\frac{2\left(2de^2-be\right)x^3-2\left(2d^2e-bd\right)x+\left(e^2x^4-(4de-b)x^2+d^2\right)\sqrt{-2de+b}}{e^2x^4-bx^2+d^2}\right)}{2\sqrt{-2de+b}},\ \frac{\arctan\left(\frac{ex}{\sqrt{2de-b}}\right)+\arctan\left(\frac{e^2x^3+(de-b)x}{\sqrt{2de-b}d}\right)}{\sqrt{2de-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(e^2*x^4 - b*x^2 + d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*(2*d*e^2 - b*e)*x^3 - 2*(2*d^2*e - b*d)*x + (e^2*x^4
- (4*d*e - b)*x^2 + d^2)*sqrt(-2*d*e + b))/(e^2*x^4 - b*x^2 + d^2
))/sqrt(-2*d*e + b), (arctan(e*x/sqrt(2*d*e - b)) + arctan((e^2*x
^3 + (d*e - b)*x)/(sqrt(2*d*e - b)*d)))/sqrt(2*d*e - b)]
```

Sympy [A] time = 1.48727, size = 110, normalized size = 1.41

$$\frac{\sqrt{\frac{1}{b-2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{\frac{1}{b-2de}}+2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right)}{2}-\frac{\sqrt{\frac{1}{b-2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(b\sqrt{\frac{1}{b-2de}}-2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(e**2*x**4-b*x**2+d**2),x)
```

```
[Out] sqrt(1/(b - 2*d*e))*log(-d/e + x**2 + x*(-b*sqrt(1/(b - 2*d*e)) +
 2*d*e*sqrt(1/(b - 2*d*e)))/e)/2 - sqrt(1/(b - 2*d*e))*log(-d/e +
 x**2 + x*(b*sqrt(1/(b - 2*d*e)) - 2*d*e*sqrt(1/(b - 2*d*e)))/e)/
2
```

GIAC/XCAS [A] time = 0.438578, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(e^2*x^4 - b*x^2 + d^2),x, algorithm="giac")
```

```
[Out] Done
```

3.37 $\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx$

Optimal. Leaf size=86

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}-\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

```
[Out] -(ArcTan[(Sqrt[2*d*e + f] - 2*e*x)/Sqrt[2*d*e - f]]/Sqrt[2*d*e -
f]) + ArcTan[(Sqrt[2*d*e + f] + 2*e*x)/Sqrt[2*d*e - f]]/Sqrt[2*d*
e - f]
```

Rubi [A] time = 0.194507, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}-\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[2*d*e + f] - 2*e*x)/Sqrt[2*d*e - f]]/Sqrt[2*d*e -
f]) + ArcTan[(Sqrt[2*d*e + f] + 2*e*x)/Sqrt[2*d*e - f]]/Sqrt[2*d*
e - f]
```

Rubi in Sympy [A] time = 18.1983, size = 75, normalized size = 0.87

$$-\frac{\operatorname{atanh}\left(\frac{2ex-\sqrt{2de+f}}{\sqrt{-2de+f}}\right)}{\sqrt{-2de+f}}-\frac{\operatorname{atanh}\left(\frac{2ex+\sqrt{2de+f}}{\sqrt{-2de+f}}\right)}{\sqrt{-2de+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(e**2*x**4-f*x**2+d**2),x)
```

```
[Out] -atanh((2*e*x - sqrt(2*d*e + f))/sqrt(-2*d*e + f))/sqrt(-2*d*e +
f) - atanh((2*e*x + sqrt(2*d*e + f))/sqrt(-2*d*e + f))/sqrt(-2*d*
e + f)
```

Mathematica [B] time = 0.180308, size = 189, normalized size = 2.2

$$\frac{\frac{\left(\sqrt{f^2-4d^2e^2}+2de+f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{-\sqrt{f^2-4d^2e^2}-f}}\right)}{\sqrt{-\sqrt{f^2-4d^2e^2}-f}}+\frac{\left(\sqrt{f^2-4d^2e^2}-2de-f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{f^2-4d^2e^2}-f}}\right)}{\sqrt{\sqrt{f^2-4d^2e^2}-f}}}{\sqrt{2}\sqrt{f^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] (((2*d*e + f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[
-f - Sqrt[-4*d^2*e^2 + f^2]]])/Sqrt[-f - Sqrt[-4*d^2*e^2 + f^2]]
+ ((-2*d*e - f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqr
t[-f + Sqrt[-4*d^2*e^2 + f^2]]])/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^2]
])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

Maple [A] time = 0.031, size = 75, normalized size = 0.9

$$-1\arctan\left(1\left(-2\,ex+\sqrt{2\,de+f}\right)\frac{1}{\sqrt{2\,de-f}}\right)\frac{1}{\sqrt{2\,de-f}}+1\arctan\left(1\left(2\,ex+\sqrt{2\,de+f}\right)\frac{1}{\sqrt{2\,de-f}}\right)\frac{1}{\sqrt{2\,de-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x)
```

```
[Out] -arctan((-2*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)
+arctan((2*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ex^2+d}{e^2x^4-fx^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(e^2*x^4 - f*x^2 + d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(e^2*x^4 - f*x^2 + d^2), x)
```

Fricas [A] time = 0.283058, size = 1, normalized size = 0.01

$$\left[\frac{\log\left(\frac{2\,(2\,de^2-ef)\,x^3-2\,(2\,d^2e-df)\,x+\left(e^2x^4-(4\,de-f)x^2+d^2\right)\sqrt{-2\,de+f}}{e^2x^4-f\,x^2+d^2}\right)}{2\,\sqrt{-2\,de+f}},-\frac{\arctan\left(-\frac{ex}{\sqrt{2\,de-f}}\right)-\arctan\left(\frac{e^2x^3+(de-f)x}{\sqrt{2\,de-f}\,d}\right)}{\sqrt{2\,de-f}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(e^2*x^4 - f*x^2 + d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*(2*d*e^2 - e*f)*x^3 - 2*(2*d^2*e - d*f)*x + (e^2*x^4
- (4*d*e - f)*x^2 + d^2)*sqrt(-2*d*e + f))/(e^2*x^4 - f*x^2 + d^2
))/sqrt(-2*d*e + f), -(arctan(-e*x/sqrt(2*d*e - f)) - arctan((e^2
*x^3 + (d*e - f)*x)/(sqrt(2*d*e - f)*d)))/sqrt(2*d*e - f)]
```

Sympy [A] time = 1.35391, size = 121, normalized size = 1.41

$$-\frac{\sqrt{-\frac{1}{2de-f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{-\frac{1}{2de-f}}+f\sqrt{-\frac{1}{2de-f}}\right)}{e}\right)}{2}+\frac{\sqrt{-\frac{1}{2de-f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{-\frac{1}{2de-f}}-f\sqrt{-\frac{1}{2de-f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(e**2*x**4-f*x**2+d**2),x)
```

```
[Out] -sqrt(-1/(2*d*e - f))*log(-d/e + x**2 + x*(-2*d*e*sqrt(-1/(2*d*e
- f)) + f*sqrt(-1/(2*d*e - f)))/e)/2 + sqrt(-1/(2*d*e - f))*log(-
d/e + x**2 + x*(2*d*e*sqrt(-1/(2*d*e - f)) - f*sqrt(-1/(2*d*e - f
)))/e)/2
```

GIAC/XCAS [A] time = 0.438522, size = 1, normalized size = 0.01

$$\mathit{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(e^2*x^4 - f*x^2 + d^2),x, algorithm="giac")
```

```
[Out] Done
```

3.38 $\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx$

Optimal. Leaf size=78

$$\frac{\log\left(x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}-\frac{\log\left(-x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}$$

```
[Out] -Log[d - Sqrt[-b + 2*d*e]*x + e*x^2]/(2*Sqrt[-b + 2*d*e]) + Log[d
 + Sqrt[-b + 2*d*e]*x + e*x^2]/(2*Sqrt[-b + 2*d*e])
```

Rubi [A] time = 0.103538, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074

$$\frac{\log\left(x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}-\frac{\log\left(-x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d - e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] -Log[d - Sqrt[-b + 2*d*e]*x + e*x^2]/(2*Sqrt[-b + 2*d*e]) + Log[d
 + Sqrt[-b + 2*d*e]*x + e*x^2]/(2*Sqrt[-b + 2*d*e])
```

Rubi in Sympy [A] time = 31.4696, size = 66, normalized size = 0.85

$$-\frac{\log\left(\frac{d}{e}+x^2-\frac{x\sqrt{-b+2de}}{e}\right)}{2\sqrt{-b+2de}}+\frac{\log\left(\frac{d}{e}+x^2+\frac{x\sqrt{-b+2de}}{e}\right)}{2\sqrt{-b+2de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-e*x**2+d)/(e**2*x**4+b*x**2+d**2),x)
```

```
[Out] -log(d/e + x**2 - x*sqrt(-b + 2*d*e)/e)/(2*sqrt(-b + 2*d*e)) + lo
g(d/e + x**2 + x*sqrt(-b + 2*d*e)/e)/(2*sqrt(-b + 2*d*e))
```

Mathematica [B] time = 0.22723, size = 182, normalized size = 2.33

$$\frac{\frac{\left(-\sqrt{b^2-4d^2e^2}+b+2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}-\frac{\left(\sqrt{b^2-4d^2e^2}+b+2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{b^2-4d^2e^2}+b}}\right)}{\sqrt{\sqrt{b^2-4d^2e^2}+b}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d - e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] (((b + 2*d*e - Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b
 - Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[b - Sqrt[b^2 - 4*d^2*e^2]] - ((b
 + 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b + S
qrt[b^2 - 4*d^2*e^2]]])/Sqrt[b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]
*Sqrt[b^2 - 4*d^2*e^2])
```

Maple [A] time = 0.022, size = 88, normalized size = 1.1

$$-\frac{1}{-4\,de+2\,b}\sqrt{2\,de-b}\ln\left(d+ex^2+x\sqrt{2\,de-b}\right)+\frac{1}{-4\,de+2\,b}\sqrt{2\,de-b}\ln\left(-ex^2+x\sqrt{2\,de-b}-d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x)
```

```
[Out] -1/(-4*d*e+2*b)*(2*d*e-b)^(1/2)*ln(d+e*x^2+x*(2*d*e-b)^(1/2))+1/(
-4*d*e+2*b)*(2*d*e-b)^(1/2)*ln(-e*x^2+x*(2*d*e-b)^(1/2)-d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{ex^2-d}{e^2x^4+bx^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 - d)/(e^2*x^4 + b*x^2 + d^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(e^2*x^4 + b*x^2 + d^2), x)
```

Fricas [A] time = 0.289879, size = 1, normalized size = 0.01

$$\left[\frac{\log\left(\frac{2\left(2\,de^2-be\right)x^3+2\left(2\,d^2e-bd\right)x+\left(e^2x^4+(4\,de-b)x^2+d^2\right)\sqrt{2\,de-b}}{e^2x^4+bx^2+d^2}\right)}{2\sqrt{2\,de-b}},\ \frac{\arctan\left(\frac{\sqrt{-2\,de+b}ex}{2\,de-b}\right)+\arctan\left(\frac{e^2x^3-(de-b)x}{\sqrt{-2\,de+b}d}\right)}{\sqrt{-2\,de+b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 - d)/(e^2*x^4 + b*x^2 + d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*(2*d*e^2 - b*e)*x^3 + 2*(2*d^2*e - b*d)*x + (e^2*x^4
+ (4*d*e - b)*x^2 + d^2)*sqrt(2*d*e - b))/(e^2*x^4 + b*x^2 + d^2)
)/sqrt(2*d*e - b), (arctan(sqrt(-2*d*e + b)*e*x/(2*d*e - b)) + ar
ctan((e^2*x^3 - (d*e - b)*x)/(sqrt(-2*d*e + b)*d)))/sqrt(-2*d*e +
 b)]
```

Sympy [A] time = 2.15032, size = 121, normalized size = 1.55

$$\frac{\sqrt{-\frac{1}{b-2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{-\frac{1}{b-2de}}+2de\sqrt{-\frac{1}{b-2de}}\right)}{e}\right)}{2}-\frac{\sqrt{-\frac{1}{b-2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(b\sqrt{-\frac{1}{b-2de}}-2de\sqrt{-\frac{1}{b-2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-e*x**2+d)/(e**2*x**4+b*x**2+d**2),x)
```

```
[Out] sqrt(-1/(b - 2*d*e))*log(d/e + x**2 + x*(-b*sqrt(-1/(b - 2*d*e))
+ 2*d*e*sqrt(-1/(b - 2*d*e)))/e)/2 - sqrt(-1/(b - 2*d*e))*log(d/e
```

```
 + x**2 + x*(b*sqrt(-1/(b - 2*d*e)) - 2*d*e*sqrt(-1/(b - 2*d*e)))
/e)/2
```

GIAC/XCAS [A] time = 0.453287, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 - d)/(e^2*x^4 + b*x^2 + d^2),x, algorithm="giac")
```

```
[Out] Done
```

3.39 $\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx$

Optimal. Leaf size=78

$$\frac{\log\left(x\sqrt{2de-f}+d+ex^2\right)}{2\sqrt{2de-f}}-\frac{\log\left(-x\sqrt{2de-f}+d+ex^2\right)}{2\sqrt{2de-f}}$$

```
[Out] -Log[d - Sqrt[2*d*e - f]*x + e*x^2]/(2*Sqrt[2*d*e - f]) + Log[d +
 Sqrt[2*d*e - f]*x + e*x^2]/(2*Sqrt[2*d*e - f])
```

Rubi [A] time = 0.0891105, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074

$$\frac{\log\left(x\sqrt{2de-f}+d+ex^2\right)}{2\sqrt{2de-f}}-\frac{\log\left(-x\sqrt{2de-f}+d+ex^2\right)}{2\sqrt{2de-f}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d - e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] -Log[d - Sqrt[2*d*e - f]*x + e*x^2]/(2*Sqrt[2*d*e - f]) + Log[d +
 Sqrt[2*d*e - f]*x + e*x^2]/(2*Sqrt[2*d*e - f])
```

Rubi in Sympy [A] time = 31.5672, size = 66, normalized size = 0.85

$$-\frac{\log\left(\frac{d}{e}+x^2-\frac{x\sqrt{2de-f}}{e}\right)}{2\sqrt{2de-f}}+\frac{\log\left(\frac{d}{e}+x^2+\frac{x\sqrt{2de-f}}{e}\right)}{2\sqrt{2de-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-e*x**2+d)/(e**2*x**4+f*x**2+d**2),x)
```

```
[Out] -log(d/e + x**2 - x*sqrt(2*d*e - f)/e)/(2*sqrt(2*d*e - f)) + log(
d/e + x**2 + x*sqrt(2*d*e - f)/e)/(2*sqrt(2*d*e - f))
```

Mathematica [B] time = 0.239329, size = 182, normalized size = 2.33

$$\frac{\frac{\left(-\sqrt{f^2-4d^2e^2}+2de+f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{f-\sqrt{f^2-4d^2e^2}}}\right)}{\sqrt{f-\sqrt{f^2-4d^2e^2}}}-\frac{\left(\sqrt{f^2-4d^2e^2}+2de+f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{f^2-4d^2e^2}+f}}\right)}{\sqrt{\sqrt{f^2-4d^2e^2}+f}}}{\sqrt{2}\sqrt{f^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d - e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] (((2*d*e + f - Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[
f - Sqrt[-4*d^2*e^2 + f^2]]])/Sqrt[f - Sqrt[-4*d^2*e^2 + f^2]] -
((2*d*e + f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f
 + Sqrt[-4*d^2*e^2 + f^2]]])/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]])/(S
```

```
qrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

Maple [A] time = 0.022, size = 69, normalized size = 0.9

$$-\frac{1}{2}\ln\left(-ex^2+x\sqrt{2de-f}-d\right)\frac{1}{\sqrt{2de-f}}+\frac{1}{2}\ln\left(d+ex^2+x\sqrt{2de-f}\right)\frac{1}{\sqrt{2de-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x)
```

```
[Out] -1/2/(2*d*e-f)^(1/2)*ln(-e*x^2+x*(2*d*e-f)^(1/2)-d)+1/2*ln(d+e*x^
2+x*(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{ex^2-d}{e^2x^4+fx^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 - d)/(e^2*x^4 + f*x^2 + d^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(e^2*x^4 + f*x^2 + d^2), x)
```

Fricas [A] time = 0.289568, size = 1, normalized size = 0.01

$$\left[\frac{\log\left(\frac{2\,(2\,de^2-ef)\,x^3+2\,(2\,d^2e-df)\,x+\left(e^2x^4+(4\,de-f)x^2+d^2\right)\sqrt{2\,de-f}}{e^2x^4+fx^2+d^2}\right)}{2\,\sqrt{2\,de-f}},-\frac{\arctan\left(-\frac{\sqrt{-2\,de+f}\,ex}{2\,de-f}\right)-\arctan\left(\frac{e^2x^3-(de-f)x}{\sqrt{-2\,de+f}\,d}\right)}{\sqrt{-2\,de+f}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 - d)/(e^2*x^4 + f*x^2 + d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*(2*d*e^2 - e*f)*x^3 + 2*(2*d^2*e - d*f)*x + (e^2*x^4
+ (4*d*e - f)*x^2 + d^2)*sqrt(2*d*e - f))/(e^2*x^4 + f*x^2 + d^2)
)/sqrt(2*d*e - f), -(arctan(-sqrt(-2*d*e + f)*e*x/(2*d*e - f)) -
arctan((e^2*x^3 - (d*e - f)*x)/(sqrt(-2*d*e + f)*d)))/sqrt(-2*d*e
 + f)]
```

Sympy [A] time = 2.27549, size = 110, normalized size = 1.41

$$-\frac{\sqrt{\frac{1}{2de-f}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{\frac{1}{2de-f}}+f\sqrt{\frac{1}{2de-f}}\right)}{e}\right)}{2}+\frac{\sqrt{\frac{1}{2de-f}}\log\left(\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{\frac{1}{2de-f}}-f\sqrt{\frac{1}{2de-f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-e*x**2+d)/(e**2*x**4+f*x**2+d**2),x)
```

```
[Out] -sqrt(1/(2*d*e - f))*log(d/e + x**2 + x*(-2*d*e*sqrt(1/(2*d*e - f
)) + f*sqrt(1/(2*d*e - f)))/e)/2 + sqrt(1/(2*d*e - f))*log(d/e +
x**2 + x*(2*d*e*sqrt(1/(2*d*e - f)) - f*sqrt(1/(2*d*e - f)))/e)/2
```

GIAC/XCAS [A] time = 0.454, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 - d)/(e^2*x^4 + f*x^2 + d^2),x, algorithm="giac")
```

```
[Out] Done
```

3.40 $\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx$

Optimal. Leaf size=70

$$\frac{\log\left(x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}-\frac{\log\left(-x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}$$

```
[Out] -Log[d - Sqrt[b + 2*d*e]*x + e*x^2]/(2*Sqrt[b + 2*d*e]) + Log[d +
 Sqrt[b + 2*d*e]*x + e*x^2]/(2*Sqrt[b + 2*d*e])
```

Rubi [A] time = 0.0832881, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{\log\left(x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}-\frac{\log\left(-x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d - e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] -Log[d - Sqrt[b + 2*d*e]*x + e*x^2]/(2*Sqrt[b + 2*d*e]) + Log[d +
 Sqrt[b + 2*d*e]*x + e*x^2]/(2*Sqrt[b + 2*d*e])
```

Rubi in Sympy [A] time = 32.3858, size = 66, normalized size = 0.94

$$-\frac{\log\left(\frac{d}{e}+x^2-\frac{x\sqrt{b+2de}}{e}\right)}{2\sqrt{b+2de}}+\frac{\log\left(\frac{d}{e}+x^2+\frac{x\sqrt{b+2de}}{e}\right)}{2\sqrt{b+2de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-e*x**2+d)/(e**2*x**4-b*x**2+d**2),x)
```

```
[Out] -log(d/e + x**2 - x*sqrt(b + 2*d*e)/e)/(2*sqrt(b + 2*d*e)) + log(
d/e + x**2 + x*sqrt(b + 2*d*e)/e)/(2*sqrt(b + 2*d*e))
```

Mathematica [B] time = 0.235477, size = 190, normalized size = 2.71

$$\frac{\frac{\left(-\sqrt{b^2-4d^2e^2}+b-2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{b^2-4d^2e^2}-b}}\right)}{\sqrt{\sqrt{b^2-4d^2e^2}-b}}-\frac{\left(\sqrt{b^2-4d^2e^2}+b-2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{-\sqrt{b^2-4d^2e^2}-b}}\right)}{\sqrt{-\sqrt{b^2-4d^2e^2}-b}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d - e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] (-(((b - 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt
[-b - Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[-b - Sqrt[b^2 - 4*d^2*e^2]])
+ ((b - 2*d*e - Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[
-b + Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]])/(
Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

Maple [A] time = 0.023, size = 61, normalized size = 0.9

$$\frac{1}{2}\ln\left(d+ex^{2}+x\sqrt{2\,de+b}\right)\frac{1}{\sqrt{2\,de+b}}-\frac{1}{2}\ln\left(-ex^{2}+x\sqrt{2\,de+b}-d\right)\frac{1}{\sqrt{2\,de+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x)
```

```
[Out] 1/2*ln(d+e*x^2+x*(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)-1/2/(2*d*e+b)^(
1/2)*ln(-e*x^2+x*(2*d*e+b)^(1/2)-d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{ex^{2}-d}{e^{2}x^{4}-bx^{2}+d^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x^2 - d)/(e^2*x^4 - b*x^2 + d^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(e^2*x^4 - b*x^2 + d^2), x)
```

Fricas [A] time = 0.277639, size = 1, normalized size = 0.01

$$\left[\frac{\log\left(\frac{2\left(2\,de^{2}+be\right)x^{3}+2\left(2\,d^{2}e+bd\right)x+\left(e^{2}x^{4}+\left(4\,de+b\right)x^{2}+d^{2}\right)\sqrt{2\,de+b}}{e^{2}x^{4}-bx^{2}+d^{2}}\right)}{2\sqrt{2\,de+b}},\ \frac{\arctan\left(\frac{\sqrt{-2\,de-b}ex}{2\,de+b}\right)+\arctan\left(\frac{e^{2}x^{3}-(de+b)x}{\sqrt{-2\,de-b}d}\right)}{\sqrt{-2\,de-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x^2 - d)/(e^2*x^4 - b*x^2 + d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*(2*d*e^2 + b*e)*x^3 + 2*(2*d^2*e + b*d)*x + (e^2*x^4
+ (4*d*e + b)*x^2 + d^2)*sqrt(2*d*e + b))/(e^2*x^4 - b*x^2 + d^2)
)/sqrt(2*d*e + b), (arctan(sqrt(-2*d*e - b)*e*x/(2*d*e + b)) + ar
ctan((e^2*x^3 - (d*e + b)*x)/(sqrt(-2*d*e - b)*d)))/sqrt(-2*d*e -
 b)]
```

Sympy [A] time = 2.234, size = 112, normalized size = 1.6

$$-\frac{\sqrt{\frac{1}{b+2de}}\log\left(\frac{d}{e}+x^{2}+\frac{x\left(-b\sqrt{\frac{1}{b+2de}}-2de\sqrt{\frac{1}{b+2de}}\right)}{e}\right)}{2}+\frac{\sqrt{\frac{1}{b+2de}}\log\left(\frac{d}{e}+x^{2}+\frac{x\left(b\sqrt{\frac{1}{b+2de}}+2de\sqrt{\frac{1}{b+2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-e*x**2+d)/(e**2*x**4-b*x**2+d**2),x)
```

```
[Out] -sqrt(1/(b + 2*d*e))*log(d/e + x**2 + x*(-b*sqrt(1/(b + 2*d*e)) -
 2*d*e*sqrt(1/(b + 2*d*e)))/e)/2 + sqrt(1/(b + 2*d*e))*log(d/e +
```

```
x**2 + x*(b*sqrt(1/(b + 2*d*e)) + 2*d*e*sqrt(1/(b + 2*d*e)))/e)/2
```

GIAC/XCAS [A] time = 0.440108, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 - d)/(e^2*x^4 - b*x^2 + d^2),x, algorithm="giac")

[Out] Done
```

3.41 $\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx$

Optimal. Leaf size=70

$$\frac{\log\left(x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}-\frac{\log\left(-x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}$$

```
[Out] -Log[d - Sqrt[2*d*e + f]*x + e*x^2]/(2*Sqrt[2*d*e + f]) + Log[d +
 Sqrt[2*d*e + f]*x + e*x^2]/(2*Sqrt[2*d*e + f])
```

Rubi [A] time = 0.0922556, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{\log\left(x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}-\frac{\log\left(-x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d - e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] -Log[d - Sqrt[2*d*e + f]*x + e*x^2]/(2*Sqrt[2*d*e + f]) + Log[d +
 Sqrt[2*d*e + f]*x + e*x^2]/(2*Sqrt[2*d*e + f])
```

Rubi in Sympy [A] time = 32.2335, size = 66, normalized size = 0.94

$$-\frac{\log\left(\frac{d}{e}+x^2-\frac{x\sqrt{2de+f}}{e}\right)}{2\sqrt{2de+f}}+\frac{\log\left(\frac{d}{e}+x^2+\frac{x\sqrt{2de+f}}{e}\right)}{2\sqrt{2de+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-e*x**2+d)/(e**2*x**4-f*x**2+d**2),x)
```

```
[Out] -log(d/e + x**2 - x*sqrt(2*d*e + f)/e)/(2*sqrt(2*d*e + f)) + log(
d/e + x**2 + x*sqrt(2*d*e + f)/e)/(2*sqrt(2*d*e + f))
```

Mathematica [B] time = 0.23142, size = 190, normalized size = 2.71

$$\frac{\frac{\left(-\sqrt{f^2-4d^2e^2}-2de+f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{f^2-4d^2e^2}-f}}\right)}{\sqrt{\sqrt{f^2-4d^2e^2}-f}}-\frac{\left(\sqrt{f^2-4d^2e^2}-2de+f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{-\sqrt{f^2-4d^2e^2}-f}}\right)}{\sqrt{-\sqrt{f^2-4d^2e^2}-f}}}{\sqrt{2}\sqrt{f^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d - e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] (-(((-2*d*e + f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sq
rt[-f - Sqrt[-4*d^2*e^2 + f^2]]])/Sqrt[-f - Sqrt[-4*d^2*e^2 + f^2
]]) + ((-2*d*e + f - Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)
/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^2]]])/Sqrt[-f + Sqrt[-4*d^2*e^2 +
```

```
f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

Maple [A] time = 0.022, size = 61, normalized size = 0.9

$$\frac{1}{2}\ln\left(d+ex^{2}+x\sqrt{2\,de+f}\right)\frac{1}{\sqrt{2\,de+f}}-\frac{1}{2}\ln\left(-ex^{2}+x\sqrt{2\,de+f}-d\right)\frac{1}{\sqrt{2\,de+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x)
```

```
[Out] 1/2*ln(d+e*x^2+x*(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)-1/2/(2*d*e+f)^(
1/2)*ln(-e*x^2+x*(2*d*e+f)^(1/2)-d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{ex^{2}-d}{e^{2}x^{4}-fx^{2}+d^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 - d)/(e^2*x^4 - f*x^2 + d^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(e^2*x^4 - f*x^2 + d^2), x)
```

Fricas [A] time = 0.29136, size = 1, normalized size = 0.01

$$\left[\frac{\log\left(\frac{2\left(2\,de^{2}+ef\right)x^{3}+2\left(2\,d^{2}e+df\right)x+\left(e^{2}x^{4}+(4\,de+f)x^{2}+d^{2}\right)\sqrt{2\,de+f}}{e^{2}x^{4}-fx^{2}+d^{2}}\right)}{2\sqrt{2\,de+f}},\ \frac{\arctan\left(\frac{\sqrt{-2\,de-f}\,ex}{2\,de+f}\right)+\arctan\left(\frac{e^{2}x^{3}-(de+f)x}{\sqrt{-2\,de-f}\,d}\right)}{\sqrt{-2\,de-f}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 - d)/(e^2*x^4 - f*x^2 + d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*(2*d*e^2 + e*f)*x^3 + 2*(2*d^2*e + d*f)*x + (e^2*x^4
+ (4*d*e + f)*x^2 + d^2)*sqrt(2*d*e + f))/(e^2*x^4 - f*x^2 + d^2)
)/sqrt(2*d*e + f), (arctan(sqrt(-2*d*e - f)*e*x/(2*d*e + f)) + ar
ctan((e^2*x^3 - (d*e + f)*x)/(sqrt(-2*d*e - f)*d)))/sqrt(-2*d*e -
 f)]
```

Sympy [A] time = 2.20474, size = 112, normalized size = 1.6

$$-\frac{\sqrt{\frac{1}{2de+f}}\log\left(\frac{d}{e}+x^{2}+\frac{x\left(-2de\sqrt{\frac{1}{2de+f}}-f\sqrt{\frac{1}{2de+f}}\right)}{e}\right)}{2}+\frac{\sqrt{\frac{1}{2de+f}}\log\left(\frac{d}{e}+x^{2}+\frac{x\left(2de\sqrt{\frac{1}{2de+f}}+f\sqrt{\frac{1}{2de+f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-e*x**2+d)/(e**2*x**4-f*x**2+d**2),x)
```

```
[Out] -sqrt(1/(2*d*e + f))*log(d/e + x**2 + x*(-2*d*e*sqrt(1/(2*d*e + f
)) - f*sqrt(1/(2*d*e + f)))/e)/2 + sqrt(1/(2*d*e + f))*log(d/e +
x**2 + x*(2*d*e*sqrt(1/(2*d*e + f)) + f*sqrt(1/(2*d*e + f)))/e)/2
```

GIAC/XCAS [A] time = 0.441338, size = 1, normalized size = 0.01

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 - d)/(e^2*x^4 - f*x^2 + d^2),x, algorithm="giac")
```

```
[Out] Done
```

3.42 $\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$

Optimal. Leaf size=134

$$\frac{e^{3/2}\log\left(\sqrt{e}x\sqrt{2cd-be}+\sqrt{c}d+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}}-\frac{e^{3/2}\log\left(-\sqrt{e}x\sqrt{2cd-be}+\sqrt{c}d+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}}$$

```
[Out] -(e^(3/2)*Log[Sqrt[c]*d - Sqrt[e]*Sqrt[2*c*d - b*e]*x + Sqrt[c]*e
*x^2])/(2*Sqrt[c]*Sqrt[2*c*d - b*e]) + (e^(3/2)*Log[Sqrt[c]*d + S
qrt[e]*Sqrt[2*c*d - b*e]*x + Sqrt[c]*e*x^2])/(2*Sqrt[c]*Sqrt[2*c*
d - b*e])
```

Rubi [A] time = 0.227989, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{e^{3/2}\log\left(\sqrt{e}x\sqrt{2cd-be}+\sqrt{c}d+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}}-\frac{e^{3/2}\log\left(-\sqrt{e}x\sqrt{2cd-be}+\sqrt{c}d+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d - e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] -(e^(3/2)*Log[Sqrt[c]*d - Sqrt[e]*Sqrt[2*c*d - b*e]*x + Sqrt[c]*e
*x^2])/(2*Sqrt[c]*Sqrt[2*c*d - b*e]) + (e^(3/2)*Log[Sqrt[c]*d + S
qrt[e]*Sqrt[2*c*d - b*e]*x + Sqrt[c]*e*x^2])/(2*Sqrt[c]*Sqrt[2*c*
d - b*e])
```

Rubi in Sympy [A] time = 46.312, size = 110, normalized size = 0.82

$$-\frac{e^{\frac{3}{2}}\log\left(\frac{d}{e}+x^2-\frac{x\sqrt{-be+2cd}}{\sqrt{c}\sqrt{e}}\right)}{2\sqrt{c}\sqrt{-be+2cd}}+\frac{e^{\frac{3}{2}}\log\left(\frac{d}{e}+x^2+\frac{x\sqrt{-be+2cd}}{\sqrt{c}\sqrt{e}}\right)}{2\sqrt{c}\sqrt{-be+2cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-e*x**2+d)/(c*d**2/e**2+b*x**2+c*x**4),x)
```

```
[Out] -e**(3/2)*log(d/e + x**2 - x*sqrt(-b*e + 2*c*d)/(sqrt(c)*sqrt(e))
)/(2*sqrt(c)*sqrt(-b*e + 2*c*d)) + e**(3/2)*log(d/e + x**2 + x*sq
rt(-b*e + 2*c*d)/(sqrt(c)*sqrt(e)))/(2*sqrt(c)*sqrt(-b*e + 2*c*d)
)
```

Mathematica [A] time = 0.271039, size = 250, normalized size = 1.87

$$\frac{e^{3/2}\left(-\frac{\left(\sqrt{b^2e^2-4c^2d^2}-be-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}}\right)}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}}-\frac{\left(\sqrt{b^2e^2-4c^2d^2}+be+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2e^2-4c^2d^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d - e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] (e^(3/2)*(-(((-2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(
Sqrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]
])/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]) - ((2*c*d + b*e + Sqrt
[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b
*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b
^2*e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^2*e^2])
```

Maple [B] time = 0.086, size = 582, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x)
```

```
[Out] -1/2*e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((b*e^2+(e^2
*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(x*c*e*2^(1/2)/((
b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2
*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*
(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(x*c*e*2^(1/2)/((b*e^2+(e^2*(b
*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d-1/2*e^2*2^(1/2)/((b*e^2
+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(x*c*e*2^(1/
2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))-1/2*e^4
/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-b*e^2+(e^2*(b*e-2
*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(x*c*e*2^(1/2)/((-b*e^2
+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e
-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e
+2*c*d))^(1/2))*c)^(1/2)*arctanh(x*c*e*2^(1/2)/((-b*e^2+(e^2*(b*e
-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d+1/2*e^2*2^(1/2)/((-b*e^2+
(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(x*c*e*2^(1/
2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ex^2 - d}{cx^4 + bx^2 + \frac{cd^2}{e^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x^2 - d)/(c*x^4 + b*x^2 + c*d^2/e^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(c*x^4 + b*x^2 + c*d^2/e^2), x)
```

Fricas [A] time = 0.275956, size = 1, normalized size = 0.01

$$\left[\frac{1}{2}\, e\sqrt{\frac{e}{2\,c^2d - bce}}\log\left(\frac{ce^2x^4 + cd^2 + \left(4\,cde - be^2\right)x^2 + 2\left(\left(2\,c^2de - bce^2\right)x^3 + \left(2\,c^2d^2 - bcde\right)x\right)\sqrt{\frac{e}{2\,c^2d - bce}}}{ce^2x^4 + be^2x^2 + cd^2}\right),\right.$$
$$-e\sqrt{-\frac{e}{2\,c^2d - bce}}\arctan\left(-\frac{ex}{(2\,cd - be)\sqrt{-\frac{e}{2\,c^2d - bce}}}\right)$$
$$\left.+\, e\sqrt{-\frac{e}{2\,c^2d - bce}}\arctan\left(-\frac{ce^2x^3 - \left(cde - be^2\right)x}{(2\,c^2d^2 - bcde)\sqrt{-\frac{e}{2\,c^2d - bce}}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x^2 - d)/(c*x^4 + b*x^2 + c*d^2/e^2),x, algorithm="fricas")
```

```
[Out] [1/2*e*sqrt(e/(2*c^2*d - b*c*e))*log((c*e^2*x^4 + c*d^2 + (4*c*d*
e - b*e^2)*x^2 + 2*((2*c^2*d*e - b*c*e^2)*x^3 + (2*c^2*d^2 - b*c*
d*e)*x)*sqrt(e/(2*c^2*d - b*c*e)))/(c*e^2*x^4 + b*e^2*x^2 + c*d^2
)), -e*sqrt(-e/(2*c^2*d - b*c*e))*arctan(-e*x/((2*c*d - b*e)*sqrt
(-e/(2*c^2*d - b*c*e)))) + e*sqrt(-e/(2*c^2*d - b*c*e))*arctan(-(
c*e^2*x^3 - (c*d*e - b*e^2)*x)/((2*c^2*d^2 - b*c*d*e)*sqrt(-e/(2*
c^2*d - b*c*e))))]
```

Sympy [A] time = 2.81985, size = 158, normalized size = 1.18

$$\frac{\sqrt{-\frac{e^3}{c(be-2cd)}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-be\sqrt{-\frac{e^3}{c(be-2cd)}}+2cd\sqrt{-\frac{e^3}{c(be-2cd)}}\right)}{e^2}\right)}{2}-\frac{\sqrt{-\frac{e^3}{c(be-2cd)}}\log\left(\frac{d}{e}+x^2+\frac{x\left(be\sqrt{-\frac{e^3}{c(be-2cd)}}-2cd\sqrt{-\frac{e^3}{c(be-2cd)}}\right)}{e^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(c*d**2/e**2+b*x**2+c*x**4),x)
```

```
[Out] sqrt(-e**3/(c*(b*e - 2*c*d)))*log(d/e + x**2 + x*(-b*e*sqrt(-e**3
/(c*(b*e - 2*c*d))) + 2*c*d*sqrt(-e**3/(c*(b*e - 2*c*d))))/e**2)/
2 - sqrt(-e**3/(c*(b*e - 2*c*d)))*log(d/e + x**2 + x*(b*e*sqrt(-e
**3/(c*(b*e - 2*c*d))) - 2*c*d*sqrt(-e**3/(c*(b*e - 2*c*d))))/e**
2)/2
```

GIAC/XCAS [A] time = 0.703189, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x^2 - d)/(c*x^4 + b*x^2 + c*d^2/e^2),x, algorithm="giac")
```

```
[Out] Done
```

3.43 $\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$

Optimal. Leaf size=130

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

```
[Out] -((e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] - 2*Sqrt[c]*Sqrt[e]*x)/Sqrt[
2*c*d + b*e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])) + (e^(3/2)*ArcTan[(Sq
rt[2*c*d - b*e] + 2*Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*e]])/(Sqrt[
c]*Sqrt[2*c*d + b*e])
```

Rubi [A] time = 0.309034, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] -((e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] - 2*Sqrt[c]*Sqrt[e]*x)/Sqrt[
2*c*d + b*e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])) + (e^(3/2)*ArcTan[(Sq
rt[2*c*d - b*e] + 2*Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*e]])/(Sqrt[
c]*Sqrt[2*c*d + b*e])
```

Rubi in Sympy [A] time = 32.108, size = 121, normalized size = 0.93

$$\frac{e^{\frac{3}{2}}\operatorname{atan}\left(\frac{2\sqrt{c}\sqrt{e}x-\sqrt{-be+2cd}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}+\frac{e^{\frac{3}{2}}\operatorname{atan}\left(\frac{2\sqrt{c}\sqrt{e}x+\sqrt{-be+2cd}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(c*d**2/e**2+b*x**2+c*x**4),x)
```

```
[Out] e**(3/2)*atan((2*sqrt(c)*sqrt(e)*x - sqrt(-b*e + 2*c*d))/sqrt(b*e
 + 2*c*d))/(sqrt(c)*sqrt(b*e + 2*c*d)) + e**(3/2)*atan((2*sqrt(c)
*sqrt(e)*x + sqrt(-b*e + 2*c*d))/sqrt(b*e + 2*c*d))/(sqrt(c)*sqrt
(b*e + 2*c*d))
```

Mathematica [A] time = 0.209863, size = 248, normalized size = 1.91

$$\frac{e^{3/2}\left(\frac{\left(\sqrt{b^2e^2-4c^2d^2}-be+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}}\right)}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}}+\frac{\left(\sqrt{b^2e^2-4c^2d^2}+be-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2e^2-4c^2d^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] (e^(3/2)*(((2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqr
t[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/
Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]] + ((-2*c*d + b*e + Sqrt[-4
*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e
+ Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*
e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^2*e^2])
```

Maple [B] time = 0.03, size = 582, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x)
```

```
[Out] 1/2*e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((b*e^2+(e^2*
(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(x*c*e*2^(1/2)/((b
*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*
(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(
b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(x*c*e*2^(1/2)/((b*e^2+(e^2*(b*
e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d+1/2*e^2*2^(1/2)/((b*e^2+
(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(x*c*e*2^(1/2
)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))+1/2*e^4/
(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-b*e^2+(e^2*(b*e-2*
c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(x*c*e*2^(1/2)/((-b*e^2+
(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e-
2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+
2*c*d))^(1/2))*c)^(1/2)*arctanh(x*c*e*2^(1/2)/((-b*e^2+(e^2*(b*e-
2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d-1/2*e^2*2^(1/2)/((-b*e^2+(
e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(x*c*e*2^(1/2
)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{cx^4 + bx^2 + \frac{cd^2}{e^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(c*x^4 + b*x^2 + c*d^2/e^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(c*x^4 + b*x^2 + c*d^2/e^2), x)
```

Fricas [A] time = 0.292006, size = 1, normalized size = 0.01

$$\left[\frac{1}{2}\, e\sqrt{-\frac{e}{2\,c^2d + bce}} \log\left(\frac{ce^2x^4 + cd^2 - \left(4\,cde + be^2\right)x^2 + 2\,\left(\left(2\,c^2de + bce^2\right)x^3 - \left(2\,c^2d^2 + bcde\right)x\right)\sqrt{-\frac{e}{2\,c^2d+bce}}}{ce^2x^4 + be^2x^2 + cd^2}\right), e\sqrt{\frac{e}{2\,c^2d \cdots}}\right.$$

$$\left. + e\sqrt{\frac{e}{2\,c^2d + bce}} \arctan\left(\frac{ce^2x^3 + \left(cde + be^2\right)x}{\left(2\,c^2d^2 + bcde\right)\sqrt{\frac{e}{2\,c^2d+bce}}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(c*x^4 + b*x^2 + c*d^2/e^2),x, algorithm="fricas")
```

```
[Out] [1/2*e*sqrt(-e/(2*c^2*d + b*c*e))*log((c*e^2*x^4 + c*d^2 - (4*c*d
*e + b*e^2)*x^2 + 2*((2*c^2*d*e + b*c*e^2)*x^3 - (2*c^2*d^2 + b*c
*d*e)*x)*sqrt(-e/(2*c^2*d + b*c*e)))/(c*e^2*x^4 + b*e^2*x^2 + c*d
^2)), e*sqrt(e/(2*c^2*d + b*c*e))*arctan(e*x/((2*c*d + b*e)*sqrt(
e/(2*c^2*d + b*c*e)))) + e*sqrt(e/(2*c^2*d + b*c*e))*arctan((c*e^
2*x^3 + (c*d*e + b*e^2)*x)/((2*c^2*d^2 + b*c*d*e)*sqrt(e/(2*c^2*d
 + b*c*e))))]
```

Sympy [A] time = 2.79102, size = 160, normalized size = 1.23

$$-\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-be\sqrt{-\frac{e^3}{c(be+2cd)}}-2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}+\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(be\sqrt{-\frac{e^3}{c(be+2cd)}}+2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(c*d**2/e**2+b*x**2+c*x**4),x)
```

```
[Out] -sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(-b*e*sqrt(-e*
*3/(c*(b*e + 2*c*d))) - 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2
)/2 + sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(b*e*sqrt
(-e**3/(c*(b*e + 2*c*d))) + 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/
e**2)/2
```

GIAC/XCAS [A] time = 0.703076, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(c*x^4 + b*x^2 + c*d^2/e^2),x, algorithm="giac")
```

```
[Out] Done
```

3.44 $\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx$

Optimal. Leaf size=130

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

```
[Out] -((e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] - 2*Sqrt[c]*Sqrt[e]*x)/Sqrt[
2*c*d + b*e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])) + (e^(3/2)*ArcTan[(Sq
rt[2*c*d - b*e] + 2*Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*e]])/(Sqrt[
c]*Sqrt[2*c*d + b*e])
```

Rubi [A] time = 0.25932, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(b*x^2 + c*(d^2/e^2 + x^4)),x]
```

```
[Out] -((e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] - 2*Sqrt[c]*Sqrt[e]*x)/Sqrt[
2*c*d + b*e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])) + (e^(3/2)*ArcTan[(Sq
rt[2*c*d - b*e] + 2*Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*e]])/(Sqrt[
c]*Sqrt[2*c*d + b*e])
```

Rubi in Sympy [A] time = 39.1256, size = 121, normalized size = 0.93

$$\frac{e^{\frac{3}{2}}\operatorname{atan}\left(\frac{2\sqrt{c}\sqrt{e}x-\sqrt{-be+2cd}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}+\frac{e^{\frac{3}{2}}\operatorname{atan}\left(\frac{2\sqrt{c}\sqrt{e}x+\sqrt{-be+2cd}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(b*x**2+c*(d**2/e**2+x**4)),x)
```

```
[Out] e**(3/2)*atan((2*sqrt(c)*sqrt(e)*x - sqrt(-b*e + 2*c*d))/sqrt(b*e
 + 2*c*d))/(sqrt(c)*sqrt(b*e + 2*c*d)) + e**(3/2)*atan((2*sqrt(c)
*sqrt(e)*x + sqrt(-b*e + 2*c*d))/sqrt(b*e + 2*c*d))/(sqrt(c)*sqrt
(b*e + 2*c*d))
```

Mathematica [A] time = 0.0714055, size = 248, normalized size = 1.91

$$\frac{e^{3/2}\left(\frac{\left(\sqrt{b^2e^2-4c^2d^2}-be+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}}\right)}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}}+\frac{\left(\sqrt{b^2e^2-4c^2d^2}+be-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2e^2-4c^2d^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/(b*x^2 + c*(d^2/e^2 + x^4)),x]
```

```
[Out] (e^(3/2)*(((2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqr
t[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/
Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]] + ((-2*c*d + b*e + Sqrt[-4
*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e
+ Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*
e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^2*e^2])
```

Maple [B] time = 0.003, size = 582, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x)
```

```
[Out] 1/2*e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((b*e^2+(e^2*
(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(x*c*e*2^(1/2)/((b
*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*
(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(
b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(x*c*e*2^(1/2)/((b*e^2+(e^2*(b*
e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d+1/2*e^2*2^(1/2)/((b*e^2+
(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(x*c*e*2^(1/2
)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))+1/2*e^4/
(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-b*e^2+(e^2*(b*e-2*
c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(x*c*e*2^(1/2)/((-b*e^2+
(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e-
2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+
2*c*d))^(1/2))*c)^(1/2)*arctanh(x*c*e*2^(1/2)/((-b*e^2+(e^2*(b*e-
2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d-1/2*e^2*2^(1/2)/((-b*e^2+(
e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(x*c*e*2^(1/2
)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{bx^2 + \left(x^4 + \frac{d^2}{e^2}\right)c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(b*x^2 + (x^4 + d^2/e^2)*c),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(b*x^2 + (x^4 + d^2/e^2)*c), x)
```

Fricas [A] time = 0.286317, size = 1, normalized size = 0.01

$$\left[\frac{1}{2}\, e\sqrt{-\frac{e}{2\,c^2d + bce}}\log\left(\frac{ce^2x^4 + cd^2 - \left(4\,cde + be^2\right)x^2 + 2\left(\left(2\,c^2de + bce^2\right)x^3 - \left(2\,c^2d^2 + bcde\right)x\right)\sqrt{-\frac{e}{2\,c^2d + bce}}}{ce^2x^4 + be^2x^2 + cd^2}\right), e\sqrt{\frac{e}{2\,c^2d \cdots}}\right.$$
$$\left. + e\sqrt{\frac{e}{2\,c^2d + bce}}\arctan\left(\frac{ce^2x^3 + \left(cde + be^2\right)x}{\left(2\,c^2d^2 + bcde\right)\sqrt{\frac{e}{2\,c^2d + bce}}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(b*x^2 + (x^4 + d^2/e^2)*c),x, algorithm="fricas")
```

```
[Out] [1/2*e*sqrt(-e/(2*c^2*d + b*c*e))*log((c*e^2*x^4 + c*d^2 - (4*c*d
*e + b*e^2)*x^2 + 2*((2*c^2*d*e + b*c*e^2)*x^3 - (2*c^2*d^2 + b*c
*d*e)*x)*sqrt(-e/(2*c^2*d + b*c*e)))/(c*e^2*x^4 + b*e^2*x^2 + c*d
^2)), e*sqrt(e/(2*c^2*d + b*c*e))*arctan(e*x/((2*c*d + b*e)*sqrt(
e/(2*c^2*d + b*c*e)))) + e*sqrt(e/(2*c^2*d + b*c*e))*arctan((c*e^
2*x^3 + (c*d*e + b*e^2)*x)/((2*c^2*d^2 + b*c*d*e)*sqrt(e/(2*c^2*d
 + b*c*e))))]
```

Sympy [A] time = 2.84113, size = 160, normalized size = 1.23

$$-\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-be\sqrt{-\frac{e^3}{c(be+2cd)}}-2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}+\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(be\sqrt{-\frac{e^3}{c(be+2cd)}}+2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(b*x**2+c*(d**2/e**2+x**4)),x)
```

```
[Out] -sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(-b*e*sqrt(-e*
*3/(c*(b*e + 2*c*d))) - 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2
)/2 + sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(b*e*sqrt
(-e**3/(c*(b*e + 2*c*d))) + 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/
e**2)/2
```

GIAC/XCAS [A] time = 0.705802, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(b*x^2 + (x^4 + d^2/e^2)*c),x, algorithm="giac")
```

```
[Out] Done
```

3.45 $\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx$

Optimal. Leaf size=29

$$\frac{1}{2}\log\left(a+bx^2+x\right)-\frac{1}{2}\log\left(a+bx^2-x\right)$$

```
[Out] -Log[a - x + b*x^2]/2 + Log[a + x + b*x^2]/2
```

Rubi [A] time = 0.051117, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{1}{2}\log\left(a+bx^2+x\right)-\frac{1}{2}\log\left(a+bx^2-x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]

[Out] -Log[a - x + b*x^2]/2 + Log[a + x + b*x^2]/2
```

Rubi in Sympy [A] time = 24.1843, size = 26, normalized size = 0.9

$$-\frac{\log\left(\frac{a}{b}+x^2-\frac{x}{b}\right)}{2}+\frac{\log\left(\frac{a}{b}+x^2+\frac{x}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-b*x**2+a)/(a**2+(2*a*b-1)*x**2+b**2*x**4),x)

[Out] -log(a/b + x**2 - x/b)/2 + log(a/b + x**2 + x/b)/2
```

Mathematica [A] time = 0.0297309, size = 29, normalized size = 1.

$$\frac{1}{2}\log\left(a+bx^2+x\right)-\frac{1}{2}\log\left(a+bx^2-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]

[Out] -Log[a - x + b*x^2]/2 + Log[a + x + b*x^2]/2
```

Maple [A] time = 0.01, size = 26, normalized size = 0.9

$$-\frac{\ln\left(bx^2+a-x\right)}{2}+\frac{\ln\left(bx^2+a+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x)
```

```
[Out] -1/2*ln(b*x^2+a-x)+1/2*ln(b*x^2+a+x)
```

Maxima [A] time = 0.733349, size = 34, normalized size = 1.17

$$\frac{1}{2}\log\left(bx^2+a+x\right)-\frac{1}{2}\log\left(bx^2+a-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*x^2 - a)/(b^2*x^4 + (2*a*b - 1)*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 + a + x) - 1/2*log(b*x^2 + a - x)
```

Fricas [A] time = 0.279564, size = 34, normalized size = 1.17

$$\frac{1}{2}\log\left(bx^2+a+x\right)-\frac{1}{2}\log\left(bx^2+a-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*x^2 - a)/(b^2*x^4 + (2*a*b - 1)*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x^2 + a + x) - 1/2*log(b*x^2 + a - x)
```

Sympy [A] time = 1.87413, size = 26, normalized size = 0.9

$$-\frac{\log\left(\frac{a}{b}+x^2-\frac{x}{b}\right)}{2}+\frac{\log\left(\frac{a}{b}+x^2+\frac{x}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)/(a**2+(2*a*b-1)*x**2+b**2*x**4),x)
```

```
[Out] -log(a/b + x**2 - x/b)/2 + log(a/b + x**2 + x/b)/2
```

GIAC/XCAS [A] time = 0.277673, size = 34, normalized size = 1.17

$$\frac{1}{2}\ln\left(bx^2+a+x\right)-\frac{1}{2}\ln\left(bx^2+a-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*x^2 - a)/(b^2*x^4 + (2*a*b - 1)*x^2 + a^2),x, algorithm="giac")
```

```
[Out] 1/2*ln(b*x^2 + a + x) - 1/2*ln(b*x^2 + a - x)
```

3.46 $\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}-\frac{\tanh^{-1}\left(\frac{2bx+1}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}$$

```
[Out] ArcTanh[(1 - 2*b*x)/Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b] - ArcTanh[(1
 + 2*b*x)/Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b]
```

Rubi [A] time = 0.129054, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$\frac{\tanh^{-1}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}-\frac{\tanh^{-1}\left(\frac{2bx+1}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
```

```
[Out] ArcTanh[(1 - 2*b*x)/Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b] - ArcTanh[(1
 + 2*b*x)/Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b]
```

Rubi in Sympy [A] time = 17.3441, size = 58, normalized size = 0.97

$$-\frac{\operatorname{atanh}\left(\frac{2bx-1}{\sqrt{-4ab+1}}\right)}{\sqrt{-4ab+1}}-\frac{\operatorname{atanh}\left(\frac{2bx+1}{\sqrt{-4ab+1}}\right)}{\sqrt{-4ab+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(a**2+(2*a*b-1)*x**2+b**2*x**4),x)
```

```
[Out] -atanh((2*b*x - 1)/sqrt(-4*a*b + 1))/sqrt(-4*a*b + 1) - atanh((2*
b*x + 1)/sqrt(-4*a*b + 1))/sqrt(-4*a*b + 1)
```

Mathematica [B] time = 0.346594, size = 138, normalized size = 2.3

$$\frac{\frac{\left(\sqrt{1-4ab}+1\right)\tan^{-1}\left(\frac{bx}{\sqrt{ab-\frac{1}{2}\sqrt{1-4ab}-\frac{1}{2}}}\right)}{\sqrt{2ab-\sqrt{1-4ab}-1}}+\frac{\left(\sqrt{1-4ab}-1\right)\tan^{-1}\left(\frac{\sqrt{2}bx}{\sqrt{2ab+\sqrt{1-4ab}-1}}\right)}{\sqrt{2ab+\sqrt{1-4ab}-1}}}{\sqrt{2-8ab}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
```

```
[Out] (((1 + Sqrt[1 - 4*a*b])*ArcTan[(b*x)/Sqrt[-1/2 + a*b - Sqrt[1 - 4
*a*b]/2]])/Sqrt[-1 + 2*a*b - Sqrt[1 - 4*a*b]] + ((-1 + Sqrt[1 - 4
*a*b])*ArcTan[(Sqrt[2]*b*x)/Sqrt[-1 + 2*a*b + Sqrt[1 - 4*a*b]]])/
Sqrt[-1 + 2*a*b + Sqrt[1 - 4*a*b]])/Sqrt[2 - 8*a*b]
```

Maple [A] time = 0.012, size = 52, normalized size = 0.9

$$1\arctan\left((2\,bx-1)\frac{1}{\sqrt{4\,ab-1}}\right)\frac{1}{\sqrt{4\,ab-1}}+1\arctan\left((2\,bx+1)\frac{1}{\sqrt{4\,ab-1}}\right)\frac{1}{\sqrt{4\,ab-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x)
```

```
[Out] 1/(4*a*b-1)^(1/2)*arctan((2*b*x-1)/(4*a*b-1)^(1/2))+1/(4*a*b-1)^(
1/2)*arctan((2*b*x+1)/(4*a*b-1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(b^2*x^4 + (2*a*b - 1)*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.295897, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{2\,(4\,ab^2-b)\,x^3-2\,(4\,a^2b-a)\,x+\left(b^2x^4-(6\,ab-1)x^2+a^2\right)\sqrt{-4\,ab+1}}{b^2x^4+(2\,ab-1)x^2+a^2}\right)}{2\,\sqrt{-4\,ab+1}},\ \frac{\arctan\left(\frac{bx}{\sqrt{4\,ab-1}}\right)+\arctan\left(\frac{b^2x^3+(3\,ab-1)x}{\sqrt{4\,ab-1}a}\right)}{\sqrt{4\,ab-1}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(b^2*x^4 + (2*a*b - 1)*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*(4*a*b^2 - b)*x^3 - 2*(4*a^2*b - a)*x + (b^2*x^4 - (6
*a*b - 1)*x^2 + a^2)*sqrt(-4*a*b + 1))/(b^2*x^4 + (2*a*b - 1)*x^2
 + a^2))/sqrt(-4*a*b + 1), (arctan(b*x/sqrt(4*a*b - 1)) + arctan(
(b^2*x^3 + (3*a*b - 1)*x)/(sqrt(4*a*b - 1)*a)))/sqrt(4*a*b - 1)]
```

Sympy [A] time = 1.22528, size = 117, normalized size = 1.95

$$-\frac{\sqrt{-\frac{1}{4ab-1}}\log\left(-\frac{a}{b}+x^2+\frac{x\left(-4ab\sqrt{-\frac{1}{4ab-1}}+\sqrt{-\frac{1}{4ab-1}}\right)}{b}\right)}{2}+\frac{\sqrt{-\frac{1}{4ab-1}}\log\left(-\frac{a}{b}+x^2+\frac{x\left(4ab\sqrt{-\frac{1}{4ab-1}}-\sqrt{-\frac{1}{4ab-1}}\right)}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(a**2+(2*a*b-1)*x**2+b**2*x**4),x)
```

```
[Out] -sqrt(-1/(4*a*b - 1))*log(-a/b + x**2 + x*(-4*a*b*sqrt(-1/(4*a*b
- 1)) + sqrt(-1/(4*a*b - 1)))/b)/2 + sqrt(-1/(4*a*b - 1))*log(-a/
b + x**2 + x*(4*a*b*sqrt(-1/(4*a*b - 1)) - sqrt(-1/(4*a*b - 1)))/
b)/2
```

GIAC/XCAS [A] time = 0.274772, size = 69, normalized size = 1.15

$$\frac{\arctan\left(\frac{2\,bx+1}{\sqrt{4\,ab-1}}\right)}{\sqrt{4\,ab-1}} + \frac{\arctan\left(\frac{2\,bx-1}{\sqrt{4\,ab-1}}\right)}{\sqrt{4\,ab-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(b^2*x^4 + (2*a*b - 1)*x^2 + a^2),x, algorithm="giac")
```

```
[Out] arctan((2*b*x + 1)/sqrt(4*a*b - 1))/sqrt(4*a*b - 1) + arctan((2*b
*x - 1)/sqrt(4*a*b - 1))/sqrt(4*a*b - 1)
```

3.47 $\int \frac{1+2x^2}{1+bx^2+4x^4} \, dx$

Optimal. Leaf size=62

$$\frac{\tan^{-1}\left(\frac{\sqrt{4-b}+4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}-\frac{\tan^{-1}\left(\frac{\sqrt{4-b}-4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

```
[Out] -(ArcTan[(Sqrt[4 - b] - 4*x)/Sqrt[4 + b]]/Sqrt[4 + b]) + ArcTan[(
Sqrt[4 - b] + 4*x)/Sqrt[4 + b]]/Sqrt[4 + b]
```

Rubi [A] time = 0.107997, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{\tan^{-1}\left(\frac{\sqrt{4-b}+4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}-\frac{\tan^{-1}\left(\frac{\sqrt{4-b}-4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[4 - b] - 4*x)/Sqrt[4 + b]]/Sqrt[4 + b]) + ArcTan[(
Sqrt[4 - b] + 4*x)/Sqrt[4 + b]]/Sqrt[4 + b]
```

Rubi in Sympy [A] time = 12.0003, size = 49, normalized size = 0.79

$$\frac{\operatorname{atan}\left(\frac{4x-\sqrt{-b+4}}{\sqrt{b+4}}\right)}{\sqrt{b+4}}+\frac{\operatorname{atan}\left(\frac{4x+\sqrt{-b+4}}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+1)/(4*x**4+b*x**2+1),x)
```

```
[Out] atan((4*x - sqrt(-b + 4))/sqrt(b + 4))/sqrt(b + 4) + atan((4*x +
sqrt(-b + 4))/sqrt(b + 4))/sqrt(b + 4)
```

Mathematica [B] time = 0.0978946, size = 126, normalized size = 2.03

$$\frac{\frac{\left(\sqrt{b^2-16}-b+4\right)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{b-\sqrt{b^2-16}}}\right)}{\sqrt{b-\sqrt{b^2-16}}}+\frac{\left(\sqrt{b^2-16}+b-4\right)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{\sqrt{b^2-16}+b}}\right)}{\sqrt{\sqrt{b^2-16}+b}}}{\sqrt{2}\sqrt{b^2-16}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

```
[Out] (((4 - b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[b - Sqrt[-1
6 + b^2]]])/Sqrt[b - Sqrt[-16 + b^2]] + ((-4 + b + Sqrt[-16 + b^2
])*ArcTan[(2*Sqrt[2]*x)/Sqrt[b + Sqrt[-16 + b^2]]])/Sqrt[b + Sqrt
[-16 + b^2]])/(Sqrt[2]*Sqrt[-16 + b^2])
```

Maple [B] time = 0.051, size = 277, normalized size = 4.5

$$-4\frac{1}{\sqrt{(b-4)(4+b)}\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\arctan\left(4\frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\right)+1\arctan\left(4\frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\right)\frac{1}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}+b\arctan\left(4\frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\right)\frac{1}{\sqrt{(b-4)(4+b)}}\frac{1}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}+4\frac{1}{\sqrt{(b-4)(4+b)}\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}\arctan\left(4\frac{x}{\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}\right)+1\arctan\left(4\frac{x}{\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}\right)\frac{1}{\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}-b\arctan\left(4\frac{x}{\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}\right)\frac{1}{\sqrt{(b-4)(4+b)}}\frac{1}{\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((2*x^2+1)/(4*x^4+b*x^2+1),x)
```

```
[Out] -4/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4
*x/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))+1/(2*((b-4)*(4+b))^(1/2)+2*
b)^(1/2)*arctan(4*x/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))+1/((b-4)*(
4+b))^(1/2)/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(2*((b-4
)*(4+b))^(1/2)+2*b)^(1/2))*b+4/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+
b))^(1/2)+2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2
))+1/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+
b))^(1/2)+2*b)^(1/2))-1/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+b))^(1/
2)+2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4+bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*x^2 + 1)/(4*x^4 + b*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + b*x^2 + 1), x)
```

Fricas [A] time = 0.278878, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{4(b+4)x^3-2(b+4)x+(4x^4-(b+8)x^2+1)\sqrt{-b-4}}{4x^4+bx^2+1}\right)}{2\sqrt{-b-4}},\ \frac{\arctan\left(\frac{4x^3+(b+2)x}{\sqrt{b+4}}\right)+\arctan\left(\frac{2x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 + b*x^2 + 1),x, algorithm="fricas")
```

```
[Out] [1/2*log((4*(b + 4)*x^3 - 2*(b + 4)*x + (4*x^4 - (b + 8)*x^2 + 1)
*sqrt(-b - 4))/(4*x^4 + b*x^2 + 1))/sqrt(-b - 4), (arctan((4*x^3
+ (b + 2)*x)/sqrt(b + 4)) + arctan(2*x/sqrt(b + 4)))/sqrt(b + 4)]
```

Sympy [A] time = 0.792382, size = 95, normalized size = 1.53

$$-\frac{\sqrt{-\frac{1}{b+4}}\log\left(x^2+x\left(-\frac{b\sqrt{-\frac{1}{b+4}}}{2}-2\sqrt{-\frac{1}{b+4}}\right)-\frac{1}{2}\right)}{2}+\frac{\sqrt{-\frac{1}{b+4}}\log\left(x^2+x\left(\frac{b\sqrt{-\frac{1}{b+4}}}{2}+2\sqrt{-\frac{1}{b+4}}\right)-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+1)/(4*x**4+b*x**2+1),x)
```

```
[Out] -sqrt(-1/(b + 4))*log(x**2 + x*(-b*sqrt(-1/(b + 4))/2 - 2*sqrt(-1
/(b + 4))) - 1/2)/2 + sqrt(-1/(b + 4))*log(x**2 + x*(b*sqrt(-1/(b
 + 4))/2 + 2*sqrt(-1/(b + 4))) - 1/2)/2
```

GIAC/XCAS [A] time = 0.325165, size = 1, normalized size = 0.02

$$\mathit{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 + b*x^2 + 1),x, algorithm="giac")
```

```
[Out] Done
```

3.48 $\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx$

Optimal. Leaf size=66

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+4}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b+4}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}$$

```
[Out] -(ArcTan[(Sqrt[4 + b] - 4*x)/Sqrt[4 - b]]/Sqrt[4 - b]) + ArcTan[(
Sqrt[4 + b] + 4*x)/Sqrt[4 - b]]/Sqrt[4 - b]
```

Rubi [A] time = 0.109298, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+4}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b+4}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^2)/(1 - b*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[4 + b] - 4*x)/Sqrt[4 - b]]/Sqrt[4 - b]) + ArcTan[(
Sqrt[4 + b] + 4*x)/Sqrt[4 - b]]/Sqrt[4 - b]
```

Rubi in Sympy [A] time = 12.5832, size = 49, normalized size = 0.74

$$\frac{\operatorname{atan}\left(\frac{4x-\sqrt{b+4}}{\sqrt{-b+4}}\right)}{\sqrt{-b+4}}+\frac{\operatorname{atan}\left(\frac{4x+\sqrt{b+4}}{\sqrt{-b+4}}\right)}{\sqrt{-b+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+1)/(4*x**4-b*x**2+1),x)
```

```
[Out] atan((4*x - sqrt(b + 4))/sqrt(-b + 4))/sqrt(-b + 4) + atan((4*x +
 sqrt(b + 4))/sqrt(-b + 4))/sqrt(-b + 4)
```

Mathematica [B] time = 0.095841, size = 134, normalized size = 2.03

$$\frac{\frac{\left(\sqrt{b^2-16}+b+4\right)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{-\sqrt{b^2-16}-b}}\right)}{\sqrt{-\sqrt{b^2-16}-b}}+\frac{\left(\sqrt{b^2-16}-b-4\right)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{\sqrt{b^2-16}-b}}\right)}{\sqrt{\sqrt{b^2-16}-b}}}{\sqrt{2}\sqrt{b^2-16}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2)/(1 - b*x^2 + 4*x^4),x]
```

```
[Out] (((4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[-b - Sqrt[-
16 + b^2]]])/Sqrt[-b - Sqrt[-16 + b^2]] + ((-4 - b + Sqrt[-16 + b
^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[-b + Sqrt[-16 + b^2]]])/Sqrt[-b +
Sqrt[-16 + b^2]])/(Sqrt[2]*Sqrt[-16 + b^2])
```

Maple [B] time = 0.034, size = 277, normalized size = 4.2

$$4\frac{1}{\sqrt{(b-4)(4+b)}\sqrt{-2\sqrt{(b-4)(4+b)}-2b}}\arctan\left(4\frac{x}{\sqrt{-2\sqrt{(b-4)(4+b)}-2b}}\right)$$
$$+1\arctan\left(4\frac{x}{\sqrt{-2\sqrt{(b-4)(4+b)}-2b}}\right)\frac{1}{\sqrt{-2\sqrt{(b-4)(4+b)}-2b}}$$
$$+b\arctan\left(4\frac{x}{\sqrt{-2\sqrt{(b-4)(4+b)}-2b}}\right)\frac{1}{\sqrt{(b-4)(4+b)}}\frac{1}{\sqrt{-2\sqrt{(b-4)(4+b)}-2b}}$$
$$-4\frac{1}{\sqrt{(b-4)(4+b)}\sqrt{2\sqrt{(b-4)(4+b)}-2b}}\arctan\left(4\frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}-2b}}\right)$$
$$+1\arctan\left(4\frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}-2b}}\right)\frac{1}{\sqrt{2\sqrt{(b-4)(4+b)}-2b}}$$
$$-b\arctan\left(4\frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}-2b}}\right)\frac{1}{\sqrt{(b-4)(4+b)}}\frac{1}{\sqrt{2\sqrt{(b-4)(4+b)}-2b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+1)/(4*x^4-b*x^2+1),x)
```

```
[Out] 4/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+b))^(1/2)-2*b)^(1/2)*arctan(4
*x/(-2*((b-4)*(4+b))^(1/2)-2*b)^(1/2))+1/(-2*((b-4)*(4+b))^(1/2)-
2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)-2*b)^(1/2))+1/((b-4
)*(4+b))^(1/2)/(-2*((b-4)*(4+b))^(1/2)-2*b)^(1/2)*arctan(4*x/(-2*
((b-4)*(4+b))^(1/2)-2*b)^(1/2))*b-4/((b-4)*(4+b))^(1/2)/(2*((b-4)
*(4+b))^(1/2)-2*b)^(1/2)*arctan(4*x/(2*((b-4)*(4+b))^(1/2)-2*b)^(
1/2))+1/(2*((b-4)*(4+b))^(1/2)-2*b)^(1/2)*arctan(4*x/(2*((b-4)*(4
+b))^(1/2)-2*b)^(1/2))-1/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(1/
2)-2*b)^(1/2)*arctan(4*x/(2*((b-4)*(4+b))^(1/2)-2*b)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4-bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - b*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - b*x^2 + 1), x)
```

Fricas [A] time = 0.294226, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{4(b-4)x^3-2(b-4)x-(4x^4+(b-8)x^2+1)\sqrt{b-4}}{4x^4-bx^2+1}\right)}{2\sqrt{b-4}}, \frac{\arctan\left(\frac{4x^3-(b-2)x}{\sqrt{-b+4}}\right)-\arctan\left(\frac{2\sqrt{-b+4}x}{b-4}\right)}{\sqrt{-b+4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - b*x^2 + 1),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(4*(b - 4)*x^3 - 2*(b - 4)*x - (4*x^4 + (b - 8)*x^2 + 1
)*sqrt(b - 4))/(4*x^4 - b*x^2 + 1))/sqrt(b - 4), (arctan((4*x^3 -
 (b - 2)*x)/sqrt(-b + 4)) - arctan(2*sqrt(-b + 4)*x/(b - 4)))/sqr
t(-b + 4)]
```

Sympy [A] time = 0.833746, size = 83, normalized size = 1.26

$$\frac{\sqrt{\frac{1}{b-4}}\log\left(x^{2}+x\left(-\frac{b\sqrt{\frac{1}{b-4}}}{2}+2\sqrt{\frac{1}{b-4}}\right)-\frac{1}{2}\right)}{2}-\frac{\sqrt{\frac{1}{b-4}}\log\left(x^{2}+x\left(\frac{b\sqrt{\frac{1}{b-4}}}{2}-2\sqrt{\frac{1}{b-4}}\right)-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+1)/(4*x**4-b*x**2+1),x)
```

```
[Out] sqrt(1/(b - 4))*log(x**2 + x*(-b*sqrt(1/(b - 4))/2 + 2*sqrt(1/(b
- 4))) - 1/2)/2 - sqrt(1/(b - 4))*log(x**2 + x*(b*sqrt(1/(b - 4))
/2 - 2*sqrt(1/(b - 4))) - 1/2)/2
```

GIAC/XCAS [A] time = 0.318356, size = 1, normalized size = 0.02

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - b*x^2 + 1),x, algorithm="giac")
```

```
[Out] Done
```

3.49 $\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}$$

```
[Out] ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[10] + ArcTan[(2*x)/Sqrt[3 +
Sqrt[5]]]/Sqrt[10]
```

Rubi [A] time = 0.114716, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[10] + ArcTan[(2*x)/Sqrt[3 +
Sqrt[5]]]/Sqrt[10]
```

Rubi in Sympy [A] time = 9.45092, size = 70, normalized size = 1.56

$$\frac{\left(-\frac{\sqrt{5}}{5}+1\right)\operatorname{atan}\left(\frac{2x}{\sqrt{-\sqrt{5}+3}}\right)}{2\sqrt{-\sqrt{5}+3}}+\frac{\left(\frac{\sqrt{5}}{5}+1\right)\operatorname{atan}\left(\frac{2x}{\sqrt{\sqrt{5}+3}}\right)}{2\sqrt{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*x**2+1)/(4*x**4+6*x**2+1),x)
```

```
[Out] (-sqrt(5)/5 + 1)*atan(2*x/sqrt(-sqrt(5) + 3))/(2*sqrt(-sqrt(5) +
3)) + (sqrt(5)/5 + 1)*atan(2*x/sqrt(sqrt(5) + 3))/(2*sqrt(sqrt(5)
 + 3))
```

Mathematica [A] time = 0.122975, size = 83, normalized size = 1.84

$$\frac{\left(\sqrt{5}-1\right)\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{2\sqrt{5\left(3-\sqrt{5}\right)}}+\frac{\left(1+\sqrt{5}\right)\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{2\sqrt{5\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

```
[Out] ((-1 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]])/(2*Sqrt[5*(3 - S
qrt[5])]) + ((1 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]])/(2*Sq
rt[5*(3 + Sqrt[5])])
```

Maple [B] time = 0.06, size = 136, normalized size = 3.

$$-\frac{2\sqrt{5}}{10\sqrt{10}-10\sqrt{2}}\arctan\left(8\frac{x}{2\sqrt{10}-2\sqrt{2}}\right)+2\frac{1}{2\sqrt{10}-2\sqrt{2}}\arctan\left(8\frac{x}{2\sqrt{10}-2\sqrt{2}}\right)+\frac{2\sqrt{5}}{10\sqrt{10}+10\sqrt{2}}\arctan\left(8\frac{x}{2\sqrt{10}+2\sqrt{2}}\right)+2\frac{1}{2\sqrt{10}+2\sqrt{2}}\arctan\left(8\frac{x}{2\sqrt{10}+2\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+1)/(4*x^4+6*x^2+1),x)
```

```
[Out] -2/5*5^(1/2)/(2*10^(1/2)-2*2^(1/2))*arctan(8*x/(2*10^(1/2)-2*2^(1
/2)))+2/(2*10^(1/2)-2*2^(1/2))*arctan(8*x/(2*10^(1/2)-2*2^(1/2)))
+2/5*5^(1/2)/(2*10^(1/2)+2*2^(1/2))*arctan(8*x/(2*10^(1/2)+2*2^(1
/2)))+2/(2*10^(1/2)+2*2^(1/2))*arctan(8*x/(2*10^(1/2)+2*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4+6x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 + 6*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + 6*x^2 + 1), x)
```

Fricas [A] time = 0.282201, size = 35, normalized size = 0.78

$$\frac{1}{10}\sqrt{10}\left(\arctan\left(\frac{2}{5}\sqrt{10}\left(x^3+2x\right)\right)+\arctan\left(\frac{1}{5}\sqrt{10}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 + 6*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(10)*(arctan(2/5*sqrt(10)*(x^3 + 2*x)) + arctan(1/5*sqrt
(10)*x))
```

Sympy [A] time = 0.257224, size = 42, normalized size = 0.93

$$\frac{\sqrt{10}\left(2\operatorname{atan}\left(\frac{\sqrt{10}x}{5}\right)+2\operatorname{atan}\left(\frac{2\sqrt{10}x^3}{5}+\frac{4\sqrt{10}x}{5}\right)\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+1)/(4*x**4+6*x**2+1),x)
```

```
[Out] sqrt(10)*(2*atan(sqrt(10)*x/5) + 2*atan(2*sqrt(10)*x**3/5 + 4*sqr
t(10)*x/5))/20
```

GIAC/XCAS [A] time = 0.275919, size = 53, normalized size = 1.18

$$\frac{1}{10}\sqrt{10}\arctan\left(\frac{4x}{\sqrt{10}+\sqrt{2}}\right)+\frac{1}{10}\sqrt{10}\arctan\left(\frac{4x}{\sqrt{10}-\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 + 6*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(10)*arctan(4*x/(sqrt(10) + sqrt(2))) + 1/10*sqrt(10)*ar
ctan(4*x/(sqrt(10) - sqrt(2)))
```

3.50 $\int \frac{1+2x^2}{1+5x^2+4x^4} \, dx$

Optimal. Leaf size=15

$$\frac{1}{3}\tan^{-1}(x)+\frac{1}{3}\tan^{-1}(2x)$$

```
[Out] ArcTan[x]/3 + ArcTan[2*x]/3
```

Rubi [A] time = 0.0241888, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{1}{3}\tan^{-1}(x)+\frac{1}{3}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^2)/(1 + 5*x^2 + 4*x^4),x]

[Out] ArcTan[x]/3 + ArcTan[2*x]/3
```

Rubi in Sympy [A] time = 7.80914, size = 10, normalized size = 0.67

$$\frac{\operatorname{atan}(x)}{3}+\frac{\operatorname{atan}(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+1)/(4*x**4+5*x**2+1),x)

[Out] atan(x)/3 + atan(2*x)/3
```

Mathematica [A] time = 0.0114525, size = 17, normalized size = 1.13

$$-\frac{1}{3}\tan^{-1}\left(\frac{3x}{2x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2)/(1 + 5*x^2 + 4*x^4),x]

[Out] -ArcTan[(3*x)/(-1 + 2*x^2)]/3
```

Maple [A] time = 0.013, size = 12, normalized size = 0.8

$$\frac{\arctan(x)}{3}+\frac{\arctan(2\,x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+1)/(4*x^4+5*x^2+1),x)
```

```
[Out] 1/3*arctan(x)+1/3*arctan(2*x)
```

Maxima [A] time = 0.805394, size = 15, normalized size = 1.

$$\frac{1}{3}\arctan(2x) + \frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2 + 1)/(4*x^4 + 5*x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(2*x) + 1/3*arctan(x)
```

Fricas [A] time = 0.284029, size = 26, normalized size = 1.73

$$\frac{1}{3}\arctan\left(\frac{4}{3}x^3 + \frac{7}{3}x\right) + \frac{1}{3}\arctan\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2 + 1)/(4*x^4 + 5*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/3*arctan(4/3*x^3 + 7/3*x) + 1/3*arctan(2/3*x)
```

Sympy [A] time = 0.214648, size = 22, normalized size = 1.47

$$\frac{\operatorname{atan}\left(\frac{2x}{3}\right)}{3} + \frac{\operatorname{atan}\left(\frac{4x^3}{3} + \frac{7x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+5*x**2+1),x)
```

```
[Out] atan(2*x/3)/3 + atan(4*x**3/3 + 7*x/3)/3
```

GIAC/XCAS [A] time = 0.270681, size = 15, normalized size = 1.

$$\frac{1}{3}\arctan(2x) + \frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2 + 1)/(4*x^4 + 5*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/3*arctan(2*x) + 1/3*arctan(x)
```

3.51 $\int \frac{1+2x^2}{1+4x^2+4x^4} dx$

Optimal. Leaf size=14

$$\frac{\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Rubi [A] time = 0.014605, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Rubi in Sympy [A] time = 4.61066, size = 14, normalized size = 1.

$$\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+1)/(4*x**4+4*x**2+1),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*x)/2
```

Mathematica [A] time = 0.00532676, size = 14, normalized size = 1.

$$\frac{\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Maple [A] time = 0.003, size = 12, normalized size = 0.9

$$\frac{\arctan\left(\sqrt{2}x\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+1)/(4*x^4+4*x^2+1),x)

[Out] 1/2*arctan(2^(1/2)*x)*2^(1/2)
```

Maxima [A] time = 0.803186, size = 15, normalized size = 1.07

$$\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 + 4*x^2 + 1),x, algorithm="maxima")

[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

Fricas [A] time = 0.280621, size = 15, normalized size = 1.07

$$\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 + 4*x^2 + 1),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

Sympy [A] time = 0.182625, size = 14, normalized size = 1.

$$\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+1)/(4*x**4+4*x**2+1),x)

[Out] sqrt(2)*atan(sqrt(2)*x)/2
```

GIAC/XCAS [A] time = 0.269831, size = 15, normalized size = 1.07

$$\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 + 4*x^2 + 1),x, algorithm="giac")

[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

3.52 $\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right)}{\sqrt{7}}-\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] -(ArcTan[(1 - 4*x)/Sqrt[7]]/Sqrt[7]) + ArcTan[(1 + 4*x)/Sqrt[7]]/
Sqrt[7]
```

Rubi [A] time = 0.0708948, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{\tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right)}{\sqrt{7}}-\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(1 - 4*x)/Sqrt[7]]/Sqrt[7]) + ArcTan[(1 + 4*x)/Sqrt[7]]/
Sqrt[7]
```

Rubi in Sympy [A] time = 8.45441, size = 42, normalized size = 1.11

$$\frac{\sqrt{7}\,\mathrm{atan}\left(\sqrt{7}\left(\frac{4x}{7}-\frac{1}{7}\right)\right)}{7}+\frac{\sqrt{7}\,\mathrm{atan}\left(\sqrt{7}\left(\frac{4x}{7}+\frac{1}{7}\right)\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+1)/(4*x**4+3*x**2+1),x)
```

```
[Out] sqrt(7)*atan(sqrt(7)*(4*x/7 - 1/7))/7 + sqrt(7)*atan(sqrt(7)*(4*x
/7 + 1/7))/7
```

Mathematica [C] time = 0.312761, size = 97, normalized size = 2.55

$$\frac{\left(\sqrt{7}-i\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(3-i\sqrt{7}\right)}}\right)}{\sqrt{42-14i\sqrt{7}}}+\frac{\left(\sqrt{7}+i\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(3+i\sqrt{7}\right)}}\right)}{\sqrt{42+14i\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

```
[Out] ((-I + Sqrt[7])*ArcTan[(2*x)/Sqrt[(3 - I*Sqrt[7])/2]])/Sqrt[42 -
(14*I)*Sqrt[7]] + ((I + Sqrt[7])*ArcTan[(2*x)/Sqrt[(3 + I*Sqrt[7]
)/2]])/Sqrt[42 + (14*I)*Sqrt[7]]
```

Maple [A] time = 0.01, size = 34, normalized size = 0.9

$$\frac{\sqrt{7}}{7}\arctan\left(\frac{(4x-1)\sqrt{7}}{7}\right)+\frac{\sqrt{7}}{7}\arctan\left(\frac{(1+4x)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+1)/(4*x^4+3*x^2+1),x)
```

```
[Out] 1/7*7^(1/2)*arctan(1/7*(4*x-1)*7^(1/2))+1/7*arctan(1/7*(1+4*x)*7^
(1/2))*7^(1/2)
```

Maxima [A] time = 0.827622, size = 45, normalized size = 1.18

$$\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x+1)\right)+\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 + 3*x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 1)) + 1/7*sqrt(7)*arctan(1/
7*sqrt(7)*(4*x - 1))
```

Fricas [A] time = 0.281389, size = 38, normalized size = 1.

$$\frac{1}{7}\sqrt{7}\left(\arctan\left(\frac{1}{7}\sqrt{7}\left(4x^3+5x\right)\right)+\arctan\left(\frac{2}{7}\sqrt{7}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 + 3*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/7*sqrt(7)*(arctan(1/7*sqrt(7)*(4*x^3 + 5*x)) + arctan(2/7*sqrt(
7)*x))
```

Sympy [A] time = 0.250976, size = 44, normalized size = 1.16

$$\frac{\sqrt{7}\left(2\operatorname{atan}\left(\frac{2\sqrt{7}x}{7}\right)+2\operatorname{atan}\left(\frac{4\sqrt{7}x^3}{7}+\frac{5\sqrt{7}x}{7}\right)\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+1)/(4*x**4+3*x**2+1),x)
```

```
[Out] sqrt(7)*(2*atan(2*sqrt(7)*x/7) + 2*atan(4*sqrt(7)*x**3/7 + 5*sqrt
(7)*x/7))/14
```

GIAC/XCAS [A] time = 0.269562, size = 45, normalized size = 1.18

$$\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x+1)\right)+\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 + 3*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 1)) + 1/7*sqrt(7)*arctan(1/
7*sqrt(7)*(4*x - 1))
```

3.53 $\int \frac{1+2x^2}{1+2x^2+4x^4} \, dx$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{2\sqrt{2}x+1}{\sqrt{3}}\right)}{\sqrt{6}}-\frac{\tan^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

```
[Out] -(ArcTan[(1 - 2*Sqrt[2]*x)/Sqrt[3]]/Sqrt[6]) + ArcTan[(1 + 2*Sqrt
[2]*x)/Sqrt[3]]/Sqrt[6]
```

Rubi [A] time = 0.079638, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{\tan^{-1}\left(\frac{2\sqrt{2}x+1}{\sqrt{3}}\right)}{\sqrt{6}}-\frac{\tan^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```


```
[Out] -(ArcTan[(1 - 2*Sqrt[2]*x)/Sqrt[3]]/Sqrt[6]) + ArcTan[(1 + 2*Sqrt
[2]*x)/Sqrt[3]]/Sqrt[6]
```

Rubi in Sympy [A] time = 9.49795, size = 49, normalized size = 1.02

$$\frac{\sqrt{6}\operatorname{atan}\left(\sqrt{6}\left(\frac{2x}{3}-\frac{\sqrt{2}}{6}\right)\right)}{6}+\frac{\sqrt{6}\operatorname{atan}\left(\sqrt{6}\left(\frac{2x}{3}+\frac{\sqrt{2}}{6}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+1)/(4*x**4+2*x**2+1),x)
```

```
[Out] sqrt(6)*atan(sqrt(6)*(2*x/3 - sqrt(2)/6))/6 + sqrt(6)*atan(sqrt(6
)*(2*x/3 + sqrt(2)/6))/6
```

Mathematica [C] time = 0.164327, size = 99, normalized size = 2.06

$$\frac{\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{2x}{\sqrt{1-i\sqrt{3}}}\right)}{2\sqrt{3\left(1-i\sqrt{3}\right)}}+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{2x}{\sqrt{1+i\sqrt{3}}}\right)}{2\sqrt{3\left(1+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

```
[Out] ((-I + Sqrt[3])*ArcTan[(2*x)/Sqrt[1 - I*Sqrt[3]]])/(2*Sqrt[3*(1 -
 I*Sqrt[3])]) + ((I + Sqrt[3])*ArcTan[(2*x)/Sqrt[1 + I*Sqrt[3]]])
/(2*Sqrt[3*(1 + I*Sqrt[3])])
```

Maple [A] time = 0.036, size = 40, normalized size = 0.8

$$\frac{\sqrt{6}}{6}\arctan\left(\frac{\left(4\,x-\sqrt{2}\right)\sqrt{6}}{6}\right)+\frac{\sqrt{6}}{6}\arctan\left(\frac{\left(4\,x+\sqrt{2}\right)\sqrt{6}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+1)/(4*x^4+2*x^2+1),x)
```

```
[Out] 1/6*6^(1/2)*arctan(1/6*(4*x-2^(1/2))*6^(1/2))+1/6*6^(1/2)*arctan(
1/6*(4*x+2^(1/2))*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^2+1}{4\,x^4+2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 + 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + 2*x^2 + 1), x)
```

Fricas [A] time = 0.280466, size = 32, normalized size = 0.67

$$\frac{1}{6}\,\sqrt{6}\left(\arctan\left(\frac{2}{3}\,\sqrt{6}\left(x^3+x\right)\right)+\arctan\left(\frac{1}{3}\,\sqrt{6}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 + 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*(arctan(2/3*sqrt(6)*(x^3 + x)) + arctan(1/3*sqrt(6)*x
))
```

Sympy [A] time = 0.245157, size = 42, normalized size = 0.88

$$\frac{\sqrt{6}\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}\right)+2\operatorname{atan}\left(\frac{2\sqrt{6}x^3}{3}+\frac{2\sqrt{6}x}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+1)/(4*x**4+2*x**2+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3) + 2*atan(2*sqrt(6)*x**3/3 + 2*sqrt(6
)*x/3))/12
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^2+1}{4\,x^4+2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 + 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + 2*x^2 + 1), x)
```

3.54 $\int \frac{1+2x^2}{1+x^2+4x^4} \, dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{3}}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] -(ArcTan[(Sqrt[3] - 4*x)/Sqrt[5]]/Sqrt[5]) + ArcTan[(Sqrt[3] + 4*
x)/Sqrt[5]]/Sqrt[5]
```

Rubi [A] time = 0.0788841, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{3}}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^2)/(1 + x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[3] - 4*x)/Sqrt[5]]/Sqrt[5]) + ArcTan[(Sqrt[3] + 4*
x)/Sqrt[5]]/Sqrt[5]
```

Rubi in Sympy [A] time = 8.5618, size = 49, normalized size = 1.07

$$\frac{\sqrt{5}\,\text{atan}\left(\sqrt{5}\left(\frac{4x}{5}-\frac{\sqrt{3}}{5}\right)\right)}{5}+\frac{\sqrt{5}\,\text{atan}\left(\sqrt{5}\left(\frac{4x}{5}+\frac{\sqrt{3}}{5}\right)\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+1)/(4*x**4+x**2+1),x)
```

```
[Out] sqrt(5)*atan(sqrt(5)*(4*x/5 - sqrt(3)/5))/5 + sqrt(5)*atan(sqrt(5
)*(4*x/5 + sqrt(3)/5))/5
```

Mathematica [C] time = 0.383883, size = 97, normalized size = 2.11

$$\frac{\left(\sqrt{15}-3i\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{15}\right)}}\right)}{\sqrt{30-30i\sqrt{15}}}+\frac{\left(\sqrt{15}+3i\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{15}\right)}}\right)}{\sqrt{30+30i\sqrt{15}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2)/(1 + x^2 + 4*x^4),x]
```

```
[Out] ((-3*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(1 - I*Sqrt[15])/2]])/Sqrt[3
0 - (30*I)*Sqrt[15]] + ((3*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(1 + I
*Sqrt[15])/2]])/Sqrt[30 + (30*I)*Sqrt[15]]
```

Maple [A] time = 0.034, size = 40, normalized size = 0.9

$$\frac{\sqrt{5}}{5}\arctan\left(\frac{\left(4x+\sqrt{3}\right)\sqrt{5}}{5}\right)+\frac{\sqrt{5}}{5}\arctan\left(\frac{\left(4x-\sqrt{3}\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((2*x^2+1)/(4*x^4+x^2+1),x)
```

```
[Out] 1/5*arctan(1/5*(4*x+3^(1/2))*5^(1/2))*5^(1/2)+1/5*5^(1/2)*arctan(
1/5*(4*x-3^(1/2))*5^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4+x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*x^2 + 1)/(4*x^4 + x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + x^2 + 1), x)
```

Fricas [A] time = 0.270759, size = 38, normalized size = 0.83

$$\frac{1}{5}\sqrt{5}\left(\arctan\left(\frac{1}{5}\sqrt{5}\left(4x^3+3x\right)\right)+\arctan\left(\frac{2}{5}\sqrt{5}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*x^2 + 1)/(4*x^4 + x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*(arctan(1/5*sqrt(5)*(4*x^3 + 3*x)) + arctan(2/5*sqrt(
5)*x))
```

Sympy [A] time = 0.247556, size = 44, normalized size = 0.96

$$\frac{\sqrt{5}\left(2\operatorname{atan}\left(\frac{2\sqrt{5}x}{5}\right)+2\operatorname{atan}\left(\frac{4\sqrt{5}x^3}{5}+\frac{3\sqrt{5}x}{5}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*x**2+1)/(4*x**4+x**2+1),x)
```

```
[Out] sqrt(5)*(2*atan(2*sqrt(5)*x/5) + 2*atan(4*sqrt(5)*x**3/5 + 3*sqrt
(5)*x/5))/10
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4+x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 + x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + x^2 + 1), x)
```

3.55 $\int \frac{1+2x^2}{1+4x^4} \, dx$

Optimal. Leaf size=21

$$\frac{1}{2}\tan^{-1}(2x+1) - \frac{1}{2}\tan^{-1}(1-2x)$$

```
[Out] -ArcTan[1 - 2*x]/2 + ArcTan[1 + 2*x]/2
```

Rubi [A] time = 0.0294308, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\frac{1}{2}\tan^{-1}(2x+1) - \frac{1}{2}\tan^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^2)/(1 + 4*x^4),x]

[Out] -ArcTan[1 - 2*x]/2 + ArcTan[1 + 2*x]/2
```

Rubi in Sympy [A] time = 5.03569, size = 15, normalized size = 0.71

$$\frac{\operatorname{atan}(2x-1)}{2} + \frac{\operatorname{atan}(2x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+1)/(4*x**4+1),x)

[Out] atan(2*x - 1)/2 + atan(2*x + 1)/2
```

Mathematica [A] time = 0.00967117, size = 17, normalized size = 0.81

$$-\frac{1}{2}\tan^{-1}\left(\frac{2x}{2x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2)/(1 + 4*x^4),x]

[Out] -ArcTan[(2*x)/(-1 + 2*x^2)]/2
```

Maple [A] time = 0.007, size = 18, normalized size = 0.9

$$\frac{\arctan(2x-1)}{2} + \frac{\arctan(1+2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+1)/(4*x^4+1),x)
```

```
[Out] 1/2*arctan(2*x-1)+1/2*arctan(1+2*x)
```

Maxima [A] time = 0.807812, size = 23, normalized size = 1.1

$$\frac{1}{2}\arctan\left(2x+1\right)+\frac{1}{2}\arctan\left(2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(2*x + 1) + 1/2*arctan(2*x - 1)
```

Fricas [A] time = 0.282668, size = 20, normalized size = 0.95

$$\frac{1}{2}\arctan\left(2x^{3}+x\right)+\frac{1}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(2*x^3 + x) + 1/2*arctan(x)
```

Sympy [A] time = 0.190449, size = 14, normalized size = 0.67

$$\frac{\operatorname{atan}\left(x\right)}{2}+\frac{\operatorname{atan}\left(2x^{3}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+1)/(4*x**4+1),x)
```

```
[Out] atan(x)/2 + atan(2*x**3 + x)/2
```

GIAC/XCAS [A] time = 0.269835, size = 62, normalized size = 2.95

$$\frac{1}{2}\arctan\left(2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{2}\arctan\left(2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(2*sqrt(2)*(1/4)^(3/4)*(2*x + sqrt(2)*(1/4)^(1/4))) + 1
/2*arctan(2*sqrt(2)*(1/4)^(3/4)*(2*x - sqrt(2)*(1/4)^(1/4)))
```

3.56 $\int \frac{1+2x^2}{1-x^2+4x^4}\,dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{5}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(Sqrt[5] - 4*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(Sqrt[5] + 4*
x)/Sqrt[3]]/Sqrt[3]
```

Rubi [A] time = 0.0779575, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{5}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^2)/(1 - x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[5] - 4*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(Sqrt[5] + 4*
x)/Sqrt[3]]/Sqrt[3]
```

Rubi in Sympy [A] time = 8.74153, size = 49, normalized size = 1.07

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{4x}{3}-\frac{\sqrt{5}}{3}\right)\right)}{3}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{4x}{3}+\frac{\sqrt{5}}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+1)/(4*x**4-x**2+1),x)
```

```
[Out] sqrt(3)*atan(sqrt(3)*(4*x/3 - sqrt(5)/3))/3 + sqrt(3)*atan(sqrt(3
)*(4*x/3 + sqrt(5)/3))/3
```

Mathematica [C] time = 0.476232, size = 101, normalized size = 2.2

$$\frac{\left(\sqrt{15}-5i\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(-1-i\sqrt{15}\right)}}\right)}{\sqrt{30\left(-1-i\sqrt{15}\right)}}+\frac{\left(\sqrt{15}+5i\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(-1+i\sqrt{15}\right)}}\right)}{\sqrt{30\left(-1+i\sqrt{15}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2)/(1 - x^2 + 4*x^4),x]
```

```
[Out] ((-5*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(-1 - I*Sqrt[15])/2]])/Sqrt[
30*(-1 - I*Sqrt[15])] + ((5*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(-1 +
 I*Sqrt[15])/2]])/Sqrt[30*(-1 + I*Sqrt[15])]
```

Maple [A] time = 0.036, size = 40, normalized size = 0.9

$$\frac{\sqrt{3}}{3}\arctan\left(\frac{\left(4x+\sqrt{5}\right)\sqrt{3}}{3}\right)+\frac{\sqrt{3}}{3}\arctan\left(\frac{\left(4x-\sqrt{5}\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+1)/(4*x^4-x^2+1),x)
```

```
[Out] 1/3*arctan(1/3*(4*x+5^(1/2))*3^(1/2))*3^(1/2)+1/3*3^(1/2)*arctan(
1/3*(4*x-5^(1/2))*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - x^2 + 1), x)
```

Fricas [A] time = 0.272119, size = 35, normalized size = 0.76

$$\frac{1}{3}\sqrt{3}\left(\arctan\left(\frac{1}{3}\sqrt{3}\left(4x^3+x\right)\right)+\arctan\left(\frac{2}{3}\sqrt{3}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*(arctan(1/3*sqrt(3)*(4*x^3 + x)) + arctan(2/3*sqrt(3)
*x))
```

Sympy [A] time = 0.250869, size = 42, normalized size = 0.91

$$\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}\right)+2\,\mathrm{atan}\left(\frac{4\sqrt{3}x^3}{3}+\frac{\sqrt{3}x}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+1)/(4*x**4-x**2+1),x)
```

```
[Out] sqrt(3)*(2*atan(2*sqrt(3)*x/3) + 2*atan(4*sqrt(3)*x**3/3 + sqrt(3
)*x/3))/6
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - x^2 + 1), x)
```

3.57 $\int \frac{1+2x^2}{1-2x^2+4x^4} \, dx$

Optimal. Leaf size=44

$$\frac{\tan^{-1}\left(2\sqrt{2}x+\sqrt{3}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{3}-2\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[Sqrt[3] - 2*Sqrt[2]*x]/Sqrt[2]) + ArcTan[Sqrt[3] + 2*Sqr
t[2]*x]/Sqrt[2]
```

Rubi [A] time = 0.0680242, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{\tan^{-1}\left(2\sqrt{2}x+\sqrt{3}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{3}-2\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[Sqrt[3] - 2*Sqrt[2]*x]/Sqrt[2]) + ArcTan[Sqrt[3] + 2*Sqr
t[2]*x]/Sqrt[2]
```

Rubi in Sympy [A] time = 8.72689, size = 46, normalized size = 1.05

$$\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\left(2x-\frac{\sqrt{6}}{2}\right)\right)}{2}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\left(2x+\frac{\sqrt{6}}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+1)/(4*x**4-2*x**2+1),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*(2*x - sqrt(6)/2))/2 + sqrt(2)*atan(sqrt(2)*
(2*x + sqrt(6)/2))/2
```

Mathematica [C] time = 0.171382, size = 99, normalized size = 2.25

$$\frac{\left(\sqrt{3}-3i\right)\tan^{-1}\left(\frac{2x}{\sqrt{-1-i\sqrt{3}}}\right)}{2\sqrt{3\left(-1-i\sqrt{3}\right)}}+\frac{\left(\sqrt{3}+3i\right)\tan^{-1}\left(\frac{2x}{\sqrt{-1+i\sqrt{3}}}\right)}{2\sqrt{3\left(-1+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

```
[Out] ((-3*I + Sqrt[3])*ArcTan[(2*x)/Sqrt[-1 - I*Sqrt[3]]])/(2*Sqrt[3*(
-1 - I*Sqrt[3])]) + ((3*I + Sqrt[3])*ArcTan[(2*x)/Sqrt[-1 + I*Sqr
t[3]]])/(2*Sqrt[3*(-1 + I*Sqrt[3])])
```

Maple [A] time = 0.039, size = 40, normalized size = 0.9

$$\frac{\sqrt{2}}{2}\arctan\left(\frac{\left(4x-\sqrt{6}\right)\sqrt{2}}{2}\right)+\frac{\sqrt{2}}{2}\arctan\left(\frac{\left(4x+\sqrt{6}\right)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+1)/(4*x^4-2*x^2+1),x)
```

```
[Out] 1/2*2^(1/2)*arctan(1/2*(4*x-6^(1/2))*2^(1/2))+1/2*2^(1/2)*arctan(
1/2*(4*x+6^(1/2))*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4-2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - 2*x^2 + 1), x)
```

Fricas [A] time = 0.289893, size = 28, normalized size = 0.64

$$\frac{1}{2}\sqrt{2}\left(\arctan\left(2\sqrt{2}x^3\right)+\arctan\left(\sqrt{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*(arctan(2*sqrt(2)*x^3) + arctan(sqrt(2)*x))
```

Sympy [A] time = 0.232445, size = 29, normalized size = 0.66

$$\frac{\sqrt{2}\left(2\operatorname{atan}\left(\sqrt{2}x\right)+2\operatorname{atan}\left(2\sqrt{2}x^3\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+1)/(4*x**4-2*x**2+1),x)
```

```
[Out] sqrt(2)*(2*atan(sqrt(2)*x) + 2*atan(2*sqrt(2)*x**3))/4
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4-2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - 2*x^2 + 1),x, algorithm="giac")

[Out] integrate((2*x^2 + 1)/(4*x^4 - 2*x^2 + 1), x)
```

3.58 $\int \frac{1+2x^2}{1-3x^2+4x^4}\,dx$

Optimal. Leaf size=23

$$\tan^{-1}\left(4x+\sqrt{7}\right)-\tan^{-1}\left(\sqrt{7}-4x\right)$$

```
[Out] -ArcTan[Sqrt[7] - 4*x] + ArcTan[Sqrt[7] + 4*x]
```

Rubi [A] time = 0.0495436, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\tan^{-1}\left(4x+\sqrt{7}\right)-\tan^{-1}\left(\sqrt{7}-4x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^2)/(1 - 3*x^2 + 4*x^4),x]

[Out] -ArcTan[Sqrt[7] - 4*x] + ArcTan[Sqrt[7] + 4*x]
```

Rubi in Sympy [A] time = 8.34717, size = 19, normalized size = 0.83

$$\operatorname{atan}\left(4x-\sqrt{7}\right)+\operatorname{atan}\left(4x+\sqrt{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+1)/(4*x**4-3*x**2+1),x)

[Out] atan(4*x - sqrt(7)) + atan(4*x + sqrt(7))
```

Mathematica [A] time = 0.0112752, size = 14, normalized size = 0.61

$$-\tan^{-1}\left(\frac{x}{2x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2)/(1 - 3*x^2 + 4*x^4),x]

[Out] -ArcTan[x/(-1 + 2*x^2)]
```

Maple [A] time = 0.035, size = 20, normalized size = 0.9

$$\arctan\left(4x-\sqrt{7}\right)+\arctan\left(4x+\sqrt{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+1)/(4*x^4-3*x^2+1),x)

[Out] arctan(4*x-7^(1/2))+arctan(4*x+7^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^2+1}{4\,x^4-3\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - 3*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - 3*x^2 + 1), x)
```

Fricas [A] time = 0.303061, size = 20, normalized size = 0.87

$$\arctan\left(4\,x^3-x\right)+\arctan\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - 3*x^2 + 1),x, algorithm="fricas")
```

```
[Out] arctan(4*x^3 - x) + arctan(2*x)
```

Sympy [A] time = 0.211849, size = 12, normalized size = 0.52

$$\operatorname{atan}\left(2x\right)+\operatorname{atan}\left(4x^3-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+1)/(4*x**4-3*x**2+1),x)
```

```
[Out] atan(2*x) + atan(4*x**3 - x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^2+1}{4\,x^4-3\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - 3*x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - 3*x^2 + 1), x)
```

3.59 $\int \frac{1+2x^2}{1-4x^2+4x^4} \, dx$

Optimal. Leaf size=11

$$\frac{x}{1-2x^2}$$

```
[Out] x/(1 - 2*x^2)
```

Rubi [A] time = 0.00988683, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{x}{1-2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^2)/(1 - 4*x^2 + 4*x^4),x]

[Out] x/(1 - 2*x^2)
```

Rubi in Sympy [A] time = 6.04925, size = 8, normalized size = 0.73

$$\frac{2x}{-4x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+1)/(4*x**4-4*x**2+1),x)

[Out] 2*x/(-4*x**2 + 2)
```

Mathematica [A] time = 0.00833844, size = 12, normalized size = 1.09

$$-\frac{x}{2x^2-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2)/(1 - 4*x^2 + 4*x^4),x]

[Out] -(x/(-1 + 2*x^2))
```

Maple [A] time = 0.009, size = 11, normalized size = 1.

$$-\frac{x}{2}\left(x^2-\frac{1}{2}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+1)/(4*x^4-4*x^2+1),x)
```

```
[Out] -1/2*x/(x^2-1/2)
```

Maxima [A] time = 0.732339, size = 16, normalized size = 1.45

$$-\frac{x}{2\,x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - 4*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -x/(2*x^2 - 1)
```

Fricas [A] time = 0.291718, size = 16, normalized size = 1.45

$$-\frac{x}{2\,x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - 4*x^2 + 1),x, algorithm="fricas")
```

```
[Out] -x/(2*x^2 - 1)
```

Sympy [A] time = 0.15684, size = 8, normalized size = 0.73

$$-\frac{x}{2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+1)/(4*x**4-4*x**2+1),x)
```

```
[Out] -x/(2*x**2 - 1)
```

GIAC/XCAS [A] time = 0.269831, size = 16, normalized size = 1.45

$$-\frac{x}{2\,x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - 4*x^2 + 1),x, algorithm="giac")
```

```
[Out] -x/(2*x^2 - 1)
```

3.60 $\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{2}\log(1-2x)+\frac{1}{2}\log(1-x)-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(2x+1)$$

```
[Out] -Log[1 - 2*x]/2 + Log[1 - x]/2 - Log[1 + x]/2 + Log[1 + 2*x]/2
```

Rubi [A] time = 0.0409623, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{1}{2}\log(1-2x)+\frac{1}{2}\log(1-x)-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^2)/(1 - 5*x^2 + 4*x^4),x]

[Out] -Log[1 - 2*x]/2 + Log[1 - x]/2 - Log[1 + x]/2 + Log[1 + 2*x]/2
```

Rubi in Sympy [A] time = 8.97766, size = 29, normalized size = 0.74

$$-\frac{\log(-2x+1)}{2}+\frac{\log(-x+1)}{2}-\frac{\log(x+1)}{2}+\frac{\log(2x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+1)/(4*x**4-5*x**2+1),x)

[Out] -log(-2*x + 1)/2 + log(-x + 1)/2 - log(x + 1)/2 + log(2*x + 1)/2
```

Mathematica [A] time = 0.00995051, size = 29, normalized size = 0.74

$$\frac{1}{2}\log\left(-2x^2+x+1\right)-\frac{1}{2}\log\left(-2x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2)/(1 - 5*x^2 + 4*x^4),x]

[Out] -Log[1 - x - 2*x^2]/2 + Log[1 + x - 2*x^2]/2
```

Maple [A] time = 0.013, size = 30, normalized size = 0.8

$$\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}-\frac{\ln(2x-1)}{2}+\frac{\ln(1+2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+1)/(4*x^4-5*x^2+1),x)
```

```
[Out] 1/2*ln(-1+x)-1/2*ln(1+x)-1/2*ln(2*x-1)+1/2*ln(1+2*x)
```

Maxima [A] time = 0.738028, size = 39, normalized size = 1.

$$\frac{1}{2}\log(2x+1)-\frac{1}{2}\log(2x-1)-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - 5*x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*log(2*x + 1) - 1/2*log(2*x - 1) - 1/2*log(x + 1) + 1/2*log(x
- 1)
```

Fricas [A] time = 0.281927, size = 34, normalized size = 0.87

$$-\frac{1}{2}\log\left(2x^2+x-1\right)+\frac{1}{2}\log\left(2x^2-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - 5*x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*x^2 + x - 1) + 1/2*log(2*x^2 - x - 1)
```

Sympy [A] time = 0.206458, size = 26, normalized size = 0.67

$$\frac{\log\left(x^2-\frac{x}{2}-\frac{1}{2}\right)}{2}-\frac{\log\left(x^2+\frac{x}{2}-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+1)/(4*x**4-5*x**2+1),x)
```

```
[Out] log(x**2 - x/2 - 1/2)/2 - log(x**2 + x/2 - 1/2)/2
```

GIAC/XCAS [A] time = 0.271918, size = 45, normalized size = 1.15

$$\frac{1}{2}\ln(|2x+1|)-\frac{1}{2}\ln(|2x-1|)-\frac{1}{2}\ln(|x+1|)+\frac{1}{2}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - 5*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(2*x + 1)) - 1/2*ln(abs(2*x - 1)) - 1/2*ln(abs(x + 1))
+ 1/2*ln(abs(x - 1))
```

3.61 $\int \frac{1+2x^2}{1-6x^2+4x^4} \, dx$

Optimal. Leaf size=44

$$\frac{\tanh^{-1}\left(\sqrt{5}-2\sqrt{2}x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(2\sqrt{2}x+\sqrt{5}\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[5] - 2*Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[5] + 2*Sqrt
[2]*x]/Sqrt[2]
```

Rubi [A] time = 0.0719869, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{\tanh^{-1}\left(\sqrt{5}-2\sqrt{2}x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(2\sqrt{2}x+\sqrt{5}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
```

```
[Out] ArcTanh[Sqrt[5] - 2*Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[5] + 2*Sqrt
[2]*x]/Sqrt[2]
```

Rubi in Sympy [A] time = 8.8038, size = 48, normalized size = 1.09

$$-\frac{\sqrt{2}\,\text{atanh}\left(\sqrt{2}\left(2x-\frac{\sqrt{10}}{2}\right)\right)}{2}-\frac{\sqrt{2}\,\text{atanh}\left(\sqrt{2}\left(2x+\frac{\sqrt{10}}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+1)/(4*x**4-6*x**2+1),x)
```

```
[Out] -sqrt(2)*atanh(sqrt(2)*(2*x - sqrt(10)/2))/2 - sqrt(2)*atanh(sqrt
(2)*(2*x + sqrt(10)/2))/2
```

Mathematica [A] time = 0.020653, size = 42, normalized size = 0.95

$$\frac{\log\left(-2x^2+\sqrt{2}x+1\right)-\log\left(2x^2+\sqrt{2}x-1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
```

```
[Out] (Log[1 + Sqrt[2]*x - 2*x^2] - Log[-1 + Sqrt[2]*x + 2*x^2])/(2*Sqr
t[2])
```

Maple [B] time = 0.048, size = 82, normalized size = 1.9

$$-\frac{\left(10+2\sqrt{5}\right)\sqrt{5}}{10\sqrt{10}+10\sqrt{2}}\mathit{Artanh}\left(8\frac{x}{2\sqrt{10}+2\sqrt{2}}\right)-\frac{2\sqrt{5}\left(-5+\sqrt{5}\right)}{10\sqrt{10}-10\sqrt{2}}\mathit{Artanh}\left(8\frac{x}{2\sqrt{10}-2\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((2*x^2+1)/(4*x^4-6*x^2+1),x)
```

```
[Out] -2/5*(5+5^(1/2))*5^(1/2)/(2*10^(1/2)+2*2^(1/2))*arctanh(8*x/(2*10
^(1/2)+2*2^(1/2)))-2/5*5^(1/2)*(-5+5^(1/2))/(2*10^(1/2)-2*2^(1/2)
)*arctanh(8*x/(2*10^(1/2)-2*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4-6x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*x^2 + 1)/(4*x^4 - 6*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - 6*x^2 + 1), x)
```

Fricas [A] time = 0.27736, size = 65, normalized size = 1.48

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{8x^3-\sqrt{2}\left(4x^4-2x^2+1\right)-4x}{4x^4-6x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*x^2 + 1)/(4*x^4 - 6*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(8*x^3 - sqrt(2)*(4*x^4 - 2*x^2 + 1) - 4*x)/(4*x
^4 - 6*x^2 + 1))
```

Sympy [A] time = 0.207952, size = 46, normalized size = 1.05

$$\frac{\sqrt{2}\log\left(x^2-\frac{\sqrt{2}x}{2}-\frac{1}{2}\right)}{4}-\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{2}-\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*x**2+1)/(4*x**4-6*x**2+1),x)
```

```
[Out] sqrt(2)*log(x**2 - sqrt(2)*x/2 - 1/2)/4 - sqrt(2)*log(x**2 + sqrt
(2)*x/2 - 1/2)/4
```

GIAC/XCAS [A] time = 0.308264, size = 104, normalized size = 2.36

$$-\frac{1}{4}\sqrt{2}\ln\left(\left|x+\frac{1}{4}\sqrt{10}+\frac{1}{4}\sqrt{2}\right|\right)+\frac{1}{4}\sqrt{2}\ln\left(\left|x+\frac{1}{4}\sqrt{10}-\frac{1}{4}\sqrt{2}\right|\right)-\frac{1}{4}\sqrt{2}\ln\left(\left|x-\frac{1}{4}\sqrt{10}+\frac{1}{4}\sqrt{2}\right|\right)+\frac{1}{4}\sqrt{2}\ln\left(\left|x-\frac{1}{4}\sqrt{10}-\frac{1}{4}\sqrt{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/(4*x^4 - 6*x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*ln(abs(x + 1/4*sqrt(10) + 1/4*sqrt(2))) + 1/4*sqrt(2
)*ln(abs(x + 1/4*sqrt(10) - 1/4*sqrt(2))) - 1/4*sqrt(2)*ln(abs(x
- 1/4*sqrt(10) + 1/4*sqrt(2))) + 1/4*sqrt(2)*ln(abs(x - 1/4*sqrt(
10) - 1/4*sqrt(2)))
```

3.62 $\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx$

Optimal. Leaf size=66

$$\frac{\log\left(\sqrt{4-b}x+2x^2+1\right)}{2\sqrt{4-b}}-\frac{\log\left(-\sqrt{4-b}x+2x^2+1\right)}{2\sqrt{4-b}}$$

```
[Out] -Log[1 - Sqrt[4 - b]*x + 2*x^2]/(2*Sqrt[4 - b]) + Log[1 + Sqrt[4
- b]*x + 2*x^2]/(2*Sqrt[4 - b])
```

Rubi [A] time = 0.0629816, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\log\left(\sqrt{4-b}x+2x^2+1\right)}{2\sqrt{4-b}}-\frac{\log\left(-\sqrt{4-b}x+2x^2+1\right)}{2\sqrt{4-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - Sqrt[4 - b]*x + 2*x^2]/(2*Sqrt[4 - b]) + Log[1 + Sqrt[4
- b]*x + 2*x^2]/(2*Sqrt[4 - b])
```

Rubi in Sympy [A] time = 16.1353, size = 53, normalized size = 0.8

$$-\frac{\log\left(x^2-\frac{x\sqrt{-b+4}}{2}+\frac{1}{2}\right)}{2\sqrt{-b+4}}+\frac{\log\left(x^2+\frac{x\sqrt{-b+4}}{2}+\frac{1}{2}\right)}{2\sqrt{-b+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2*x**2+1)/(4*x**4+b*x**2+1),x)
```

```
[Out] -log(x**2 - x*sqrt(-b + 4)/2 + 1/2)/(2*sqrt(-b + 4)) + log(x**2 +
 x*sqrt(-b + 4)/2 + 1/2)/(2*sqrt(-b + 4))
```

Mathematica [A] time = 0.122538, size = 127, normalized size = 1.92

$$\frac{\frac{\left(-\sqrt{b^2-16}+b+4\right)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{b-\sqrt{b^2-16}}}\right)}{\sqrt{b-\sqrt{b^2-16}}}-\frac{\left(\sqrt{b^2-16}+b+4\right)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{\sqrt{b^2-16}+b}}\right)}{\sqrt{\sqrt{b^2-16}+b}}}{\sqrt{2}\sqrt{b^2-16}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

```
[Out] (((4 + b - Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[b - Sqrt[-1
6 + b^2]]])/Sqrt[b - Sqrt[-16 + b^2]] - ((4 + b + Sqrt[-16 + b^2]
)*ArcTan[(2*Sqrt[2]*x)/Sqrt[b + Sqrt[-16 + b^2]]])/Sqrt[b + Sqrt[
-16 + b^2]])/(Sqrt[2]*Sqrt[-16 + b^2])
```

Maple [B] time = 0.022, size = 279, normalized size = 4.2

$$-4\frac{1}{\sqrt{(b-4)(4+b)}\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\arctan\left(4\frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\right)$$
$$-1\arctan\left(4\frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\right)\frac{1}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}$$
$$-b\arctan\left(4\frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\right)\frac{1}{\sqrt{(b-4)(4+b)}}\frac{1}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}$$
$$+4\frac{1}{\sqrt{(b-4)(4+b)}\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}\arctan\left(4\frac{x}{\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}\right)$$
$$-1\arctan\left(4\frac{x}{\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}\right)\frac{1}{\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}$$
$$+b\arctan\left(4\frac{x}{\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}\right)\frac{1}{\sqrt{(b-4)(4+b)}}\frac{1}{\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((-2*x^2+1)/(4*x^4+b*x^2+1),x)
```

```
[Out] -4/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4
*x/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))-1/(2*((b-4)*(4+b))^(1/2)+2*
b)^(1/2)*arctan(4*x/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))-1/((b-4)*(
4+b))^(1/2)/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(2*((b-4
)*(4+b))^(1/2)+2*b)^(1/2))*b+4/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+
b))^(1/2)+2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2
))-1/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+
b))^(1/2)+2*b)^(1/2))+1/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+b))^(1/
2)+2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x^2-1}{4x^4+bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(2*x^2 - 1)/(4*x^4 + b*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 + b*x^2 + 1), x)
```

Fricas [A] time = 0.290522, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{4(b-4)x^3+2(b-4)x-(4x^4-(b-8)x^2+1)\sqrt{-b+4}}{4x^4+bx^2+1}\right)}{2\sqrt{-b+4}}, \frac{\arctan\left(\frac{4x^3+(b-2)x}{\sqrt{b-4}}\right)-\arctan\left(\frac{2x}{\sqrt{b-4}}\right)}{\sqrt{b-4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 + b*x^2 + 1),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(4*(b - 4)*x^3 + 2*(b - 4)*x - (4*x^4 - (b - 8)*x^2 + 1
)*sqrt(-b + 4))/(4*x^4 + b*x^2 + 1))/sqrt(-b + 4), (arctan((4*x^3
 + (b - 2)*x)/sqrt(b - 4)) - arctan(2*x/sqrt(b - 4)))/sqrt(b - 4)
]
```

Sympy [A] time = 0.783422, size = 94, normalized size = 1.42

$$\frac{\sqrt{-\frac{1}{b-4}}\log\left(x^{2}+x\left(-\frac{b\sqrt{-\frac{1}{b-4}}}{2}+2\sqrt{-\frac{1}{b-4}}\right)+\frac{1}{2}\right)}{2}-\frac{\sqrt{-\frac{1}{b-4}}\log\left(x^{2}+x\left(\frac{b\sqrt{-\frac{1}{b-4}}}{2}-2\sqrt{-\frac{1}{b-4}}\right)+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x**2+1)/(4*x**4+b*x**2+1),x)
```

```
[Out] sqrt(-1/(b - 4))*log(x**2 + x*(-b*sqrt(-1/(b - 4))/2 + 2*sqrt(-1/
(b - 4))) + 1/2)/2 - sqrt(-1/(b - 4))*log(x**2 + x*(b*sqrt(-1/(b
- 4))/2 - 2*sqrt(-1/(b - 4))) + 1/2)/2
```

GIAC/XCAS [A] time = 0.326921, size = 1, normalized size = 0.02

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 + b*x^2 + 1),x, algorithm="giac")
```

```
[Out] Done
```

3.63 $\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}$$

```
[Out] ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[2] - ArcTan[(2*x)/Sqrt[3 + S
qrt[5]]]/Sqrt[2]
```

Rubi [A] time = 0.0741989, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[2] - ArcTan[(2*x)/Sqrt[3 + S
qrt[5]]]/Sqrt[2]
```

Rubi in Sympy [A] time = 9.42177, size = 68, normalized size = 1.48

$$-\frac{\left(-\sqrt{5}+1\right)\operatorname{atan}\left(\frac{2x}{\sqrt{-\sqrt{5}+3}}\right)}{2\sqrt{-\sqrt{5}+3}}-\frac{\left(1+\sqrt{5}\right)\operatorname{atan}\left(\frac{2x}{\sqrt{\sqrt{5}+3}}\right)}{2\sqrt{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2*x**2+1)/(4*x**4+6*x**2+1),x)
```

```
[Out] -(-sqrt(5) + 1)*atan(2*x/sqrt(-sqrt(5) + 3))/(2*sqrt(-sqrt(5) + 3
)) - (1 + sqrt(5))*atan(2*x/sqrt(sqrt(5) + 3))/(2*sqrt(sqrt(5) +
3))
```

Mathematica [A] time = 0.122399, size = 84, normalized size = 1.83

$$\frac{-\left(\sqrt{5}-5\right)\sqrt{3+\sqrt{5}}\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)-\sqrt{3-\sqrt{5}}\left(5+\sqrt{5}\right)\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

```
[Out] (-((-5 + Sqrt[5])*Sqrt[3 + Sqrt[5]]*ArcTan[(2*x)/Sqrt[3 - Sqrt[5]
]]) - Sqrt[3 - Sqrt[5]]*(5 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 + Sqrt[
5]]])/(4*Sqrt[5])
```

Maple [B] time = 0.019, size = 136, normalized size = 3.

$$2\frac{\sqrt{5}}{2\sqrt{10}-2\sqrt{2}}\arctan\left(8\frac{x}{2\sqrt{10}-2\sqrt{2}}\right)-2\frac{1}{2\sqrt{10}-2\sqrt{2}}\arctan\left(8\frac{x}{2\sqrt{10}-2\sqrt{2}}\right)-2\frac{\sqrt{5}}{2\sqrt{10}+2\sqrt{2}}\arctan\left(8\frac{x}{2\sqrt{10}+2\sqrt{2}}\right)-2\frac{1}{2\sqrt{10}+2\sqrt{2}}\arctan\left(8\frac{x}{2\sqrt{10}+2\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((-2*x^2+1)/(4*x^4+6*x^2+1),x)
```

```
[Out] 2*5^(1/2)/(2*10^(1/2)-2*2^(1/2))*arctan(8*x/(2*10^(1/2)-2*2^(1/2)
))-2/(2*10^(1/2)-2*2^(1/2))*arctan(8*x/(2*10^(1/2)-2*2^(1/2)))-2*
5^(1/2)/(2*10^(1/2)+2*2^(1/2))*arctan(8*x/(2*10^(1/2)+2*2^(1/2)))
-2/(2*10^(1/2)+2*2^(1/2))*arctan(8*x/(2*10^(1/2)+2*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{2x^2-1}{4x^4+6x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(2*x^2 - 1)/(4*x^4 + 6*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 + 6*x^2 + 1), x)
```

Fricas [A] time = 0.275214, size = 34, normalized size = 0.74

$$\frac{1}{2}\sqrt{2}\left(\arctan\left(2\sqrt{2}(x^3+x)\right)-\arctan\left(\sqrt{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(2*x^2 - 1)/(4*x^4 + 6*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*(arctan(2*sqrt(2)*(x^3 + x)) - arctan(sqrt(2)*x))
```

Sympy [A] time = 0.246948, size = 39, normalized size = 0.85

$$-\frac{\sqrt{2}\left(2\operatorname{atan}\left(\sqrt{2}x\right)-2\operatorname{atan}\left(2\sqrt{2}x^3+2\sqrt{2}x\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((-2*x**2+1)/(4*x**4+6*x**2+1),x)
```

```
[Out] -sqrt(2)*(2*atan(sqrt(2)*x) - 2*atan(2*sqrt(2)*x**3 + 2*sqrt(2)*x
))/4
```

GIAC/XCAS [A] time = 0.27799, size = 53, normalized size = 1.15

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{4x}{\sqrt{10}+\sqrt{2}}\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{4x}{\sqrt{10}-\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 + 6*x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(4*x/(sqrt(10) + sqrt(2))) + 1/2*sqrt(2)*arcta
n(4*x/(sqrt(10) - sqrt(2)))
```

3.64 $\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx$

Optimal. Leaf size=9

$$\tan^{-1}(2x) - \tan^{-1}(x)$$

```
[Out] -ArcTan[x] + ArcTan[2*x]
```

Rubi [A] time = 0.0225844, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\tan^{-1}(2x) - \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 5*x^2 + 4*x^4),x]

[Out] -ArcTan[x] + ArcTan[2*x]
```

Rubi in Sympy [A] time = 7.9074, size = 7, normalized size = 0.78

$$-\operatorname{atan}(x) + \operatorname{atan}(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-2*x**2+1)/(4*x**4+5*x**2+1),x)

[Out] -atan(x) + atan(2*x)
```

Mathematica [A] time = 0.0114487, size = 12, normalized size = 1.33

$$\tan^{-1}\left(\frac{x}{2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 5*x^2 + 4*x^4),x]

[Out] ArcTan[x/(1 + 2*x^2)]
```

Maple [A] time = 0.009, size = 10, normalized size = 1.1

$$-\arctan(x) + \arctan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+5*x^2+1),x)

[Out] -arctan(x)+arctan(2*x)
```

Maxima [A] time = 0.840871, size = 12, normalized size = 1.33

$$\arctan(2x) - \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 + 5*x^2 + 1),x, algorithm="maxima")
```

```
[Out] arctan(2*x) - arctan(x)
```

Fricas [A] time = 0.31455, size = 23, normalized size = 2.56

$$\arctan\left(4x^3 + 3x\right) - \arctan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 + 5*x^2 + 1),x, algorithm="fricas")
```

```
[Out] arctan(4*x^3 + 3*x) - arctan(2*x)
```

Sympy [A] time = 0.214699, size = 14, normalized size = 1.56

$$-\operatorname{atan}(2x) + \operatorname{atan}\left(4x^3 + 3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x**2+1)/(4*x**4+5*x**2+1),x)
```

```
[Out] -atan(2*x) + atan(4*x**3 + 3*x)
```

GIAC/XCAS [A] time = 0.268504, size = 12, normalized size = 1.33

$$\arctan(2x) - \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 + 5*x^2 + 1),x, algorithm="giac")
```

```
[Out] arctan(2*x) - arctan(x)
```

3.65 $\int \frac{1-2x^2}{1+4x^2+4x^4} \, dx$

Optimal. Leaf size=11

$$\frac{x}{2x^2+1}$$

```
[Out] x/(1 + 2*x^2)
```

Rubi [A] time = 0.0103355, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{x}{2x^2+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

```
[Out] x/(1 + 2*x^2)
```

Rubi in Sympy [A] time = 6.09194, size = 8, normalized size = 0.73

$$\frac{2x}{4x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2*x**2+1)/(4*x**4+4*x**2+1),x)
```

```
[Out] 2*x/(4*x**2 + 2)
```

Mathematica [A] time = 0.00683356, size = 11, normalized size = 1.

$$\frac{x}{2x^2+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

```
[Out] x/(1 + 2*x^2)
```

Maple [A] time = 0.008, size = 11, normalized size = 1.

$$\frac{x}{2}\left(x^2+\frac{1}{2}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2*x^2+1)/(4*x^4+4*x^2+1),x)
```

```
[Out] 1/2*x/(x^2+1/2)
```

Maxima [A] time = 0.73177, size = 15, normalized size = 1.36

$$\frac{x}{2\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 + 4*x^2 + 1),x, algorithm="maxima")
```

```
[Out] x/(2*x^2 + 1)
```

Fricas [A] time = 0.274797, size = 15, normalized size = 1.36

$$\frac{x}{2\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 + 4*x^2 + 1),x, algorithm="fricas")
```

```
[Out] x/(2*x^2 + 1)
```

Sympy [A] time = 0.154271, size = 7, normalized size = 0.64

$$\frac{x}{2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x**2+1)/(4*x**4+4*x**2+1),x)
```

```
[Out] x/(2*x**2 + 1)
```

GIAC/XCAS [A] time = 0.27068, size = 15, normalized size = 1.36

$$\frac{x}{2\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 + 4*x^2 + 1),x, algorithm="giac")
```

```
[Out] x/(2*x^2 + 1)
```

3.66 $\int \frac{1-2x^2}{1+3x^2+4x^4} \, dx$

Optimal. Leaf size=29

$$\frac{1}{2}\log\left(2x^2+x+1\right)-\frac{1}{2}\log\left(2x^2-x+1\right)$$

```
[Out] -Log[1 - x + 2*x^2]/2 + Log[1 + x + 2*x^2]/2
```

Rubi [A] time = 0.0326194, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{1}{2}\log\left(2x^2+x+1\right)-\frac{1}{2}\log\left(2x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 2*x^2)/(1 + 3*x^2 + 4*x^4),x]

[Out] -Log[1 - x + 2*x^2]/2 + Log[1 + x + 2*x^2]/2
```

Rubi in Sympy [A] time = 11.4104, size = 26, normalized size = 0.9

$$-\frac{\log\left(x^2-\frac{x}{2}+\frac{1}{2}\right)}{2}+\frac{\log\left(x^2+\frac{x}{2}+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2*x**2+1)/(4*x**4+3*x**2+1),x)

[Out] -log(x**2 - x/2 + 1/2)/2 + log(x**2 + x/2 + 1/2)/2
```

Mathematica [A] time = 0.00986284, size = 29, normalized size = 1.

$$\frac{1}{2}\log\left(2x^2+x+1\right)-\frac{1}{2}\log\left(2x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 2*x^2)/(1 + 3*x^2 + 4*x^4),x]

[Out] -Log[1 - x + 2*x^2]/2 + Log[1 + x + 2*x^2]/2
```

Maple [A] time = 0.005, size = 26, normalized size = 0.9

$$-\frac{\ln\left(2\,x^2-x+1\right)}{2}+\frac{\ln\left(2\,x^2+x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2*x^2+1)/(4*x^4+3*x^2+1),x)
```

```
[Out] -1/2*ln(2*x^2-x+1)+1/2*ln(2*x^2+x+1)
```

Maxima [A] time = 0.744593, size = 34, normalized size = 1.17

$$\frac{1}{2}\log\left(2x^2+x+1\right)-\frac{1}{2}\log\left(2x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 + 3*x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)
```

Fricas [A] time = 0.282175, size = 34, normalized size = 1.17

$$\frac{1}{2}\log\left(2x^2+x+1\right)-\frac{1}{2}\log\left(2x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 + 3*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)
```

Sympy [A] time = 0.19189, size = 26, normalized size = 0.9

$$-\frac{\log\left(x^2-\frac{x}{2}+\frac{1}{2}\right)}{2}+\frac{\log\left(x^2+\frac{x}{2}+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x**2+1)/(4*x**4+3*x**2+1),x)
```

```
[Out] -log(x**2 - x/2 + 1/2)/2 + log(x**2 + x/2 + 1/2)/2
```

GIAC/XCAS [A] time = 0.271278, size = 34, normalized size = 1.17

$$\frac{1}{2}\ln\left(2x^2+x+1\right)-\frac{1}{2}\ln\left(2x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 + 3*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*ln(2*x^2 + x + 1) - 1/2*ln(2*x^2 - x + 1)
```

3.67 $\int \frac{1-2x^2}{1+2x^2+4x^4}\, dx$

Optimal. Leaf size=50

$$\frac{\log\left(2x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(2x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}$$

```
[Out] -Log[1 - Sqrt[2]*x + 2*x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*x + 2*x
^2]/(2*Sqrt[2])
```

Rubi [A] time = 0.0480019, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\log\left(2x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(2x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - Sqrt[2]*x + 2*x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*x + 2*x
^2]/(2*Sqrt[2])
```

Rubi in Sympy [A] time = 13.5464, size = 46, normalized size = 0.92

$$-\frac{\sqrt{2}\log\left(x^2-\frac{\sqrt{2}x}{2}+\frac{1}{2}\right)}{4}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{2}+\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2*x**2+1)/(4*x**4+2*x**2+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x/2 + 1/2)/4 + sqrt(2)*log(x**2 + sqr
t(2)*x/2 + 1/2)/4
```

Mathematica [A] time = 0.0199631, size = 42, normalized size = 0.84

$$\frac{\log\left(2x^2+\sqrt{2}x+1\right)-\log\left(-2x^2+\sqrt{2}x-1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[2]*x - 2*x^2] + Log[1 + Sqrt[2]*x + 2*x^2])/(2*Sq
rt[2])
```

Maple [A] time = 0.014, size = 39, normalized size = 0.8

$$-\frac{\ln\left(1+2\,x^2-\sqrt{2}x\right)\sqrt{2}}{4}+\frac{\ln\left(1+2\,x^2+\sqrt{2}x\right)\sqrt{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((-2*x^2+1)/(4*x^4+2*x^2+1),x)
```

```
[Out] -1/4*ln(1+2*x^2-2^(1/2)*x)*2^(1/2)+1/4*ln(1+2*x^2+2^(1/2)*x)*2^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,x^2-1}{4\,x^4+2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(2*x^2 - 1)/(4*x^4 + 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 + 2*x^2 + 1), x)
```

Fricas [A] time = 0.280369, size = 62, normalized size = 1.24

$$\frac{1}{4}\,\sqrt{2}\log\left(\frac{8\,x^3+\sqrt{2}\left(4\,x^4+6\,x^2+1\right)+4\,x}{4\,x^4+2\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(2*x^2 - 1)/(4*x^4 + 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((8*x^3 + sqrt(2)*(4*x^4 + 6*x^2 + 1) + 4*x)/(4*x^
4 + 2*x^2 + 1))
```

Sympy [A] time = 0.199765, size = 46, normalized size = 0.92

$$-\frac{\sqrt{2}\log\left(x^2-\frac{\sqrt{2}x}{2}+\frac{1}{2}\right)}{4}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{2}+\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((-2*x**2+1)/(4*x**4+2*x**2+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x/2 + 1/2)/4 + sqrt(2)*log(x**2 + sqr
t(2)*x/2 + 1/2)/4
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,x^2-1}{4\,x^4+2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(2*x^2 - 1)/(4*x^4 + 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^2 - 1)/(4*x^4 + 2*x^2 + 1), x)
```

3.68 $\int \frac{1-2x^2}{1+x^2+4x^4}\,dx$

Optimal. Leaf size=50

$$\frac{\log\left(2x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(2x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}$$

```
[Out] -Log[1 - Sqrt[3]*x + 2*x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x + 2*x
^2]/(2*Sqrt[3])
```

Rubi [A] time = 0.0457355, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{\log\left(2x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(2x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - Sqrt[3]*x + 2*x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x + 2*x
^2]/(2*Sqrt[3])
```

Rubi in Sympy [A] time = 13.8858, size = 46, normalized size = 0.92

$$-\frac{\sqrt{3}\log\left(x^2-\frac{\sqrt{3}x}{2}+\frac{1}{2}\right)}{6}+\frac{\sqrt{3}\log\left(x^2+\frac{\sqrt{3}x}{2}+\frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-2*x**2+1)/(4*x**4+x**2+1),x)
```

```
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x/2 + 1/2)/6 + sqrt(3)*log(x**2 + sqr
t(3)*x/2 + 1/2)/6
```

Mathematica [A] time = 0.0221911, size = 42, normalized size = 0.84

$$\frac{\log\left(2x^2+\sqrt{3}x+1\right)-\log\left(-2x^2+\sqrt{3}x-1\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + x^2 + 4*x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[3]*x - 2*x^2] + Log[1 + Sqrt[3]*x + 2*x^2])/(2*Sq
rt[3])
```

Maple [A] time = 0.015, size = 39, normalized size = 0.8

$$-\frac{\ln\left(1+2\,x^2-x\sqrt{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+2\,x^2+x\sqrt{3}\right)\sqrt{3}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((-2*x^2+1)/(4*x^4+x^2+1),x)
```

```
[Out] -1/6*ln(1+2*x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+2*x^2+x*3^(1/2))*3^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x^2-1}{4x^4+x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(2*x^2 - 1)/(4*x^4 + x^2 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 + x^2 + 1), x)
```

Fricas [A] time = 0.278298, size = 59, normalized size = 1.18

$$\frac{1}{6}\sqrt{3}\log\left(\frac{12x^3+\sqrt{3}\left(4x^4+7x^2+1\right)+6x}{4x^4+x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(2*x^2 - 1)/(4*x^4 + x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((12*x^3 + sqrt(3)*(4*x^4 + 7*x^2 + 1) + 6*x)/(4*x
^4 + x^2 + 1))
```

Sympy [A] time = 0.200585, size = 46, normalized size = 0.92

$$-\frac{\sqrt{3}\log\left(x^2-\frac{\sqrt{3}x}{2}+\frac{1}{2}\right)}{6}+\frac{\sqrt{3}\log\left(x^2+\frac{\sqrt{3}x}{2}+\frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((-2*x**2+1)/(4*x**4+x**2+1),x)
```

```
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x/2 + 1/2)/6 + sqrt(3)*log(x**2 + sqr
t(3)*x/2 + 1/2)/6
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2x^2-1}{4x^4+x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(2*x^2 - 1)/(4*x^4 + x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^2 - 1)/(4*x^4 + x^2 + 1), x)
```

3.69 $\int \frac{1-2x^2}{1+4x^4} \, dx$

Optimal. Leaf size=31

$$\frac{1}{4}\log\left(2x^2+2x+1\right)-\frac{1}{4}\log\left(2x^2-2x+1\right)$$

```
[Out] -Log[1 - 2*x + 2*x^2]/4 + Log[1 + 2*x + 2*x^2]/4
```

Rubi [A] time = 0.0293277, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{1}{4}\log\left(2x^2+2x+1\right)-\frac{1}{4}\log\left(2x^2-2x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 2*x^2)/(1 + 4*x^4),x]

[Out] -Log[1 - 2*x + 2*x^2]/4 + Log[1 + 2*x + 2*x^2]/4
```

Rubi in Sympy [A] time = 9.23885, size = 26, normalized size = 0.84

$$-\frac{\log\left(2x^2-2x+1\right)}{4}+\frac{\log\left(2x^2+2x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2*x**2+1)/(4*x**4+1),x)

[Out] -log(2*x**2 - 2*x + 1)/4 + log(2*x**2 + 2*x + 1)/4
```

Mathematica [A] time = 0.00748568, size = 31, normalized size = 1.

$$\frac{1}{4}\log\left(2x^2+2x+1\right)-\frac{1}{4}\log\left(2x^2-2x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 2*x^2)/(1 + 4*x^4),x]

[Out] -Log[1 - 2*x + 2*x^2]/4 + Log[1 + 2*x + 2*x^2]/4
```

Maple [A] time = 0.005, size = 28, normalized size = 0.9

$$-\frac{\ln\left(2\,x^2-2\,x+1\right)}{4}+\frac{\ln\left(2\,x^2+2\,x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2*x^2+1)/(4*x^4+1),x)
```

```
[Out] -1/4*ln(2*x^2-2*x+1)+1/4*ln(2*x^2+2*x+1)
```

Maxima [A] time = 0.741158, size = 36, normalized size = 1.16

$$\frac{1}{4}\log\left(2x^2+2x+1\right)-\frac{1}{4}\log\left(2x^2-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/4*log(2*x^2 + 2*x + 1) - 1/4*log(2*x^2 - 2*x + 1)
```

Fricas [A] time = 0.279919, size = 36, normalized size = 1.16

$$\frac{1}{4}\log\left(2x^2+2x+1\right)-\frac{1}{4}\log\left(2x^2-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/4*log(2*x^2 + 2*x + 1) - 1/4*log(2*x^2 - 2*x + 1)
```

Sympy [A] time = 0.191743, size = 22, normalized size = 0.71

$$-\frac{\log\left(x^2-x+\frac{1}{2}\right)}{4}+\frac{\log\left(x^2+x+\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x**2+1)/(4*x**4+1),x)
```

```
[Out] -log(x**2 - x + 1/2)/4 + log(x**2 + x + 1/2)/4
```

GIAC/XCAS [A] time = 0.275379, size = 46, normalized size = 1.48

$$\frac{1}{4}\ln\left(x^2+\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{4}\ln\left(x^2-\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/4*ln(x^2 + sqrt(2)*(1/4)^(1/4)*x + 1/2) - 1/4*ln(x^2 - sqrt(2)*
(1/4)^(1/4)*x + 1/2)
```

3.70 $\int \frac{1-2x^2}{1-x^2+4x^4}\,dx$

Optimal. Leaf size=50

$$\frac{\log\left(2x^2+\sqrt{5}x+1\right)}{2\sqrt{5}}-\frac{\log\left(2x^2-\sqrt{5}x+1\right)}{2\sqrt{5}}$$

```
[Out] -Log[1 - Sqrt[5]*x + 2*x^2]/(2*Sqrt[5]) + Log[1 + Sqrt[5]*x + 2*x
^2]/(2*Sqrt[5])
```

Rubi [A] time = 0.0476567, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\log\left(2x^2+\sqrt{5}x+1\right)}{2\sqrt{5}}-\frac{\log\left(2x^2-\sqrt{5}x+1\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - Sqrt[5]*x + 2*x^2]/(2*Sqrt[5]) + Log[1 + Sqrt[5]*x + 2*x
^2]/(2*Sqrt[5])
```

Rubi in Sympy [A] time = 13.8308, size = 46, normalized size = 0.92

$$-\frac{\sqrt{5}\log\left(x^2-\frac{\sqrt{5}x}{2}+\frac{1}{2}\right)}{10}+\frac{\sqrt{5}\log\left(x^2+\frac{\sqrt{5}x}{2}+\frac{1}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-2*x**2+1)/(4*x**4-x**2+1),x)
```

```
[Out] -sqrt(5)*log(x**2 - sqrt(5)*x/2 + 1/2)/10 + sqrt(5)*log(x**2 + sq
rt(5)*x/2 + 1/2)/10
```

Mathematica [A] time = 0.0236739, size = 42, normalized size = 0.84

$$\frac{\log\left(2x^2+\sqrt{5}x+1\right)-\log\left(-2x^2+\sqrt{5}x-1\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - x^2 + 4*x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[5]*x - 2*x^2] + Log[1 + Sqrt[5]*x + 2*x^2])/(2*Sq
rt[5])
```

Maple [A] time = 0.015, size = 39, normalized size = 0.8

$$-\frac{\ln\left(1+2\,x^2-x\sqrt{5}\right)\sqrt{5}}{10}+\frac{\ln\left(1+2\,x^2+x\sqrt{5}\right)\sqrt{5}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((-2*x^2+1)/(4*x^4-x^2+1),x)
```

```
[Out] -1/10*ln(1+2*x^2-x*5^(1/2))*5^(1/2)+1/10*ln(1+2*x^2+x*5^(1/2))*5^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,x^2-1}{4\,x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(2*x^2 - 1)/(4*x^4 - x^2 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 - x^2 + 1), x)
```

Fricas [A] time = 0.270817, size = 62, normalized size = 1.24

$$\frac{1}{10}\,\sqrt{5}\log\left(\frac{20\,x^3+\sqrt{5}\left(4\,x^4+9\,x^2+1\right)+10\,x}{4\,x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(2*x^2 - 1)/(4*x^4 - x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*log((20*x^3 + sqrt(5)*(4*x^4 + 9*x^2 + 1) + 10*x)/(4
*x^4 - x^2 + 1))
```

Sympy [A] time = 0.196494, size = 46, normalized size = 0.92

$$-\frac{\sqrt{5}\log\left(x^2-\frac{\sqrt{5}x}{2}+\frac{1}{2}\right)}{10}+\frac{\sqrt{5}\log\left(x^2+\frac{\sqrt{5}x}{2}+\frac{1}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((-2*x**2+1)/(4*x**4-x**2+1),x)
```

```
[Out] -sqrt(5)*log(x**2 - sqrt(5)*x/2 + 1/2)/10 + sqrt(5)*log(x**2 + sq
rt(5)*x/2 + 1/2)/10
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,x^2-1}{4\,x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(2*x^2 - 1)/(4*x^4 - x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^2 - 1)/(4*x^4 - x^2 + 1), x)
```

3.71 $\int \frac{1-2x^2}{1-2x^2+4x^4}\,dx$

Optimal. Leaf size=50

$$\frac{\log\left(2x^2+\sqrt{6}x+1\right)}{2\sqrt{6}}-\frac{\log\left(2x^2-\sqrt{6}x+1\right)}{2\sqrt{6}}$$

```
[Out] -Log[1 - Sqrt[6]*x + 2*x^2]/(2*Sqrt[6]) + Log[1 + Sqrt[6]*x + 2*x
^2]/(2*Sqrt[6])
```

Rubi [A] time = 0.0513051, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\log\left(2x^2+\sqrt{6}x+1\right)}{2\sqrt{6}}-\frac{\log\left(2x^2-\sqrt{6}x+1\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - Sqrt[6]*x + 2*x^2]/(2*Sqrt[6]) + Log[1 + Sqrt[6]*x + 2*x
^2]/(2*Sqrt[6])
```

Rubi in Sympy [A] time = 15.0116, size = 46, normalized size = 0.92

$$-\frac{\sqrt{6}\log\left(x^2-\frac{\sqrt{6}x}{2}+\frac{1}{2}\right)}{12}+\frac{\sqrt{6}\log\left(x^2+\frac{\sqrt{6}x}{2}+\frac{1}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2*x**2+1)/(4*x**4-2*x**2+1),x)
```

```
[Out] -sqrt(6)*log(x**2 - sqrt(6)*x/2 + 1/2)/12 + sqrt(6)*log(x**2 + sq
rt(6)*x/2 + 1/2)/12
```

Mathematica [A] time = 0.0310131, size = 42, normalized size = 0.84

$$\frac{\log\left(2x^2+\sqrt{6}x+1\right)-\log\left(-2x^2+\sqrt{6}x-1\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[6]*x - 2*x^2] + Log[1 + Sqrt[6]*x + 2*x^2])/(2*Sq
rt[6])
```

Maple [A] time = 0.015, size = 39, normalized size = 0.8

$$-\frac{\ln\left(1+2\,x^2-x\sqrt{6}\right)\sqrt{6}}{12}+\frac{\ln\left(1+2\,x^2+x\sqrt{6}\right)\sqrt{6}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2*x^2+1)/(4*x^4-2*x^2+1),x)
```

```
[Out] -1/12*ln(1+2*x^2-x*6^(1/2))*6^(1/2)+1/12*ln(1+2*x^2+x*6^(1/2))*6^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,x^2-1}{4\,x^4-2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 - 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 - 2*x^2 + 1), x)
```

Fricas [A] time = 0.282287, size = 62, normalized size = 1.24

$$\frac{1}{12}\,\sqrt{6}\log\left(\frac{24\,x^3+\sqrt{6}\left(4\,x^4+10\,x^2+1\right)+12\,x}{4\,x^4-2\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 - 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*log((24*x^3 + sqrt(6)*(4*x^4 + 10*x^2 + 1) + 12*x)/(
4*x^4 - 2*x^2 + 1))
```

Sympy [A] time = 0.205065, size = 46, normalized size = 0.92

$$-\frac{\sqrt{6}\log\left(x^2-\frac{\sqrt{6}x}{2}+\frac{1}{2}\right)}{12}+\frac{\sqrt{6}\log\left(x^2+\frac{\sqrt{6}x}{2}+\frac{1}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x**2+1)/(4*x**4-2*x**2+1),x)
```

```
[Out] -sqrt(6)*log(x**2 - sqrt(6)*x/2 + 1/2)/12 + sqrt(6)*log(x**2 + sq
rt(6)*x/2 + 1/2)/12
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,x^2-1}{4\,x^4-2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 - 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^2 - 1)/(4*x^4 - 2*x^2 + 1), x)
```

3.72 $\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx$

Optimal. Leaf size=50

$$\frac{\log\left(2x^2+\sqrt{7}x+1\right)}{2\sqrt{7}}-\frac{\log\left(2x^2-\sqrt{7}x+1\right)}{2\sqrt{7}}$$

```
[Out] -Log[1 - Sqrt[7]*x + 2*x^2]/(2*Sqrt[7]) + Log[1 + Sqrt[7]*x + 2*x
^2]/(2*Sqrt[7])
```

Rubi [A] time = 0.0500175, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\log\left(2x^2+\sqrt{7}x+1\right)}{2\sqrt{7}}-\frac{\log\left(2x^2-\sqrt{7}x+1\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - Sqrt[7]*x + 2*x^2]/(2*Sqrt[7]) + Log[1 + Sqrt[7]*x + 2*x
^2]/(2*Sqrt[7])
```

Rubi in Sympy [A] time = 13.8782, size = 46, normalized size = 0.92

$$-\frac{\sqrt{7}\log\left(x^2-\frac{\sqrt{7}x}{2}+\frac{1}{2}\right)}{14}+\frac{\sqrt{7}\log\left(x^2+\frac{\sqrt{7}x}{2}+\frac{1}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-2*x**2+1)/(4*x**4-3*x**2+1),x)
```

```
[Out] -sqrt(7)*log(x**2 - sqrt(7)*x/2 + 1/2)/14 + sqrt(7)*log(x**2 + sq
rt(7)*x/2 + 1/2)/14
```

Mathematica [A] time = 0.0260793, size = 42, normalized size = 0.84

$$\frac{\log\left(2x^2+\sqrt{7}x+1\right)-\log\left(-2x^2+\sqrt{7}x-1\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[7]*x - 2*x^2] + Log[1 + Sqrt[7]*x + 2*x^2])/(2*Sq
rt[7])
```

Maple [A] time = 0.015, size = 39, normalized size = 0.8

$$-\frac{\ln\left(1+2x^2-x\sqrt{7}\right)\sqrt{7}}{14}+\frac{\ln\left(1+2x^2+x\sqrt{7}\right)\sqrt{7}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((-2*x^2+1)/(4*x^4-3*x^2+1),x)
```

```
[Out] -1/14*ln(1+2*x^2-x*7^(1/2))*7^(1/2)+1/14*ln(1+2*x^2+x*7^(1/2))*7^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,x^2-1}{4\,x^4-3\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(2*x^2 - 1)/(4*x^4 - 3*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 - 3*x^2 + 1), x)
```

Fricas [A] time = 0.30237, size = 62, normalized size = 1.24

$$\frac{1}{14}\,\sqrt{7}\log\left(\frac{28\,x^3+\sqrt{7}\left(4\,x^4+11\,x^2+1\right)+14\,x}{4\,x^4-3\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(2*x^2 - 1)/(4*x^4 - 3*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/14*sqrt(7)*log((28*x^3 + sqrt(7)*(4*x^4 + 11*x^2 + 1) + 14*x)/(
4*x^4 - 3*x^2 + 1))
```

Sympy [A] time = 0.201535, size = 46, normalized size = 0.92

$$-\frac{\sqrt{7}\log\left(x^2-\frac{\sqrt{7}x}{2}+\frac{1}{2}\right)}{14}+\frac{\sqrt{7}\log\left(x^2+\frac{\sqrt{7}x}{2}+\frac{1}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((-2*x**2+1)/(4*x**4-3*x**2+1),x)
```

```
[Out] -sqrt(7)*log(x**2 - sqrt(7)*x/2 + 1/2)/14 + sqrt(7)*log(x**2 + sq
rt(7)*x/2 + 1/2)/14
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,x^2-1}{4\,x^4-3\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(2*x^2 - 1)/(4*x^4 - 3*x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^2 - 1)/(4*x^4 - 3*x^2 + 1), x)
```

3.73 $\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx$

Optimal. Leaf size=14

$$\frac{\tanh^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[2]*x]/Sqrt[2]
```

Rubi [A] time = 0.0143112, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{\tanh^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 2*x^2)/(1 - 4*x^2 + 4*x^4),x]

[Out] ArcTanh[Sqrt[2]*x]/Sqrt[2]
```

Rubi in Sympy [A] time = 4.57746, size = 14, normalized size = 1.

$$\frac{\sqrt{2}\,\text{atanh}\left(\sqrt{2}x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2*x**2+1)/(4*x**4-4*x**2+1),x)

[Out] sqrt(2)*atanh(sqrt(2)*x)/2
```

Mathematica [B] time = 0.0112848, size = 32, normalized size = 2.29

$$\frac{\log\left(2x+\sqrt{2}\right)-\log\left(\sqrt{2}-2x\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 2*x^2)/(1 - 4*x^2 + 4*x^4),x]

[Out] (-Log[Sqrt[2] - 2*x] + Log[Sqrt[2] + 2*x])/(2*Sqrt[2])
```

Maple [A] time = 0.003, size = 12, normalized size = 0.9

$$\frac{\textit{Artanh}\left(\sqrt{2}x\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2*x^2+1)/(4*x^4-4*x^2+1),x)
```

```
[Out] 1/2*arctanh(2^(1/2)*x)*2^(1/2)
```

Maxima [A] time = 0.826188, size = 35, normalized size = 2.5

$$-\frac{1}{4}\sqrt{2}\log\left(\frac{2\left(2x-\sqrt{2}\right)}{4x+2\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 - 4*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*log(2*(2*x - sqrt(2))/((2*sqrt(2)) + 4*x))
```

Fricas [A] time = 0.280236, size = 42, normalized size = 3.

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\sqrt{2}\left(2x^2+1\right)+4x}{2x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 - 4*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((sqrt(2)*(2*x^2 + 1) + 4*x)/(2*x^2 - 1))
```

Sympy [A] time = 0.16878, size = 32, normalized size = 2.29

$$-\frac{\sqrt{2}\log\left(x-\frac{\sqrt{2}}{2}\right)}{4}+\frac{\sqrt{2}\log\left(x+\frac{\sqrt{2}}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x**2+1)/(4*x**4-4*x**2+1),x)
```

```
[Out] -sqrt(2)*log(x - sqrt(2)/2)/4 + sqrt(2)*log(x + sqrt(2)/2)/4
```

GIAC/XCAS [A] time = 0.27209, size = 39, normalized size = 2.79

$$\frac{1}{4}\sqrt{2}\ln\left(\left|x+\frac{1}{2}\sqrt{2}\right|\right)-\frac{1}{4}\sqrt{2}\ln\left(\left|x-\frac{1}{2}\sqrt{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 - 4*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*ln(abs(x + 1/2*sqrt(2))) - 1/4*sqrt(2)*ln(abs(x - 1/2
*sqrt(2)))
```

3.74 $\int \frac{1-2x^2}{1-5x^2+4x^4} \, dx$

Optimal. Leaf size=39

$$-\frac{1}{6}\log(1-2x)-\frac{1}{6}\log(1-x)+\frac{1}{6}\log(x+1)+\frac{1}{6}\log(2x+1)$$

```
[Out] -Log[1 - 2*x]/6 - Log[1 - x]/6 + Log[1 + x]/6 + Log[1 + 2*x]/6
```

Rubi [A] time = 0.0397329, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{1}{6}\log(1-2x)-\frac{1}{6}\log(1-x)+\frac{1}{6}\log(x+1)+\frac{1}{6}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 2*x^2)/(1 - 5*x^2 + 4*x^4),x]

[Out] -Log[1 - 2*x]/6 - Log[1 - x]/6 + Log[1 + x]/6 + Log[1 + 2*x]/6
```

Rubi in Sympy [A] time = 9.08806, size = 29, normalized size = 0.74

$$-\frac{\log(-2x+1)}{6}-\frac{\log(-x+1)}{6}+\frac{\log(x+1)}{6}+\frac{\log(2x+1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2*x**2+1)/(4*x**4-5*x**2+1),x)

[Out] -log(-2*x + 1)/6 - log(-x + 1)/6 + log(x + 1)/6 + log(2*x + 1)/6
```

Mathematica [A] time = 0.00988012, size = 31, normalized size = 0.79

$$\frac{1}{6}\log\left(2x^2+3x+1\right)-\frac{1}{6}\log\left(2x^2-3x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 2*x^2)/(1 - 5*x^2 + 4*x^4),x]

[Out] -Log[1 - 3*x + 2*x^2]/6 + Log[1 + 3*x + 2*x^2]/6
```

Maple [A] time = 0.011, size = 30, normalized size = 0.8

$$-\frac{\ln(-1+x)}{6}+\frac{\ln(1+x)}{6}-\frac{\ln(2x-1)}{6}+\frac{\ln(1+2x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2*x^2+1)/(4*x^4-5*x^2+1),x)
```

```
[Out] -1/6*ln(-1+x)+1/6*ln(1+x)-1/6*ln(2*x-1)+1/6*ln(1+2*x)
```

Maxima [A] time = 0.747022, size = 39, normalized size = 1.

$$\frac{1}{6}\log\left(2x+1\right)-\frac{1}{6}\log\left(2x-1\right)+\frac{1}{6}\log\left(x+1\right)-\frac{1}{6}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 - 5*x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/6*log(2*x + 1) - 1/6*log(2*x - 1) + 1/6*log(x + 1) - 1/6*log(x
- 1)
```

Fricas [A] time = 0.28148, size = 36, normalized size = 0.92

$$\frac{1}{6}\log\left(2x^2+3x+1\right)-\frac{1}{6}\log\left(2x^2-3x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 - 5*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/6*log(2*x^2 + 3*x + 1) - 1/6*log(2*x^2 - 3*x + 1)
```

Sympy [A] time = 0.199873, size = 29, normalized size = 0.74

$$-\frac{\log\left(x^2-\frac{3x}{2}+\frac{1}{2}\right)}{6}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x**2+1)/(4*x**4-5*x**2+1),x)
```

```
[Out] -log(x**2 - 3*x/2 + 1/2)/6 + log(x**2 + 3*x/2 + 1/2)/6
```

GIAC/XCAS [A] time = 0.271201, size = 45, normalized size = 1.15

$$\frac{1}{6}\ln\left(|2x+1|\right)-\frac{1}{6}\ln\left(|2x-1|\right)+\frac{1}{6}\ln\left(|x+1|\right)-\frac{1}{6}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 - 5*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/6*ln(abs(2*x + 1)) - 1/6*ln(abs(2*x - 1)) + 1/6*ln(abs(x + 1))
- 1/6*ln(abs(x - 1))
```

3.75 $\int \frac{1-2x^2}{1-6x^2+4x^4}\,dx$

Optimal. Leaf size=48

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{2}x+1}{\sqrt{5}}\right)}{\sqrt{10}}-\frac{\tanh^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}$$

```
[Out] -(ArcTanh[(1 - 2*Sqrt[2]*x)/Sqrt[5]]/Sqrt[10]) + ArcTanh[(1 + 2*S
qrt[2]*x)/Sqrt[5]]/Sqrt[10]
```

Rubi [A] time = 0.086317, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{2}x+1}{\sqrt{5}}\right)}{\sqrt{10}}-\frac{\tanh^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTanh[(1 - 2*Sqrt[2]*x)/Sqrt[5]]/Sqrt[10]) + ArcTanh[(1 + 2*S
qrt[2]*x)/Sqrt[5]]/Sqrt[10]
```

Rubi in Sympy [A] time = 8.77798, size = 53, normalized size = 1.1

$$-\frac{\sqrt{10}\,\operatorname{atanh}\left(\sqrt{10}\left(-\frac{2x}{5}-\frac{\sqrt{2}}{10}\right)\right)}{10}-\frac{\sqrt{10}\,\operatorname{atanh}\left(\sqrt{10}\left(-\frac{2x}{5}+\frac{\sqrt{2}}{10}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2*x**2+1)/(4*x**4-6*x**2+1),x)
```

```
[Out] -sqrt(10)*atanh(sqrt(10)*(-2*x/5 - sqrt(2)/10))/10 - sqrt(10)*ata
nh(sqrt(10)*(-2*x/5 + sqrt(2)/10))/10
```

Mathematica [A] time = 0.0306739, size = 42, normalized size = 0.88

$$\frac{\log\left(2x^2+\sqrt{10}x+1\right)-\log\left(-2x^2+\sqrt{10}x-1\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[10]*x - 2*x^2] + Log[1 + Sqrt[10]*x + 2*x^2])/(2*
Sqrt[10])
```

Maple [B] time = 0.018, size = 82, normalized size = 1.7

$$\frac{2\sqrt{5}\left(\sqrt{5}+1\right)}{10\sqrt{10}+10\sqrt{2}}Artanh\left(8\frac{x}{2\sqrt{10}+2\sqrt{2}}\right)+\frac{\left(-2+2\sqrt{5}\right)\sqrt{5}}{10\sqrt{10}-10\sqrt{2}}Artanh\left(8\frac{x}{2\sqrt{10}-2\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2*x^2+1)/(4*x^4-6*x^2+1),x)
```

```
[Out] 2/5*5^(1/2)*(5^(1/2)+1)/(2*10^(1/2)+2*2^(1/2))*arctanh(8*x/(2*10^
(1/2)+2*2^(1/2)))+2/5*(5^(1/2)-1)*5^(1/2)/(2*10^(1/2)-2*2^(1/2))*
arctanh(8*x/(2*10^(1/2)-2*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,x^2-1}{4\,x^4-6\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 - 6*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 - 6*x^2 + 1), x)
```

Fricas [A] time = 0.275535, size = 62, normalized size = 1.29

$$\frac{1}{20}\,\sqrt{10}\log\left(\frac{40\,x^3+\sqrt{10}\left(4\,x^4+14\,x^2+1\right)+20\,x}{4\,x^4-6\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 - 6*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(10)*log((40*x^3 + sqrt(10)*(4*x^4 + 14*x^2 + 1) + 20*x)
/(4*x^4 - 6*x^2 + 1))
```

Sympy [A] time = 0.207049, size = 46, normalized size = 0.96

$$-\frac{\sqrt{10}\log\left(x^2-\frac{\sqrt{10}x}{2}+\frac{1}{2}\right)}{20}+\frac{\sqrt{10}\log\left(x^2+\frac{\sqrt{10}x}{2}+\frac{1}{2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x**2+1)/(4*x**4-6*x**2+1),x)
```

```
[Out] -sqrt(10)*log(x**2 - sqrt(10)*x/2 + 1/2)/20 + sqrt(10)*log(x**2 +
 sqrt(10)*x/2 + 1/2)/20
```

GIAC/XCAS [A] time = 0.308836, size = 104, normalized size = 2.17

$$\frac{1}{20}\,\sqrt{10}\ln\left(\left|x+\frac{1}{4}\,\sqrt{10}+\frac{1}{4}\,\sqrt{2}\right|\right)+\frac{1}{20}\,\sqrt{10}\ln\left(\left|x+\frac{1}{4}\,\sqrt{10}-\frac{1}{4}\,\sqrt{2}\right|\right)-\frac{1}{20}\,\sqrt{10}\ln\left(\left|x-\frac{1}{4}\,\sqrt{10}+\frac{1}{4}\,\sqrt{2}\right|\right)-\frac{1}{20}\,\sqrt{10}\ln\left(\left|x-\frac{1}{4}\,\sqrt{10}-\frac{1}{4}\,\sqrt{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 - 1)/(4*x^4 - 6*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(10)*ln(abs(x + 1/4*sqrt(10) + 1/4*sqrt(2))) + 1/20*sqrt
(10)*ln(abs(x + 1/4*sqrt(10) - 1/4*sqrt(2))) - 1/20*sqrt(10)*ln(a
bs(x - 1/4*sqrt(10) + 1/4*sqrt(2))) - 1/20*sqrt(10)*ln(abs(x - 1/
4*sqrt(10) - 1/4*sqrt(2)))
```

3.76 $\int \frac{1+x^2}{1+bx^2+x^4}\,dx$

Optimal. Leaf size=62

$$\frac{\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}$$

```
[Out] -(ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]) + ArcTan[(
Sqrt[2 - b] + 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]
```

Rubi [A] time = 0.104, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]) + ArcTan[(
Sqrt[2 - b] + 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]
```

Rubi in Sympy [A] time = 10.9647, size = 49, normalized size = 0.79

$$\frac{\operatorname{atan}\left(\frac{2x-\sqrt{-b+2}}{\sqrt{b+2}}\right)}{\sqrt{b+2}}+\frac{\operatorname{atan}\left(\frac{2x+\sqrt{-b+2}}{\sqrt{b+2}}\right)}{\sqrt{b+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/(x**4+b*x**2+1),x)
```

```
[Out] atan((2*x - sqrt(-b + 2))/sqrt(b + 2))/sqrt(b + 2) + atan((2*x +
sqrt(-b + 2))/sqrt(b + 2))/sqrt(b + 2)
```

Mathematica [A] time = 0.0935912, size = 124, normalized size = 2.

$$\frac{\frac{\left(\sqrt{b^2-4}-b+2\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{b^2-4}}}\right)}{\sqrt{b-\sqrt{b^2-4}}}+\frac{\left(\sqrt{b^2-4}+b-2\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{\sqrt{b^2-4}+b}}}{\sqrt{2}\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] (((2 - b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 +
b^2]]])/Sqrt[b - Sqrt[-4 + b^2]] + ((-2 + b + Sqrt[-4 + b^2])*Arc
Tan[(Sqrt[2]*x)/Sqrt[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2
]])/(Sqrt[2]*Sqrt[-4 + b^2])
```

Maple [B] time = 0.049, size = 277, normalized size = 4.5

$$2\frac{1}{\sqrt{(b-2)(2+b)}\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\arctan\left(2\frac{x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)$$
$$+1\arctan\left(2\frac{x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)\frac{1}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}$$
$$-b\arctan\left(2\frac{x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)\frac{1}{\sqrt{(b-2)(2+b)}}\frac{1}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}$$
$$-2\frac{1}{\sqrt{(b-2)(2+b)}\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)$$
$$+1\arctan\left(2\frac{x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)\frac{1}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}$$
$$+b\arctan\left(2\frac{x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)\frac{1}{\sqrt{(b-2)(2+b)}}\frac{1}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^4+b*x^2+1),x)
```

```
[Out] 2/((b-2)*(2+b))^(1/2)/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2
*x/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))+1/(-2*((b-2)*(2+b))^(1/2)+
2*b)^(1/2)*arctan(2*x/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))-1/((b-2
)*(2+b))^(1/2)/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*
((b-2)*(2+b))^(1/2)+2*b)^(1/2))*b-2/((b-2)*(2+b))^(1/2)/(2*((b-2)
*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((b-2)*(2+b))^(1/2)+2*b)^(
1/2))+1/(2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((b-2)*(2
+b))^(1/2)+2*b)^(1/2))+1/((b-2)*(2+b))^(1/2)/(2*((b-2)*(2+b))^(1/
2)+2*b)^(1/2)*arctan(2*x/(2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{x^4+bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + b*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + b*x^2 + 1), x)
```

Fricas [A] time = 0.289962, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{2\,(b+2)x^3-2\,(b+2)x+\left(x^4-(b+4)x^2+1\right)\sqrt{-b-2}}{x^4+bx^2+1}\right)}{2\sqrt{-b-2}},\ \frac{\arctan\left(\frac{x^3+(b+1)x}{\sqrt{b+2}}\right)+\arctan\left(\frac{x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + b*x^2 + 1),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*(b + 2)*x^3 - 2*(b + 2)*x + (x^4 - (b + 4)*x^2 + 1)*s
qrt(-b - 2))/(x^4 + b*x^2 + 1))/sqrt(-b - 2), (arctan((x^3 + (b +
 1)*x)/sqrt(b + 2)) + arctan(x/sqrt(b + 2)))/sqrt(b + 2)]
```

Sympy [A] time = 0.734525, size = 88, normalized size = 1.42

$$-\frac{\sqrt{-\frac{1}{b+2}}\log\left(x^2+x\left(-b\sqrt{-\frac{1}{b+2}}-2\sqrt{-\frac{1}{b+2}}\right)-1\right)}{2}+\frac{\sqrt{-\frac{1}{b+2}}\log\left(x^2+x\left(b\sqrt{-\frac{1}{b+2}}+2\sqrt{-\frac{1}{b+2}}\right)-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**4+b*x**2+1),x)
```

```
[Out] -sqrt(-1/(b + 2))*log(x**2 + x*(-b*sqrt(-1/(b + 2)) - 2*sqrt(-1/(
b + 2))) - 1)/2 + sqrt(-1/(b + 2))*log(x**2 + x*(b*sqrt(-1/(b + 2
)) + 2*sqrt(-1/(b + 2))) - 1)/2
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + b*x^2 + 1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.77 $\int \frac{1+x^2}{1+5x^2+x^4} \, dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{7}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)}{\sqrt{7}}$$

```
[Out] ArcTan[Sqrt[2/(5 + Sqrt[21])]*x]/Sqrt[7] + ArcTan[Sqrt[(5 + Sqrt[
21])/2]*x]/Sqrt[7]
```

Rubi [A] time = 0.154938, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{7}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/(1 + 5*x^2 + x^4),x]
```

```
[Out] ArcTan[Sqrt[2/(5 + Sqrt[21])]*x]/Sqrt[7] + ArcTan[Sqrt[(5 + Sqrt[
21])/2]*x]/Sqrt[7]
```

Rubi in Sympy [A] time = 8.99053, size = 87, normalized size = 1.78

$$\frac{\sqrt{2}\left(-\frac{\sqrt{21}}{14}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{-\sqrt{21}+5}}\right)}{\sqrt{-\sqrt{21}+5}}+\frac{\sqrt{2}\left(\frac{\sqrt{21}}{14}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{21}+5}}\right)}{\sqrt{\sqrt{21}+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/(x**4+5*x**2+1),x)
```

```
[Out] sqrt(2)*(-sqrt(21)/14 + 1/2)*atan(sqrt(2)*x/sqrt(-sqrt(21) + 5))/
sqrt(-sqrt(21) + 5) + sqrt(2)*(sqrt(21)/14 + 1/2)*atan(sqrt(2)*x/
sqrt(sqrt(21) + 5))/sqrt(sqrt(21) + 5)
```

Mathematica [A] time = 0.231082, size = 83, normalized size = 1.69

$$\frac{\left(\sqrt{21}-3\right)\tan^{-1}\left(\sqrt{\frac{2}{5-\sqrt{21}}}x\right)}{\sqrt{42\left(5-\sqrt{21}\right)}}+\frac{\left(3+\sqrt{21}\right)\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{42\left(5+\sqrt{21}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/(1 + 5*x^2 + x^4),x]
```

```
[Out] ((-3 + Sqrt[21])*ArcTan[Sqrt[2/(5 - Sqrt[21])]*x])/Sqrt[42*(5 - S
qrt[21])] + ((3 + Sqrt[21])*ArcTan[Sqrt[2/(5 + Sqrt[21])]*x])/Sqr
t[42*(5 + Sqrt[21])]
```

Maple [B] time = 0.058, size = 136, normalized size = 2.8

$$\frac{2\sqrt{21}}{14\sqrt{7}+14\sqrt{3}}\arctan\left(4\frac{x}{2\sqrt{7}+2\sqrt{3}}\right)+2\frac{1}{2\sqrt{7}+2\sqrt{3}}\arctan\left(4\frac{x}{2\sqrt{7}+2\sqrt{3}}\right)-\frac{2\sqrt{21}}{14\sqrt{7}-14\sqrt{3}}\arctan\left(4\frac{x}{2\sqrt{7}-2\sqrt{3}}\right)+2\frac{1}{2\sqrt{7}-2\sqrt{3}}\arctan\left(4\frac{x}{2\sqrt{7}-2\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^4+5*x^2+1),x)
```

```
[Out] 2/7*21^(1/2)/(2*7^(1/2)+2*3^(1/2))*arctan(4*x/(2*7^(1/2)+2*3^(1/2
)))+2/(2*7^(1/2)+2*3^(1/2))*arctan(4*x/(2*7^(1/2)+2*3^(1/2)))-2/7
*21^(1/2)/(2*7^(1/2)-2*3^(1/2))*arctan(4*x/(2*7^(1/2)-2*3^(1/2)))
+2/(2*7^(1/2)-2*3^(1/2))*arctan(4*x/(2*7^(1/2)-2*3^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{x^4+5x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + 5*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + 5*x^2 + 1), x)
```

Fricas [A] time = 0.264699, size = 35, normalized size = 0.71

$$\frac{1}{7}\sqrt{7}\left(\arctan\left(\frac{1}{7}\sqrt{7}\left(x^3+6x\right)\right)+\arctan\left(\frac{1}{7}\sqrt{7}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + 5*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/7*sqrt(7)*(arctan(1/7*sqrt(7)*(x^3 + 6*x)) + arctan(1/7*sqrt(7)
*x))
```

Sympy [A] time = 0.236008, size = 41, normalized size = 0.84

$$\frac{\sqrt{7}\left(2\operatorname{atan}\left(\frac{\sqrt{7}x}{7}\right)+2\operatorname{atan}\left(\frac{\sqrt{7}x^3}{7}+\frac{6\sqrt{7}x}{7}\right)\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**4+5*x**2+1),x)
```

```
[Out] sqrt(7)*(2*atan(sqrt(7)*x/7) + 2*atan(sqrt(7)*x**3/7 + 6*sqrt(7)*
x/7))/14
```

<u>GIAC/XCAS</u> [A] time = 0.277339, size = 35, normalized size = 0.71

$$\frac{1}{14}\sqrt{7}\left(\pi\,\mathrm{sign}\left(x\right)+2\,\arctan\left(\frac{\sqrt{7}\left(x^{2}-1\right)}{7\,x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + 5*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/14*sqrt(7)*(pi*sign(x) + 2*arctan(1/7*sqrt(7)*(x^2 - 1)/x))
```

3.78 $\int \frac{1+x^2}{1+4x^2+x^4} \, dx$

Optimal. Leaf size=43

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}$$

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[6] + ArcTan[x/Sqrt[2 + Sqrt[3]]]
/Sqrt[6]
```

Rubi [A] time = 0.0892349, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + 4*x^2 + x^4),x]
```

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[6] + ArcTan[x/Sqrt[2 + Sqrt[3]]]
/Sqrt[6]
```

Rubi in Sympy [A] time = 7.87974, size = 66, normalized size = 1.53

$$\frac{\left(-\frac{\sqrt{3}}{6}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{x}{\sqrt{-\sqrt{3}+2}}\right)}{\sqrt{-\sqrt{3}+2}}+\frac{\left(\frac{\sqrt{3}}{6}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{x}{\sqrt{\sqrt{3}+2}}\right)}{\sqrt{\sqrt{3}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2+1)/(x**4+4*x**2+1),x)
```

```
[Out] (-sqrt(3)/6 + 1/2)*atan(x/sqrt(-sqrt(3) + 2))/sqrt(-sqrt(3) + 2)
+ (sqrt(3)/6 + 1/2)*atan(x/sqrt(sqrt(3) + 2))/sqrt(sqrt(3) + 2)
```

Mathematica [A] time = 0.109381, size = 81, normalized size = 1.88

$$\frac{\left(\sqrt{3}-1\right)\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\left(1+\sqrt{3}\right)\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + 4*x^2 + x^4),x]
```

```
[Out] ((-1 + Sqrt[3])*ArcTan[x/Sqrt[2 - Sqrt[3]]])/(2*Sqrt[3*(2 - Sqrt[
3])]) + ((1 + Sqrt[3])*ArcTan[x/Sqrt[2 + Sqrt[3]]])/(2*Sqrt[3*(2
+ Sqrt[3])])
```

Maple [B] time = 0.048, size = 110, normalized size = 2.6

$$\frac{\sqrt{3}}{3\sqrt{6}+3\sqrt{2}}\arctan\left(2\frac{x}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{\sqrt{6}+\sqrt{2}}\arctan\left(2\frac{x}{\sqrt{6}+\sqrt{2}}\right) -\frac{\sqrt{3}}{3\sqrt{6}-3\sqrt{2}}\arctan\left(2\frac{x}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{\sqrt{6}-\sqrt{2}}\arctan\left(2\frac{x}{\sqrt{6}-\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^4+4*x^2+1),x)
```

```
[Out] 1/3*3^(1/2)/(6^(1/2)+2^(1/2))*arctan(2*x/(6^(1/2)+2^(1/2)))+1/(6^
(1/2)+2^(1/2))*arctan(2*x/(6^(1/2)+2^(1/2)))-1/3*3^(1/2)/(6^(1/2)
-2^(1/2))*arctan(2*x/(6^(1/2)-2^(1/2)))+1/(6^(1/2)-2^(1/2))*arcta
n(2*x/(6^(1/2)-2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{x^4+4x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + 4*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + 4*x^2 + 1), x)
```

Fricas [A] time = 0.293339, size = 35, normalized size = 0.81

$$\frac{1}{6}\sqrt{6}\left(\arctan\left(\frac{1}{6}\sqrt{6}\left(x^3+5x\right)\right)+\arctan\left(\frac{1}{6}\sqrt{6}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + 4*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*(arctan(1/6*sqrt(6)*(x^3 + 5*x)) + arctan(1/6*sqrt(6)
*x))
```

Sympy [A] time = 0.238659, size = 41, normalized size = 0.95

$$\frac{\sqrt{6}\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{6}\right)+2\operatorname{atan}\left(\frac{\sqrt{6}x^3}{6}+\frac{5\sqrt{6}x}{6}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**4+4*x**2+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/6) + 2*atan(sqrt(6)*x**3/6 + 5*sqrt(6)*
x/6))/12
```

<u>GIAC/XCAS</u> [A] time = 0.279035, size = 35, normalized size = 0.81

$$\frac{1}{12}\sqrt{6}\left(\pi\operatorname{sign}(x) + 2\arctan\left(\frac{\sqrt{6}(x^2-1)}{6x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + 4*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*(pi*sign(x) + 2*arctan(1/6*sqrt(6)*(x^2 - 1)/x))
```

3.79 $\int \frac{1+x^2}{1+3x^2+x^4}\,dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{5}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt{5}}$$

```
[Out] ArcTan[Sqrt[2/(3 + Sqrt[5])]*x]/Sqrt[5] + ArcTan[Sqrt[(3 + Sqrt[5
])/2]*x]/Sqrt[5]
```

Rubi [A] time = 0.122879, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{5}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] ArcTan[Sqrt[2/(3 + Sqrt[5])]*x]/Sqrt[5] + ArcTan[Sqrt[(3 + Sqrt[5
])/2]*x]/Sqrt[5]
```

Rubi in Sympy [A] time = 8.94797, size = 87, normalized size = 1.78

$$\frac{\sqrt{2}\left(-\frac{\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{-\sqrt{5}+3}}\right)}{\sqrt{-\sqrt{5}+3}}+\frac{\sqrt{2}\left(\frac{\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{5}+3}}\right)}{\sqrt{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2+1)/(x**4+3*x**2+1),x)
```

```
[Out] sqrt(2)*(-sqrt(5)/10 + 1/2)*atan(sqrt(2)*x/sqrt(-sqrt(5) + 3))/sq
rt(-sqrt(5) + 3) + sqrt(2)*(sqrt(5)/10 + 1/2)*atan(sqrt(2)*x/sqrt
(sqrt(5) + 3))/sqrt(sqrt(5) + 3)
```

Mathematica [A] time = 0.164721, size = 83, normalized size = 1.69

$$\frac{\left(\sqrt{5}-1\right)\tan^{-1}\left(\sqrt{\frac{2}{3-\sqrt{5}}}x\right)}{\sqrt{10\left(3-\sqrt{5}\right)}}+\frac{\left(1+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] ((-1 + Sqrt[5])*ArcTan[Sqrt[2/(3 - Sqrt[5])]*x])/Sqrt[10*(3 - Sqr
t[5])] + ((1 + Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x])/Sqrt[10*
(3 + Sqrt[5])]
```

Maple [B] time = 0.042, size = 104, normalized size = 2.1

$$\frac{2\sqrt{5}}{10\sqrt{5}+10}\arctan\left(4\frac{x}{2\sqrt{5}+2}\right)+2\frac{1}{2\sqrt{5}+2}\arctan\left(4\frac{x}{2\sqrt{5}+2}\right) - \frac{2\sqrt{5}}{-10+10\sqrt{5}}\arctan\left(4\frac{x}{-2+2\sqrt{5}}\right)+2\frac{1}{-2+2\sqrt{5}}\arctan\left(4\frac{x}{-2+2\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^4+3*x^2+1),x)
```

```
[Out] 2/5*5^(1/2)/(2*5^(1/2)+2)*arctan(4*x/(2*5^(1/2)+2))+2/(2*5^(1/2)+
2)*arctan(4*x/(2*5^(1/2)+2))-2/5*5^(1/2)/(-2+2*5^(1/2))*arctan(4*
x/(-2+2*5^(1/2)))+2/(-2+2*5^(1/2))*arctan(4*x/(-2+2*5^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{x^4+3x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + 3*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + 3*x^2 + 1), x)
```

Fricas [A] time = 0.280968, size = 35, normalized size = 0.71

$$\frac{1}{5}\sqrt{5}\left(\arctan\left(\frac{1}{5}\sqrt{5}\left(x^3+4x\right)\right)+\arctan\left(\frac{1}{5}\sqrt{5}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + 3*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*(arctan(1/5*sqrt(5)*(x^3 + 4*x)) + arctan(1/5*sqrt(5)
*x))
```

Sympy [A] time = 0.237262, size = 41, normalized size = 0.84

$$\frac{\sqrt{5}\left(2\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)+2\operatorname{atan}\left(\frac{\sqrt{5}x^3}{5}+\frac{4\sqrt{5}x}{5}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**4+3*x**2+1),x)
```

```
[Out] sqrt(5)*(2*atan(sqrt(5)*x/5) + 2*atan(sqrt(5)*x**3/5 + 4*sqrt(5)*
x/5))/10
```

GIAC/XCAS [A] time = 0.2728, size = 35, normalized size = 0.71

$$\frac{1}{10}\sqrt{5}\left(\pi\operatorname{sign}\left(x\right)+2\arctan\left(\frac{\sqrt{5}\left(x^{2}-1\right)}{5\,x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + 3*x^2 + 1),x, algorithm="giac")

[Out] 1/10*sqrt(5)*(pi*sign(x) + 2*arctan(1/5*sqrt(5)*(x^2 - 1)/x))
```

3.80 $\int \frac{1+x^2}{1+2x^2+x^4}\,dx$

Optimal. Leaf size=2

$$\tan^{-1}(x)$$

```
[Out] ArcTan[x]
```

Rubi [A] time = 0.00498854, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/(1 + 2*x^2 + x^4),x]

[Out] ArcTan[x]
```

Rubi in Sympy [A] time = 2.76924, size = 2, normalized size = 1.

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/(x**4+2*x**2+1),x)

[Out] atan(x)
```

Mathematica [A] time = 0.00360941, size = 2, normalized size = 1.

$$\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/(1 + 2*x^2 + x^4),x]

[Out] ArcTan[x]
```

Maple [A] time = 0.002, size = 3, normalized size = 1.5

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^4+2*x^2+1),x)

[Out] arctan(x)
```

Maxima [A] time = 0.836428, size = 3, normalized size = 1.5

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

Fricas [A] time = 0.283217, size = 3, normalized size = 1.5

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] arctan(x)
```

Sympy [A] time = 0.151909, size = 2, normalized size = 1.

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**4+2*x**2+1),x)
```

```
[Out] atan(x)
```

GIAC/XCAS [A] time = 0.268526, size = 3, normalized size = 1.5

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] arctan(x)
```

3.81 $\int \frac{1+x^2}{1+x^2+x^4} \, dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/
Sqrt[3]
```

Rubi [A] time = 0.0595223, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/(1 + x^2 + x^4),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/
Sqrt[3]
```

Rubi in Sympy [A] time = 7.14213, size = 42, normalized size = 1.11

$$\frac{\sqrt{3}\,\text{atan}\left(\sqrt{3}\left(\frac{2x}{3} - \frac{1}{3}\right)\right)}{3} + \frac{\sqrt{3}\,\text{atan}\left(\sqrt{3}\left(\frac{2x}{3} + \frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/(x**4+x**2+1),x)
```

```
[Out] sqrt(3)*atan(sqrt(3)*(2*x/3 - 1/3))/3 + sqrt(3)*atan(sqrt(3)*(2*x
/3 + 1/3))/3
```

Mathematica [C] time = 0.320642, size = 99, normalized size = 2.61

$$\frac{\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{\sqrt{6\left(1-i\sqrt{3}\right)}} + \frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{\sqrt{6\left(1+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/(1 + x^2 + x^4),x]
```

```
[Out] ((-I + Sqrt[3])*ArcTan[x/Sqrt[(1 - I*Sqrt[3])/2]])/Sqrt[6*(1 - I*
Sqrt[3])] + ((I + Sqrt[3])*ArcTan[x/Sqrt[(1 + I*Sqrt[3])/2]])/Sqr
t[6*(1 + I*Sqrt[3])]
```

Maple [A] time = 0.007, size = 34, normalized size = 0.9

$$\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^4+x^2+1),x)
```

```
[Out] 1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/3*3^(1/2)*arctan(1/3*(2
*x-1)*3^(1/2))
```

Maxima [A] time = 0.834928, size = 45, normalized size = 1.18

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x - 1))
```

Fricas [A] time = 0.286355, size = 35, normalized size = 0.92

$$\frac{1}{3}\sqrt{3}\left(\arctan\left(\frac{1}{3}\sqrt{3}\left(x^3+2x\right)\right)+\arctan\left(\frac{1}{3}\sqrt{3}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*(arctan(1/3*sqrt(3)*(x^3 + 2*x)) + arctan(1/3*sqrt(3)
*x))
```

Sympy [A] time = 0.225736, size = 41, normalized size = 1.08

$$\frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\operatorname{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**4+x**2+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*
x/3))/6
```

GIAC/XCAS [A] time = 0.273355, size = 35, normalized size = 0.92

$$\frac{1}{6}\sqrt{3}\left(\pi\operatorname{sign}(x)+2\arctan\left(\frac{\sqrt{3}\left(x^2-1\right)}{3x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + x^2 + 1),x, algorithm="giac")

[Out] 1/6*sqrt(3)*(pi*sign(x) + 2*arctan(1/3*sqrt(3)*(x^2 - 1)/x))
```

3.82 $\int \frac{1+x^2}{1+x^4}\,dx$

Optimal. Leaf size=35

$$\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/Sqrt[2]
```

Rubi [A] time = 0.0421558, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + x^4),x]

[Out] -(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/Sqrt[2]
```

Rubi in Sympy [A] time = 4.60352, size = 32, normalized size = 0.91

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2+1)/(x**4+1),x)

[Out] sqrt(2)*atan(sqrt(2)*x - 1)/2 + sqrt(2)*atan(sqrt(2)*x + 1)/2
```

Mathematica [A] time = 0.0205835, size = 30, normalized size = 0.86

$$\frac{\tan^{-1}\left(\sqrt{2}x+1\right)-\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + x^4),x]

[Out] (-ArcTan[1 - Sqrt[2]*x] + ArcTan[1 + Sqrt[2]*x])/Sqrt[2]
```

Maple [B] time = 0.006, size = 88, normalized size = 2.5

$$\frac{\arctan\left(\sqrt{2}x-1\right)\sqrt{2}}{2}+\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2+\sqrt{2}x}{1+x^2-\sqrt{2}x}\right)+\frac{\arctan\left(1+\sqrt{2}x\right)\sqrt{2}}{2}+\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2-\sqrt{2}x}{1+x^2+\sqrt{2}x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^4+1),x)
```

```
[Out] 1/2*arctan(2^(1/2)*x-1)*2^(1/2)+1/8*2^(1/2)*ln((1+x^2+2^(1/2)*x)/
(1+x^2-2^(1/2)*x))+1/2*arctan(1+2^(1/2)*x)*2^(1/2)+1/8*2^(1/2)*ln
((1+x^2-2^(1/2)*x)/(1+x^2+2^(1/2)*x))
```

Maxima [A] time = 0.819567, size = 53, normalized size = 1.51

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*sqrt(2)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2)))
```

Fricas [A] time = 0.274277, size = 32, normalized size = 0.91

$$\frac{1}{2}\sqrt{2}\left(\arctan\left(\frac{1}{2}\sqrt{2}\left(x^3+x\right)\right)+\arctan\left(\frac{1}{2}\sqrt{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*(arctan(1/2*sqrt(2)*(x^3 + x)) + arctan(1/2*sqrt(2)*x
))
```

Sympy [A] time = 0.213157, size = 39, normalized size = 1.11

$$\frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}x^3}{2}+\frac{\sqrt{2}x}{2}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**4+1),x)
```

```
[Out] sqrt(2)*(2*atan(sqrt(2)*x/2) + 2*atan(sqrt(2)*x**3/2 + sqrt(2)*x/
2))/4
```

GIAC/XCAS [A] time = 0.26763, size = 53, normalized size = 1.51

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*sqrt(2)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2)))
```

3.83 $\int \frac{1+x^2}{1-x^2+x^4} dx$

Optimal. Leaf size=23

$$\tan^{-1}\left(2x+\sqrt{3}\right)-\tan^{-1}\left(\sqrt{3}-2x\right)$$

```
[Out] -ArcTan[Sqrt[3] - 2*x] + ArcTan[Sqrt[3] + 2*x]
```

Rubi [A] time = 0.036119, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\tan^{-1}\left(2x+\sqrt{3}\right)-\tan^{-1}\left(\sqrt{3}-2x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/(1 - x^2 + x^4),x]

[Out] -ArcTan[Sqrt[3] - 2*x] + ArcTan[Sqrt[3] + 2*x]
```

Rubi in Sympy [A] time = 7.65397, size = 19, normalized size = 0.83

$$\operatorname{atan}\left(2x-\sqrt{3}\right)+\operatorname{atan}\left(2x+\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/(x**4-x**2+1),x)

[Out] atan(2*x - sqrt(3)) + atan(2*x + sqrt(3))
```

Mathematica [A] time = 0.0107325, size = 12, normalized size = 0.52

$$-\tan^{-1}\left(\frac{x}{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/(1 - x^2 + x^4),x]

[Out] -ArcTan[x/(-1 + x^2)]
```

Maple [A] time = 0.018, size = 20, normalized size = 0.9

$$\arctan\left(2x-\sqrt{3}\right)+\arctan\left(2x+\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^4-x^2+1),x)

[Out] arctan(2*x-3^(1/2))+arctan(2*x+3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + 1}{x^4 - x^2 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 - x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 - x^2 + 1), x)
```

Fricas [A] time = 0.284387, size = 9, normalized size = 0.39

$$\arctan\left(x^3\right) + \arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 - x^2 + 1),x, algorithm="fricas")
```

```
[Out] arctan(x^3) + arctan(x)
```

Sympy [A] time = 0.191684, size = 7, normalized size = 0.3

$$\operatorname{atan}\left(x\right) + \operatorname{atan}\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**4-x**2+1),x)
```

```
[Out] atan(x) + atan(x**3)
```

GIAC/XCAS [A] time = 0.273567, size = 41, normalized size = 1.78

$$\frac{1}{4}\,\pi\,\mathrm{sign}\left(x\right) + \frac{1}{2}\,\arctan\left(\frac{x^4 - 3\,x^2 + 1}{2\,\left(x^3 - x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 - x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/4*pi*sign(x) + 1/2*arctan(1/2*(x^4 - 3*x^2 + 1)/(x^3 - x))
```

3.84 $\int \frac{1+x^2}{1-2x^2+x^4} \, dx$

Optimal. Leaf size=11

$$\frac{x}{1-x^2}$$

```
[Out] x/(1 - x^2)
```

Rubi [A] time = 0.0072089, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{x}{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] x/(1 - x^2)
```

Rubi in Sympy [A] time = 5.04906, size = 5, normalized size = 0.45

$$\frac{x}{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2+1)/(x**4-2*x**2+1),x)
```

```
[Out] x/(-x**2 + 1)
```

Mathematica [A] time = 0.00631134, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] -(x/(-1 + x^2))
```

Maple [A] time = 0.008, size = 16, normalized size = 1.5

$$-\frac{1}{-2+2x}-\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4-2*x^2+1),x)
```

```
[Out] -1/2/(-1+x)-1/2/(1+x)
```

Maxima [A] time = 0.736606, size = 14, normalized size = 1.27

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 - 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -x/(x^2 - 1)
```

Fricas [A] time = 0.260342, size = 14, normalized size = 1.27

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 - 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] -x/(x^2 - 1)
```

Sympy [A] time = 0.14502, size = 7, normalized size = 0.64

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**4-2*x**2+1),x)
```

```
[Out] -x/(x**2 - 1)
```

GIAC/XCAS [A] time = 0.267557, size = 15, normalized size = 1.36

$$-\frac{1}{x-\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 - 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/(x - 1/x)
```

3.85 $\int \frac{1+x^2}{1-3x^2+x^4} \, dx$

Optimal. Leaf size=65

$$\frac{1}{2}\log\left(-2x-\sqrt{5}+1\right)+\frac{1}{2}\log\left(-2x+\sqrt{5}+1\right)-\frac{1}{2}\log\left(2x-\sqrt{5}+1\right)-\frac{1}{2}\log\left(2x+\sqrt{5}+1\right)$$

```
[Out] Log[1 - Sqrt[5] - 2*x]/2 + Log[1 + Sqrt[5] - 2*x]/2 - Log[1 - Sqr
t[5] + 2*x]/2 - Log[1 + Sqrt[5] + 2*x]/2
```

Rubi [A] time = 0.0686159, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{1}{2}\log\left(-2x-\sqrt{5}+1\right)+\frac{1}{2}\log\left(-2x+\sqrt{5}+1\right)-\frac{1}{2}\log\left(2x-\sqrt{5}+1\right)-\frac{1}{2}\log\left(2x+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/(1 - 3*x^2 + x^4),x]
```

```
[Out] Log[1 - Sqrt[5] - 2*x]/2 + Log[1 + Sqrt[5] - 2*x]/2 - Log[1 - Sqr
t[5] + 2*x]/2 - Log[1 + Sqrt[5] + 2*x]/2
```

Rubi in Sympy [A] time = 8.29486, size = 53, normalized size = 0.82

$$\frac{\log\left(-2x+1+\sqrt{5}\right)}{2}+\frac{\log\left(-2x-\sqrt{5}+1\right)}{2}-\frac{\log\left(2x+1+\sqrt{5}\right)}{2}-\frac{\log\left(2x-\sqrt{5}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/(x**4-3*x**2+1),x)
```

```
[Out] log(-2*x + 1 + sqrt(5))/2 + log(-2*x - sqrt(5) + 1)/2 - log(2*x +
 1 + sqrt(5))/2 - log(2*x - sqrt(5) + 1)/2
```

Mathematica [A] time = 0.00883409, size = 29, normalized size = 0.45

$$\frac{1}{2}\log\left(-x^2+x+1\right)-\frac{1}{2}\log\left(-x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/(1 - 3*x^2 + x^4),x]
```

```
[Out] -Log[1 - x - x^2]/2 + Log[1 + x - x^2]/2
```

Maple [A] time = 0.007, size = 22, normalized size = 0.3

$$\frac{\ln\left(x^2-x-1\right)}{2}-\frac{\ln\left(x^2+x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^4-3*x^2+1),x)

[Out] 1/2*ln(x^2-x-1)-1/2*ln(x^2+x-1)
```

Maxima [A] time = 0.765275, size = 28, normalized size = 0.43

$$-\frac{1}{2}\log\left(x^2+x-1\right)+\frac{1}{2}\log\left(x^2-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 - 3*x^2 + 1),x, algorithm="maxima")

[Out] -1/2*log(x^2 + x - 1) + 1/2*log(x^2 - x - 1)
```

Fricas [A] time = 0.28306, size = 28, normalized size = 0.43

$$-\frac{1}{2}\log\left(x^2+x-1\right)+\frac{1}{2}\log\left(x^2-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 - 3*x^2 + 1),x, algorithm="fricas")

[Out] -1/2*log(x^2 + x - 1) + 1/2*log(x^2 - x - 1)
```

Sympy [A] time = 0.190241, size = 19, normalized size = 0.29

$$\frac{\log\left(x^2-x-1\right)}{2}-\frac{\log\left(x^2+x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**4-3*x**2+1),x)

[Out] log(x**2 - x - 1)/2 - log(x**2 + x - 1)/2
```

GIAC/XCAS [A] time = 0.272327, size = 58, normalized size = 0.89

$$-\frac{1}{4}\ln\left(\left|x+\frac{1}{x-\frac{1}{x}}-\frac{1}{x}+2\right|\right)+\frac{1}{4}\ln\left(\left|x+\frac{1}{x-\frac{1}{x}}-\frac{1}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 - 3*x^2 + 1),x, algorithm="giac")

[Out] -1/4*ln(abs(x + 1/(x - 1/x) - 1/x + 2)) + 1/4*ln(abs(x + 1/(x - 1
/x) - 1/x - 2))
```

3.86 $\int \frac{1+x^2}{1-4x^2+x^4}\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\sqrt{3}-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{2}x+\sqrt{3}\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[3] - Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[3] + Sqrt[2]*
x]/Sqrt[2]
```

Rubi [A] time = 0.0643275, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\tanh^{-1}\left(\sqrt{3}-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{2}x+\sqrt{3}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 4*x^2 + x^4),x]
```

```
[Out] ArcTanh[Sqrt[3] - Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[3] + Sqrt[2]*
x]/Sqrt[2]
```

Rubi in Sympy [A] time = 7.97172, size = 44, normalized size = 1.02

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\left(x-\frac{\sqrt{6}}{2}\right)\right)}{2}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\left(x+\frac{\sqrt{6}}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2+1)/(x**4-4*x**2+1),x)
```

```
[Out] -sqrt(2)*atanh(sqrt(2)*(x - sqrt(6)/2))/2 - sqrt(2)*atanh(sqrt(2)
*(x + sqrt(6)/2))/2
```

Mathematica [A] time = 0.019222, size = 40, normalized size = 0.93

$$\frac{\log\left(-x^2+\sqrt{2}x+1\right)-\log\left(x^2+\sqrt{2}x-1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 4*x^2 + x^4),x]
```

```
[Out] (Log[1 + Sqrt[2]*x - x^2] - Log[-1 + Sqrt[2]*x + x^2])/(2*Sqrt[2]
)
```

Maple [B] time = 0.043, size = 70, normalized size = 1.6

$$-\frac{\left(-3+\sqrt{3}\right)\sqrt{3}}{3\sqrt{6}-3\sqrt{2}}\mathit{Artanh}\left(2\,\frac{x}{\sqrt{6}-\sqrt{2}}\right)-\frac{\sqrt{3}\left(\sqrt{3}+3\right)}{3\sqrt{6}+3\sqrt{2}}\mathit{Artanh}\left(2\,\frac{x}{\sqrt{6}+\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^4-4*x^2+1),x)
```

```
[Out] -1/3*(-3+3^(1/2))*3^(1/2)/(6^(1/2)-2^(1/2))*arctanh(2*x/(6^(1/2)-
2^(1/2)))-1/3*3^(1/2)*(3^(1/2)+3)/(6^(1/2)+2^(1/2))*arctanh(2*x/(
6^(1/2)+2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{x^4-4x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 - 4*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 - 4*x^2 + 1), x)
```

Fricas [A] time = 0.307083, size = 53, normalized size = 1.23

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{4x^3-\sqrt{2}\left(x^4+1\right)-4x}{x^4-4x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 - 4*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(4*x^3 - sqrt(2)*(x^4 + 1) - 4*x)/(x^4 - 4*x^2 +
 1))
```

Sympy [A] time = 0.193082, size = 39, normalized size = 0.91

$$\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x-1\right)}{4}-\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**4-4*x**2+1),x)
```

```
[Out] sqrt(2)*log(x**2 - sqrt(2)*x - 1)/4 - sqrt(2)*log(x**2 + sqrt(2)*
x - 1)/4
```

GIAC/XCAS [A] time = 0.280613, size = 53, normalized size = 1.23

$$\frac{1}{4}\sqrt{2}\ln\left(\frac{\left|2x-2\sqrt{2}-\frac{2}{x}\right|}{\left|2x+2\sqrt{2}-\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 - 4*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*ln(abs(2*x - 2*sqrt(2) - 2/x)/abs(2*x + 2*sqrt(2) - 2
/x))
```

3.87 $\int \frac{1+x^2}{1-5x^2+x^4} \, dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2x+\sqrt{7}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTanh[(Sqrt[7] - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTanh[(Sqrt[7] + 2*x
)/Sqrt[3]]/Sqrt[3]
```

Rubi [A] time = 0.0762049, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\tanh^{-1}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2x+\sqrt{7}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 5*x^2 + x^4),x]
```

```
[Out] ArcTanh[(Sqrt[7] - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTanh[(Sqrt[7] + 2*x
)/Sqrt[3]]/Sqrt[3]
```

Rubi in Sympy [A] time = 7.77546, size = 51, normalized size = 1.11

$$-\frac{\sqrt{3}\operatorname{atanh}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{\sqrt{7}}{3}\right)\right)}{3}-\frac{\sqrt{3}\operatorname{atanh}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{\sqrt{7}}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2+1)/(x**4-5*x**2+1),x)
```

```
[Out] -sqrt(3)*atanh(sqrt(3)*(2*x/3 - sqrt(7)/3))/3 - sqrt(3)*atanh(sqr
t(3)*(2*x/3 + sqrt(7)/3))/3
```

Mathematica [A] time = 0.0189088, size = 40, normalized size = 0.87

$$\frac{\log\left(-x^2+\sqrt{3}x+1\right)-\log\left(x^2+\sqrt{3}x-1\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 5*x^2 + x^4),x]
```

```
[Out] (Log[1 + Sqrt[3]*x - x^2] - Log[-1 + Sqrt[3]*x + x^2])/(2*Sqrt[3]
)
```

Maple [B] time = 0.045, size = 82, normalized size = 1.8

$$-\frac{\left(14+2\sqrt{21}\right)\sqrt{21}}{42\sqrt{7}+42\sqrt{3}}\mathit{Artanh}\left(4\frac{x}{2\sqrt{7}+2\sqrt{3}}\right)-\frac{2\sqrt{21}\left(-7+\sqrt{21}\right)}{42\sqrt{7}-42\sqrt{3}}\mathit{Artanh}\left(4\frac{x}{2\sqrt{7}-2\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^4-5*x^2+1),x)
```

```
[Out] -2/21*(7+21^(1/2))*21^(1/2)/(2*7^(1/2)+2*3^(1/2))*arctanh(4*x/(2*
7^(1/2)+2*3^(1/2)))-2/21*21^(1/2)*(-7+21^(1/2))/(2*7^(1/2)-2*3^(1
/2))*arctanh(4*x/(2*7^(1/2)-2*3^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{x^4-5\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 - 5*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 - 5*x^2 + 1), x)
```

Fricas [A] time = 0.288182, size = 57, normalized size = 1.24

$$\frac{1}{6}\,\sqrt{3}\log\left(-\frac{6\,x^3-\sqrt{3}\left(x^4+x^2+1\right)-6\,x}{x^4-5\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 - 5*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(6*x^3 - sqrt(3)*(x^4 + x^2 + 1) - 6*x)/(x^4 - 5
*x^2 + 1))
```

Sympy [A] time = 0.199197, size = 39, normalized size = 0.85

$$\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x-1\right)}{6}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**4-5*x**2+1),x)
```

```
[Out] sqrt(3)*log(x**2 - sqrt(3)*x - 1)/6 - sqrt(3)*log(x**2 + sqrt(3)*
x - 1)/6
```

GIAC/XCAS [A] time = 0.279095, size = 53, normalized size = 1.15

$$\frac{1}{6}\,\sqrt{3}\ln\left(\frac{\left|2\,x-2\,\sqrt{3}-\frac{2}{x}\right|}{\left|2\,x+2\,\sqrt{3}-\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 - 5*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*ln(abs(2*x - 2*sqrt(3) - 2/x)/abs(2*x + 2*sqrt(3) - 2
/x))
```

3.88 $\int \frac{1-x^2}{1+bx^2+x^4}\,dx$

Optimal. Leaf size=62

$$\frac{\log\left(\sqrt{2-b}x+x^2+1\right)}{2\sqrt{2-b}}-\frac{\log\left(-\sqrt{2-b}x+x^2+1\right)}{2\sqrt{2-b}}$$

```
[Out] -Log[1 - Sqrt[2 - b]*x + x^2]/(2*Sqrt[2 - b]) + Log[1 + Sqrt[2 -
b]*x + x^2]/(2*Sqrt[2 - b])
```

Rubi [A] time = 0.0556812, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{\log\left(\sqrt{2-b}x+x^2+1\right)}{2\sqrt{2-b}}-\frac{\log\left(-\sqrt{2-b}x+x^2+1\right)}{2\sqrt{2-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] -Log[1 - Sqrt[2 - b]*x + x^2]/(2*Sqrt[2 - b]) + Log[1 + Sqrt[2 -
b]*x + x^2]/(2*Sqrt[2 - b])
```

Rubi in Sympy [A] time = 13.903, size = 46, normalized size = 0.74

$$-\frac{\log\left(x^2-x\sqrt{-b+2}+1\right)}{2\sqrt{-b+2}}+\frac{\log\left(x^2+x\sqrt{-b+2}+1\right)}{2\sqrt{-b+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)/(x**4+b*x**2+1),x)
```

```
[Out] -log(x**2 - x*sqrt(-b + 2) + 1)/(2*sqrt(-b + 2)) + log(x**2 + x*s
qrt(-b + 2) + 1)/(2*sqrt(-b + 2))
```

Mathematica [B] time = 0.125112, size = 125, normalized size = 2.02

$$\frac{\frac{\left(-\sqrt{b^2-4}+b+2\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{b^2-4}}}\right)}{\sqrt{b-\sqrt{b^2-4}}}-\frac{\left(\sqrt{b^2-4}+b+2\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{\sqrt{b^2-4}+b}}}{\sqrt{2}\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] (((2 + b - Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 +
b^2]]])/Sqrt[b - Sqrt[-4 + b^2]] - ((2 + b + Sqrt[-4 + b^2])*ArcT
an[(Sqrt[2]*x)/Sqrt[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]
])/(Sqrt[2]*Sqrt[-4 + b^2])
```

Maple [B] time = 0.024, size = 279, normalized size = 4.5

$$2\frac{1}{\sqrt{(b-2)(2+b)}\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\arctan\left(2\frac{x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)$$
$$-1\arctan\left(2\frac{x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)\frac{1}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}$$
$$+b\arctan\left(2\frac{x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)\frac{1}{\sqrt{(b-2)(2+b)}}\frac{1}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}$$
$$-2\frac{1}{\sqrt{(b-2)(2+b)}\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)$$
$$-1\arctan\left(2\frac{x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)\frac{1}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}$$
$$-b\arctan\left(2\frac{x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)\frac{1}{\sqrt{(b-2)(2+b)}}\frac{1}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((-x^2+1)/(x^4+b*x^2+1),x)
```

```
[Out] 2/((b-2)*(2+b))^(1/2)/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2
*x/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))-1/(-2*((b-2)*(2+b))^(1/2)+
2*b)^(1/2)*arctan(2*x/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))+1/((b-2
)*(2+b))^(1/2)/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*
((b-2)*(2+b))^(1/2)+2*b)^(1/2))*b-2/((b-2)*(2+b))^(1/2)/(2*((b-2)
*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((b-2)*(2+b))^(1/2)+2*b)^(
1/2))-1/(2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((b-2)*(2
+b))^(1/2)+2*b)^(1/2))-1/((b-2)*(2+b))^(1/2)/(2*((b-2)*(2+b))^(1/
2)+2*b)^(1/2)*arctan(2*x/(2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2-1}{x^4+bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x^2 - 1)/(x^4 + b*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 + b*x^2 + 1), x)
```

Fricas [A] time = 0.292533, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{2(b-2)x^3+2(b-2)x-(x^4-(b-4)x^2+1)\sqrt{-b+2}}{x^4+bx^2+1}\right)}{2\sqrt{-b+2}},\ \frac{\arctan\left(\frac{x^3+(b-1)x}{\sqrt{b-2}}\right)-\arctan\left(\frac{x}{\sqrt{b-2}}\right)}{\sqrt{b-2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 + b*x^2 + 1),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*(b - 2)*x^3 + 2*(b - 2)*x - (x^4 - (b - 4)*x^2 + 1)*
sqrt(-b + 2))/(x^4 + b*x^2 + 1))/sqrt(-b + 2), (arctan((x^3 + (b
- 1)*x)/sqrt(b - 2)) - arctan(x/sqrt(b - 2)))/sqrt(b - 2)]
```

Sympy [A] time = 0.736163, size = 87, normalized size = 1.4

$$\frac{\sqrt{-\frac{1}{b-2}}\log\left(x^2+x\left(-b\sqrt{-\frac{1}{b-2}}+2\sqrt{-\frac{1}{b-2}}\right)+1\right)}{2}-\frac{\sqrt{-\frac{1}{b-2}}\log\left(x^2+x\left(b\sqrt{-\frac{1}{b-2}}-2\sqrt{-\frac{1}{b-2}}\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)/(x**4+b*x**2+1),x)
```

```
[Out] sqrt(-1/(b - 2))*log(x**2 + x*(-b*sqrt(-1/(b - 2)) + 2*sqrt(-1/(b
 - 2))) + 1)/2 - sqrt(-1/(b - 2))*log(x**2 + x*(b*sqrt(-1/(b - 2)
) - 2*sqrt(-1/(b - 2))) + 1)/2
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 + b*x^2 + 1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.89 $\int \frac{1-x^2}{1+5x^2+x^4} \, dx$

Optimal. Leaf size=50

$$\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[Sqrt[2/(5 + Sqrt[21])]*x]/Sqrt[3]) + ArcTan[Sqrt[(5 + Sq
rt[21])/2]*x]/Sqrt[3]
```

Rubi [A] time = 0.128619, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + 5*x^2 + x^4),x]
```

```
[Out] -(ArcTan[Sqrt[2/(5 + Sqrt[21])]*x]/Sqrt[3]) + ArcTan[Sqrt[(5 + Sq
rt[21])/2]*x]/Sqrt[3]
```

Rubi in Sympy [A] time = 9.36661, size = 88, normalized size = 1.76

$$-\frac{\sqrt{2}\left(-\frac{\sqrt{21}}{6}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{-\sqrt{21}+5}}\right)}{\sqrt{-\sqrt{21}+5}}-\frac{\sqrt{2}\left(\frac{1}{2}+\frac{\sqrt{21}}{6}\right)\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{21}+5}}\right)}{\sqrt{\sqrt{21}+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-x**2+1)/(x**4+5*x**2+1),x)
```

```
[Out] -sqrt(2)*(-sqrt(21)/6 + 1/2)*atan(sqrt(2)*x/sqrt(-sqrt(21) + 5))/
sqrt(-sqrt(21) + 5) - sqrt(2)*(1/2 + sqrt(21)/6)*atan(sqrt(2)*x/s
qrt(sqrt(21) + 5))/sqrt(sqrt(21) + 5)
```

Mathematica [A] time = 0.216824, size = 87, normalized size = 1.74

$$\frac{\left(7-\sqrt{21}\right)\tan^{-1}\left(\sqrt{\frac{2}{5-\sqrt{21}}}x\right)}{\sqrt{42\left(5-\sqrt{21}\right)}}+\frac{\left(-7-\sqrt{21}\right)\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{42\left(5+\sqrt{21}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + 5*x^2 + x^4),x]
```

```
[Out] ((7 - Sqrt[21])*ArcTan[Sqrt[2/(5 - Sqrt[21])]*x])/Sqrt[42*(5 - Sq
rt[21])] + ((-7 - Sqrt[21])*ArcTan[Sqrt[2/(5 + Sqrt[21])]*x])/Sqr
t[42*(5 + Sqrt[21])]
```

Maple [B] time = 0.017, size = 136, normalized size = 2.7

$$-\frac{2\sqrt{21}}{6\sqrt{7}+6\sqrt{3}}\arctan\left(4\frac{x}{2\sqrt{7}+2\sqrt{3}}\right)-2\frac{1}{2\sqrt{7}+2\sqrt{3}}\arctan\left(4\frac{x}{2\sqrt{7}+2\sqrt{3}}\right)+\frac{2\sqrt{21}}{6\sqrt{7}-6\sqrt{3}}\arctan\left(4\frac{x}{2\sqrt{7}-2\sqrt{3}}\right)-2\frac{1}{2\sqrt{7}-2\sqrt{3}}\arctan\left(4\frac{x}{2\sqrt{7}-2\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((-x^2+1)/(x^4+5*x^2+1),x)
```

```
[Out] -2/3*21^(1/2)/(2*7^(1/2)+2*3^(1/2))*arctan(4*x/(2*7^(1/2)+2*3^(1/
2)))-2/(2*7^(1/2)+2*3^(1/2))*arctan(4*x/(2*7^(1/2)+2*3^(1/2)))+2/
3*21^(1/2)/(2*7^(1/2)-2*3^(1/2))*arctan(4*x/(2*7^(1/2)-2*3^(1/2))
)-2/(2*7^(1/2)-2*3^(1/2))*arctan(4*x/(2*7^(1/2)-2*3^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2-1}{x^4+5\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x^2 - 1)/(x^4 + 5*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 + 5*x^2 + 1), x)
```

Fricas [A] time = 0.273551, size = 38, normalized size = 0.76

$$\frac{1}{3}\sqrt{3}\left(\arctan\left(\frac{1}{3}\sqrt{3}\left(x^3+4\,x\right)\right)-\arctan\left(\frac{1}{3}\sqrt{3}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x^2 - 1)/(x^4 + 5*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*(arctan(1/3*sqrt(3)*(x^3 + 4*x)) - arctan(1/3*sqrt(3)
*x))
```

Sympy [A] time = 0.254755, size = 42, normalized size = 0.84

$$-\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right)-2\,\mathrm{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{4\sqrt{3}x}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((-x**2+1)/(x**4+5*x**2+1),x)
```

```
[Out] -sqrt(3)*(2*atan(sqrt(3)*x/3) - 2*atan(sqrt(3)*x**3/3 + 4*sqrt(3)
*x/3))/6
```

GIAC/XCAS [A] time = 0.277505, size = 35, normalized size = 0.7

$$\frac{1}{6}\sqrt{3}\left(\pi\operatorname{sign}\left(x\right)-2\arctan\left(\frac{\sqrt{3}\left(x^{2}+1\right)}{3x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 + 5*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(pi*sign(x) - 2*arctan(1/3*sqrt(3)*(x^2 + 1)/x))
```

3.90 $\int \frac{1-x^2}{1+4x^2+x^4}\,dx$

Optimal. Leaf size=44

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}$$

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[2] - ArcTan[x/Sqrt[2 + Sqrt[3]]]
/Sqrt[2]
```

Rubi [A] time = 0.0655146, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^2)/(1 + 4*x^2 + x^4),x]
```

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[2] - ArcTan[x/Sqrt[2 + Sqrt[3]]]
/Sqrt[2]
```

Rubi in Sympy [A] time = 8.18728, size = 68, normalized size = 1.55

$$-\frac{\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{x}{\sqrt{-\sqrt{3}+2}}\right)}{\sqrt{-\sqrt{3}+2}}-\frac{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\operatorname{atan}\left(\frac{x}{\sqrt{\sqrt{3}+2}}\right)}{\sqrt{\sqrt{3}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)/(x**4+4*x**2+1),x)
```

```
[Out] -(-sqrt(3)/2 + 1/2)*atan(x/sqrt(-sqrt(3) + 2))/sqrt(-sqrt(3) + 2)
 - (1/2 + sqrt(3)/2)*atan(x/sqrt(sqrt(3) + 2))/sqrt(sqrt(3) + 2)
```

Mathematica [A] time = 0.125505, size = 82, normalized size = 1.86

$$\frac{-\left(\sqrt{3}-3\right)\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)-\sqrt{2-\sqrt{3}}\left(3+\sqrt{3}\right)\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^2)/(1 + 4*x^2 + x^4),x]
```

```
[Out] (-((-3 + Sqrt[3])*Sqrt[2 + Sqrt[3]]*ArcTan[x/Sqrt[2 - Sqrt[3]]])
- Sqrt[2 - Sqrt[3]]*(3 + Sqrt[3])*ArcTan[x/Sqrt[2 + Sqrt[3]]])/(2
*Sqrt[3])
```

Maple [B] time = 0.017, size = 111, normalized size = 2.5

$$-\frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}\arctan\left(2\frac{x}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{\sqrt{6}+\sqrt{2}}\arctan\left(2\frac{x}{\sqrt{6}+\sqrt{2}}\right)+\frac{\sqrt{3}}{\sqrt{6}-\sqrt{2}}\arctan\left(2\frac{x}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{\sqrt{6}-\sqrt{2}}\arctan\left(2\frac{x}{\sqrt{6}-\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)/(x^4+4*x^2+1),x)
```

```
[Out] -3^(1/2)/(6^(1/2)+2^(1/2))*arctan(2*x/(6^(1/2)+2^(1/2)))-1/(6^(1/
2)+2^(1/2))*arctan(2*x/(6^(1/2)+2^(1/2)))+3^(1/2)/(6^(1/2)-2^(1/2
))*arctan(2*x/(6^(1/2)-2^(1/2)))-1/(6^(1/2)-2^(1/2))*arctan(2*x/(
6^(1/2)-2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-1}{x^4+4x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 + 4*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 + 4*x^2 + 1), x)
```

Fricas [A] time = 0.282871, size = 38, normalized size = 0.86

$$\frac{1}{2}\sqrt{2}\left(\arctan\left(\frac{1}{2}\sqrt{2}\left(x^3+3x\right)\right)-\arctan\left(\frac{1}{2}\sqrt{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 + 4*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*(arctan(1/2*sqrt(2)*(x^3 + 3*x)) - arctan(1/2*sqrt(2)
*x))
```

Sympy [A] time = 0.242035, size = 42, normalized size = 0.95

$$-\frac{\sqrt{2}\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)-2\operatorname{atan}\left(\frac{\sqrt{2}x^3}{2}+\frac{3\sqrt{2}x}{2}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)/(x**4+4*x**2+1),x)
```

```
[Out] -sqrt(2)*(2*atan(sqrt(2)*x/2) - 2*atan(sqrt(2)*x**3/2 + 3*sqrt(2)
*x/2))/4
```

GIAC/XCAS [A] time = 0.275714, size = 35, normalized size = 0.8

$$\frac{1}{4}\sqrt{2}\left(\pi\operatorname{sign}(x) - 2\arctan\left(\frac{\sqrt{2}(x^2+1)}{2x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 + 4*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(pi*sign(x) - 2*arctan(1/2*sqrt(2)*(x^2 + 1)/x))
```

3.91 $\int \frac{1-x^2}{1+3x^2+x^4}\,dx$

Optimal. Leaf size=39

$$\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)-\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

```
[Out] -ArcTan[Sqrt[2/(3 + Sqrt[5])]*x] + ArcTan[Sqrt[(3 + Sqrt[5])/2]*x
]
```

Rubi [A] time = 0.0922117, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)-\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[2/(3 + Sqrt[5])]*x] + ArcTan[Sqrt[(3 + Sqrt[5])/2]*x
]
```

Rubi in Sympy [A] time = 9.2912, size = 88, normalized size = 2.26

$$-\frac{\sqrt{2}\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{-\sqrt{5}+3}}\right)}{\sqrt{-\sqrt{5}+3}}-\frac{\sqrt{2}\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{5}+3}}\right)}{\sqrt{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)/(x**4+3*x**2+1),x)
```

```
[Out] -sqrt(2)*(-sqrt(5)/2 + 1/2)*atan(sqrt(2)*x/sqrt(-sqrt(5) + 3))/sq
rt(-sqrt(5) + 3) - sqrt(2)*(1/2 + sqrt(5)/2)*atan(sqrt(2)*x/sqrt(
sqrt(5) + 3))/sqrt(sqrt(5) + 3)
```

Mathematica [A] time = 0.0100228, size = 10, normalized size = 0.26

$$\tan^{-1}\left(\frac{x}{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] ArcTan[x/(1 + x^2)]
```

Maple [B] time = 0.017, size = 104, normalized size = 2.7

$$-2\,\frac{\sqrt{5}}{2\sqrt{5}+2}\arctan\left(4\,\frac{x}{2\sqrt{5}+2}\right)-2\,\frac{1}{2\sqrt{5}+2}\arctan\left(4\,\frac{x}{2\sqrt{5}+2}\right)$$
$$+2\,\frac{\sqrt{5}}{-2+2\sqrt{5}}\arctan\left(4\,\frac{x}{-2+2\sqrt{5}}\right)-2\,\frac{1}{-2+2\sqrt{5}}\arctan\left(4\,\frac{x}{-2+2\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((-x^2+1)/(x^4+3*x^2+1),x)
```

```
[Out] -2*5^(1/2)/(2*5^(1/2)+2)*arctan(4*x/(2*5^(1/2)+2))-2/(2*5^(1/2)+2
)*arctan(4*x/(2*5^(1/2)+2))+2*5^(1/2)/(-2+2*5^(1/2))*arctan(4*x/(
-2+2*5^(1/2)))-2/(-2+2*5^(1/2))*arctan(4*x/(-2+2*5^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - 1}{x^4 + 3x^2 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x^2 - 1)/(x^4 + 3*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 + 3*x^2 + 1), x)
```

Fricas [A] time = 0.264358, size = 18, normalized size = 0.46

$$\arctan\left(x^3 + 2x\right) - \arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x^2 - 1)/(x^4 + 3*x^2 + 1),x, algorithm="fricas")
```

```
[Out] arctan(x^3 + 2*x) - arctan(x)
```

Sympy [A] time = 0.201657, size = 10, normalized size = 0.26

$$-\operatorname{atan}(x) + \operatorname{atan}\left(x^3 + 2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((-x**2+1)/(x**4+3*x**2+1),x)
```

```
[Out] -atan(x) + atan(x**3 + 2*x)
```

GIAC/XCAS [A] time = 0.274356, size = 35, normalized size = 0.9

$$\frac{1}{4}\pi\operatorname{sign}(x) - \frac{1}{2}\arctan\left(\frac{x^4 + x^2 + 1}{2(x^3 + x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x^2 - 1)/(x^4 + 3*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/4*pi*sign(x) - 1/2*arctan(1/2*(x^4 + x^2 + 1)/(x^3 + x))
```

3.92 $\int \frac{1-x^2}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=9

$$\frac{x}{x^2 + 1}$$

```
[Out] x/(1 + x^2)
```

Rubi [A] time = 0.00740825, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{x}{x^2 + 1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + 2*x^2 + x^4),x]

[Out] x/(1 + x^2)
```

Rubi in Sympy [A] time = 5.4246, size = 5, normalized size = 0.56

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-x**2+1)/(x**4+2*x**2+1),x)

[Out] x/(x**2 + 1)
```

Mathematica [A] time = 0.00695003, size = 9, normalized size = 1.

$$\frac{x}{x^2 + 1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + 2*x^2 + x^4),x]

[Out] x/(1 + x^2)
```

Maple [A] time = 0.008, size = 10, normalized size = 1.1

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+2*x^2+1),x)

[Out] x/(x^2+1)
```

Maxima [A] time = 0.736815, size = 12, normalized size = 1.33

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 + 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] x/(x^2 + 1)
```

Fricas [A] time = 0.293635, size = 12, normalized size = 1.33

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 + 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] x/(x^2 + 1)
```

Sympy [A] time = 0.156196, size = 5, normalized size = 0.56

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)/(x**4+2*x**2+1),x)
```

```
[Out] x/(x**2 + 1)
```

GIAC/XCAS [A] time = 0.270084, size = 9, normalized size = 1.

$$\frac{1}{x + \frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 + 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/(x + 1/x)
```

3.93 $\int \frac{1-x^2}{1+x^2+x^4} \, dx$

Optimal. Leaf size=25

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

```
[Out] -Log[1 - x + x^2]/2 + Log[1 + x + x^2]/2
```

Rubi [A] time = 0.0256313, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^2)/(1 + x^2 + x^4),x]

[Out] -Log[1 - x + x^2]/2 + Log[1 + x + x^2]/2
```

Rubi in Sympy [A] time = 9.75505, size = 19, normalized size = 0.76

$$-\frac{\log\left(x^2-x+1\right)}{2}+\frac{\log\left(x^2+x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)/(x**4+x**2+1),x)

[Out] -log(x**2 - x + 1)/2 + log(x**2 + x + 1)/2
```

Mathematica [A] time = 0.00822612, size = 25, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^2)/(1 + x^2 + x^4),x]

[Out] -Log[1 - x + x^2]/2 + Log[1 + x + x^2]/2
```

Maple [A] time = 0.005, size = 22, normalized size = 0.9

$$-\frac{\ln\left(x^2-x+1\right)}{2}+\frac{\ln\left(x^2+x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)/(x^4+x^2+1),x)
```

```
[Out] -1/2*ln(x^2-x+1)+1/2*ln(x^2+x+1)
```

Maxima [A] time = 0.750881, size = 28, normalized size = 1.12

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 + x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^2 + x + 1) - 1/2*log(x^2 - x + 1)
```

Fricas [A] time = 0.275494, size = 28, normalized size = 1.12

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 + x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 + x + 1) - 1/2*log(x^2 - x + 1)
```

Sympy [A] time = 0.187626, size = 19, normalized size = 0.76

$$-\frac{\log\left(x^2-x+1\right)}{2}+\frac{\log\left(x^2+x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)/(x**4+x**2+1),x)
```

```
[Out] -log(x**2 - x + 1)/2 + log(x**2 + x + 1)/2
```

GIAC/XCAS [A] time = 0.272406, size = 47, normalized size = 1.88

$$\frac{1}{4}\ln\left(\left|x+\frac{1}{x+\frac{1}{x}}+\frac{1}{x}+2\right|\right)-\frac{1}{4}\ln\left(\left|x+\frac{1}{x+\frac{1}{x}}+\frac{1}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 + x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/4*ln(abs(x + 1/(x + 1/x) + 1/x + 2)) - 1/4*ln(abs(x + 1/(x + 1/
x) + 1/x - 2))
```

3.94 $\int \frac{1-x^2}{1+x^4}\,dx$

Optimal. Leaf size=46

$$\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}$$

```
[Out] -Log[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/
(2*Sqrt[2])
```

Rubi [A] time = 0.0373196, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + x^4),x]
```

```
[Out] -Log[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/
(2*Sqrt[2])
```

Rubi in Sympy [A] time = 9.39688, size = 39, normalized size = 0.85

$$-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{4}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-x**2+1)/(x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/4 + sqrt(2)*log(x**2 + sqrt(2)
*x + 1)/4
```

Mathematica [A] time = 0.0172064, size = 40, normalized size = 0.87

$$\frac{\log\left(x^2+\sqrt{2}x+1\right)-\log\left(-x^2+\sqrt{2}x-1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[2]*x - x^2] + Log[1 + Sqrt[2]*x + x^2])/(2*Sqrt[2
])
```

Maple [A] time = 0.003, size = 62, normalized size = 1.4

$$\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2+\sqrt{2}x}{1+x^2-\sqrt{2}x}\right)-\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2-\sqrt{2}x}{1+x^2+\sqrt{2}x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)/(x^4+1),x)
```

```
[Out] 1/8*2^(1/2)*ln((1+x^2+2^(1/2)*x)/(1+x^2-2^(1/2)*x))-1/8*2^(1/2)*l
n((1+x^2-2^(1/2)*x)/(1+x^2+2^(1/2)*x))
```

Maxima [A] time = 0.811743, size = 46, normalized size = 1.

$$\frac{1}{4}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{4}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/4*sqrt(2)*log(x^2 - sqrt
(2)*x + 1)
```

Fricas [A] time = 0.278512, size = 50, normalized size = 1.09

$$\frac{1}{4}\sqrt{2}\log\left(\frac{4\,x^3+\sqrt{2}\left(x^4+4\,x^2+1\right)+4\,x}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((4*x^3 + sqrt(2)*(x^4 + 4*x^2 + 1) + 4*x)/(x^4 +
1))
```

Sympy [A] time = 0.182918, size = 39, normalized size = 0.85

$$-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{4}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)/(x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/4 + sqrt(2)*log(x**2 + sqrt(2)
*x + 1)/4
```

GIAC/XCAS [A] time = 0.273028, size = 46, normalized size = 1.

$$\frac{1}{4}\sqrt{2}\ln\left(x^2+\sqrt{2}x+1\right)-\frac{1}{4}\sqrt{2}\ln\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*ln(x^2 + sqrt(2)*x + 1) - 1/4*sqrt(2)*ln(x^2 - sqrt(2
)*x + 1)
```

3.95 $\int \frac{1-x^2}{1-x^2+x^4} \, dx$

Optimal. Leaf size=46

$$\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}$$

```
[Out] -Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/
(2*Sqrt[3])
```

Rubi [A] time = 0.0400427, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^2)/(1 - x^2 + x^4),x]
```

```
[Out] -Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/
(2*Sqrt[3])
```

Rubi in Sympy [A] time = 12.8177, size = 39, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{6}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)/(x**4-x**2+1),x)
```

```
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x + 1)/6 + sqrt(3)*log(x**2 + sqrt(3)
*x + 1)/6
```

Mathematica [A] time = 0.02027, size = 40, normalized size = 0.87

$$\frac{\log\left(x^2+\sqrt{3}x+1\right)-\log\left(-x^2+\sqrt{3}x-1\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^2)/(1 - x^2 + x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[3]*x - x^2] + Log[1 + Sqrt[3]*x + x^2])/(2*Sqrt[3
])
```

Maple [A] time = 0.014, size = 35, normalized size = 0.8

$$-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)/(x^4-x^2+1),x)
```

```
[Out] -1/6*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 - x^2 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

Fricas [A] time = 0.290388, size = 57, normalized size = 1.24

$$\frac{1}{6}\sqrt{3}\log\left(\frac{6x^3+\sqrt{3}\left(x^4+5x^2+1\right)+6x}{x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 - x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((6*x^3 + sqrt(3)*(x^4 + 5*x^2 + 1) + 6*x)/(x^4 -
x^2 + 1))
```

Sympy [A] time = 0.197104, size = 39, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{6}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)/(x**4-x**2+1),x)
```

```
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x + 1)/6 + sqrt(3)*log(x**2 + sqrt(3)
*x + 1)/6
```

GIAC/XCAS [A] time = 0.27022, size = 53, normalized size = 1.15

$$-\frac{1}{6}\sqrt{3}\ln\left(\frac{\left|2x-2\sqrt{3}+\frac{2}{x}\right|}{\left|2x+2\sqrt{3}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 - x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*ln(abs(2*x - 2*sqrt(3) + 2/x)/abs(2*x + 2*sqrt(3) +
2/x))
```

3.96 $\int \frac{1-x^2}{1-2x^2+x^4} \, dx$

Optimal. Leaf size=2

$$\tanh^{-1}(x)$$

```
[Out] ArcTanh[x]
```

Rubi [A] time = 0.00582081, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] ArcTanh[x]
```

Rubi in Sympy [A] time = 3.88464, size = 2, normalized size = 1.

$$\operatorname{atanh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)/(x**4-2*x**2+1),x)
```

```
[Out] atanh(x)
```

Mathematica [B] time = 0.00318543, size = 19, normalized size = 9.5

$$\frac{1}{2}\log(x+1) - \frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] -Log[1 - x]/2 + Log[1 + x]/2
```

Maple [A] time = 0.001, size = 3, normalized size = 1.5

$$\mathit{Artanh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)/(x^4-2*x^2+1),x)
```

```
[Out] arctanh(x)
```

Maxima [A] time = 0.735299, size = 18, normalized size = 9.

$$\frac{1}{2}\,\log\left(x+1\right)-\frac{1}{2}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 - 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [A] time = 0.280422, size = 18, normalized size = 9.

$$\frac{1}{2}\,\log\left(x+1\right)-\frac{1}{2}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 - 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [A] time = 0.166231, size = 12, normalized size = 6.

$$-\frac{\log\left(x-1\right)}{2}+\frac{\log\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)/(x**4-2*x**2+1),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2
```

GIAC/XCAS [A] time = 0.269551, size = 20, normalized size = 10.

$$\frac{1}{2}\,\ln\left(|x+1|\right)-\frac{1}{2}\,\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 - 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(x + 1)) - 1/2*ln(abs(x - 1))
```

3.97 $\int \frac{1-x^2}{1-3x^2+x^4}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{2x+1}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] -(ArcTanh[(1 - 2*x)/Sqrt[5]]/Sqrt[5]) + ArcTanh[(1 + 2*x)/Sqrt[5]
]/Sqrt[5]
```

Rubi [A] time = 0.0637345, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{\tanh^{-1}\left(\frac{2x+1}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^2)/(1 - 3*x^2 + x^4),x]
```

```
[Out] -(ArcTanh[(1 - 2*x)/Sqrt[5]]/Sqrt[5]) + ArcTanh[(1 + 2*x)/Sqrt[5]
]/Sqrt[5]
```

Rubi in Sympy [A] time = 7.90534, size = 46, normalized size = 1.21

$$-\frac{\sqrt{5}\,\operatorname{atanh}\left(\sqrt{5}\left(-\frac{2x}{5}-\frac{1}{5}\right)\right)}{5}-\frac{\sqrt{5}\,\operatorname{atanh}\left(\sqrt{5}\left(-\frac{2x}{5}+\frac{1}{5}\right)\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)/(x**4-3*x**2+1),x)
```

```
[Out] -sqrt(5)*atanh(sqrt(5)*(-2*x/5 - 1/5))/5 - sqrt(5)*atanh(sqrt(5)*
(-2*x/5 + 1/5))/5
```

Mathematica [A] time = 0.0230205, size = 40, normalized size = 1.05

$$\frac{\log\left(x^2+\sqrt{5}x+1\right)-\log\left(-x^2+\sqrt{5}x-1\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^2)/(1 - 3*x^2 + x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[5]*x - x^2] + Log[1 + Sqrt[5]*x + x^2])/(2*Sqrt[5
])
```

Maple [A] time = 0.005, size = 34, normalized size = 0.9

$$\frac{\sqrt{5}}{5}\mathit{Artanh}\left(\frac{(2\,x-1)\sqrt{5}}{5}\right)+\frac{\sqrt{5}}{5}\mathit{Artanh}\left(\frac{(1+2\,x)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((-x^2+1)/(x^4-3*x^2+1),x)
```

```
[Out] 1/5*5^(1/2)*arctanh(1/5*(2*x-1)*5^(1/2))+1/5*arctanh(1/5*(1+2*x)*
5^(1/2))*5^(1/2)
```

Maxima [A] time = 0.828843, size = 74, normalized size = 1.95

$$-\frac{1}{10}\sqrt{5}\log\left(\frac{2x-\sqrt{5}+1}{2x+\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{5}\log\left(\frac{2x-\sqrt{5}-1}{2x+\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x^2 - 1)/(x^4 - 3*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/10
*sqrt(5)*log((2*x - sqrt(5) - 1)/(2*x + sqrt(5) - 1))
```

Fricas [A] time = 0.287555, size = 57, normalized size = 1.5

$$\frac{1}{10}\sqrt{5}\log\left(\frac{10x^3+\sqrt{5}\left(x^4+7x^2+1\right)+10x}{x^4-3x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x^2 - 1)/(x^4 - 3*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*log((10*x^3 + sqrt(5)*(x^4 + 7*x^2 + 1) + 10*x)/(x^4
 - 3*x^2 + 1))
```

Sympy [A] time = 0.205419, size = 39, normalized size = 1.03

$$-\frac{\sqrt{5}\log\left(x^2-\sqrt{5}x+1\right)}{10}+\frac{\sqrt{5}\log\left(x^2+\sqrt{5}x+1\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((-x**2+1)/(x**4-3*x**2+1),x)
```

```
[Out] -sqrt(5)*log(x**2 - sqrt(5)*x + 1)/10 + sqrt(5)*log(x**2 + sqrt(5
)*x + 1)/10
```

GIAC/XCAS [A] time = 0.270349, size = 53, normalized size = 1.39

$$-\frac{1}{10}\sqrt{5}\ln\left(\frac{\left|2x-2\sqrt{5}+\frac{2}{x}\right|}{\left|2x+2\sqrt{5}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x^2 - 1)/(x^4 - 3*x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5)*ln(abs(2*x - 2*sqrt(5) + 2/x)/abs(2*x + 2*sqrt(5) +
 2/x))
```

3.98 $\int \frac{1-x^2}{1-4x^2+x^4}\,dx$

Optimal. Leaf size=47

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x+1}{\sqrt{3}}\right)}{\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{1-\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

```
[Out] -(ArcTanh[(1 - Sqrt[2]*x)/Sqrt[3]]/Sqrt[6]) + ArcTanh[(1 + Sqrt[2
]*x)/Sqrt[3]]/Sqrt[6]
```

Rubi [A] time = 0.0785488, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x+1}{\sqrt{3}}\right)}{\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{1-\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 - 4*x^2 + x^4),x]
```

```
[Out] -(ArcTanh[(1 - Sqrt[2]*x)/Sqrt[3]]/Sqrt[6]) + ArcTanh[(1 + Sqrt[2
]*x)/Sqrt[3]]/Sqrt[6]
```

Rubi in Sympy [A] time = 8.3762, size = 49, normalized size = 1.04

$$-\frac{\sqrt{6}\,\mathrm{atanh}\left(\sqrt{6}\left(-\frac{x}{3}-\frac{\sqrt{2}}{6}\right)\right)}{6}-\frac{\sqrt{6}\,\mathrm{atanh}\left(\sqrt{6}\left(-\frac{x}{3}+\frac{\sqrt{2}}{6}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-x**2+1)/(x**4-4*x**2+1),x)
```

```
[Out] -sqrt(6)*atanh(sqrt(6)*(-x/3 - sqrt(2)/6))/6 - sqrt(6)*atanh(sqrt
(6)*(-x/3 + sqrt(2)/6))/6
```

Mathematica [A] time = 0.0305654, size = 40, normalized size = 0.85

$$\frac{\log\left(x^2+\sqrt{6}x+1\right)-\log\left(-x^2+\sqrt{6}x-1\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 - 4*x^2 + x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[6]*x - x^2] + Log[1 + Sqrt[6]*x + x^2])/(2*Sqrt[6
])
```

Maple [A] time = 0.016, size = 70, normalized size = 1.5

$$\frac{\left(\sqrt{3}-1\right)\sqrt{3}}{3\sqrt{6}-3\sqrt{2}}\mathit{Artanh}\left(2\frac{x}{\sqrt{6}-\sqrt{2}}\right)+\frac{\left(1+\sqrt{3}\right)\sqrt{3}}{3\sqrt{6}+3\sqrt{2}}\mathit{Artanh}\left(2\frac{x}{\sqrt{6}+\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)/(x^4-4*x^2+1),x)
```

```
[Out] 1/3*(3^(1/2)-1)*3^(1/2)/(6^(1/2)-2^(1/2))*arctanh(2*x/(6^(1/2)-2^
(1/2)))+1/3*(1+3^(1/2))*3^(1/2)/(6^(1/2)+2^(1/2))*arctanh(2*x/(6^
(1/2)+2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - 1}{x^4 - 4x^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 - 4*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 - 4*x^2 + 1), x)
```

Fricas [A] time = 0.269136, size = 57, normalized size = 1.21

$$\frac{1}{12}\sqrt{6}\log\left(\frac{12x^3 + \sqrt{6}\left(x^4 + 8x^2 + 1\right) + 12x}{x^4 - 4x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 - 4*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*log((12*x^3 + sqrt(6)*(x^4 + 8*x^2 + 1) + 12*x)/(x^4
 - 4*x^2 + 1))
```

Sympy [A] time = 0.217941, size = 39, normalized size = 0.83

$$-\frac{\sqrt{6}\log\left(x^2 - \sqrt{6}x + 1\right)}{12} + \frac{\sqrt{6}\log\left(x^2 + \sqrt{6}x + 1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)/(x**4-4*x**2+1),x)
```

```
[Out] -sqrt(6)*log(x**2 - sqrt(6)*x + 1)/12 + sqrt(6)*log(x**2 + sqrt(6
)*x + 1)/12
```

GIAC/XCAS [A] time = 0.277794, size = 53, normalized size = 1.13

$$-\frac{1}{12}\sqrt{6}\ln\left(\frac{\left|2x - 2\sqrt{6} + \frac{2}{x}\right|}{\left|2x + 2\sqrt{6} + \frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 - 4*x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*ln(abs(2*x - 2*sqrt(6) + 2/x)/abs(2*x + 2*sqrt(6) +
 2/x))
```

3.99 $\int \frac{1-x^2}{1-5x^2+x^4}\,dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{2x+\sqrt{3}}{\sqrt{7}}\right)}{\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] -(ArcTanh[(Sqrt[3] - 2*x)/Sqrt[7]]/Sqrt[7]) + ArcTanh[(Sqrt[3] +
2*x)/Sqrt[7]]/Sqrt[7]
```

Rubi [A] time = 0.0746466, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{\tanh^{-1}\left(\frac{2x+\sqrt{3}}{\sqrt{7}}\right)}{\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^2)/(1 - 5*x^2 + x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[3] - 2*x)/Sqrt[7]]/Sqrt[7]) + ArcTanh[(Sqrt[3] +
2*x)/Sqrt[7]]/Sqrt[7]
```

Rubi in Sympy [A] time = 8.04649, size = 53, normalized size = 1.15

$$-\frac{\sqrt{7}\,\text{atanh}\left(\sqrt{7}\left(-\frac{2x}{7}-\frac{\sqrt{3}}{7}\right)\right)}{7}-\frac{\sqrt{7}\,\text{atanh}\left(\sqrt{7}\left(-\frac{2x}{7}+\frac{\sqrt{3}}{7}\right)\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)/(x**4-5*x**2+1),x)
```

```
[Out] -sqrt(7)*atanh(sqrt(7)*(-2*x/7 - sqrt(3)/7))/7 - sqrt(7)*atanh(sq
rt(7)*(-2*x/7 + sqrt(3)/7))/7
```

Mathematica [A] time = 0.0237645, size = 40, normalized size = 0.87

$$\frac{\log\left(x^2+\sqrt{7}x+1\right)-\log\left(-x^2+\sqrt{7}x-1\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^2)/(1 - 5*x^2 + x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[7]*x - x^2] + Log[1 + Sqrt[7]*x + x^2])/(2*Sqrt[7
])
```

Maple [B] time = 0.018, size = 82, normalized size = 1.8

$$\frac{\left(6+2\sqrt{21}\right)\sqrt{21}}{42\sqrt{7}+42\sqrt{3}}\mathit{Artanh}\left(4\frac{x}{2\sqrt{7}+2\sqrt{3}}\right)+\frac{\left(-6+2\sqrt{21}\right)\sqrt{21}}{42\sqrt{7}-42\sqrt{3}}\mathit{Artanh}\left(4\frac{x}{2\sqrt{7}-2\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)/(x^4-5*x^2+1),x)
```

```
[Out] 2/21*(3+21^(1/2))*21^(1/2)/(2*7^(1/2)+2*3^(1/2))*arctanh(4*x/(2*7
^(1/2)+2*3^(1/2)))+2/21*(-3+21^(1/2))*21^(1/2)/(2*7^(1/2)-2*3^(1/
2))*arctanh(4*x/(2*7^(1/2)-2*3^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-1}{x^4-5\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 - 5*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 - 5*x^2 + 1), x)
```

Fricas [A] time = 0.285358, size = 57, normalized size = 1.24

$$\frac{1}{14}\,\sqrt{7}\log\left(\frac{14\,x^3+\sqrt{7}\left(x^4+9\,x^2+1\right)+14\,x}{x^4-5\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 - 5*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/14*sqrt(7)*log((14*x^3 + sqrt(7)*(x^4 + 9*x^2 + 1) + 14*x)/(x^4
 - 5*x^2 + 1))
```

Sympy [A] time = 0.202356, size = 39, normalized size = 0.85

$$-\frac{\sqrt{7}\log\left(x^2-\sqrt{7}x+1\right)}{14}+\frac{\sqrt{7}\log\left(x^2+\sqrt{7}x+1\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)/(x**4-5*x**2+1),x)
```

```
[Out] -sqrt(7)*log(x**2 - sqrt(7)*x + 1)/14 + sqrt(7)*log(x**2 + sqrt(7
)*x + 1)/14
```

GIAC/XCAS [A] time = 0.279486, size = 53, normalized size = 1.15

$$-\frac{1}{14}\,\sqrt{7}\ln\left(\frac{\left|2\,x-2\,\sqrt{7}+\frac{2}{x}\right|}{\left|2\,x+2\,\sqrt{7}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^4 - 5*x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/14*sqrt(7)*ln(abs(2*x - 2*sqrt(7) + 2/x)/abs(2*x + 2*sqrt(7) +
 2/x))
```

3.100 $\int -\frac{1+3x^2}{1+2x^2+9x^4}\,dx$

Optimal. Leaf size=43

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] ArcTan[(1 - 3*x)/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + 3*x)/Sqrt[2]]
/(2*Sqrt[2])
```

Rubi [A] time = 0.0685218, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[-((1 + 3*x^2)/(1 + 2*x^2 + 9*x^4)),x]
```

```
[Out] ArcTan[(1 - 3*x)/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + 3*x)/Sqrt[2]]
/(2*Sqrt[2])
```

Rubi in Sympy [A] time = 8.38436, size = 42, normalized size = 0.98

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(-\frac{3x}{2}+\frac{1}{2}\right)\right)}{4}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3x}{2}+\frac{1}{2}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-3*x**2-1)/(9*x**4+2*x**2+1),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*(-3*x/2 + 1/2))/4 - sqrt(2)*atan(sqrt(2)*(3*
x/2 + 1/2))/4
```

Mathematica [C] time = 0.166557, size = 99, normalized size = 2.3

$$-\frac{\left(\sqrt{2}-i\right)\tan^{-1}\left(\frac{3x}{\sqrt{1-2i\sqrt{2}}}\right)}{2\sqrt{2\left(1-2i\sqrt{2}\right)}}-\frac{\left(\sqrt{2}+i\right)\tan^{-1}\left(\frac{3x}{\sqrt{1+2i\sqrt{2}}}\right)}{2\sqrt{2\left(1+2i\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[-((1 + 3*x^2)/(1 + 2*x^2 + 9*x^4)),x]
```

```
[Out] -((-I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 - (2*I)*Sqrt[2]]])/(2*Sqrt[2
*(1 - (2*I)*Sqrt[2])]) - ((I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 + (2*
I)*Sqrt[2]]])/(2*Sqrt[2*(1 + (2*I)*Sqrt[2])])
```

Maple [A] time = 0.01, size = 34, normalized size = 0.8

$$-\frac{\sqrt{2}}{4}\arctan\left(\frac{(6x+2)\sqrt{2}}{4}\right)-\frac{\sqrt{2}}{4}\arctan\left(\frac{(6x-2)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3*x^2-1)/(9*x^4+2*x^2+1),x)
```

```
[Out] -1/4*2^(1/2)*arctan(1/4*(6*x+2)*2^(1/2))-1/4*2^(1/2)*arctan(1/4*(
6*x-2)*2^(1/2))
```

Maxima [A] time = 0.840625, size = 45, normalized size = 1.05

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(3*x^2 + 1)/(9*x^4 + 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1
/2*sqrt(2)*(3*x - 1))
```

Fricas [A] time = 0.280519, size = 38, normalized size = 0.88

$$-\frac{1}{4}\sqrt{2}\left(\arctan\left(\frac{1}{4}\sqrt{2}\left(9x^3+5x\right)\right)+\arctan\left(\frac{3}{4}\sqrt{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(3*x^2 + 1)/(9*x^4 + 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*(arctan(1/4*sqrt(2)*(9*x^3 + 5*x)) + arctan(3/4*sqrt
(2)*x))
```

Sympy [A] time = 0.267402, size = 46, normalized size = 1.07

$$-\frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{3\sqrt{2}x}{4}\right)+2\,\mathrm{atan}\left(\frac{9\sqrt{2}x^3}{4}+\frac{5\sqrt{2}x}{4}\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*x**2-1)/(9*x**4+2*x**2+1),x)
```

```
[Out] -sqrt(2)*(2*atan(3*sqrt(2)*x/4) + 2*atan(9*sqrt(2)*x**3/4 + 5*sqr
t(2)*x/4))/8
```

GIAC/XCAS [A] time = 0.269518, size = 45, normalized size = 1.05

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(3*x^2 + 1)/(9*x^4 + 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1
/2*sqrt(2)*(3*x - 1))
```

3.101 $\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx$

Optimal. Leaf size=43

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] ArcTan[(1 - 3*x)/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + 3*x)/Sqrt[2]]
/(2*Sqrt[2])
```

Rubi [A] time = 0.0614777, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 3*x^2)/(-1 - 2*x^2 - 9*x^4),x]
```

```
[Out] ArcTan[(1 - 3*x)/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + 3*x)/Sqrt[2]]
/(2*Sqrt[2])
```

Rubi in Sympy [A] time = 8.43228, size = 44, normalized size = 1.02

$$-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3x}{2}-\frac{1}{2}\right)\right)}{4}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3x}{2}+\frac{1}{2}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+1)/(-9*x**4-2*x**2-1),x)
```

```
[Out] -sqrt(2)*atan(sqrt(2)*(3*x/2 - 1/2))/4 - sqrt(2)*atan(sqrt(2)*(3*
x/2 + 1/2))/4
```

Mathematica [C] time = 0.0356167, size = 99, normalized size = 2.3

$$-\frac{\left(\sqrt{2}-i\right)\tan^{-1}\left(\frac{3x}{\sqrt{1-2i\sqrt{2}}}\right)}{2\sqrt{2\left(1-2i\sqrt{2}\right)}}-\frac{\left(\sqrt{2}+i\right)\tan^{-1}\left(\frac{3x}{\sqrt{1+2i\sqrt{2}}}\right)}{2\sqrt{2\left(1+2i\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 3*x^2)/(-1 - 2*x^2 - 9*x^4),x]
```

```
[Out] -((-I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 - (2*I)*Sqrt[2]]])/(2*Sqrt[2
*(1 - (2*I)*Sqrt[2])]) - ((I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 + (2*
I)*Sqrt[2]]])/(2*Sqrt[2*(1 + (2*I)*Sqrt[2])])
```

Maple [A] time = 0.004, size = 34, normalized size = 0.8

$$-\frac{\sqrt{2}}{4}\arctan\left(\frac{(6x+2)\sqrt{2}}{4}\right)-\frac{\sqrt{2}}{4}\arctan\left(\frac{(6x-2)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((3*x^2+1)/(-9*x^4-2*x^2-1),x)
```

```
[Out] -1/4*2^(1/2)*arctan(1/4*(6*x+2)*2^(1/2))-1/4*2^(1/2)*arctan(1/4*(
6*x-2)*2^(1/2))
```

Maxima [A] time = 0.846034, size = 45, normalized size = 1.05

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(3*x^2 + 1)/(9*x^4 + 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1
/2*sqrt(2)*(3*x - 1))
```

Fricas [A] time = 0.281668, size = 38, normalized size = 0.88

$$-\frac{1}{4}\sqrt{2}\left(\arctan\left(\frac{1}{4}\sqrt{2}\left(9x^3+5x\right)\right)+\arctan\left(\frac{3}{4}\sqrt{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(3*x^2 + 1)/(9*x^4 + 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*(arctan(1/4*sqrt(2)*(9*x^3 + 5*x)) + arctan(3/4*sqrt
(2)*x))
```

Sympy [A] time = 0.275033, size = 46, normalized size = 1.07

$$-\frac{\sqrt{2}\left(2\operatorname{atan}\left(\frac{3\sqrt{2}x}{4}\right)+2\operatorname{atan}\left(\frac{9\sqrt{2}x^3}{4}+\frac{5\sqrt{2}x}{4}\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((3*x**2+1)/(-9*x**4-2*x**2-1),x)
```

```
[Out] -sqrt(2)*(2*atan(3*sqrt(2)*x/4) + 2*atan(9*sqrt(2)*x**3/4 + 5*sqr
t(2)*x/4))/8
```

GIAC/XCAS [A] time = 0.269567, size = 45, normalized size = 1.05

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(3*x^2 + 1)/(9*x^4 + 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1
/2*sqrt(2)*(3*x - 1))
```

3.102 $\int \frac{3+2x^2}{1-2x^2+x^4}\,dx$

Optimal. Leaf size=21

$$\frac{5x}{2(1-x^2)}+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] (5*x)/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rubi [A] time = 0.0160887, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{5x}{2(1-x^2)}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 2*x^2)/(1 - 2*x^2 + x^4),x]

[Out] (5*x)/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rubi in Sympy [A] time = 5.50197, size = 14, normalized size = 0.67

$$\frac{5x}{2(-x^2+1)}+\frac{\operatorname{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+3)/(x**4-2*x**2+1),x)

[Out] 5*x/(2*(-x**2 + 1)) + atanh(x)/2
```

Mathematica [A] time = 0.0165703, size = 27, normalized size = 1.29

$$\frac{1}{4}\left(-\frac{10x}{x^2-1}-\log(1-x)+\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 2*x^2)/(1 - 2*x^2 + x^4),x]

[Out] ((-10*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

Maple [A] time = 0.013, size = 28, normalized size = 1.3

$$-\frac{5}{-4+4x}-\frac{\ln(-1+x)}{4}-\frac{5}{4+4x}+\frac{\ln(1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+3)/(x^4-2*x^2+1),x)
```

```
[Out] -5/4/(-1+x)-1/4*ln(-1+x)-5/4/(1+x)+1/4*ln(1+x)
```

Maxima [A] time = 0.753515, size = 31, normalized size = 1.48

$$-\frac{5\,x}{2\,(x^2-1)}+\frac{1}{4}\,\log\left(x+1\right)-\frac{1}{4}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 3)/(x^4 - 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -5/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [A] time = 0.28267, size = 46, normalized size = 2.19

$$\frac{\left(x^2-1\right)\,\log\left(x+1\right)-\left(x^2-1\right)\,\log\left(x-1\right)-10\,x}{4\,(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 3)/(x^4 - 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 10*x)/(x^2 - 1
)
```

Sympy [A] time = 0.193748, size = 22, normalized size = 1.05

$$-\frac{5x}{2x^2-2}-\frac{\log\left(x-1\right)}{4}+\frac{\log\left(x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+3)/(x**4-2*x**2+1),x)
```

```
[Out] -5*x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

GIAC/XCAS [A] time = 0.268233, size = 34, normalized size = 1.62

$$-\frac{5\,x}{2\,(x^2-1)}+\frac{1}{4}\ln\left(|x+1|\right)-\frac{1}{4}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 3)/(x^4 - 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] -5/2*x/(x^2 - 1) + 1/4*ln(abs(x + 1)) - 1/4*ln(abs(x - 1))
```

3.103 $\int \frac{2+3x^2}{5-8x^2+3x^4}\,dx$

Optimal. Leaf size=28

$$\frac{5}{2}\tanh^{-1}(x)-\frac{7}{2}\sqrt{\frac{3}{5}}\tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)$$

```
[Out] (5*ArcTanh[x])/2 - (7*Sqrt[3/5]*ArcTanh[Sqrt[3/5]*x])/2
```

Rubi [A] time = 0.0313043, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{5}{2}\tanh^{-1}(x)-\frac{7}{2}\sqrt{\frac{3}{5}}\tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(5 - 8*x^2 + 3*x^4),x]

[Out] (5*ArcTanh[x])/2 - (7*Sqrt[3/5]*ArcTanh[Sqrt[3/5]*x])/2
```

Rubi in Sympy [A] time = 8.6189, size = 24, normalized size = 0.86

$$\frac{5\operatorname{atanh}(x)}{2}-\frac{7\sqrt{15}\operatorname{atanh}\left(\frac{\sqrt{15}x}{5}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/(3*x**4-8*x**2+5),x)

[Out] 5*atanh(x)/2 - 7*sqrt(15)*atanh(sqrt(15)*x/5)/10
```

Mathematica [A] time = 0.0304294, size = 53, normalized size = 1.89

$$\frac{1}{20}\left(7\sqrt{15}\log\left(\sqrt{15}-3x\right)-25\log(1-x)+25\log(x+1)-7\sqrt{15}\log\left(3x+\sqrt{15}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/(5 - 8*x^2 + 3*x^4),x]

[Out] (7*Sqrt[15]*Log[Sqrt[15] - 3*x] - 25*Log[1 - x] + 25*Log[1 + x] -
 7*Sqrt[15]*Log[Sqrt[15] + 3*x])/20
```

Maple [A] time = 0.01, size = 26, normalized size = 0.9

$$-\frac{5\ln(-1+x)}{4}-\frac{7\sqrt{15}}{10}Artanh\left(\frac{x\sqrt{15}}{5}\right)+\frac{5\ln(1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/(3*x^4-8*x^2+5),x)
```

```
[Out] -5/4*ln(-1+x)-7/10*arctanh(1/5*x*15^(1/2))*15^(1/2)+5/4*ln(1+x)
```

Maxima [A] time = 0.847103, size = 51, normalized size = 1.82

$$\frac{7}{20}\sqrt{15}\log\left(\frac{3x-\sqrt{15}}{3x+\sqrt{15}}\right)+\frac{5}{4}\log(x+1)-\frac{5}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(3*x^4 - 8*x^2 + 5),x, algorithm="maxima")
```

```
[Out] 7/20*sqrt(15)*log((3*x - sqrt(15))/(3*x + sqrt(15))) + 5/4*log(x
+ 1) - 5/4*log(x - 1)
```

Fricas [A] time = 0.286182, size = 78, normalized size = 2.79

$$\frac{1}{20}\sqrt{5}\left(5\sqrt{5}\log(x+1)-5\sqrt{5}\log(x-1)+7\sqrt{3}\log\left(\frac{\sqrt{5}(3x^2+5)-10\sqrt{3}x}{3x^2-5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(3*x^4 - 8*x^2 + 5),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*(5*sqrt(5)*log(x + 1) - 5*sqrt(5)*log(x - 1) + 7*sqr
t(3)*log((sqrt(5)*(3*x^2 + 5) - 10*sqrt(3)*x)/(3*x^2 - 5)))
```

Sympy [A] time = 1.85561, size = 53, normalized size = 1.89

$$-\frac{5\log(x-1)}{4}+\frac{5\log(x+1)}{4}+\frac{7\sqrt{15}\log\left(x-\frac{\sqrt{15}}{3}\right)}{20}-\frac{7\sqrt{15}\log\left(x+\frac{\sqrt{15}}{3}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/(3*x**4-8*x**2+5),x)
```

```
[Out] -5*log(x - 1)/4 + 5*log(x + 1)/4 + 7*sqrt(15)*log(x - sqrt(15)/3)
/20 - 7*sqrt(15)*log(x + sqrt(15)/3)/20
```

GIAC/XCAS [A] time = 0.269819, size = 59, normalized size = 2.11

$$\frac{7}{20}\sqrt{15}\ln\left(\frac{\left|6x-2\sqrt{15}\right|}{\left|6x+2\sqrt{15}\right|}\right)+\frac{5}{4}\ln(|x+1|)-\frac{5}{4}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(3*x^4 - 8*x^2 + 5),x, algorithm="giac")
```

```
[Out] 7/20*sqrt(15)*ln(abs(6*x - 2*sqrt(15))/abs(6*x + 2*sqrt(15))) + 5
/4*ln(abs(x + 1)) - 5/4*ln(abs(x - 1))
```

3.104 $\int \frac{d+ex^2}{5-8x^2+3x^4} dx$

Optimal. Leaf size=36

$$\frac{1}{2}(d+e)\tanh^{-1}(x)-\frac{(3d+5e)\tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)}{2\sqrt{15}}$$

```
[Out] ((d + e)*ArcTanh[x])/2 - ((3*d + 5*e)*ArcTanh[Sqrt[3/5]*x])/(2*Sq
rt[15])
```

Rubi [A] time = 0.0744802, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{1}{2}(d+e)\tanh^{-1}(x)-\frac{(3d+5e)\tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)}{2\sqrt{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

```
[Out] ((d + e)*ArcTanh[x])/2 - ((3*d + 5*e)*ArcTanh[Sqrt[3/5]*x])/(2*Sq
rt[15])
```

Rubi in Sympy [A] time = 9.37891, size = 32, normalized size = 0.89

$$\left(\frac{d}{2}+\frac{e}{2}\right)\operatorname{atanh}(x)-\frac{\sqrt{15}\,(3d+5e)\operatorname{atanh}\left(\frac{\sqrt{15}x}{5}\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((e*x**2+d)/(3*x**4-8*x**2+5),x)
```

```
[Out] (d/2 + e/2)*atanh(x) - sqrt(15)*(3*d + 5*e)*atanh(sqrt(15)*x/5)/3
0
```

Mathematica [A] time = 0.0665744, size = 72, normalized size = 2.

$$\frac{1}{60}\left(\sqrt{15}(3d+5e)\log\left(\sqrt{15}-3x\right)-15(d+e)\log(1-x)+15(d+e)\log(x+1)-\sqrt{15}(3d+5e)\log\left(3x+\sqrt{15}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

```
[Out] (Sqrt[15]*(3*d + 5*e)*Log[Sqrt[15] - 3*x] - 15*(d + e)*Log[1 - x]
 + 15*(d + e)*Log[1 + x] - Sqrt[15]*(3*d + 5*e)*Log[Sqrt[15] + 3*
x])/60
```

Maple [B] time = 0.009, size = 56, normalized size = 1.6

$$-\frac{\ln(-1+x)\,d}{4}-\frac{\ln(-1+x)\,e}{4}-\frac{\sqrt{15}d}{10}Artanh\left(\frac{x\sqrt{15}}{5}\right)-\frac{\sqrt{15}e}{6}Artanh\left(\frac{x\sqrt{15}}{5}\right)+\frac{\ln(1+x)\,d}{4}+\frac{\ln(1+x)\,e}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((e*x^2+d)/(3*x^4-8*x^2+5),x)
```

```
[Out] -1/4*ln(-1+x)*d-1/4*ln(-1+x)*e-1/10*15^(1/2)*arctanh(1/5*x*15^(1/
2))*d-1/6*15^(1/2)*arctanh(1/5*x*15^(1/2))*e+1/4*ln(1+x)*d+1/4*ln
(1+x)*e
```

Maxima [A] time = 0.846735, size = 69, normalized size = 1.92

$$\frac{1}{60}\,\sqrt{15}(3\,d+5\,e)\log\left(\frac{3\,x-\sqrt{15}}{3\,x+\sqrt{15}}\right)+\frac{1}{4}\,(d+e)\log\left(x+1\right)-\frac{1}{4}\,(d+e)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((e*x^2 + d)/(3*x^4 - 8*x^2 + 5),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(15)*(3*d + 5*e)*log((3*x - sqrt(15))/(3*x + sqrt(15)))
+ 1/4*(d + e)*log(x + 1) - 1/4*(d + e)*log(x - 1)
```

Fricas [A] time = 0.29432, size = 85, normalized size = 2.36

$$\frac{1}{60}\,\sqrt{15}\left(\sqrt{15}(d+e)\log\left(x+1\right)-\sqrt{15}(d+e)\log\left(x-1\right)+(3\,d+5\,e)\log\left(\frac{\sqrt{15}\left(3\,x^{2}+5\right)-30\,x}{3\,x^{2}-5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((e*x^2 + d)/(3*x^4 - 8*x^2 + 5),x, algorithm="fricas")
```

```
[Out] 1/60*sqrt(15)*(sqrt(15)*(d + e)*log(x + 1) - sqrt(15)*(d + e)*log
(x - 1) + (3*d + 5*e)*log((sqrt(15)*(3*x^2 + 5) - 30*x)/(3*x^2 -
5)))
```

Sympy [A] time = 3.33006, size = 474, normalized size = 13.17

$$\frac{(d+e)\log\left(x+\frac{-51d^{3}(d+e)-180d^{2}e(d+e)-225de^{2}(d+e)+60d(d+e)^{3}-100e^{3}(d+e)+75e(d+e)^{3}}{9d^{4}+24d^{3}e-40de^{3}-25e^{4}}\right)}{4}-\frac{(d+e)\log\left(x+\frac{51d^{3}(d+e)+180d^{2}e(d+e)+225de^{2}(d+e)-60d(d+e)^{3}+100e^{3}(d+e)-75e(d+e)^{3}}{9d^{4}+24d^{3}e-40de^{3}-25e^{4}}\right)}{4}+\frac{\sqrt{15}\,(3d+5e)\log\left(x+\frac{-\frac{17\sqrt{15}d^{3}(3d+5e)}{5}-12\sqrt{15}d^{2}e(3d+5e)-15\sqrt{15}de^{2}(3d+5e)+\frac{4\sqrt{15}d(3d+5e)^{3}}{15}-\frac{20\sqrt{15}e^{3}(3d+5e)}{3}+\frac{\sqrt{15}e(3d+5e)^{3}}{3}}{9d^{4}+24d^{3}e-40de^{3}-25e^{4}}\right)}{60}-\frac{\sqrt{15}\,(3d+5e)\log\left(x+\frac{\frac{17\sqrt{15}d^{3}(3d+5e)}{5}+12\sqrt{15}d^{2}e(3d+5e)+15\sqrt{15}de^{2}(3d+5e)-\frac{4\sqrt{15}d(3d+5e)^{3}}{15}+\frac{20\sqrt{15}e^{3}(3d+5e)}{3}-\frac{\sqrt{15}e(3d+5e)^{3}}{3}}{9d^{4}+24d^{3}e-40de^{3}-25e^{4}}\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((e*x**2+d)/(3*x**4-8*x**2+5),x)
```

```
[Out] (d + e)*log(x + (-51*d**3*(d + e) - 180*d**2*e*(d + e) - 225*d*e*
*2*(d + e) + 60*d*(d + e)**3 - 100*e**3*(d + e) + 75*e*(d + e)**3
)/(9*d**4 + 24*d**3*e - 40*d*e**3 - 25*e**4))/4 - (d + e)*log(x +
```

```
 (51*d**3*(d + e) + 180*d**2*e*(d + e) + 225*d*e**2*(d + e) - 60*
d*(d + e)**3 + 100*e**3*(d + e) - 75*e*(d + e)**3)/(9*d**4 + 24*d
**3*e - 40*d*e**3 - 25*e**4))/4 + sqrt(15)*(3*d + 5*e)*log(x + (-
17*sqrt(15)*d**3*(3*d + 5*e)/5 - 12*sqrt(15)*d**2*e*(3*d + 5*e) -
 15*sqrt(15)*d*e**2*(3*d + 5*e) + 4*sqrt(15)*d*(3*d + 5*e)**3/15
- 20*sqrt(15)*e**3*(3*d + 5*e)/3 + sqrt(15)*e*(3*d + 5*e)**3/3)/(
9*d**4 + 24*d**3*e - 40*d*e**3 - 25*e**4))/60 - sqrt(15)*(3*d + 5
*e)*log(x + (17*sqrt(15)*d**3*(3*d + 5*e)/5 + 12*sqrt(15)*d**2*e*
(3*d + 5*e) + 15*sqrt(15)*d*e**2*(3*d + 5*e) - 4*sqrt(15)*d*(3*d
+ 5*e)**3/15 + 20*sqrt(15)*e**3*(3*d + 5*e)/3 - sqrt(15)*e*(3*d +
 5*e)**3/3)/(9*d**4 + 24*d**3*e - 40*d*e**3 - 25*e**4))/60
```

GIAC/XCAS [A] time = 0.270001, size = 81, normalized size = 2.25

$$\frac{1}{60}\sqrt{15}(3\,d+5\,e)\ln\left(\frac{\left|6\,x-2\,\sqrt{15}\right|}{\left|6\,x+2\,\sqrt{15}\right|}\right)+\frac{1}{4}\,(d+e)\ln\left(\left|x+1\right|\right)-\frac{1}{4}\,(d+e)\ln\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(3*x^4 - 8*x^2 + 5),x, algorithm="giac")
```

```
[Out] 1/60*sqrt(15)*(3*d + 5*e)*ln(abs(6*x - 2*sqrt(15))/abs(6*x + 2*sq
rt(15))) + 1/4*(d + e)*ln(abs(x + 1)) - 1/4*(d + e)*ln(abs(x - 1)
)
```

3.105 $\int \frac{3+x^2}{1+3x^2+x^4}\,dx$

Optimal. Leaf size=74

$$\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{10}}-\frac{1}{10}\sqrt{180-80\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

```
[Out] -(Sqrt[180 - 80*Sqrt[5]]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x])/10 + ((
3 + Sqrt[5])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x])/(2*Sqrt[10])
```

Rubi [A] time = 0.132275, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{10}}-\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] -(Sqrt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x]) + ((3
+ Sqrt[5])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x])/(2*Sqrt[10])
```

Rubi in Sympy [A] time = 9.24372, size = 90, normalized size = 1.22

$$\frac{\sqrt{2}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{-\sqrt{5}+3}}\right)}{\sqrt{-\sqrt{5}+3}}+\frac{\sqrt{2}\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{5}+3}}\right)}{\sqrt{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+3)/(x**4+3*x**2+1),x)
```

```
[Out] sqrt(2)*(1/2 + 3*sqrt(5)/10)*atan(sqrt(2)*x/sqrt(-sqrt(5) + 3))/s
qrt(-sqrt(5) + 3) + sqrt(2)*(-3*sqrt(5)/10 + 1/2)*atan(sqrt(2)*x/
sqrt(sqrt(5) + 3))/sqrt(sqrt(5) + 3)
```

Mathematica [A] time = 0.158851, size = 73, normalized size = 0.99

$$\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)-\left(3-\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] (-((3 - Sqrt[5])^(3/2)*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x]) + (3 + Sq
rt[5])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x])/(2*Sqrt[10])
```

Maple [B] time = 0.03, size = 104, normalized size = 1.4

$$2\,\frac{1}{2\sqrt{5}+2}\arctan\left(4\,\frac{x}{2\sqrt{5}+2}\right)-\frac{6\sqrt{5}}{10\sqrt{5}+10}\arctan\left(4\,\frac{x}{2\sqrt{5}+2}\right)+2\,\frac{1}{-2+2\sqrt{5}}\arctan\left(4\,\frac{x}{-2+2\sqrt{5}}\right)+\frac{6\sqrt{5}}{-10+10\sqrt{5}}\arctan\left(4\,\frac{x}{-2+2\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+3)/(x^4+3*x^2+1),x)
```

```
[Out] 2/(2*5^(1/2)+2)*arctan(4*x/(2*5^(1/2)+2))-6/5*5^(1/2)/(2*5^(1/2)+
2)*arctan(4*x/(2*5^(1/2)+2))+2/(-2+2*5^(1/2))*arctan(4*x/(-2+2*5^
(1/2)))+6/5*5^(1/2)/(-2+2*5^(1/2))*arctan(4*x/(-2+2*5^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+3}{x^4+3\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3)/(x^4 + 3*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 3)/(x^4 + 3*x^2 + 1), x)
```

Fricas [A] time = 0.290024, size = 196, normalized size = 2.65

$$\frac{2}{5}\sqrt{\sqrt{5}\left(9\sqrt{5}-20\right)}\arctan\left(\frac{\sqrt{\sqrt{5}\left(9\sqrt{5}-20\right)}\left(3\sqrt{5}+7\right)}{2\left(\sqrt{5}\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2\,x^2+3)+5\right)}+\sqrt{5}x\right)}\right)+\frac{2}{5}\sqrt{\sqrt{5}\left(9\sqrt{5}+20\right)}\arctan\left(\frac{\sqrt{\sqrt{5}\left(9\sqrt{5}+20\right)}\left(3\sqrt{5}-7\right)}{2\left(\sqrt{5}\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2\,x^2+3)-5\right)}+\sqrt{5}x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3)/(x^4 + 3*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(sqrt(5)*(9*sqrt(5) - 20))*arctan(1/2*sqrt(sqrt(5)*(9*sqr
t(5) - 20))*(3*sqrt(5) + 7)/(sqrt(5)*sqrt(1/10)*sqrt(sqrt(5)*(sqr
t(5)*(2*x^2 + 3) + 5)) + sqrt(5)*x)) + 2/5*sqrt(sqrt(5)*(9*sqrt(5
) + 20))*arctan(1/2*sqrt(sqrt(5)*(9*sqrt(5) + 20))*(3*sqrt(5) - 7
)/(sqrt(5)*sqrt(1/10)*sqrt(sqrt(5)*(sqrt(5)*(2*x^2 + 3) - 5)) + s
qrt(5)*x))
```

Sympy [A] time = 0.500868, size = 46, normalized size = 0.62

$$2\left(\frac{\sqrt{5}}{5}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2x}{-1+\sqrt{5}}\right)-2\left(-\frac{\sqrt{5}}{5}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2x}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+3)/(x**4+3*x**2+1),x)
```

```
[Out] 2*(sqrt(5)/5 + 1/2)*atan(2*x/(-1 + sqrt(5))) - 2*(-sqrt(5)/5 + 1/
2)*atan(2*x/(1 + sqrt(5)))
```

GIAC/XCAS [A] time = 0.269401, size = 55, normalized size = 0.74

$$\frac{1}{5}\left(2\sqrt{5}-5\right)\arctan\left(\frac{2x}{\sqrt{5}+1}\right)+\frac{1}{5}\left(2\sqrt{5}+5\right)\arctan\left(\frac{2x}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3)/(x^4 + 3*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/5*(2*sqrt(5) - 5)*arctan(2*x/(sqrt(5) + 1)) + 1/5*(2*sqrt(5) +
5)*arctan(2*x/(sqrt(5) - 1))
```

3.106 $\int \frac{a+bx^2}{1+x^2+x^4} \, dx$

Optimal. Leaf size=83

$$-\frac{1}{4}(a-b)\log\left(x^2-x+1\right)+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)-\frac{(a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(a+b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -((a + b)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((a + b)*ArcTa
n[(1 + 2*x)/Sqrt[3]])/(2*Sqrt[3]) - ((a - b)*Log[1 - x + x^2])/4
+ ((a - b)*Log[1 + x + x^2])/4
```

Rubi [A] time = 0.124618, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$

$$-\frac{1}{4}(a-b)\log\left(x^2-x+1\right)+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)-\frac{(a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(a+b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(1 + x^2 + x^4),x]
```

```
[Out] -((a + b)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((a + b)*ArcTa
n[(1 + 2*x)/Sqrt[3]])/(2*Sqrt[3]) - ((a - b)*Log[1 - x + x^2])/4
+ ((a - b)*Log[1 + x + x^2])/4
```

Rubi in Sympy [A] time = 19.396, size = 80, normalized size = 0.96

$$-\left(\frac{a}{4}-\frac{b}{4}\right)\log\left(x^2-x+1\right)+\left(\frac{a}{4}-\frac{b}{4}\right)\log\left(x^2+x+1\right)+\frac{\sqrt{3}(a+b)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}+\frac{\sqrt{3}(a+b)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)/(x**4+x**2+1),x)
```

```
[Out] -(a/4 - b/4)*log(x**2 - x + 1) + (a/4 - b/4)*log(x**2 + x + 1) +
sqrt(3)*(a + b)*atan(sqrt(3)*(2*x/3 - 1/3))/6 + sqrt(3)*(a + b)*a
tan(sqrt(3)*(2*x/3 + 1/3))/6
```

Mathematica [C] time = 0.209343, size = 97, normalized size = 1.17

$$\frac{\left(2ia+\left(\sqrt{3}-i\right)b\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)}{\sqrt{6+6i\sqrt{3}}}+\frac{\left(\left(\sqrt{3}+i\right)b-2ia\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)}{\sqrt{6-6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)/(1 + x^2 + x^4),x]
```

```
[Out] (((2*I)*a + (-I + Sqrt[3])*b)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[
6 + (6*I)*Sqrt[3]] + (((-2*I)*a + (I + Sqrt[3])*b)*ArcTan[((I + S
```

```
qrt[3])*x)/2])/Sqrt[6 - (6*I)*Sqrt[3]]
```

Maple [A] time = 0.007, size = 114, normalized size = 1.4

$$\frac{\ln\left(x^2+x+1\right)a}{4}-\frac{\ln\left(x^2+x+1\right)b}{4}+\frac{\sqrt{3}a}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}b}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\ln\left(x^2-x+1\right)a}{4}+\frac{\ln\left(x^2-x+1\right)b}{4}+\frac{\sqrt{3}a}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}b}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(x^4+x^2+1),x)
```

```
[Out] 1/4*ln(x^2+x+1)*a-1/4*ln(x^2+x+1)*b+1/6*3^(1/2)*arctan(1/3*(1+2*x
)*3^(1/2))*a+1/6*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*b-1/4*ln(x^2
-x+1)*a+1/4*ln(x^2-x+1)*b+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
*a+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*b
```

Maxima [A] time = 0.861314, size = 93, normalized size = 1.12

$$\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)-\frac{1}{4}(a-b)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(x^4 + x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(
a + b)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(a - b)*log(x^2 + x +
1) - 1/4*(a - b)*log(x^2 - x + 1)
```

Fricas [A] time = 0.294693, size = 99, normalized size = 1.19

$$\frac{1}{12}\sqrt{3}\left(\sqrt{3}(a-b)\log\left(x^2+x+1\right)-\sqrt{3}(a-b)\log\left(x^2-x+1\right)+2(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+2(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(x^4 + x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*(sqrt(3)*(a - b)*log(x^2 + x + 1) - sqrt(3)*(a - b)*
log(x^2 - x + 1) + 2*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 2*(a
 + b)*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 2.95709, size = 740, normalized size = 8.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(x**4+x**2+1),x)
```

```
[Out] (-a/4 + b/4 - sqrt(3)*I*(a + b)/12)*log(x + (2*a**3*(-a/4 + b/4 -
 sqrt(3)*I*(a + b)/12) + 6*a**2*b*(-a/4 + b/4 - sqrt(3)*I*(a + b)
/12) - 12*a*b**2*(-a/4 + b/4 - sqrt(3)*I*(a + b)/12) + 24*a*(-a/4
 + b/4 - sqrt(3)*I*(a + b)/12)**3 + 2*b**3*(-a/4 + b/4 - sqrt(3)*
I*(a + b)/12) - 48*b*(-a/4 + b/4 - sqrt(3)*I*(a + b)/12)**3)/(a**
4 - a**3*b + a*b**3 - b**4)) + (-a/4 + b/4 + sqrt(3)*I*(a + b)/12
)*log(x + (2*a**3*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12) + 6*a**2*b*
(-a/4 + b/4 + sqrt(3)*I*(a + b)/12) - 12*a*b**2*(-a/4 + b/4 + sqr
t(3)*I*(a + b)/12) + 24*a*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12)**3
+ 2*b**3*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12) - 48*b*(-a/4 + b/4 +
 sqrt(3)*I*(a + b)/12)**3)/(a**4 - a**3*b + a*b**3 - b**4)) + (a/
4 - b/4 - sqrt(3)*I*(a + b)/12)*log(x + (2*a**3*(a/4 - b/4 - sqrt
(3)*I*(a + b)/12) + 6*a**2*b*(a/4 - b/4 - sqrt(3)*I*(a + b)/12) -
 12*a*b**2*(a/4 - b/4 - sqrt(3)*I*(a + b)/12) + 24*a*(a/4 - b/4 -
 sqrt(3)*I*(a + b)/12)**3 + 2*b**3*(a/4 - b/4 - sqrt(3)*I*(a + b)
/12) - 48*b*(a/4 - b/4 - sqrt(3)*I*(a + b)/12)**3)/(a**4 - a**3*b
 + a*b**3 - b**4)) + (a/4 - b/4 + sqrt(3)*I*(a + b)/12)*log(x + (
2*a**3*(a/4 - b/4 + sqrt(3)*I*(a + b)/12) + 6*a**2*b*(a/4 - b/4 +
 sqrt(3)*I*(a + b)/12) - 12*a*b**2*(a/4 - b/4 + sqrt(3)*I*(a + b)
/12) + 24*a*(a/4 - b/4 + sqrt(3)*I*(a + b)/12)**3 + 2*b**3*(a/4 -
 b/4 + sqrt(3)*I*(a + b)/12) - 48*b*(a/4 - b/4 + sqrt(3)*I*(a + b
)/12)**3)/(a**4 - a**3*b + a*b**3 - b**4))
```

GIAC/XCAS [A] time = 0.271006, size = 93, normalized size = 1.12

$$\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}(a-b)\ln\left(x^2+x+1\right)-\frac{1}{4}(a-b)\ln\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(x^4 + x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(
a + b)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(a - b)*ln(x^2 + x + 1
) - 1/4*(a - b)*ln(x^2 - x + 1)
```

3.107 $\int \frac{a+bx^2}{(1+x^2+x^4)^2}\,dx$

Optimal. Leaf size=119

$$-\frac{1}{8}(2a-b)\log\left(x^2-x+1\right)+\frac{1}{8}(2a-b)\log\left(x^2+x+1\right)+\frac{x\left(x^2(-(a-2b))+a+b\right)}{6\left(x^4+x^2+1\right)}-\frac{(4a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4a+b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{12\sqrt{3}}$$

```
[Out] (x*(a + b - (a - 2*b)*x^2))/(6*(1 + x^2 + x^4)) - ((4*a + b)*ArcT
an[(1 - 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((4*a + b)*ArcTan[(1 + 2*x)
/Sqrt[3]])/(12*Sqrt[3]) - ((2*a - b)*Log[1 - x + x^2])/8 + ((2*a
- b)*Log[1 + x + x^2])/8
```

Rubi [A] time = 0.188073, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{1}{8}(2a-b)\log\left(x^2-x+1\right)+\frac{1}{8}(2a-b)\log\left(x^2+x+1\right)+\frac{x\left(x^2(-(a-2b))+a+b\right)}{6\left(x^4+x^2+1\right)}-\frac{(4a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4a+b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{12\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (x*(a + b - (a - 2*b)*x^2))/(6*(1 + x^2 + x^4)) - ((4*a + b)*ArcT
an[(1 - 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((4*a + b)*ArcTan[(1 + 2*x)
/Sqrt[3]])/(12*Sqrt[3]) - ((2*a - b)*Log[1 - x + x^2])/8 + ((2*a
- b)*Log[1 + x + x^2])/8
```

Rubi in Sympy [A] time = 30.9515, size = 109, normalized size = 0.92

$$\frac{x\left(a+b-x^2(a-2b)\right)}{6\left(x^4+x^2+1\right)}-\left(\frac{a}{4}-\frac{b}{8}\right)\log\left(x^2-x+1\right)+\left(\frac{a}{4}-\frac{b}{8}\right)\log\left(x^2+x+1\right)+\frac{\sqrt{3}(4a+b)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{36}+\frac{\sqrt{3}(4a+b)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(x**4+x**2+1)**2,x)
```

```
[Out] x*(a + b - x**2*(a - 2*b))/(6*(x**4 + x**2 + 1)) - (a/4 - b/8)*lo
g(x**2 - x + 1) + (a/4 - b/8)*log(x**2 + x + 1) + sqrt(3)*(4*a +
b)*atan(sqrt(3)*(2*x/3 - 1/3))/36 + sqrt(3)*(4*a + b)*atan(sqrt(3
)*(2*x/3 + 1/3))/36
```

Mathematica [C] time = 0.412382, size = 147, normalized size = 1.24

$$\frac{x\left(-ax^2+a+2bx^2+b\right)}{6\left(x^4+x^2+1\right)}-\frac{\left(\left(\sqrt{3}-11i\right)a-2\left(\sqrt{3}-2i\right)b\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)}{6\sqrt{6+6i\sqrt{3}}}-\frac{\left(\left(\sqrt{3}+11i\right)a-2\left(\sqrt{3}+2i\right)b\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)}{6\sqrt{6-6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (x*(a + b - a*x^2 + 2*b*x^2))/(6*(1 + x^2 + x^4)) - (((-11*I + Sq
rt[3])*a - 2*(-2*I + Sqrt[3])*b)*ArcTan[((-I + Sqrt[3])*x)/2])/(6
*Sqrt[6 + (6*I)*Sqrt[3]]) - (((11*I + Sqrt[3])*a - 2*(2*I + Sqrt[
3])*b)*ArcTan[((I + Sqrt[3])*x)/2])/(6*Sqrt[6 - (6*I)*Sqrt[3]])
```

Maple [A] time = 0.016, size = 168, normalized size = 1.4

$$\frac{1}{4x^2+4x+4}\left(\left(-\frac{a}{3}+\frac{2b}{3}\right)x-\frac{2a}{3}+\frac{b}{3}\right)+\frac{\ln\left(x^2+x+1\right)a}{4}-\frac{\ln\left(x^2+x+1\right)b}{8}+\frac{\sqrt{3}a}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}b}{36}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{1}{4x^2-4x+4}\left(\left(\frac{a}{3}-\frac{2b}{3}\right)x-\frac{2a}{3}+\frac{b}{3}\right)-\frac{\ln\left(x^2-x+1\right)a}{4}+\frac{\ln\left(x^2-x+1\right)b}{8}+\frac{\sqrt{3}a}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}b}{36}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(x^4+x^2+1)^2,x)
```

```
[Out] 1/4*((-1/3*a+2/3*b)*x-2/3*a+1/3*b)/(x^2+x+1)+1/4*ln(x^2+x+1)*a-1/
8*ln(x^2+x+1)*b+1/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*a+1/36*3^
(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*b-1/4*((1/3*a-2/3*b)*x-2/3*a+1/
3*b)/(x^2-x+1)-1/4*ln(x^2-x+1)*a+1/8*ln(x^2-x+1)*b+1/9*3^(1/2)*ar
ctan(1/3*(2*x-1)*3^(1/2))*a+1/36*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/
2))*b
```

Maxima [A] time = 0.842297, size = 142, normalized size = 1.19

$$\frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}(2a-b)\log\left(x^2+x+1\right)-\frac{1}{8}(2a-b)\log\left(x^2-x+1\right)-\frac{(a-2b)x^3-(a+b)x}{6\left(x^4+x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(x^4 + x^2 + 1)^2,x, algorithm="maxima")
```

```
[Out] 1/36*sqrt(3)*(4*a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(
3)*(4*a + b)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*a - b)*log(x^
2 + x + 1) - 1/8*(2*a - b)*log(x^2 - x + 1) - 1/6*((a - 2*b)*x^3
- (a + b)*x)/(x^4 + x^2 + 1)
```

Fricas [A] time = 0.298028, size = 261, normalized size = 2.19

$$\sqrt{3}\left(3\sqrt{3}\left((2a-b)x^4+(2a-b)x^2+2a-b\right)\log\left(x^2+x+1\right)-3\sqrt{3}\left((2a-b)x^4+(2a-b)x^2+2a-b\right)\log\left(x^2-x+1\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(x^4 + x^2 + 1)^2,x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(3)*(3*sqrt(3)*((2*a - b)*x^4 + (2*a - b)*x^2 + 2*a - b)
*log(x^2 + x + 1) - 3*sqrt(3)*((2*a - b)*x^4 + (2*a - b)*x^2 + 2*
a - b)*log(x^2 - x + 1) + 2*((4*a + b)*x^4 + (4*a + b)*x^2 + 4*a
+ b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 2*((4*a + b)*x^4 + (4*a + b)
*x^2 + 4*a + b)*arctan(1/3*sqrt(3)*(2*x - 1)) - 4*sqrt(3)*((a - 2
*b)*x^3 - (a + b)*x))/(x^4 + x^2 + 1)
```

Sympy [A] time = 4.76565, size = 876, normalized size = 7.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(x**4+x**2+1)**2,x)
```

```
[Out] -(x**3*(a - 2*b) + x*(-a - b))/(6*x**4 + 6*x**2 + 6) + (-a/4 + b/
8 - sqrt(3)*I*(4*a + b)/72)*log(x + (76*a**3*(-a/4 + b/8 - sqrt(3
)*I*(4*a + b)/72) + 948*a**2*b*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/
72) - 816*a*b**2*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72) + 12096*a*
(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72)**3 + 148*b**3*(-a/4 + b/8 -
 sqrt(3)*I*(4*a + b)/72) - 8640*b*(-a/4 + b/8 - sqrt(3)*I*(4*a +
b)/72)**3)/(248*a**4 - 262*a**3*b + 75*a**2*b**2 + 11*a*b**3 - 7*
b**4)) + (-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72)*log(x + (76*a**3*(
-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72) + 948*a**2*b*(-a/4 + b/8 + s
qrt(3)*I*(4*a + b)/72) - 816*a*b**2*(-a/4 + b/8 + sqrt(3)*I*(4*a
+ b)/72) + 12096*a*(-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72)**3 + 148
*b**3*(-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72) - 8640*b*(-a/4 + b/8
+ sqrt(3)*I*(4*a + b)/72)**3)/(248*a**4 - 262*a**3*b + 75*a**2*b*
*2 + 11*a*b**3 - 7*b**4)) + (a/4 - b/8 - sqrt(3)*I*(4*a + b)/72)*
log(x + (76*a**3*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72) + 948*a**2*
b*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72) - 816*a*b**2*(a/4 - b/8 -
sqrt(3)*I*(4*a + b)/72) + 12096*a*(a/4 - b/8 - sqrt(3)*I*(4*a + b
)/72)**3 + 148*b**3*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72) - 8640*b
*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72)**3)/(248*a**4 - 262*a**3*b
+ 75*a**2*b**2 + 11*a*b**3 - 7*b**4)) + (a/4 - b/8 + sqrt(3)*I*(4
*a + b)/72)*log(x + (76*a**3*(a/4 - b/8 + sqrt(3)*I*(4*a + b)/72)
 + 948*a**2*b*(a/4 - b/8 + sqrt(3)*I*(4*a + b)/72) - 816*a*b**2*(
a/4 - b/8 + sqrt(3)*I*(4*a + b)/72) + 12096*a*(a/4 - b/8 + sqrt(3
)*I*(4*a + b)/72)**3 + 148*b**3*(a/4 - b/8 + sqrt(3)*I*(4*a + b)/
72) - 8640*b*(a/4 - b/8 + sqrt(3)*I*(4*a + b)/72)**3)/(248*a**4 -
 262*a**3*b + 75*a**2*b**2 + 11*a*b**3 - 7*b**4))
```

GIAC/XCAS [A] time = 0.271991, size = 147, normalized size = 1.24

$$\frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}(2a-b)\ln\left(x^2+x+1\right)-\frac{1}{8}(2a-b)\ln\left(x^2-x+1\right)-\frac{ax^3-2bx^3-ax-bx}{6\left(x^4+x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(x^4 + x^2 + 1)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(4*a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(
3)*(4*a + b)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*a - b)*ln(x^2
 + x + 1) - 1/8*(2*a - b)*ln(x^2 - x + 1) - 1/6*(a*x^3 - 2*b*x^3
- a*x - b*x)/(x^4 + x^2 + 1)
```

3.108 $\int \frac{a+bx^2}{2+x^2+x^4}\,dx$

Optimal. Leaf size=234

$$-\frac{\left(a-\sqrt{2}b\right)\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\left(a-\sqrt{2}b\right)\log\left(x^2+\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}-\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(a+\sqrt{2}b\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{1+2\sqrt{2}}}\right)+\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(a+\sqrt{2}b\right)\tan^{-1}\left(\frac{2x+\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)$$

```
[Out] -(Sqrt[(-1 + 2*Sqrt[2])/14]*(a + Sqrt[2]*b)*ArcTan[(Sqrt[-1 + 2*S
qrt[2]] - 2*x)/Sqrt[1 + 2*Sqrt[2]]])/2 + (Sqrt[(-1 + 2*Sqrt[2])/1
4]*(a + Sqrt[2]*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2
*Sqrt[2]]])/2 - ((a - Sqrt[2]*b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2
]]*x + x^2])/(4*Sqrt[2*(-1 + 2*Sqrt[2])]) + ((a - Sqrt[2]*b)*Log[
Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(4*Sqrt[2*(-1 + 2*Sqrt[2
])])
```

Rubi [A] time = 0.477237, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$-\frac{\left(a-\sqrt{2}b\right)\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\left(a-\sqrt{2}b\right)\log\left(x^2+\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}-\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(a+\sqrt{2}b\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{1+2\sqrt{2}}}\right)+\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(a+\sqrt{2}b\right)\tan^{-1}\left(\frac{2x+\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(2 + x^2 + x^4),x]
```

```
[Out] -(Sqrt[(-1 + 2*Sqrt[2])/14]*(a + Sqrt[2]*b)*ArcTan[(Sqrt[-1 + 2*S
qrt[2]] - 2*x)/Sqrt[1 + 2*Sqrt[2]]])/2 + (Sqrt[(-1 + 2*Sqrt[2])/1
4]*(a + Sqrt[2]*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2
*Sqrt[2]]])/2 - ((a - Sqrt[2]*b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2
]]*x + x^2])/(4*Sqrt[2*(-1 + 2*Sqrt[2])]) + ((a - Sqrt[2]*b)*Log[
Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(4*Sqrt[2*(-1 + 2*Sqrt[2
])])
```

Rubi in Sympy [A] time = 33.0675, size = 212, normalized size = 0.91

$$-\frac{\sqrt{2}\left(a-\sqrt{2}b\right)\log\left(x^2-x\sqrt{-1+2\sqrt{2}}+\sqrt{2}\right)}{8\sqrt{-1+2\sqrt{2}}}+\frac{\sqrt{2}\left(a-\sqrt{2}b\right)\log\left(x^2+x\sqrt{-1+2\sqrt{2}}+\sqrt{2}\right)}{8\sqrt{-1+2\sqrt{2}}}+\frac{\sqrt{2}\left(a+\sqrt{2}b\right)\operatorname{atan}\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)}{4\sqrt{1+2\sqrt{2}}}+\frac{\sqrt{2}\left(a+\sqrt{2}b\right)\operatorname{atan}\left(\frac{2x+\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)}{4\sqrt{1+2\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(x**4+x**2+2),x)
```

```
[Out] -sqrt(2)*(a - sqrt(2)*b)*log(x**2 - x*sqrt(-1 + 2*sqrt(2)) + sqrt
(2))/(8*sqrt(-1 + 2*sqrt(2))) + sqrt(2)*(a - sqrt(2)*b)*log(x**2
+ x*sqrt(-1 + 2*sqrt(2)) + sqrt(2))/(8*sqrt(-1 + 2*sqrt(2))) + sq
rt(2)*(a + sqrt(2)*b)*atan((2*x - sqrt(-1 + 2*sqrt(2)))/sqrt(1 +
2*sqrt(2)))/(4*sqrt(1 + 2*sqrt(2))) + sqrt(2)*(a + sqrt(2)*b)*ata
n((2*x + sqrt(-1 + 2*sqrt(2)))/sqrt(1 + 2*sqrt(2)))/(4*sqrt(1 + 2
*sqrt(2)))
```

Mathematica [C] time = 0.19488, size = 111, normalized size = 0.47

$$\frac{\left(\left(\sqrt{7}+i\right)b-2ia\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{\sqrt{14-14i\sqrt{7}}}+\frac{\left(2ia+\left(\sqrt{7}-i\right)b\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{\sqrt{14+14i\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(2 + x^2 + x^4),x]
```

```
[Out] (((-2*I)*a + (I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/
Sqrt[14 - (14*I)*Sqrt[7]] + (((2*I)*a + (-I + Sqrt[7])*b)*ArcTan[
x/Sqrt[(1 + I*Sqrt[7])/2]])/Sqrt[14 + (14*I)*Sqrt[7]]
```

Maple [B] time = 0.103, size = 710, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(x^4+x^2+2),x)
```

```
[Out] -1/56*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)
*2^(1/2)*a+1/14*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1
/2))^(1/2)*2^(1/2)*b-1/14*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*
(-1+2*2^(1/2))^(1/2)*a+1/28*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2)
)*(-1+2*2^(1/2))^(1/2)*b-1/28/(1+2*2^(1/2))^(1/2)*arctan((2*x-(-1
+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))*2^(1/2)*a+
1/7/(1+2*2^(1/2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^
(1/2))^(1/2))*(-1+2*2^(1/2))*2^(1/2)*b-1/7/(1+2*2^(1/2))^(1/2)*ar
ctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2
))*a+1/14/(1+2*2^(1/2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(
1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))*b+1/2/(1+2*2^(1/2))^(1/2)*arct
an((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*2^(1/2)*a+1/56
*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*2^(1
/2)*a-1/14*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^
(1/2)*2^(1/2)*b+1/14*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2
*2^(1/2))^(1/2)*a-1/28*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1
+2*2^(1/2))^(1/2)*b-1/28/(1+2*2^(1/2))^(1/2)*arctan((2*x+(-1+2*2^
(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))*2^(1/2)*a+1/7/(
1+2*2^(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2)
)^(1/2))*(-1+2*2^(1/2))*2^(1/2)*b-1/7/(1+2*2^(1/2))^(1/2)*arctan(
(2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))*a+
1/14/(1+2*2^(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2
^(1/2))^(1/2))*(-1+2*2^(1/2))*b+1/2/(1+2*2^(1/2))^(1/2)*arctan((2
*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*2^(1/2)*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{x^4 + x^2 + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(x^4 + x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/(x^4 + x^2 + 2), x)
```

Fricas [A] time = 0.788866, size = 7285, normalized size = 31.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(x^4 + x^2 + 2),x, algorithm="fricas")
```

```
[Out] -1/4*(28*sqrt(2)*(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)^(1
/4)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*arctan(7*(a^4 - 2*a^3*b + 5*a^2
*b^2 - 4*a*b^3 + 4*b^4)^(1/4)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*(sqrt
(2)*(2*a^3 - 3*a^2*b + 5*a*b^2 - 2*b^3) + sqrt(a^4 - 2*a^3*b + 5*
a^2*b^2 - 4*a*b^3 + 4*b^4)*(a - 4*b))/(sqrt(1/2)*(98^(1/4)*sqrt(7
)*sqrt(2)*(a^6 - 9*a^5*b + 10*a^4*b^2 - 20*a^2*b^4 + 36*a*b^5 - 8
*b^6) + 4*98^(1/4)*sqrt(7)*(a^4 - a^3*b + 2*a*b^3 - 4*b^4)*sqrt(a
^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4))*sqrt((4*a^4 - 8*a^3*
b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 + sqrt(2)*sqrt(a^4 - 2*a^3*b +
 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(9*a^4 - 32*
a^3*b + 108*a^2*b^2 - 64*a*b^3 + 36*b^4 + 4*sqrt(2)*sqrt(a^4 - 2*
a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2)))*sqrt
((4*sqrt(2)*(11*a^10 - 97*a^9*b + 535*a^8*b^2 - 1579*a^7*b^3 + 36
54*a^6*b^4 - 5740*a^5*b^5 + 7308*a^4*b^6 - 6316*a^3*b^7 + 4280*a^
2*b^8 - 1552*a*b^9 + 352*b^10)*x^2 + 2*(25*a^8 - 289*a^7*b + 1064
*a^6*b^2 - 3038*a^5*b^3 + 4760*a^4*b^4 - 6076*a^3*b^5 + 4256*a^2*
b^6 - 2312*a*b^7 + 400*b^8)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*
b^3 + 4*b^4)*x^2 + (a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)^
(1/4)*(98^(1/4)*sqrt(2)*(25*a^7 - 308*a^6*b + 1050*a^5*b^2 - 3080
*a^4*b^3 + 3500*a^3*b^4 - 3696*a^2*b^5 + 1400*a*b^6 - 352*b^7)*sq
rt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*x + 2*98^(1/4)*(2
2*a^9 - 197*a^8*b + 1143*a^7*b^2 - 3024*a^6*b^3 + 6678*a^5*b^4 -
8680*a^4*b^5 + 9492*a^3*b^6 - 5632*a^2*b^7 + 2664*a*b^8 - 400*b^9
)*x)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 + sqr
t(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a
*b + 2*b^2))/(9*a^4 - 32*a^3*b + 108*a^2*b^2 - 64*a*b^3 + 36*b^4
+ 4*sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^
2 - 8*a*b + 2*b^2))) + 2*(44*a^8 - 344*a^7*b + 1708*a^6*b^2 - 392
0*a^5*b^3 + 7280*a^4*b^4 - 7840*a^3*b^5 + 6832*a^2*b^6 - 2752*a*b
^7 + 704*b^8 + sqrt(2)*(25*a^6 - 264*a^5*b + 750*a^4*b^2 - 1760*a
^3*b^3 + 1500*a^2*b^4 - 1056*a*b^5 + 200*b^6)*sqrt(a^4 - 2*a^3*b
+ 5*a^2*b^2 - 4*a*b^3 + 4*b^4))*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 -
4*a*b^3 + 4*b^4))/(2*sqrt(2)*(11*a^10 - 97*a^9*b + 535*a^8*b^2 -
1579*a^7*b^3 + 3654*a^6*b^4 - 5740*a^5*b^5 + 7308*a^4*b^6 - 6316*
a^3*b^7 + 4280*a^2*b^8 - 1552*a*b^9 + 352*b^10) + (25*a^8 - 289*a
^7*b + 1064*a^6*b^2 - 3038*a^5*b^3 + 4760*a^4*b^4 - 6076*a^3*b^5
+ 4256*a^2*b^6 - 2312*a*b^7 + 400*b^8)*sqrt(a^4 - 2*a^3*b + 5*a^2
*b^2 - 4*a*b^3 + 4*b^4))) + (98^(1/4)*sqrt(7)*sqrt(2)*(a^6 - 9*a^
5*b + 10*a^4*b^2 - 20*a^2*b^4 + 36*a*b^5 - 8*b^6)*x + 4*98^(1/4)*
sqrt(7)*(a^4 - a^3*b + 2*a*b^3 - 4*b^4)*sqrt(a^4 - 2*a^3*b + 5*a^
2*b^2 - 4*a*b^3 + 4*b^4)*x)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 -
16*a*b^3 + 16*b^4 + sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*
b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(9*a^4 - 32*a^3*b + 108*a^2*b
^2 - 64*a*b^3 + 36*b^4 + 4*sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2
 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))) - 7*(a^4 - 2*a^3*b +
5*a^2*b^2 - 4*a*b^3 + 4*b^4)^(1/4)*(sqrt(7)*sqrt(2)*(a^4*b - a^3*
```

```
b^2 + 2*a*b^4 - 4*b^5) - sqrt(7)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 -
 4*a*b^3 + 4*b^4)*(a^3 - 2*a*b^2)))) + 28*sqrt(2)*(a^4 - 2*a^3*b
+ 5*a^2*b^2 - 4*a*b^3 + 4*b^4)^(1/4)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4
)*arctan(7*(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)^(1/4)*sq
rt(a^4 - 4*a^2*b^2 + 4*b^4)*(sqrt(2)*(2*a^3 - 3*a^2*b + 5*a*b^2 -
 2*b^3) + sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a -
4*b))/(sqrt(1/2)*(98^(1/4)*sqrt(7)*sqrt(2)*(a^6 - 9*a^5*b + 10*a^
4*b^2 - 20*a^2*b^4 + 36*a*b^5 - 8*b^6) + 4*98^(1/4)*sqrt(7)*(a^4
- a^3*b + 2*a*b^3 - 4*b^4)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b
^3 + 4*b^4))*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b
^4 + sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a
^2 - 8*a*b + 2*b^2))/(9*a^4 - 32*a^3*b + 108*a^2*b^2 - 64*a*b^3 +
 36*b^4 + 4*sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*
b^4)*(a^2 - 8*a*b + 2*b^2)))*sqrt((4*sqrt(2)*(11*a^10 - 97*a^9*b
+ 535*a^8*b^2 - 1579*a^7*b^3 + 3654*a^6*b^4 - 5740*a^5*b^5 + 7308
*a^4*b^6 - 6316*a^3*b^7 + 4280*a^2*b^8 - 1552*a*b^9 + 352*b^10)*x
^2 + 2*(25*a^8 - 289*a^7*b + 1064*a^6*b^2 - 3038*a^5*b^3 + 4760*a
^4*b^4 - 6076*a^3*b^5 + 4256*a^2*b^6 - 2312*a*b^7 + 400*b^8)*sqrt
(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*x^2 - (a^4 - 2*a^3*
b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)^(1/4)*(98^(1/4)*sqrt(2)*(25*a^7
- 308*a^6*b + 1050*a^5*b^2 - 3080*a^4*b^3 + 3500*a^3*b^4 - 3696*a
^2*b^5 + 1400*a*b^6 - 352*b^7)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4
*a*b^3 + 4*b^4)*x + 2*98^(1/4)*(22*a^9 - 197*a^8*b + 1143*a^7*b^2
 - 3024*a^6*b^3 + 6678*a^5*b^4 - 8680*a^4*b^5 + 9492*a^3*b^6 - 56
32*a^2*b^7 + 2664*a*b^8 - 400*b^9)*x)*sqrt((4*a^4 - 8*a^3*b + 20*
a^2*b^2 - 16*a*b^3 + 16*b^4 + sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*
b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(9*a^4 - 32*a^3*b +
 108*a^2*b^2 - 64*a*b^3 + 36*b^4 + 4*sqrt(2)*sqrt(a^4 - 2*a^3*b +
 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))) + 2*(44*a^8
 - 344*a^7*b + 1708*a^6*b^2 - 3920*a^5*b^3 + 7280*a^4*b^4 - 7840*
a^3*b^5 + 6832*a^2*b^6 - 2752*a*b^7 + 704*b^8 + sqrt(2)*(25*a^6 -
 264*a^5*b + 750*a^4*b^2 - 1760*a^3*b^3 + 1500*a^2*b^4 - 1056*a*b
^5 + 200*b^6)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4))*
sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4))/(2*sqrt(2)*(11
*a^10 - 97*a^9*b + 535*a^8*b^2 - 1579*a^7*b^3 + 3654*a^6*b^4 - 57
40*a^5*b^5 + 7308*a^4*b^6 - 6316*a^3*b^7 + 4280*a^2*b^8 - 1552*a*
b^9 + 352*b^10) + (25*a^8 - 289*a^7*b + 1064*a^6*b^2 - 3038*a^5*b
^3 + 4760*a^4*b^4 - 6076*a^3*b^5 + 4256*a^2*b^6 - 2312*a*b^7 + 40
0*b^8)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4))) + (98^
(1/4)*sqrt(7)*sqrt(2)*(a^6 - 9*a^5*b + 10*a^4*b^2 - 20*a^2*b^4 +
36*a*b^5 - 8*b^6)*x + 4*98^(1/4)*sqrt(7)*(a^4 - a^3*b + 2*a*b^3 -
 4*b^4)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*x)*sqrt
((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 + sqrt(2)*sqrt
(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^
2))/(9*a^4 - 32*a^3*b + 108*a^2*b^2 - 64*a*b^3 + 36*b^4 + 4*sqrt(
2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b
 + 2*b^2))) + 7*(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)^(1/
4)*(sqrt(7)*sqrt(2)*(a^4*b - a^3*b^2 + 2*a*b^4 - 4*b^5) - sqrt(7)
*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^3 - 2*a*b^2
)))) - (a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)^(1/4)*(sqrt(
7)*sqrt(2)*(a^2 - 8*a*b + 2*b^2) + 4*sqrt(7)*sqrt(a^4 - 2*a^3*b +
 5*a^2*b^2 - 4*a*b^3 + 4*b^4))*log(56*sqrt(2)*(11*a^10 - 97*a^9*b
 + 535*a^8*b^2 - 1579*a^7*b^3 + 3654*a^6*b^4 - 5740*a^5*b^5 + 730
8*a^4*b^6 - 6316*a^3*b^7 + 4280*a^2*b^8 - 1552*a*b^9 + 352*b^10)*
x^2 + 28*(25*a^8 - 289*a^7*b + 1064*a^6*b^2 - 3038*a^5*b^3 + 4760
*a^4*b^4 - 6076*a^3*b^5 + 4256*a^2*b^6 - 2312*a*b^7 + 400*b^8)*sq
rt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*x^2 + 14*(a^4 - 2
*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)^(1/4)*(98^(1/4)*sqrt(2)*(25
*a^7 - 308*a^6*b + 1050*a^5*b^2 - 3080*a^4*b^3 + 3500*a^3*b^4 - 3
696*a^2*b^5 + 1400*a*b^6 - 352*b^7)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^
2 - 4*a*b^3 + 4*b^4)*x + 2*98^(1/4)*(22*a^9 - 197*a^8*b + 1143*a^
7*b^2 - 3024*a^6*b^3 + 6678*a^5*b^4 - 8680*a^4*b^5 + 9492*a^3*b^6
 - 5632*a^2*b^7 + 2664*a*b^8 - 400*b^9)*x)*sqrt((4*a^4 - 8*a^3*b
+ 20*a^2*b^2 - 16*a*b^3 + 16*b^4 + sqrt(2)*sqrt(a^4 - 2*a^3*b + 5
*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(9*a^4 - 32*a^
3*b + 108*a^2*b^2 - 64*a*b^3 + 36*b^4 + 4*sqrt(2)*sqrt(a^4 - 2*a^
3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))) + 28*(
44*a^8 - 344*a^7*b + 1708*a^6*b^2 - 3920*a^5*b^3 + 7280*a^4*b^4 -
 7840*a^3*b^5 + 6832*a^2*b^6 - 2752*a*b^7 + 704*b^8 + sqrt(2)*(25
*a^6 - 264*a^5*b + 750*a^4*b^2 - 1760*a^3*b^3 + 1500*a^2*b^4 - 10
56*a*b^5 + 200*b^6)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*
b^4))*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)) + (a^4 -
```

```
 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)^(1/4)*(sqrt(7)*sqrt(2)*(a
^2 - 8*a*b + 2*b^2) + 4*sqrt(7)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 -
4*a*b^3 + 4*b^4))*log(56*sqrt(2)*(11*a^10 - 97*a^9*b + 535*a^8*b^
2 - 1579*a^7*b^3 + 3654*a^6*b^4 - 5740*a^5*b^5 + 7308*a^4*b^6 - 6
316*a^3*b^7 + 4280*a^2*b^8 - 1552*a*b^9 + 352*b^10)*x^2 + 28*(25*
a^8 - 289*a^7*b + 1064*a^6*b^2 - 3038*a^5*b^3 + 4760*a^4*b^4 - 60
76*a^3*b^5 + 4256*a^2*b^6 - 2312*a*b^7 + 400*b^8)*sqrt(a^4 - 2*a^
3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*x^2 - 14*(a^4 - 2*a^3*b + 5*a^
2*b^2 - 4*a*b^3 + 4*b^4)^(1/4)*(98^(1/4)*sqrt(2)*(25*a^7 - 308*a^
6*b + 1050*a^5*b^2 - 3080*a^4*b^3 + 3500*a^3*b^4 - 3696*a^2*b^5 +
 1400*a*b^6 - 352*b^7)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 +
 4*b^4)*x + 2*98^(1/4)*(22*a^9 - 197*a^8*b + 1143*a^7*b^2 - 3024*
a^6*b^3 + 6678*a^5*b^4 - 8680*a^4*b^5 + 9492*a^3*b^6 - 5632*a^2*b
^7 + 2664*a*b^8 - 400*b^9)*x)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2
- 16*a*b^3 + 16*b^4 + sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*
a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(9*a^4 - 32*a^3*b + 108*a^2
*b^2 - 64*a*b^3 + 36*b^4 + 4*sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b
^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))) + 28*(44*a^8 - 344*
a^7*b + 1708*a^6*b^2 - 3920*a^5*b^3 + 7280*a^4*b^4 - 7840*a^3*b^5
 + 6832*a^2*b^6 - 2752*a*b^7 + 704*b^8 + sqrt(2)*(25*a^6 - 264*a^
5*b + 750*a^4*b^2 - 1760*a^3*b^3 + 1500*a^2*b^4 - 1056*a*b^5 + 20
0*b^6)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4))*sqrt(a^
4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)))/((98^(1/4)*sqrt(7)*s
qrt(2)*(a^2 - 8*a*b + 2*b^2) + 4*98^(1/4)*sqrt(7)*sqrt(a^4 - 2*a^
3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4))*sqrt((4*a^4 - 8*a^3*b + 20*a^
2*b^2 - 16*a*b^3 + 16*b^4 + sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^
2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(9*a^4 - 32*a^3*b + 1
08*a^2*b^2 - 64*a*b^3 + 36*b^4 + 4*sqrt(2)*sqrt(a^4 - 2*a^3*b + 5
*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))))
```

Sympy [A] time = 3.20968, size = 122, normalized size = 0.52

$$\operatorname{RootSum}\left(1568t^4 + t^2\left(-28a^2 + 224ab - 56b^2\right) + a^4 - 2a^3b + 5a^2b^2 - 4ab^3 + 4b^4, \left(t \mapsto t\log\left(x + \frac{112t^3a - 448t^3b + 6ta^3 +}{a^4 - a^3b +}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(x**4+x**2+2),x)
```

```
[Out] RootSum(1568*_t**4 + _t**2*(-28*a**2 + 224*a*b - 56*b**2) + a**4
- 2*a**3*b + 5*a**2*b**2 - 4*a*b**3 + 4*b**4, Lambda(_t, _t*log(x
 + (112*_t**3*a - 448*_t**3*b + 6*_t*a**3 + 12*_t*a**2*b - 48*_t*
a*b**2 + 8*_t*b**3)/(a**4 - a**3*b + 2*a*b**3 - 4*b**4))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{x^4 + x^2 + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(x^4 + x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/(x^4 + x^2 + 2), x)
```

3.109 $\int \frac{a+bx^2}{(2+x^2+x^4)^2} dx$

Optimal. Leaf size=316

$$-\frac{\left(\sqrt{2}(a-4b)+11a-2b\right)\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{112\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\left(\left(11+\sqrt{2}\right)a-2\left(2\sqrt{2}b+b\right)\right)\log\left(x^2+\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{112\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{x\left(x^2(-(a-4b))+3a+2b\right)}{28\left(x^4+x^2+2\right)}-\frac{1}{56}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{1+2\sqrt{2}}}\right)+\frac{1}{56}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\tan^{-1}\left(\frac{2x+\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)$$

```
[Out] (x*(3*a + 2*b - (a - 4*b)*x^2))/(28*(2 + x^2 + x^4)) - (Sqrt[(-1
+ 2*Sqrt[2])/14]*((11 - Sqrt[2])*a - (2 - 4*Sqrt[2])*b)*ArcTan[(S
qrt[-1 + 2*Sqrt[2]] - 2*x)/Sqrt[1 + 2*Sqrt[2]]])/56 + (Sqrt[(-1 +
 2*Sqrt[2])/14]*((11 - Sqrt[2])*a - (2 - 4*Sqrt[2])*b)*ArcTan[(Sq
rt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]]])/56 - ((11*a + Sqr
t[2]*(a - 4*b) - 2*b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2]]*x + x^2]
)/(112*Sqrt[2*(-1 + 2*Sqrt[2])]) + (((11 + Sqrt[2])*a - 2*(b + 2*
Sqrt[2]*b))*Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(112*Sqr
t[2*(-1 + 2*Sqrt[2])])
```

Rubi [A] time = 0.763603, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{\left(\sqrt{2}(a-4b)+11a-2b\right)\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{112\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\left(\left(11+\sqrt{2}\right)a-2\left(2\sqrt{2}b+b\right)\right)\log\left(x^2+\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{112\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{x\left(x^2(-(a-4b))+3a+2b\right)}{28\left(x^4+x^2+2\right)}-\frac{1}{56}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{1+2\sqrt{2}}}\right)+\frac{1}{56}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\tan^{-1}\left(\frac{2x+\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(2 + x^2 + x^4)^2,x]
```


```
[Out] (x*(3*a + 2*b - (a - 4*b)*x^2))/(28*(2 + x^2 + x^4)) - (Sqrt[(-1
+ 2*Sqrt[2])/14]*((11 - Sqrt[2])*a - (2 - 4*Sqrt[2])*b)*ArcTan[(S
qrt[-1 + 2*Sqrt[2]] - 2*x)/Sqrt[1 + 2*Sqrt[2]]])/56 + (Sqrt[(-1 +
 2*Sqrt[2])/14]*((11 - Sqrt[2])*a - (2 - 4*Sqrt[2])*b)*ArcTan[(Sq
rt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]]])/56 - ((11*a + Sqr
t[2]*(a - 4*b) - 2*b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2]]*x + x^2]
)/(112*Sqrt[2*(-1 + 2*Sqrt[2])]) + (((11 + Sqrt[2])*a - 2*(b + 2*
Sqrt[2]*b))*Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(112*Sqr
t[2*(-1 + 2*Sqrt[2])])
```

Rubi in Sympy [A] time = 49.4675, size = 275, normalized size = 0.87

$$\frac{x\left(3a+2b-x^2(a-4b)\right)}{28\left(x^4+x^2+2\right)}+\frac{\sqrt{2}\left(11a-2b-\sqrt{2}(a-4b)\right)\operatorname{atan}\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)}{112\sqrt{1+2\sqrt{2}}}+\frac{\sqrt{2}\left(11a-2b-\sqrt{2}(a-4b)\right)\operatorname{atan}\left(\frac{2x+\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)}{112\sqrt{1+2\sqrt{2}}}-\frac{\sqrt{2}\left(11a-2b+\sqrt{2}(a-4b)\right)\log\left(x^2-x\sqrt{-1+2\sqrt{2}}+\sqrt{2}\right)}{224\sqrt{-1+2\sqrt{2}}}+\frac{\sqrt{2}\left(11a-2b+\sqrt{2}(a-4b)\right)\log\left(x^2+x\sqrt{-1+2\sqrt{2}}+\sqrt{2}\right)}{224\sqrt{-1+2\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(x**4+x**2+2)**2,x)
```

```
[Out] x*(3*a + 2*b - x**2*(a - 4*b))/(28*(x**4 + x**2 + 2)) + sqrt(2)*(
11*a - 2*b - sqrt(2)*(a - 4*b))*atan((2*x - sqrt(-1 + 2*sqrt(2)))
/sqrt(1 + 2*sqrt(2)))/(112*sqrt(1 + 2*sqrt(2))) + sqrt(2)*(11*a -
 2*b - sqrt(2)*(a - 4*b))*atan((2*x + sqrt(-1 + 2*sqrt(2)))/sqrt(
1 + 2*sqrt(2)))/(112*sqrt(1 + 2*sqrt(2))) - sqrt(2)*(11*a - 2*b +
 sqrt(2)*(a - 4*b))*log(x**2 - x*sqrt(-1 + 2*sqrt(2)) + sqrt(2))/
(224*sqrt(-1 + 2*sqrt(2))) + sqrt(2)*(11*a - 2*b + sqrt(2)*(a - 4
*b))*log(x**2 + x*sqrt(-1 + 2*sqrt(2)) + sqrt(2))/(224*sqrt(-1 +
2*sqrt(2)))
```

Mathematica [C] time = 0.354002, size = 165, normalized size = 0.52

$$\frac{2b\left(2x^3+x\right)-ax\left(x^2-3\right)}{28\left(x^4+x^2+2\right)}-\frac{\left(\left(\sqrt{7}+23i\right)a-4\left(\sqrt{7}+2i\right)b\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{28\sqrt{14-14i\sqrt{7}}}-\frac{\left(\left(\sqrt{7}-23i\right)a-4\left(\sqrt{7}-2i\right)b\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{28\sqrt{14+14i\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(2 + x^2 + x^4)^2,x]
```

```
[Out] (-(a*x*(-3 + x^2)) + 2*b*(x + 2*x^3))/(28*(2 + x^2 + x^4)) - (((2
3*I + Sqrt[7])*a - 4*(2*I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 - I*Sqrt
[7])/2]])/(28*Sqrt[14 - (14*I)*Sqrt[7]]) - (((-23*I + Sqrt[7])*a
- 4*(-2*I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 + I*Sqrt[7])/2]])/(28*Sq
rt[14 + (14*I)*Sqrt[7]])
```

Maple [B] time = 0.316, size = 1506, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(x^4+x^2+2)^2,x)
```

```
[Out] 11/56/(22*2^(1/2)+25)^(1/2)*arctan((2*(1+2*2^(1/2))*x-(1+2*2^(1/2
))*(-1+2*2^(1/2))^(1/2))/(22*2^(1/2)+25)^(1/2))*2^(1/2)*a-1/28/(2
2*2^(1/2)+25)^(1/2)*arctan((2*(1+2*2^(1/2))*x-(1+2*2^(1/2))*(-1+2
*2^(1/2))^(1/2))/(22*2^(1/2)+25)^(1/2))*2^(1/2)*b-53/392/(22*2^(1
/2)+25)^(1/2)*arctan((2*(1+2*2^(1/2))*x-(1+2*2^(1/2))*(-1+2*2^(1/
2))^(1/2))/(22*2^(1/2)+25)^(1/2))*(-1+2*2^(1/2))*a+11/98/(22*2^(1
/2)+25)^(1/2)*arctan((2*(1+2*2^(1/2))*x-(1+2*2^(1/2))*(-1+2*2^(1/
2))^(1/2))/(22*2^(1/2)+25)^(1/2))*(-1+2*2^(1/2))*b+53/784/(1+2*2^
(1/2))*ln((1+2*2^(1/2))*(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2)))*(-1
+2*2^(1/2))^(1/2)*a-11/196/(1+2*2^(1/2))*ln((1+2*2^(1/2))*(x^2+2^
(1/2)+x*(-1+2*2^(1/2))^(1/2)))*(-1+2*2^(1/2))^(1/2)*b+11/56/(22*2
^(1/2)+25)^(1/2)*arctan((2*(1+2*2^(1/2))*x+(1+2*2^(1/2))*(-1+2*2^
(1/2))^(1/2))/(22*2^(1/2)+25)^(1/2))*2^(1/2)*a-1/28/(22*2^(1/2)+2
5)^(1/2)*arctan((2*(1+2*2^(1/2))*x+(1+2*2^(1/2))*(-1+2*2^(1/2))^(
1/2))/(22*2^(1/2)+25)^(1/2))*2^(1/2)*b-53/392/(22*2^(1/2)+25)^(1/
2)*arctan((2*(1+2*2^(1/2))*x+(1+2*2^(1/2))*(-1+2*2^(1/2))^(1/2))/
(22*2^(1/2)+25)^(1/2))*(-1+2*2^(1/2))*a+11/98/(22*2^(1/2)+25)^(1/
2)*arctan((2*(1+2*2^(1/2))*x+(1+2*2^(1/2))*(-1+2*2^(1/2))^(1/2))/
(22*2^(1/2)+25)^(1/2))*(-1+2*2^(1/2))*b-53/784/(1+2*2^(1/2))*ln(-
(1+2*2^(1/2))*(x*(-1+2*2^(1/2))^(1/2)-x^2-2^(1/2)))*(-1+2*2^(1/2)
)^(1/2)*a+11/196/(1+2*2^(1/2))*ln(-(1+2*2^(1/2))*(x*(-1+2*2^(1/2)
)^(1/2)-x^2-2^(1/2)))*(-1+2*2^(1/2))^(1/2)*b+11/14/(22*2^(1/2)+25
)^(1/2)*arctan((2*(1+2*2^(1/2))*x+(1+2*2^(1/2))*(-1+2*2^(1/2))^(1
/2))/(22*2^(1/2)+25)^(1/2))*a-1/7/(22*2^(1/2)+25)^(1/2)*arctan((2
*(1+2*2^(1/2))*x+(1+2*2^(1/2))*(-1+2*2^(1/2))^(1/2))/(22*2^(1/2)+
25)^(1/2))*b+1/784*((-14*a-28*2^(1/2)*a+112*b*2^(1/2)+56*b)/(1+2*
2^(1/2))*x+1/(1+2*2^(1/2))*(-1+2*2^(1/2))^(1/2)*(-70*a-42*2^(1/2)
*a+56*b*2^(1/2)+28*b))/(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))-1/784
*(-(-14*a-28*2^(1/2)*a+112*b*2^(1/2)+56*b)/(1+2*2^(1/2))*x+1/(1+2
*2^(1/2))*(-1+2*2^(1/2))^(1/2)*(-70*a-42*2^(1/2)*a+56*b*2^(1/2)+2
8*b))/(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))-107/1568/(1+2*2^(1/2))
*ln(-(1+2*2^(1/2))*(x*(-1+2*2^(1/2))^(1/2)-x^2-2^(1/2)))*(-1+2*2^
(1/2))^(1/2)*2^(1/2)*a+25/784/(1+2*2^(1/2))*ln(-(1+2*2^(1/2))*(x*
(-1+2*2^(1/2))^(1/2)-x^2-2^(1/2)))*(-1+2*2^(1/2))^(1/2)*2^(1/2)*b
-107/784/(22*2^(1/2)+25)^(1/2)*arctan((2*(1+2*2^(1/2))*x-(1+2*2^(
1/2))*(-1+2*2^(1/2))^(1/2))/(22*2^(1/2)+25)^(1/2))*(-1+2*2^(1/2))
*2^(1/2)*a+25/392/(22*2^(1/2)+25)^(1/2)*arctan((2*(1+2*2^(1/2))*x
-(1+2*2^(1/2))*(-1+2*2^(1/2))^(1/2))/(22*2^(1/2)+25)^(1/2))*(-1+2
*2^(1/2))*2^(1/2)*b+107/1568/(1+2*2^(1/2))*ln((1+2*2^(1/2))*(x^2+
2^(1/2)+x*(-1+2*2^(1/2))^(1/2)))*(-1+2*2^(1/2))^(1/2)*2^(1/2)*a-2
5/784/(1+2*2^(1/2))*ln((1+2*2^(1/2))*(x^2+2^(1/2)+x*(-1+2*2^(1/2)
)^(1/2)))*(-1+2*2^(1/2))^(1/2)*2^(1/2)*b-107/784/(22*2^(1/2)+25)^
(1/2)*arctan((2*(1+2*2^(1/2))*x+(1+2*2^(1/2))*(-1+2*2^(1/2))^(1/2
))/(22*2^(1/2)+25)^(1/2))*(-1+2*2^(1/2))*2^(1/2)*a+25/392/(22*2^(
1/2)+25)^(1/2)*arctan((2*(1+2*2^(1/2))*x+(1+2*2^(1/2))*(-1+2*2^(1
/2))^(1/2))/(22*2^(1/2)+25)^(1/2))*(-1+2*2^(1/2))*2^(1/2)*b+11/14
/(22*2^(1/2)+25)^(1/2)*arctan((2*(1+2*2^(1/2))*x-(1+2*2^(1/2))*(-
1+2*2^(1/2))^(1/2))/(22*2^(1/2)+25)^(1/2))*a-1/7/(22*2^(1/2)+25)^
(1/2)*arctan((2*(1+2*2^(1/2))*x-(1+2*2^(1/2))*(-1+2*2^(1/2))^(1/2
))/(22*2^(1/2)+25)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(a-4\,b)x^3-(3\,a+2\,b)x}{28\,(x^4+x^2+2)}+\frac{1}{28}\int-\frac{(a-4\,b)x^2-11\,a+2\,b}{x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(x^4 + x^2 + 2)^2,x, algorithm="maxima")
```

```
[Out] -1/28*((a - 4*b)*x^3 - (3*a + 2*b)*x)/(x^4 + x^2 + 2) + 1/28*inte
grate(-((a - 4*b)*x^2 - 11*a + 2*b)/(x^4 + x^2 + 2), x)
```

Fricas [A] time = 0.998068, size = 8505, normalized size = 26.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(x^4 + x^2 + 2)^2,x, algorithm="fricas")
```

```
[Out] -1/112*sqrt(2)*(196*sqrt(2)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2
 - 2332*a*b^3 + 484*b^4)^(1/4)*sqrt(289*a^4 - 136*a^3*b - 120*a^2
*b^2 + 32*a*b^3 + 16*b^4)*(x^4 + x^2 + 2)*arctan(28*(4489*a^4 - 7
102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(1/4)*(sqrt(2)*s
qrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)*(1541*
a^3 - 1755*a^2*b + 930*a*b^2 - 176*b^3) + 3*sqrt(4489*a^4 - 7102*
a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*sqrt(289*a^4 - 136*a
^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)*(5*a - 6*b))/(sqrt(2)*(39
2^(1/4)*sqrt(7)*sqrt(2)*(240329*a^6 - 734151*a^5*b + 681330*a^4*b
^2 - 226840*a^3*b^3 - 39840*a^2*b^4 + 50064*a*b^5 - 8800*b^6) + 8
*392^(1/4)*sqrt(7)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 23
32*a*b^3 + 484*b^4)*(1139*a^4 - 1169*a^3*b + 318*a^2*b^2 + 124*a*
b^3 - 88*b^4))*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18
656*a*b^3 + 3872*b^4 + sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*
a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(1
88169*a^4 - 407880*a^3*b + 409608*a^2*b^2 - 160224*a*b^3 + 25488*
b^4 + 8*sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*
a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2)))*sqrt((16*sqrt(2)
*(83374811993*a^10 - 429180060917*a^9*b + 1046369963313*a^8*b^2 -
 1547097761283*a^7*b^3 + 1544290335042*a^6*b^4 - 1087904445516*a^
5*b^5 + 549199183848*a^4*b^6 - 196180671696*a^3*b^7 + 47524805856
*a^2*b^8 - 7029577280*a*b^9 + 484356224*b^10)*x^2 + 4*(6721648705
*a^8 - 31143421331*a^7*b + 61552270234*a^6*b^2 - 71728777316*a^5*
b^3 + 53344828600*a^4*b^4 - 26127862544*a^3*b^5 + 8187157216*a^2*
b^6 - 1511762624*a*b^7 + 124220800*b^8)*sqrt(4489*a^4 - 7102*a^3*
b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^2 + sqrt(2)*(4489*a^4
 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(1/4)*(3*392^
(1/4)*sqrt(2)*(417379787*a^7 - 1854968766*a^6*b + 3371736900*a^5*
b^2 - 3504565960*a^4*b^3 + 2146853520*a^3*b^4 - 797847456*a^2*b^5
 + 163604672*a*b^6 - 14438784*b^7)*sqrt(4489*a^4 - 7102*a^3*b + 5
757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x + 2*392^(1/4)*(61475256581*
a^9 - 306522254343*a^8*b + 709992386538*a^7*b^2 - 962885785428*a^
6*b^3 + 856950314136*a^5*b^4 - 517845594000*a^4*b^5 + 21342808905
6*a^3*b^6 - 57595247040*a^2*b^7 + 9271881600*a*b^8 - 673012736*b^
9)*x)*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3
 + 3872*b^4 + sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 -
 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(188169*a^4
 - 407880*a^3*b + 409608*a^2*b^2 - 160224*a*b^3 + 25488*b^4 + 8*s
qrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 4
84*b^4)*(211*a^2 - 428*a*b + 100*b^2))) + 4*(9955201432*a^8 - 433
70370320*a^7*b + 87362920120*a^6*b^2 - 101379092192*a^5*b^3 + 755
11142560*a^4*b^4 - 36877686272*a^3*b^5 + 11609417344*a^2*b^6 - 21
31897856*a*b^7 + 176129536*b^8 + sqrt(2)*(100323115*a^6 - 3854671
08*a^5*b + 580826940*a^4*b^2 - 484547360*a^3*b^3 + 222173520*a^2*
b^4 - 55113792*a*b^5 + 5646400*b^6)*sqrt(4489*a^4 - 7102*a^3*b +
5757*a^2*b^2 - 2332*a*b^3 + 484*b^4))*sqrt(4489*a^4 - 7102*a^3*b
+ 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4))/(4*sqrt(2)*(83374811993*a
^10 - 429180060917*a^9*b + 1046369963313*a^8*b^2 - 1547097761283*
a^7*b^3 + 1544290335042*a^6*b^4 - 1087904445516*a^5*b^5 + 5491991
83848*a^4*b^6 - 196180671696*a^3*b^7 + 47524805856*a^2*b^8 - 7029
577280*a*b^9 + 484356224*b^10) + (6721648705*a^8 - 31143421331*a^
7*b + 61552270234*a^6*b^2 - 71728777316*a^5*b^3 + 53344828600*a^4
*b^4 - 26127862544*a^3*b^5 + 8187157216*a^2*b^6 - 1511762624*a*b^
7 + 124220800*b^8)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 23
32*a*b^3 + 484*b^4))) + 2*sqrt(2)*(392^(1/4)*sqrt(7)*sqrt(2)*(240
329*a^6 - 734151*a^5*b + 681330*a^4*b^2 - 226840*a^3*b^3 - 39840*
a^2*b^4 + 50064*a*b^5 - 8800*b^6)*x + 8*392^(1/4)*sqrt(7)*sqrt(44
89*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(1139*
a^4 - 1169*a^3*b + 318*a^2*b^2 + 124*a*b^3 - 88*b^4)*x)*sqrt((359
12*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 + s
qrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 4
84*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(188169*a^4 - 407880*a^3*b
```

```
 + 409608*a^2*b^2 - 160224*a*b^3 + 25488*b^4 + 8*sqrt(2)*sqrt(448
9*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^
2 - 428*a*b + 100*b^2))) + 28*(4489*a^4 - 7102*a^3*b + 5757*a^2*b
^2 - 2332*a*b^3 + 484*b^4)^(1/4)*(sqrt(7)*sqrt(2)*(1139*a^5 - 572
5*a^4*b + 4994*a^3*b^2 - 1148*a^2*b^3 - 584*a*b^4 + 352*b^5) + sq
rt(7)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 48
4*b^4)*(187*a^3 - 78*a^2*b - 36*a*b^2 + 8*b^3)))) + 196*sqrt(2)*(
4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(1/4
)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)*(x^
4 + x^2 + 2)*arctan(28*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 23
32*a*b^3 + 484*b^4)^(1/4)*(sqrt(2)*sqrt(289*a^4 - 136*a^3*b - 120
*a^2*b^2 + 32*a*b^3 + 16*b^4)*(1541*a^3 - 1755*a^2*b + 930*a*b^2
- 176*b^3) + 3*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a
*b^3 + 484*b^4)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3
 + 16*b^4)*(5*a - 6*b))/(sqrt(2)*(392^(1/4)*sqrt(7)*sqrt(2)*(2403
29*a^6 - 734151*a^5*b + 681330*a^4*b^2 - 226840*a^3*b^3 - 39840*a
^2*b^4 + 50064*a*b^5 - 8800*b^6) + 8*392^(1/4)*sqrt(7)*sqrt(4489*
a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(1139*a^4
 - 1169*a^3*b + 318*a^2*b^2 + 124*a*b^3 - 88*b^4))*sqrt((35912*a^
4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 + sqrt(2
)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^
4)*(211*a^2 - 428*a*b + 100*b^2))/(188169*a^4 - 407880*a^3*b + 40
9608*a^2*b^2 - 160224*a*b^3 + 25488*b^4 + 8*sqrt(2)*sqrt(4489*a^4
 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 4
28*a*b + 100*b^2)))*sqrt((16*sqrt(2)*(83374811993*a^10 - 42918006
0917*a^9*b + 1046369963313*a^8*b^2 - 1547097761283*a^7*b^3 + 1544
290335042*a^6*b^4 - 1087904445516*a^5*b^5 + 549199183848*a^4*b^6
- 196180671696*a^3*b^7 + 47524805856*a^2*b^8 - 7029577280*a*b^9 +
 484356224*b^10)*x^2 + 4*(6721648705*a^8 - 31143421331*a^7*b + 61
552270234*a^6*b^2 - 71728777316*a^5*b^3 + 53344828600*a^4*b^4 - 2
6127862544*a^3*b^5 + 8187157216*a^2*b^6 - 1511762624*a*b^7 + 1242
20800*b^8)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3
 + 484*b^4)*x^2 - sqrt(2)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 -
 2332*a*b^3 + 484*b^4)^(1/4)*(3*392^(1/4)*sqrt(2)*(417379787*a^7
- 1854968766*a^6*b + 3371736900*a^5*b^2 - 3504565960*a^4*b^3 + 21
46853520*a^3*b^4 - 797847456*a^2*b^5 + 163604672*a*b^6 - 14438784
*b^7)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 48
4*b^4)*x + 2*392^(1/4)*(61475256581*a^9 - 306522254343*a^8*b + 70
9992386538*a^7*b^2 - 962885785428*a^6*b^3 + 856950314136*a^5*b^4
- 517845594000*a^4*b^5 + 213428089056*a^3*b^6 - 57595247040*a^2*b
^7 + 9271881600*a*b^8 - 673012736*b^9)*x)*sqrt((35912*a^4 - 56816
*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 + sqrt(2)*sqrt(44
89*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a
^2 - 428*a*b + 100*b^2))/(188169*a^4 - 407880*a^3*b + 409608*a^2*
b^2 - 160224*a*b^3 + 25488*b^4 + 8*sqrt(2)*sqrt(4489*a^4 - 7102*a
^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b +
100*b^2))) + 4*(9955201432*a^8 - 43370370320*a^7*b + 87362920120*
a^6*b^2 - 101379092192*a^5*b^3 + 75511142560*a^4*b^4 - 3687768627
2*a^3*b^5 + 11609417344*a^2*b^6 - 2131897856*a*b^7 + 176129536*b^
8 + sqrt(2)*(100323115*a^6 - 385467108*a^5*b + 580826940*a^4*b^2
- 484547360*a^3*b^3 + 222173520*a^2*b^4 - 55113792*a*b^5 + 564640
0*b^6)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 4
84*b^4))*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 +
 484*b^4))/(4*sqrt(2)*(83374811993*a^10 - 429180060917*a^9*b + 10
46369963313*a^8*b^2 - 1547097761283*a^7*b^3 + 1544290335042*a^6*b
^4 - 1087904445516*a^5*b^5 + 549199183848*a^4*b^6 - 196180671696*
a^3*b^7 + 47524805856*a^2*b^8 - 7029577280*a*b^9 + 484356224*b^10
) + (6721648705*a^8 - 31143421331*a^7*b + 61552270234*a^6*b^2 - 7
1728777316*a^5*b^3 + 53344828600*a^4*b^4 - 26127862544*a^3*b^5 +
8187157216*a^2*b^6 - 1511762624*a*b^7 + 124220800*b^8)*sqrt(4489*
a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4))) + 2*sqr
t(2)*(392^(1/4)*sqrt(7)*sqrt(2)*(240329*a^6 - 734151*a^5*b + 6813
30*a^4*b^2 - 226840*a^3*b^3 - 39840*a^2*b^4 + 50064*a*b^5 - 8800*
b^6)*x + 8*392^(1/4)*sqrt(7)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^
2*b^2 - 2332*a*b^3 + 484*b^4)*(1139*a^4 - 1169*a^3*b + 318*a^2*b^
2 + 124*a*b^3 - 88*b^4)*x)*sqrt((35912*a^4 - 56816*a^3*b + 46056*
a^2*b^2 - 18656*a*b^3 + 3872*b^4 + sqrt(2)*sqrt(4489*a^4 - 7102*a
^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b +
100*b^2))/(188169*a^4 - 407880*a^3*b + 409608*a^2*b^2 - 160224*a*
b^3 + 25488*b^4 + 8*sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2
*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))) - 28
*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(1
```

/4)*(sqrt(7)*sqrt(2)*(1139*a^5 - 5725*a^4*b + 4994*a^3*b^2 - 1148
*a^2*b^3 - 584*a*b^4 + 352*b^5) + sqrt(7)*sqrt(4489*a^4 - 7102*a^
3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(187*a^3 - 78*a^2*b -
36*a*b^2 + 8*b^3)))) - (4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 23
32*a*b^3 + 484*b^4)^(1/4)*(sqrt(7)*sqrt(2)*((211*a^2 - 428*a*b +
100*b^2)*x^4 + (211*a^2 - 428*a*b + 100*b^2)*x^2 + 422*a^2 - 856*
a*b + 200*b^2) + 8*sqrt(7)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*
b^2 - 2332*a*b^3 + 484*b^4)*(x^4 + x^2 + 2))*log(224*sqrt(2)*(833
74811993*a^10 - 429180060917*a^9*b + 1046369963313*a^8*b^2 - 1547
097761283*a^7*b^3 + 1544290335042*a^6*b^4 - 1087904445516*a^5*b^5
 + 549199183848*a^4*b^6 - 196180671696*a^3*b^7 + 47524805856*a^2*
b^8 - 7029577280*a*b^9 + 484356224*b^10)*x^2 + 56*(6721648705*a^8
 - 31143421331*a^7*b + 61552270234*a^6*b^2 - 71728777316*a^5*b^3
+ 53344828600*a^4*b^4 - 26127862544*a^3*b^5 + 8187157216*a^2*b^6
- 1511762624*a*b^7 + 124220800*b^8)*sqrt(4489*a^4 - 7102*a^3*b +
5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^2 + 14*sqrt(2)*(4489*a^4 -
 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(1/4)*(3*392^(
1/4)*sqrt(2)*(417379787*a^7 - 1854968766*a^6*b + 3371736900*a^5*b
^2 - 3504565960*a^4*b^3 + 2146853520*a^3*b^4 - 797847456*a^2*b^5
+ 163604672*a*b^6 - 14438784*b^7)*sqrt(4489*a^4 - 7102*a^3*b + 57
57*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x + 2*392^(1/4)*(61475256581*a
^9 - 306522254343*a^8*b + 709992386538*a^7*b^2 - 962885785428*a^6
*b^3 + 856950314136*a^5*b^4 - 517845594000*a^4*b^5 + 213428089056
*a^3*b^6 - 57595247040*a^2*b^7 + 9271881600*a*b^8 - 673012736*b^9
)*x)*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3
+ 3872*b^4 + sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 -
2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(188169*a^4
- 407880*a^3*b + 409608*a^2*b^2 - 160224*a*b^3 + 25488*b^4 + 8*sq
rt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 48
4*b^4)*(211*a^2 - 428*a*b + 100*b^2))) + 56*(9955201432*a^8 - 433
70370320*a^7*b + 87362920120*a^6*b^2 - 101379092192*a^5*b^3 + 755
11142560*a^4*b^4 - 36877686272*a^3*b^5 + 11609417344*a^2*b^6 - 21
31897856*a*b^7 + 176129536*b^8 + sqrt(2)*(100323115*a^6 - 3854671
08*a^5*b + 580826940*a^4*b^2 - 484547360*a^3*b^3 + 222173520*a^2*
b^4 - 55113792*a*b^5 + 5646400*b^6)*sqrt(4489*a^4 - 7102*a^3*b +
5757*a^2*b^2 - 2332*a*b^3 + 484*b^4))*sqrt(4489*a^4 - 7102*a^3*b
+ 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)) + (4489*a^4 - 7102*a^3*b
+ 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(1/4)*(sqrt(7)*sqrt(2)*((2
11*a^2 - 428*a*b + 100*b^2)*x^4 + (211*a^2 - 428*a*b + 100*b^2)*x
^2 + 422*a^2 - 856*a*b + 200*b^2) + 8*sqrt(7)*sqrt(4489*a^4 - 710
2*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(x^4 + x^2 + 2))*l
og(224*sqrt(2)*(83374811993*a^10 - 429180060917*a^9*b + 104636996
3313*a^8*b^2 - 1547097761283*a^7*b^3 + 1544290335042*a^6*b^4 - 10
87904445516*a^5*b^5 + 549199183848*a^4*b^6 - 196180671696*a^3*b^7
 + 47524805856*a^2*b^8 - 7029577280*a*b^9 + 484356224*b^10)*x^2 +
 56*(6721648705*a^8 - 31143421331*a^7*b + 61552270234*a^6*b^2 - 7
1728777316*a^5*b^3 + 53344828600*a^4*b^4 - 26127862544*a^3*b^5 +
8187157216*a^2*b^6 - 1511762624*a*b^7 + 124220800*b^8)*sqrt(4489*
a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^2 - 14*
sqrt(2)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*
b^4)^(1/4)*(3*392^(1/4)*sqrt(2)*(417379787*a^7 - 1854968766*a^6*b
 + 3371736900*a^5*b^2 - 3504565960*a^4*b^3 + 2146853520*a^3*b^4 -
 797847456*a^2*b^5 + 163604672*a*b^6 - 14438784*b^7)*sqrt(4489*a^
4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x + 2*392^(
1/4)*(61475256581*a^9 - 306522254343*a^8*b + 709992386538*a^7*b^2
 - 962885785428*a^6*b^3 + 856950314136*a^5*b^4 - 517845594000*a^4
*b^5 + 213428089056*a^3*b^6 - 57595247040*a^2*b^7 + 9271881600*a*
b^8 - 673012736*b^9)*x)*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2
*b^2 - 18656*a*b^3 + 3872*b^4 + sqrt(2)*sqrt(4489*a^4 - 7102*a^3*
b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100
*b^2))/(188169*a^4 - 407880*a^3*b + 409608*a^2*b^2 - 160224*a*b^3
 + 25488*b^4 + 8*sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^
2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))) + 56*(9
955201432*a^8 - 43370370320*a^7*b + 87362920120*a^6*b^2 - 1013790
92192*a^5*b^3 + 75511142560*a^4*b^4 - 36877686272*a^3*b^5 + 11609
417344*a^2*b^6 - 2131897856*a*b^7 + 176129536*b^8 + sqrt(2)*(1003
23115*a^6 - 385467108*a^5*b + 580826940*a^4*b^2 - 484547360*a^3*b
^3 + 222173520*a^2*b^4 - 55113792*a*b^5 + 5646400*b^6)*sqrt(4489*
a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4))*sqrt(448
9*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)) + 2*sq
rt(2)*(392^(1/4)*sqrt(7)*sqrt(2)*((211*a^3 - 1272*a^2*b + 1812*a*
b^2 - 400*b^3)*x^3 - (633*a^3 - 862*a^2*b - 556*a*b^2 + 200*b^3)*

```
x) + 8*392^(1/4)*sqrt(7)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^
2 - 2332*a*b^3 + 484*b^4)*((a - 4*b)*x^3 - (3*a + 2*b)*x))*sqrt((
35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4
+ sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3
+ 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(188169*a^4 - 407880*a^
3*b + 409608*a^2*b^2 - 160224*a*b^3 + 25488*b^4 + 8*sqrt(2)*sqrt(
4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211
*a^2 - 428*a*b + 100*b^2))))/((392^(1/4)*sqrt(7)*sqrt(2)*((211*a^
2 - 428*a*b + 100*b^2)*x^4 + (211*a^2 - 428*a*b + 100*b^2)*x^2 +
422*a^2 - 856*a*b + 200*b^2) + 8*392^(1/4)*sqrt(7)*sqrt(4489*a^4
- 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(x^4 + x^2 +
2))*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 +
 3872*b^4 + sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2
332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(188169*a^4 -
 407880*a^3*b + 409608*a^2*b^2 - 160224*a*b^3 + 25488*b^4 + 8*sqr
t(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484
*b^4)*(211*a^2 - 428*a*b + 100*b^2))))
```

Sympy [A] time = 4.92416, size = 167, normalized size = 0.53

$$-\frac{x^3(a-4b)+x(-3a-2b)}{28x^4+28x^2+56}$$
$$+\mathrm{RootSum}\left(240945152t^4+t^2\left(-1157968a^2+2348864ab-548800b^2\right)+4489a^4-7102a^3b+5757a^2b^2-2332ab^3+484b^4,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(x**4+x**2+2)**2,x)
```

```
[Out] -(x**3*(a - 4*b) + x*(-3*a - 2*b))/(28*x**4 + 28*x**2 + 56) + Roo
tSum(240945152*_t**4 + _t**2*(-1157968*a**2 + 2348864*a*b - 54880
0*b**2) + 4489*a**4 - 7102*a**3*b + 5757*a**2*b**2 - 2332*a*b**3
+ 484*b**4, Lambda(_t, _t*log(x + (2634240*_t**3*a - 3161088*_t**
3*b + 11996*_t*a**3 + 12792*_t*a**2*b - 21936*_t*a*b**2 + 4384*_t
*b**3)/(1139*a**4 - 1169*a**3*b + 318*a**2*b**2 + 124*a*b**3 - 88
*b**4))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+a}{(x^4+x^2+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(x^4 + x^2 + 2)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/(x^4 + x^2 + 2)^2, x)
```

3.110 $\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx$

Optimal. Leaf size=160

$$-\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)+\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}$$

```
[Out] -ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2 +
Sqrt[2]]) + ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(
2*Sqrt[2 + Sqrt[2]]) - (Sqrt[1 + 1/Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt
[2]]*x + x^2])/4 + (Sqrt[1 + 1/Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]
*x + x^2])/4
```

Rubi [A] time = 0.342478, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$-\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)+\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[2] - x^2)/(1 - Sqrt[2]*x^2 + x^4),x]
```

```
[Out] -ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2 +
Sqrt[2]]) + ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(
2*Sqrt[2 + Sqrt[2]]) - (Sqrt[(2 + Sqrt[2])/2]*Log[1 - Sqrt[2 + Sq
rt[2]]*x + x^2])/4 + (Sqrt[(2 + Sqrt[2])/2]*Log[1 + Sqrt[2 + Sqrt
[2]]*x + x^2])/4
```

Rubi in Sympy [A] time = 29.8419, size = 240, normalized size = 1.5

$$-\frac{\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)\log\left(x^2-x\sqrt{\sqrt{2}+2}+1\right)}{2\sqrt{\sqrt{2}+2}}+\frac{\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)\log\left(x^2+x\sqrt{\sqrt{2}+2}+1\right)}{2\sqrt{\sqrt{2}+2}}+\frac{\left(-\frac{\left(1+\sqrt{2}\right)\sqrt{\sqrt{2}+2}}{2}+\sqrt{2}\sqrt{\sqrt{2}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)}{\sqrt{-\sqrt{2}+2}\sqrt{\sqrt{2}+2}}+\frac{\left(-\frac{\left(1+\sqrt{2}\right)\sqrt{\sqrt{2}+2}}{2}+\sqrt{2}\sqrt{\sqrt{2}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)}{\sqrt{-\sqrt{2}+2}\sqrt{\sqrt{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+2**(1/2))/(1+x**4-x**2*2**(1/2)),x)
```

```
[Out] -(1/2 + sqrt(2)/2)*log(x**2 - x*sqrt(sqrt(2) + 2) + 1)/(2*sqrt(sq
rt(2) + 2)) + (1/2 + sqrt(2)/2)*log(x**2 + x*sqrt(sqrt(2) + 2) +
1)/(2*sqrt(sqrt(2) + 2)) + (-(1 + sqrt(2))*sqrt(sqrt(2) + 2)/2 +
sqrt(2)*sqrt(sqrt(2) + 2))*atan((2*x - sqrt(sqrt(2) + 2))/sqrt(-s
qrt(2) + 2))/(sqrt(-sqrt(2) + 2)*sqrt(sqrt(2) + 2)) + (-(1 + sqrt
(2))*sqrt(sqrt(2) + 2)/2 + sqrt(2)*sqrt(sqrt(2) + 2))*atan((2*x +
 sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2))/(sqrt(-sqrt(2) + 2)*sqrt(
```

```
sqrt(2) + 2))
```

Mathematica [C] time = 0.0694501, size = 53, normalized size = 0.33

$$\frac{\sqrt{-1-i}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1-i}}\right)+\sqrt{-1+i}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+i}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[2] - x^2)/(1 - Sqrt[2]*x^2 + x^4),x]
```

```
[Out] (Sqrt[-1 - I]*ArcTan[(2^(1/4)*x)/Sqrt[-1 - I]] + Sqrt[-1 + I]*Arc
Tan[(2^(1/4)*x)/Sqrt[-1 + I]])/2^(3/4)
```

Maple [A] time = 0.089, size = 199, normalized size = 1.2

$$-\frac{\sqrt{2}\sqrt{2+\sqrt{2}}\ln\left(1+x^2-x\sqrt{2+\sqrt{2}}\right)}{8}-\frac{1}{2\sqrt{2-\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)+\frac{\sqrt{2}}{2\sqrt{2-\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)+\frac{\sqrt{2}\sqrt{2+\sqrt{2}}\ln\left(1+x^2+x\sqrt{2+\sqrt{2}}\right)}{8}-\frac{1}{2\sqrt{2-\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)+\frac{\sqrt{2}}{2\sqrt{2-\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+2^(1/2))/(1+x^4-2^(1/2)*x^2),x)
```

```
[Out] -1/8*2^(1/2)*(2+2^(1/2))^(1/2)*ln(1+x^2-x*(2+2^(1/2))^(1/2))-1/2/
(2-2^(1/2))^(1/2)*arctan((2*x-(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2
))+1/2/(2-2^(1/2))^(1/2)*arctan((2*x-(2+2^(1/2))^(1/2))/(2-2^(1/2
))^(1/2))*2^(1/2)+1/8*2^(1/2)*(2+2^(1/2))^(1/2)*ln(1+x^2+x*(2+2^(
1/2))^(1/2))-1/2/(2-2^(1/2))^(1/2)*arctan((2*x+(2+2^(1/2))^(1/2))
/(2-2^(1/2))^(1/2))+1/2/(2-2^(1/2))^(1/2)*arctan((2*x+(2+2^(1/2))
^(1/2))/(2-2^(1/2))^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2-\sqrt{2}}{x^4-\sqrt{2}x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - sqrt(2))/(x^4 - sqrt(2)*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - sqrt(2))/(x^4 - sqrt(2)*x^2 + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - sqrt(2))/(x^4 - sqrt(2)*x^2 + 1),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2**(1/2))/(1+x**4-x**2*2**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 - \sqrt{2}}{x^4 - \sqrt{2}x^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - sqrt(2))/(x^4 - sqrt(2)*x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - sqrt(2))/(x^4 - sqrt(2)*x^2 + 1), x)
```

3.111 $\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx$

Optimal. Leaf size=172

$$-\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)+\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}$$

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2 -
Sqrt[2]]) + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(
2*Sqrt[2 - Sqrt[2]]) - (Sqrt[1 - 1/Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt
[2]]*x + x^2])/4 + (Sqrt[1 - 1/Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]
*x + x^2])/4
```

Rubi [A] time = 0.339839, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$-\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)+\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[2] + x^2)/(1 + Sqrt[2]*x^2 + x^4),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2 -
Sqrt[2]]) + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(
2*Sqrt[2 - Sqrt[2]]) - (Sqrt[(2 - Sqrt[2])/2]*Log[1 - Sqrt[2 - Sq
rt[2]]*x + x^2])/4 + (Sqrt[(2 - Sqrt[2])/2]*Log[1 + Sqrt[2 - Sqrt
[2]]*x + x^2])/4
```

Rubi in Sympy [A] time = 28.8526, size = 240, normalized size = 1.4

$$\frac{\left(-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)\log\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)}{2\sqrt{-\sqrt{2}+2}}-\frac{\left(-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)\log\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right)}{2\sqrt{-\sqrt{2}+2}}+\frac{\left(\frac{\left(-\sqrt{2}+1\right)\sqrt{-\sqrt{2}+2}}{2}+\sqrt{2}\sqrt{-\sqrt{2}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)}{\sqrt{-\sqrt{2}+2}\sqrt{\sqrt{2}+2}}+\frac{\left(\frac{\left(-\sqrt{2}+1\right)\sqrt{-\sqrt{2}+2}}{2}+\sqrt{2}\sqrt{-\sqrt{2}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)}{\sqrt{-\sqrt{2}+2}\sqrt{\sqrt{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+2**(1/2))/(1+x**4+x**2*2**(1/2)),x)
```

```
[Out] (-sqrt(2)/2 + 1/2)*log(x**2 - x*sqrt(-sqrt(2) + 2) + 1)/(2*sqrt(-
sqrt(2) + 2)) - (-sqrt(2)/2 + 1/2)*log(x**2 + x*sqrt(-sqrt(2) + 2
```

```
) + 1)/(2*sqrt(-sqrt(2) + 2)) + ((-sqrt(2) + 1)*sqrt(-sqrt(2) + 2
)/2 + sqrt(2)*sqrt(-sqrt(2) + 2))*atan((2*x - sqrt(-sqrt(2) + 2))
/sqrt(sqrt(2) + 2))/(sqrt(-sqrt(2) + 2)*sqrt(sqrt(2) + 2)) + ((-s
qrt(2) + 1)*sqrt(-sqrt(2) + 2)/2 + sqrt(2)*sqrt(-sqrt(2) + 2))*at
an((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))/(sqrt(-sqrt(2) +
 2)*sqrt(sqrt(2) + 2))
```

Mathematica [C] time = 0.0545709, size = 53, normalized size = 0.31

$$\frac{\sqrt{1-i}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1-i}}\right)+\sqrt{1+i}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+i}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[2] + x^2)/(1 + Sqrt[2]*x^2 + x^4),x]
```

```
[Out] (Sqrt[1 - I]*ArcTan[(2^(1/4)*x)/Sqrt[1 - I]] + Sqrt[1 + I]*ArcTan
[(2^(1/4)*x)/Sqrt[1 + I]])/2^(3/4)
```

Maple [A] time = 0.083, size = 199, normalized size = 1.2

$$-\frac{\sqrt{2}\sqrt{2-\sqrt{2}}\ln\left(1+x^2-x\sqrt{2-\sqrt{2}}\right)}{8}+\frac{\sqrt{2}}{2\sqrt{2+\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)+\frac{1}{2\sqrt{2+\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)+\frac{\sqrt{2}\sqrt{2-\sqrt{2}}\ln\left(1+x^2+x\sqrt{2-\sqrt{2}}\right)}{8}+\frac{\sqrt{2}}{2\sqrt{2+\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)+\frac{1}{2\sqrt{2+\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+2^(1/2))/(1+x^4+2^(1/2)*x^2),x)
```

```
[Out] -1/8*2^(1/2)*(2-2^(1/2))^(1/2)*ln(1+x^2-x*(2-2^(1/2))^(1/2))+1/2/
(2+2^(1/2))^(1/2)*arctan((2*x-(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2
))*2^(1/2)+1/2/(2+2^(1/2))^(1/2)*arctan((2*x-(2-2^(1/2))^(1/2))/(
2+2^(1/2))^(1/2))+1/8*2^(1/2)*(2-2^(1/2))^(1/2)*ln(1+x^2+x*(2-2^(
1/2))^(1/2))+1/2/(2+2^(1/2))^(1/2)*arctan((2*x+(2-2^(1/2))^(1/2))
/(2+2^(1/2))^(1/2))*2^(1/2)+1/2/(2+2^(1/2))^(1/2)*arctan((2*x+(2-
2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+\sqrt{2}}{x^4+\sqrt{2}x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + sqrt(2))/(x^4 + sqrt(2)*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + sqrt(2))/(x^4 + sqrt(2)*x^2 + 1), x)
```

Fricas [A] time = 0.338771, size = 595, normalized size = 3.46

$$\frac{\sqrt{2}\left(\left(\sqrt{2}-2\right)\log\left(-\frac{34\,x^2+\sqrt{2}\left(41\,\sqrt{2}x-58\,x\right)\sqrt{\frac{\sqrt{2}-2}{2\,\sqrt{2}-3}}-24\,\sqrt{2}\left(x^2+1\right)+34}{2\left(12\,\sqrt{2}-17\right)}\right)-\left(\sqrt{2}-2\right)\log\left(-\frac{34\,x^2-\sqrt{2}\left(41\,\sqrt{2}x-58\,x\right)\sqrt{\frac{\sqrt{2}-2}{2\,\sqrt{2}-3}}-24\,\sqrt{2}\left(x^2+1\right)+34}{2\left(12\,\sqrt{2}-17\right)}\right)+ \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + sqrt(2))/(x^4 + sqrt(2)*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*((sqrt(2) - 2)*log(-1/2*(34*x^2 + sqrt(2)*(41*sqrt(2)
*x - 58*x)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) - 24*sqrt(2)*(x^2
+ 1) + 34)/(12*sqrt(2) - 17)) - (sqrt(2) - 2)*log(-1/2*(34*x^2 -
sqrt(2)*(41*sqrt(2)*x - 58*x)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3))
 - 24*sqrt(2)*(x^2 + 1) + 34)/(12*sqrt(2) - 17)) + 4*sqrt(2)*arct
an(sqrt(2)/(sqrt(2)*sqrt(1/2)*(sqrt(2) - 2)*sqrt(-(34*x^2 + sqrt(
2)*(41*sqrt(2)*x - 58*x)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) - 24
*sqrt(2)*(x^2 + 1) + 34)/(12*sqrt(2) - 17))*sqrt((sqrt(2) - 2)/(2
*sqrt(2) - 3)) + sqrt(2)*(sqrt(2)*x - 2*x)*sqrt((sqrt(2) - 2)/(2*
sqrt(2) - 3)) - sqrt(2) + 2)) + 4*sqrt(2)*arctan(sqrt(2)/(sqrt(2)
*sqrt(1/2)*(sqrt(2) - 2)*sqrt(-(34*x^2 - sqrt(2)*(41*sqrt(2)*x -
58*x)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) - 24*sqrt(2)*(x^2 + 1)
+ 34)/(12*sqrt(2) - 17))*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) + sq
rt(2)*(sqrt(2)*x - 2*x)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) + sqr
t(2) - 2)))/((sqrt(2) - 1)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+2**(1/2))/(1+x**4+x**2*2**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+\sqrt{2}}{x^4+\sqrt{2}x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + sqrt(2))/(x^4 + sqrt(2)*x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate((x^2 + sqrt(2))/(x^4 + sqrt(2)*x^2 + 1), x)
```

3.112 $\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx$

Optimal. Leaf size=160

$$-\frac{\left(1+\sqrt{2}\right)\log\left(-\sqrt{2-b}x+x^2+1\right)}{4\sqrt{2-b}}+\frac{\left(1+\sqrt{2}\right)\log\left(\sqrt{2-b}x+x^2+1\right)}{4\sqrt{2-b}}+\frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}-\frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}$$

```
[Out] ((1 - Sqrt[2])*ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]])/(2*Sqrt[2
 + b]) - ((1 - Sqrt[2])*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/
(2*Sqrt[2 + b]) - ((1 + Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/(4
*Sqrt[2 - b]) + ((1 + Sqrt[2])*Log[1 + Sqrt[2 - b]*x + x^2])/(4*S
qrt[2 - b])
```

Rubi [A] time = 0.266838, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{\left(1+\sqrt{2}\right)\log\left(-\sqrt{2-b}x+x^2+1\right)}{4\sqrt{2-b}}+\frac{\left(1+\sqrt{2}\right)\log\left(\sqrt{2-b}x+x^2+1\right)}{4\sqrt{2-b}}+\frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}-\frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[2] - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] ((1 - Sqrt[2])*ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]])/(2*Sqrt[2
 + b]) - ((1 - Sqrt[2])*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/
(2*Sqrt[2 + b]) - ((1 + Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/(4
*Sqrt[2 - b]) + ((1 + Sqrt[2])*Log[1 + Sqrt[2 - b]*x + x^2])/(4*S
qrt[2 - b])
```

Rubi in Sympy [A] time = 34.5998, size = 134, normalized size = 0.84

$$-\frac{\left(-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{-b+2}}{\sqrt{b+2}}\right)}{\sqrt{b+2}}-\frac{\left(-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{-b+2}}{\sqrt{b+2}}\right)}{\sqrt{b+2}}-\frac{\left(\frac{1}{4}+\frac{\sqrt{2}}{4}\right)\log\left(x^2-x\sqrt{-b+2}+1\right)}{\sqrt{-b+2}}+\frac{\left(\frac{1}{4}+\frac{\sqrt{2}}{4}\right)\log\left(x^2+x\sqrt{-b+2}+1\right)}{\sqrt{-b+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+2**(1/2))/(x**4+b*x**2+1),x)
```

```
[Out] -(-sqrt(2)/2 + 1/2)*atan((2*x - sqrt(-b + 2))/sqrt(b + 2))/sqrt(b
 + 2) - (-sqrt(2)/2 + 1/2)*atan((2*x + sqrt(-b + 2))/sqrt(b + 2))
/sqrt(b + 2) - (1/4 + sqrt(2)/4)*log(x**2 - x*sqrt(-b + 2) + 1)/s
qrt(-b + 2) + (1/4 + sqrt(2)/4)*log(x**2 + x*sqrt(-b + 2) + 1)/sq
rt(-b + 2)
```

Mathematica [A] time = 0.145487, size = 137, normalized size = 0.86

$$\frac{\frac{\left(-\sqrt{b^2-4}+b+2\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{b^2-4}}}\right)}{\sqrt{b-\sqrt{b^2-4}}}-\frac{\left(\sqrt{b^2-4}+b+2\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{\sqrt{b^2-4}+b}}}{\sqrt{2}\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[2] - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] (((2*Sqrt[2] + b - Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sq
rt[-4 + b^2]]])/Sqrt[b - Sqrt[-4 + b^2]] - ((2*Sqrt[2] + b + Sqrt
[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b + Sqrt[-4 + b^2]]])/Sqrt[b
+ Sqrt[-4 + b^2]])/(Sqrt[2]*Sqrt[-4 + b^2])
```

Maple [B] time = 0.029, size = 285, normalized size = 1.8

$$-1\arctan\left(2\frac{x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)\frac{1}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}$$
$$+b\arctan\left(2\frac{x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)\frac{1}{\sqrt{(b-2)(2+b)}}\frac{1}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}$$
$$+2\frac{\sqrt{2}}{\sqrt{(b-2)(2+b)}\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\arctan\left(2\frac{x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)$$
$$-1\arctan\left(2\frac{x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)\frac{1}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}$$
$$-b\arctan\left(2\frac{x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)\frac{1}{\sqrt{(b-2)(2+b)}}\frac{1}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}$$
$$-2\frac{\sqrt{2}}{\sqrt{(b-2)(2+b)}\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+2^(1/2))/(x^4+b*x^2+1),x)
```

```
[Out] -1/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((b-2)*(2+b)
)^(1/2)+2*b)^(1/2))+1/((b-2)*(2+b))^(1/2)/(-2*((b-2)*(2+b))^(1/2)
+2*b)^(1/2)*arctan(2*x/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))*b+2/((
b-2)*(2+b))^(1/2)/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(
-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))*2^(1/2)-1/(2*((b-2)*(2+b))^(1/
2)+2*b)^(1/2)*arctan(2*x/(2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))-1/((b
-2)*(2+b))^(1/2)/(2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*
((b-2)*(2+b))^(1/2)+2*b)^(1/2))*b-2/((b-2)*(2+b))^(1/2)/(2*((b-2)
*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((b-2)*(2+b))^(1/2)+2*b)^(
1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2-\sqrt{2}}{x^4+bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - sqrt(2))/(x^4 + b*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - sqrt(2))/(x^4 + b*x^2 + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - sqrt(2))/(x^4 + b*x^2 + 1),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 6.29414, size = 332, normalized size = 2.08

$$-\operatorname{RootSum}\left(t^4\left(16b^4-128b^2+256\right)+t^2\left(12b^3+16\sqrt{2}b^2-48b-64\sqrt{2}\right)+2b^2+6\sqrt{2}b+9,\left(t\mapsto t\log\left(\frac{t^3\left(64b^{12}+672\sqrt{2}\right.}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+2**(1/2))/(x**4+b*x**2+1),x)
```

```
[Out] -RootSum(_t**4*(16*b**4 - 128*b**2 + 256) + _t**2*(12*b**3 + 16*s
qrt(2)*b**2 - 48*b - 64*sqrt(2)) + 2*b**2 + 6*sqrt(2)*b + 9, Lamb
da(_t, _t*log(_t**3*(64*b**12 + 672*sqrt(2)*b**11 + 5760*b**10 +
12064*sqrt(2)*b**9 + 17744*b**8 - 27480*sqrt(2)*b**7 - 154608*b**
6 - 141376*sqrt(2)*b**5 - 69072*b**4 + 61704*sqrt(2)*b**3 + 78192
*b**2 - 2592*sqrt(2)*b - 15552)/(8*b**10 + 88*sqrt(2)*b**9 + 828*
b**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b
**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729) + _t*(16*b**7 + 116*sq
rt(2)*b**6 + 668*b**5 + 942*sqrt(2)*b**4 + 1226*b**3 + 144*sqrt(2
)*b**2 - 378*b - 108*sqrt(2))/(4*b**6 + 28*sqrt(2)*b**5 + 152*b**
4 + 192*sqrt(2)*b**3 + 189*b**2 - 27*sqrt(2)*b - 81) + x)))
```

GIAC/XCAS [A] time = 0.316526, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - sqrt(2))/(x^4 + b*x^2 + 1),x, algorithm="giac")
```

```
[Out] Done
```

3.113 $\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx$

Optimal. Leaf size=160

$$\frac{\left(1-\sqrt{2}\right)\log\left(-\sqrt{2-b}x+x^2+1\right)}{4\sqrt{2-b}}-\frac{\left(1-\sqrt{2}\right)\log\left(\sqrt{2-b}x+x^2+1\right)}{4\sqrt{2-b}}-\frac{\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}+\frac{\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}$$

```
[Out] -((1 + Sqrt[2])*ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]])/(2*Sqrt[
2 + b]) + ((1 + Sqrt[2])*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])
/(2*Sqrt[2 + b]) + ((1 - Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/(
4*Sqrt[2 - b]) - ((1 - Sqrt[2])*Log[1 + Sqrt[2 - b]*x + x^2])/(4*
Sqrt[2 - b])
```

Rubi [A] time = 0.230413, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{\left(1-\sqrt{2}\right)\log\left(-\sqrt{2-b}x+x^2+1\right)}{4\sqrt{2-b}}-\frac{\left(1-\sqrt{2}\right)\log\left(\sqrt{2-b}x+x^2+1\right)}{4\sqrt{2-b}}-\frac{\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}+\frac{\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[2] + x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] -((1 + Sqrt[2])*ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]])/(2*Sqrt[
2 + b]) + ((1 + Sqrt[2])*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])
/(2*Sqrt[2 + b]) + ((1 - Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/(
4*Sqrt[2 - b]) - ((1 - Sqrt[2])*Log[1 + Sqrt[2 - b]*x + x^2])/(4*
Sqrt[2 - b])
```

Rubi in Sympy [A] time = 34.3218, size = 134, normalized size = 0.84

$$\frac{\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{-b+2}}{\sqrt{b+2}}\right)}{\sqrt{b+2}}+\frac{\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{-b+2}}{\sqrt{b+2}}\right)}{\sqrt{b+2}}+\frac{\left(-\frac{\sqrt{2}}{4}+\frac{1}{4}\right)\log\left(x^2-x\sqrt{-b+2}+1\right)}{\sqrt{-b+2}}-\frac{\left(-\frac{\sqrt{2}}{4}+\frac{1}{4}\right)\log\left(x^2+x\sqrt{-b+2}+1\right)}{\sqrt{-b+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+2**(1/2))/(x**4+b*x**2+1),x)
```

```
[Out] (1/2 + sqrt(2)/2)*atan((2*x - sqrt(-b + 2))/sqrt(b + 2))/sqrt(b +
 2) + (1/2 + sqrt(2)/2)*atan((2*x + sqrt(-b + 2))/sqrt(b + 2))/sq
rt(b + 2) + (-sqrt(2)/4 + 1/4)*log(x**2 - x*sqrt(-b + 2) + 1)/sqr
t(-b + 2) - (-sqrt(2)/4 + 1/4)*log(x**2 + x*sqrt(-b + 2) + 1)/sqr
t(-b + 2)
```

Mathematica [A] time = 0.0935016, size = 136, normalized size = 0.85

$$\frac{\frac{\left(\sqrt{b^2-4}-b+2\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{b^2-4}}}\right)}{\sqrt{b-\sqrt{b^2-4}}}+\frac{\left(\sqrt{b^2-4}+b-2\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{\sqrt{b^2-4}+b}}}{\sqrt{2}\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[2] + x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] (((2*Sqrt[2] - b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sq
rt[-4 + b^2]]])/Sqrt[b - Sqrt[-4 + b^2]] + ((-2*Sqrt[2] + b + Sqr
t[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b + Sqrt[-4 + b^2]]])/Sqrt[b
 + Sqrt[-4 + b^2]])/(Sqrt[2]*Sqrt[-4 + b^2])
```

Maple [B] time = 0.026, size = 283, normalized size = 1.8

$$1\arctan\left(2\frac{x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)\frac{1}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}$$
$$-b\arctan\left(2\frac{x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)\frac{1}{\sqrt{(b-2)(2+b)}}\frac{1}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}$$
$$+2\frac{\sqrt{2}}{\sqrt{(b-2)(2+b)}\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\arctan\left(2\frac{x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)$$
$$+1\arctan\left(2\frac{x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)\frac{1}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}$$
$$+b\arctan\left(2\frac{x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)\frac{1}{\sqrt{(b-2)(2+b)}}\frac{1}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}$$
$$-2\frac{\sqrt{2}}{\sqrt{(b-2)(2+b)}\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+2^(1/2))/(x^4+b*x^2+1),x)
```

```
[Out] 1/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((b-2)*(2+b))
^(1/2)+2*b)^(1/2))-1/((b-2)*(2+b))^(1/2)/(-2*((b-2)*(2+b))^(1/2)+
2*b)^(1/2)*arctan(2*x/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))*b+2/((b
-2)*(2+b))^(1/2)/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-
2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))*2^(1/2)+1/(2*((b-2)*(2+b))^(1/2
)+2*b)^(1/2)*arctan(2*x/(2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))+1/((b-
2)*(2+b))^(1/2)/(2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*(
(b-2)*(2+b))^(1/2)+2*b)^(1/2))*b-2/((b-2)*(2+b))^(1/2)/(2*((b-2)*
(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((b-2)*(2+b))^(1/2)+2*b)^(1
/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+\sqrt{2}}{x^4+bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + sqrt(2))/(x^4 + b*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + sqrt(2))/(x^4 + b*x^2 + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + sqrt(2))/(x^4 + b*x^2 + 1),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 6.3439, size = 330, normalized size = 2.06

$$\mathrm{RootSum}\left(t^4\left(16b^4-128b^2+256\right)+t^2\left(12b^3-16\sqrt{2}b^2-48b+64\sqrt{2}\right)+2b^2-6\sqrt{2}b+9,\left(t\mapsto t\log\left(\frac{t^3\left(64b^{12}-672\sqrt{2}b^{11}\right.}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+2**(1/2))/(x**4+b*x**2+1),x)
```

```
[Out] RootSum(_t**4*(16*b**4 - 128*b**2 + 256) + _t**2*(12*b**3 - 16*sq
rt(2)*b**2 - 48*b + 64*sqrt(2)) + 2*b**2 - 6*sqrt(2)*b + 9, Lambd
a(_t, _t*log(_t**3*(64*b**12 - 672*sqrt(2)*b**11 + 5760*b**10 - 1
2064*sqrt(2)*b**9 + 17744*b**8 + 27480*sqrt(2)*b**7 - 154608*b**6
 + 141376*sqrt(2)*b**5 - 69072*b**4 - 61704*sqrt(2)*b**3 + 78192*
b**2 + 2592*sqrt(2)*b - 15552)/(8*b**10 - 88*sqrt(2)*b**9 + 828*b
**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b*
*4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729) + _t*(16*b**7 - 116*sqr
t(2)*b**6 + 668*b**5 - 942*sqrt(2)*b**4 + 1226*b**3 - 144*sqrt(2)
*b**2 - 378*b + 108*sqrt(2))/(4*b**6 - 28*sqrt(2)*b**5 + 152*b**4
 - 192*sqrt(2)*b**3 + 189*b**2 + 27*sqrt(2)*b - 81) + x)))
```

GIAC/XCAS [A] time = 0.317026, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + sqrt(2))/(x^4 + b*x^2 + 1),x, algorithm="giac")
```

```
[Out] Done
```

3.114 $\int \frac{2a-x^2}{a^2-ax^2+x^4} \, dx$

Optimal. Leaf size=114

$$-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{a}}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt{a}}+\sqrt{3}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTan[Sqrt[3] - (2*x)/Sqrt[a]]/(2*Sqrt[a]) + ArcTan[Sqrt[3] + (
2*x)/Sqrt[a]]/(2*Sqrt[a]) - (Sqrt[3]*Log[a - Sqrt[3]*Sqrt[a]*x +
x^2])/(4*Sqrt[a]) + (Sqrt[3]*Log[a + Sqrt[3]*Sqrt[a]*x + x^2])/(4
*Sqrt[a])
```

Rubi [A] time = 0.158471, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{a}}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt{a}}+\sqrt{3}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2*a - x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[3] - (2*x)/Sqrt[a]]/(2*Sqrt[a]) + ArcTan[Sqrt[3] + (
2*x)/Sqrt[a]]/(2*Sqrt[a]) - (Sqrt[3]*Log[a - Sqrt[3]*Sqrt[a]*x +
x^2])/(4*Sqrt[a]) + (Sqrt[3]*Log[a + Sqrt[3]*Sqrt[a]*x + x^2])/(4
*Sqrt[a])
```

Rubi in Sympy [A] time = 36.9162, size = 128, normalized size = 1.12

$$-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{a}}-\frac{\operatorname{atan}\left(\frac{\sqrt{3}\left(\sqrt{a}-\frac{2\sqrt{3}x}{3}\right)}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\operatorname{atan}\left(\frac{\sqrt{3}\left(\sqrt{a}+\frac{2\sqrt{3}x}{3}\right)}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+2*a)/(x**4-a*x**2+a**2),x)
```

```
[Out] -sqrt(3)*log(-sqrt(3)*sqrt(a)*x + a + x**2)/(4*sqrt(a)) + sqrt(3)
*log(sqrt(3)*sqrt(a)*x + a + x**2)/(4*sqrt(a)) - atan(sqrt(3)*(sq
rt(a) - 2*sqrt(3)*x/3)/sqrt(a))/(2*sqrt(a)) + atan(sqrt(3)*(sqrt(
a) + 2*sqrt(3)*x/3)/sqrt(a))/(2*sqrt(a))
```

Mathematica [C] time = 0.31148, size = 115, normalized size = 1.01

$$\frac{\sqrt[4]{-1}\left(\sqrt{\sqrt{3}-i}\left(\sqrt{3}-3i\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt{a}}\right)-\sqrt{\sqrt{3}+i}\left(\sqrt{3}+3i\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt{a}}\right)\right)}{2\sqrt{6}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*a - x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*(-(Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTan[((1 + I)*
x)/(Sqrt[-I + Sqrt[3]]*Sqrt[a])]) + Sqrt[-I + Sqrt[3]]*(-3*I + Sq
rt[3])*ArcTanh[((1 + I)*x)/(Sqrt[I + Sqrt[3]]*Sqrt[a])]))/(2*Sqrt
[6]*Sqrt[a])
```

Maple [A] time = 0.037, size = 92, normalized size = 0.8

$$\frac{\sqrt{3}}{4}\ln\left(a+x^2+\sqrt{3}\sqrt{a}x\right)\frac{1}{\sqrt{a}}+\frac{1}{2}\arctan\left(1\left(2x+\sqrt{3}\sqrt{a}\right)\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}$$
$$-\frac{\sqrt{3}}{4}\ln\left(\sqrt{3}\sqrt{a}x-x^2-a\right)\frac{1}{\sqrt{a}}-\frac{1}{2}\arctan\left(1\left(\sqrt{3}\sqrt{a}-2x\right)\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+2*a)/(x^4-a*x^2+a^2),x)
```

```
[Out] 1/4*ln(a+x^2+3^(1/2)*a^(1/2)*x)*3^(1/2)/a^(1/2)+1/2/a^(1/2)*arcta
n((2*x+3^(1/2)*a^(1/2))/a^(1/2))-1/4/a^(1/2)*3^(1/2)*ln(3^(1/2)*a
^(1/2)*x-x^2-a)-1/2/a^(1/2)*arctan((3^(1/2)*a^(1/2)-2*x)/a^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-2a}{x^4-ax^2+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 2*a)/(x^4 - a*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*a)/(x^4 - a*x^2 + a^2), x)
```

Fricas [A] time = 0.314934, size = 926, normalized size = 8.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 2*a)/(x^4 - a*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(3)*a*(a^(-2))^(3/4)*arctan(sqrt(3)*a^2*(a^(-2))^(3/4
)/(2*(2*a*sqrt(a^(-2)) + 1)*sqrt((40*a*sqrt(a^(-2))*x^2 + 41*x^2
+ (121*a^2*sqrt(a^(-2))*x + 122*a*x)*sqrt((a*sqrt(a^(-2)) + 2)/(4
*a*sqrt(a^(-2)) + 5))*(a^(-2))^(1/4) + (40*a^3*sqrt(a^(-2)) + 41*
a^2)*sqrt(a^(-2)))/(40*a*sqrt(a^(-2)) + 41))*sqrt((a*sqrt(a^(-2))
 + 2)/(4*a*sqrt(a^(-2)) + 5)) + 2*(2*a*sqrt(a^(-2))*x + x)*sqrt((
a*sqrt(a^(-2)) + 2)/(4*a*sqrt(a^(-2)) + 5)) + (2*a^2*sqrt(a^(-2))
 + a)*(a^(-2))^(1/4))) + 4*sqrt(3)*a*(a^(-2))^(3/4)*arctan(sqrt(3
)*a^2*(a^(-2))^(3/4)/(2*(2*a*sqrt(a^(-2)) + 1)*sqrt((40*a*sqrt(a^
(-2))*x^2 + 41*x^2 - (121*a^2*sqrt(a^(-2))*x + 122*a*x)*sqrt((a*s
qrt(a^(-2)) + 2)/(4*a*sqrt(a^(-2)) + 5))*(a^(-2))^(1/4) + (40*a^3
*sqrt(a^(-2)) + 41*a^2)*sqrt(a^(-2)))/(40*a*sqrt(a^(-2)) + 41))*s
qrt((a*sqrt(a^(-2)) + 2)/(4*a*sqrt(a^(-2)) + 5)) + 2*(2*a*sqrt(a^
(-2))*x + x)*sqrt((a*sqrt(a^(-2)) + 2)/(4*a*sqrt(a^(-2)) + 5)) -
(2*a^2*sqrt(a^(-2)) + a)*(a^(-2))^(1/4))) - (2*a*sqrt(a^(-2)) + 1
)*(a^(-2))^(1/4)*log(80*a*sqrt(a^(-2))*x^2 + 82*x^2 + 2*(121*a^2*
sqrt(a^(-2))*x + 122*a*x)*sqrt((a*sqrt(a^(-2)) + 2)/(4*a*sqrt(a^(
-2)) + 5))*(a^(-2))^(1/4) + 2*(40*a^3*sqrt(a^(-2)) + 41*a^2)*sqrt
```

```
(a^(-2))) + (2*a*sqrt(a^(-2)) + 1)*(a^(-2))^(1/4)*log(80*a*sqrt(a
^(-2))*x^2 + 82*x^2 - 2*(121*a^2*sqrt(a^(-2))*x + 122*a*x)*sqrt((
a*sqrt(a^(-2)) + 2)/(4*a*sqrt(a^(-2)) + 5))*(a^(-2))^(1/4) + 2*(4
0*a^3*sqrt(a^(-2)) + 41*a^2)*sqrt(a^(-2))))/((2*a*sqrt(a^(-2)) +
1)*sqrt((a*sqrt(a^(-2)) + 2)/(4*a*sqrt(a^(-2)) + 5)))
```

Sympy [A] time = 1.34961, size = 27, normalized size = 0.24

$$-\operatorname{RootSum}\left(16t^4a^2 - 4t^2a + 1, \left(t \mapsto t \log\left(-2ta + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+2*a)/(x**4-a*x**2+a**2),x)
```

```
[Out] -RootSum(16*_t**4*a**2 - 4*_t**2*a + 1, Lambda(_t, _t*log(-2*_t*a
 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 - 2\,a}{x^4 - ax^2 + a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 2*a)/(x^4 - a*x^2 + a^2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*a)/(x^4 - a*x^2 + a^2), x)
```

3.115 $\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx$

Optimal. Leaf size=122

$$-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt[4]{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt[4]{a}}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2\sqrt[4]{a}}$$

```
[Out] -ArcTan[Sqrt[3] - (2*x)/a^(1/4)]/(2*a^(1/4)) + ArcTan[Sqrt[3] + (
2*x)/a^(1/4)]/(2*a^(1/4)) - (Sqrt[3]*Log[Sqrt[a] - Sqrt[3]*a^(1/4
)*x + x^2])/(4*a^(1/4)) + (Sqrt[3]*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*
x + x^2])/(4*a^(1/4))
```

Rubi [A] time = 0.165027, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt[4]{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt[4]{a}}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2*Sqrt[a] - x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[3] - (2*x)/a^(1/4)]/(2*a^(1/4)) + ArcTan[Sqrt[3] + (
2*x)/a^(1/4)]/(2*a^(1/4)) - (Sqrt[3]*Log[Sqrt[a] - Sqrt[3]*a^(1/4
)*x + x^2])/(4*a^(1/4)) + (Sqrt[3]*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*
x + x^2])/(4*a^(1/4))
```

Rubi in Sympy [A] time = 39.6504, size = 134, normalized size = 1.1

$$-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt[4]{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt[4]{a}}-\frac{\operatorname{atan}\left(\frac{\sqrt{3}\left(\sqrt[4]{a}-\frac{2\sqrt{3}x}{3}\right)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}+\frac{\operatorname{atan}\left(\frac{\sqrt{3}\left(\sqrt[4]{a}+\frac{2\sqrt{3}x}{3}\right)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+2*a**(1/2))/(a+x**4-x**2*a**(1/2)),x)
```

```
[Out] -sqrt(3)*log(-sqrt(3)*a**(1/4)*x + sqrt(a) + x**2)/(4*a**(1/4)) +
 sqrt(3)*log(sqrt(3)*a**(1/4)*x + sqrt(a) + x**2)/(4*a**(1/4)) -
atan(sqrt(3)*(a**(1/4) - 2*sqrt(3)*x/3)/a**(1/4))/(2*a**(1/4)) +
atan(sqrt(3)*(a**(1/4) + 2*sqrt(3)*x/3)/a**(1/4))/(2*a**(1/4))
```

Mathematica [C] time = 0.303101, size = 115, normalized size = 0.94

$$\frac{\sqrt[4]{-1}\left(\sqrt{\sqrt{3}-i}\left(\sqrt{3}-3i\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt[4]{a}}\right)-\sqrt{\sqrt{3}+i}\left(\sqrt{3}+3i\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt[4]{a}}\right)\right)}{2\sqrt{6}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*Sqrt[a] - x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*(-(Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTan[((1 + I)*
x)/(Sqrt[-I + Sqrt[3]]*a^(1/4))]) + Sqrt[-I + Sqrt[3]]*(-3*I + Sq
rt[3])*ArcTanh[((1 + I)*x)/(Sqrt[I + Sqrt[3]]*a^(1/4))]))/(2*Sqrt
[6]*a^(1/4))
```

Maple [A] time = 0.05, size = 96, normalized size = 0.8

$$-\frac{\sqrt{3}}{4}\ln\left(\sqrt{3}\sqrt[4]{a}x-x^{2}-\sqrt{a}\right)\frac{1}{\sqrt[4]{a}}-\frac{1}{2}\arctan\left(1\left(\sqrt{3}\sqrt[4]{a}-2\,x\right)\frac{1}{\sqrt[4]{a}}\right)\frac{1}{\sqrt[4]{a}}+\frac{\sqrt{3}}{4}\ln\left(x^{2}+\sqrt{3}\sqrt[4]{a}x+\sqrt{a}\right)\frac{1}{\sqrt[4]{a}}+\frac{1}{2}\arctan\left(1\left(2\,x+\sqrt{3}\sqrt[4]{a}\right)\frac{1}{\sqrt[4]{a}}\right)\frac{1}{\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+2*a^(1/2))/(a+x^4-a^(1/2)*x^2),x)
```

```
[Out] -1/4/a^(1/4)*3^(1/2)*ln(3^(1/2)*a^(1/4)*x-x^2-a^(1/2))-1/2/a^(1/4
)*arctan((3^(1/2)*a^(1/4)-2*x)/a^(1/4))+1/4*ln(x^2+3^(1/2)*a^(1/4
)*x+a^(1/2))*3^(1/2)/a^(1/4)+1/2/a^(1/4)*arctan((2*x+3^(1/2)*a^(1
/4))/a^(1/4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^{2}-2\sqrt{a}}{x^{4}-\sqrt{a}x^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 2*sqrt(a))/(x^4 - sqrt(a)*x^2 + a),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*sqrt(a))/(x^4 - sqrt(a)*x^2 + a), x)
```

Fricas [A] time = 0.313, size = 350, normalized size = 2.87

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}\log\left(\sqrt{\frac{1}{2}}a\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}+\sqrt{a}x\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}\log\left(-\sqrt{\frac{1}{2}}a\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}+\sqrt{a}x\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}-\sqrt{a}}{a}}\log\left(\sqrt{\frac{1}{2}}a\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}-\sqrt{a}}{a}}+\sqrt{a}x\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}-\sqrt{a}}{a}}\log\left(-\sqrt{\frac{1}{2}}a\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}-\sqrt{a}}{a}}+\sqrt{a}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 2*sqrt(a))/(x^4 - sqrt(a)*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt((sqrt(3)*a*sqrt(-1/a) + sqrt(a))/a)*log(sqrt(1
/2)*a*sqrt((sqrt(3)*a*sqrt(-1/a) + sqrt(a))/a) + sqrt(a)*x) - 1/2
*sqrt(1/2)*sqrt((sqrt(3)*a*sqrt(-1/a) + sqrt(a))/a)*log(-sqrt(1/2
)*a*sqrt((sqrt(3)*a*sqrt(-1/a) + sqrt(a))/a) + sqrt(a)*x) + 1/2*s
qrt(1/2)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a)*log(sqrt(1/2)*
a*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a) + sqrt(a)*x) - 1/2*sq
rt(1/2)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a)*log(-sqrt(1/2)*
a*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a) + sqrt(a)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+2*a**(1/2))/(a+x**4-x**2*a**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - 2*sqrt(a))/(x^4 - sqrt(a)*x^2 + a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.116 $\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx$

Optimal. Leaf size=124

$$-\frac{\log\left(b^{2/3}-\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}$$

```
[Out] -(Sqrt[3]*ArcTan[(b^(1/3) - 2*x)/(Sqrt[3]*b^(1/3))])/(2*b^(1/3))
+ (Sqrt[3]*ArcTan[(b^(1/3) + 2*x)/(Sqrt[3]*b^(1/3))])/(2*b^(1/3))
 - Log[b^(2/3) - b^(1/3)*x + x^2]/(4*b^(1/3)) + Log[b^(2/3) + b^(
1/3)*x + x^2]/(4*b^(1/3))
```

Rubi [A] time = 0.162047, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156

$$-\frac{\log\left(b^{2/3}-\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2*b^(2/3) + x^2)/(b^(4/3) + b^(2/3)*x^2 + x^4),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(b^(1/3) - 2*x)/(Sqrt[3]*b^(1/3))])/(2*b^(1/3))
+ (Sqrt[3]*ArcTan[(b^(1/3) + 2*x)/(Sqrt[3]*b^(1/3))])/(2*b^(1/3))
 - Log[b^(2/3) - b^(1/3)*x + x^2]/(4*b^(1/3)) + Log[b^(2/3) + b^(
1/3)*x + x^2]/(4*b^(1/3))
```

Rubi in Sympy [A] time = 32.4405, size = 117, normalized size = 0.94

$$-\frac{\log\left(b^{\frac{2}{3}}-\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}+\frac{\log\left(b^{\frac{2}{3}}+\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{b}}{3}-\frac{2x}{3}\right)}{\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{b}}{3}+\frac{2x}{3}\right)}{\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*b**(2/3)+x**2)/(b**(4/3)+b**(2/3)*x**2+x**4),x)
```

```
[Out] -log(b**(2/3) - b**(1/3)*x + x**2)/(4*b**(1/3)) + log(b**(2/3) +
b**(1/3)*x + x**2)/(4*b**(1/3)) - sqrt(3)*atan(sqrt(3)*(b**(1/3)/
3 - 2*x/3)/b**(1/3))/(2*b**(1/3)) + sqrt(3)*atan(sqrt(3)*(b**(1/3
)/3 + 2*x/3)/b**(1/3))/(2*b**(1/3))
```

Mathematica [C] time = 0.22584, size = 115, normalized size = 0.93

$$\frac{\sqrt[4]{-1}\left(\sqrt{\sqrt{3}-i}\left(\sqrt{3}-3i\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt[3]{b}}\right)-\sqrt{\sqrt{3}+i}\left(\sqrt{3}+3i\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt[3]{b}}\right)\right)}{2\sqrt{6}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*b^(2/3) + x^2)/(b^(4/3) + b^(2/3)*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*(Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTan[((1 + I)*
x)/(Sqrt[I + Sqrt[3]]*b^(1/3))] - Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3
])*ArcTanh[((1 + I)*x)/(Sqrt[-I + Sqrt[3]]*b^(1/3))]))/(2*Sqrt[6]
*b^(1/3))
```

Maple [A] time = 0.021, size = 89, normalized size = 0.7

$$\frac{1}{4}\ln\left(b^{\frac{2}{3}}+\sqrt[3]{b}x+x^{2}\right)\frac{1}{\sqrt[3]{b}}+\frac{\sqrt{3}}{2}\arctan\left(\frac{\sqrt{3}}{3}\left(\sqrt[3]{b}+2\,x\right)\frac{1}{\sqrt[3]{b}}\right)\frac{1}{\sqrt[3]{b}}-\frac{1}{4}\ln\left(b^{\frac{2}{3}}-\sqrt[3]{b}x+x^{2}\right)\frac{1}{\sqrt[3]{b}}+\frac{\sqrt{3}}{2}\arctan\left(\frac{\sqrt{3}}{3}\left(-\sqrt[3]{b}+2\,x\right)\frac{1}{\sqrt[3]{b}}\right)\frac{1}{\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x)
```

```
[Out] 1/4*ln(b^(2/3)+b^(1/3)*x+x^2)/b^(1/3)+1/2*arctan(1/3*(b^(1/3)+2*x
)/b^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)-1/4*ln(b^(2/3)-b^(1/3)*x+x^2)/
b^(1/3)+1/2*3^(1/2)/b^(1/3)*arctan(1/3*(-b^(1/3)+2*x)*3^(1/2)/b^(
1/3))
```

Maxima [A] time = 0.816896, size = 170, normalized size = 1.37

$$-\frac{i\sqrt{3}\log\left(\frac{2\,x-i\,\sqrt{3}b^{\frac{1}{3}}+b^{\frac{1}{3}}}{2\,x+i\,\sqrt{3}b^{\frac{1}{3}}+b^{\frac{1}{3}}}\right)}{4\,b^{\frac{1}{3}}}-\frac{i\sqrt{3}\log\left(\frac{2\,x-i\,\sqrt{3}b^{\frac{1}{3}}-b^{\frac{1}{3}}}{2\,x+i\,\sqrt{3}b^{\frac{1}{3}}-b^{\frac{1}{3}}}\right)}{4\,b^{\frac{1}{3}}}+\frac{\log\left(x^{2}+b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4\,b^{\frac{1}{3}}}-\frac{\log\left(x^{2}-b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4\,b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*b^(2/3))/(x^4 + b^(2/3)*x^2 + b^(4/3)),x, algorithm="maxima")
```

```
[Out] -1/4*I*sqrt(3)*log((2*x - I*sqrt(3)*b^(1/3) + b^(1/3))/(2*x + I*s
qrt(3)*b^(1/3) + b^(1/3)))/b^(1/3) - 1/4*I*sqrt(3)*log((2*x - I*s
qrt(3)*b^(1/3) - b^(1/3))/(2*x + I*sqrt(3)*b^(1/3) - b^(1/3)))/b^
(1/3) + 1/4*log(x^2 + b^(1/3)*x + b^(2/3))/b^(1/3) - 1/4*log(x^2
- b^(1/3)*x + b^(2/3))/b^(1/3)
```

Fricas [A] time = 0.323741, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{3}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2\,b^{\frac{1}{3}}x^{2}+\sqrt{3}\left(2\,bx+b^{\frac{4}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+2\,b^{\frac{2}{3}}x-b}{b^{\frac{1}{3}}x^{2}+b^{\frac{2}{3}}x+b}\right)+\sqrt{3}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2\,b^{\frac{1}{3}}x^{2}+\sqrt{3}\left(2\,bx-b^{\frac{4}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-2\,b^{\frac{2}{3}}x-b}{b^{\frac{1}{3}}x^{2}-b^{\frac{2}{3}}x+b}\right)+b^{\frac{2}{3}}\log\left(b^{\frac{1}{3}}x^{2}+b^{\frac{2}{3}}\right.}{4\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*b^(2/3))/(x^4 + b^(2/3)*x^2 + b^(4/3)),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*b*sqrt(-1/b^(2/3))*log((2*b^(1/3)*x^2 + sqrt(3)*(2*
b*x + b^(4/3))*sqrt(-1/b^(2/3)) + 2*b^(2/3)*x - b)/(b^(1/3)*x^2 +
 b^(2/3)*x + b)) + sqrt(3)*b*sqrt(-1/b^(2/3))*log((2*b^(1/3)*x^2
```

```
+ sqrt(3)*(2*b*x - b^(4/3))*sqrt(-1/b^(2/3)) - 2*b^(2/3)*x - b)/(
b^(1/3)*x^2 - b^(2/3)*x + b)) + b^(2/3)*log(b^(1/3)*x^2 + b^(2/3)
*x + b) - b^(2/3)*log(b^(1/3)*x^2 - b^(2/3)*x + b))/b, 1/4*(2*sqr
t(3)*b^(2/3)*arctan(1/3*sqrt(3)*(2*b^(1/3)*x + b^(2/3))/b^(2/3))
- 2*sqrt(3)*b^(2/3)*arctan(-1/3*sqrt(3)*(2*b^(1/3)*x - b^(2/3))/b
^(2/3)) + b^(2/3)*log(b^(1/3)*x^2 + b^(2/3)*x + b) - b^(2/3)*log(
b^(1/3)*x^2 - b^(2/3)*x + b))/b]
```

Sympy [A] time = 1.41341, size = 143, normalized size = 1.15

$$\frac{\left(-\frac{1}{4}-\frac{\sqrt{3}i}{4}\right)\log\left(2\sqrt[3]{b}\left(-\frac{1}{4}-\frac{\sqrt{3}i}{4}\right)+x\right)+\left(-\frac{1}{4}+\frac{\sqrt{3}i}{4}\right)\log\left(2\sqrt[3]{b}\left(-\frac{1}{4}+\frac{\sqrt{3}i}{4}\right)+x\right)+\left(\frac{1}{4}-\frac{\sqrt{3}i}{4}\right)\log\left(2\sqrt[3]{b}\left(\frac{1}{4}-\frac{\sqrt{3}i}{4}\right)+x\right)+\left(\frac{1}{4}\right.}{\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*b**(2/3)+x**2)/(b**(4/3)+b**(2/3)*x**2+x**4),x)
```

```
[Out] ((-1/4 - sqrt(3)*I/4)*log(2*b**(1/3)*(-1/4 - sqrt(3)*I/4) + x) +
(-1/4 + sqrt(3)*I/4)*log(2*b**(1/3)*(-1/4 + sqrt(3)*I/4) + x) + (
1/4 - sqrt(3)*I/4)*log(2*b**(1/3)*(1/4 - sqrt(3)*I/4) + x) + (1/4
 + sqrt(3)*I/4)*log(2*b**(1/3)*(1/4 + sqrt(3)*I/4) + x))/b**(1/3)
```

GIAC/XCAS [A] time = 0.281628, size = 124, normalized size = 1.

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+b^{\frac{1}{3}}\right)}{3|b|^{\frac{1}{3}}}\right)}{2|b|^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-b^{\frac{1}{3}}\right)}{3|b|^{\frac{1}{3}}}\right)}{2|b|^{\frac{1}{3}}}+\frac{\ln\left(x^2+b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}}-\frac{\ln\left(x^2-b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 2*b^(2/3))/(x^4 + b^(2/3)*x^2 + b^(4/3)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + b^(1/3))/abs(b)^(1/3))/abs(
b)^(1/3) + 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - b^(1/3))/abs(b)^
(1/3))/abs(b)^(1/3) + 1/4*ln(x^2 + b^(1/3)*x + b^(2/3))/b^(1/3) -
 1/4*ln(x^2 - b^(1/3)*x + b^(2/3))/b^(1/3)
```

3.117 $\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx$

Optimal. Leaf size=136

$$-\frac{(A-aB)\log\left(-\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{3}a^{3/2}}+\frac{(A-aB)\log\left(\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{3}a^{3/2}}-\frac{(aB+A)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{(aB+A)\tan^{-1}\left(\frac{2x}{\sqrt{a}}+\sqrt{3}\right)}{2a^{3/2}}$$

```
[Out] -((A + a*B)*ArcTan[Sqrt[3] - (2*x)/Sqrt[a]])/(2*a^(3/2)) + ((A +
a*B)*ArcTan[Sqrt[3] + (2*x)/Sqrt[a]])/(2*a^(3/2)) - ((A - a*B)*Lo
g[a - Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[3]*a^(3/2)) + ((A - a*B)*
Log[a + Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[3]*a^(3/2))
```

Rubi [A] time = 0.230314, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$-\frac{(A-aB)\log\left(-\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{3}a^{3/2}}+\frac{(A-aB)\log\left(\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{3}a^{3/2}}-\frac{(aB+A)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{(aB+A)\tan^{-1}\left(\frac{2x}{\sqrt{a}}+\sqrt{3}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] -((A + a*B)*ArcTan[Sqrt[3] - (2*x)/Sqrt[a]])/(2*a^(3/2)) + ((A +
a*B)*ArcTan[Sqrt[3] + (2*x)/Sqrt[a]])/(2*a^(3/2)) - ((A - a*B)*Lo
g[a - Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[3]*a^(3/2)) + ((A - a*B)*
Log[a + Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[3]*a^(3/2))
```

Rubi in Sympy [A] time = 41.742, size = 148, normalized size = 1.09

$$-\frac{\sqrt{3}(A-Ba)\log\left(-\sqrt{3}\sqrt{a}x+a+x^2\right)}{12a^{\frac{3}{2}}}+\frac{\sqrt{3}(A-Ba)\log\left(\sqrt{3}\sqrt{a}x+a+x^2\right)}{12a^{\frac{3}{2}}}-\frac{(A+Ba)\operatorname{atan}\left(\frac{\sqrt{3}\left(\sqrt{a}-\frac{2\sqrt{3}x}{3}\right)}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}+\frac{(A+Ba)\operatorname{atan}\left(\frac{\sqrt{3}\left(\sqrt{a}+\frac{2\sqrt{3}x}{3}\right)}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(x**4-a*x**2+a**2),x)
```

```
[Out] -sqrt(3)*(A - B*a)*log(-sqrt(3)*sqrt(a)*x + a + x**2)/(12*a**(3/2
)) + sqrt(3)*(A - B*a)*log(sqrt(3)*sqrt(a)*x + a + x**2)/(12*a**(
3/2)) - (A + B*a)*atan(sqrt(3)*(sqrt(a) - 2*sqrt(3)*x/3)/sqrt(a))
/(2*a**(3/2)) + (A + B*a)*atan(sqrt(3)*(sqrt(a) + 2*sqrt(3)*x/3)/
sqrt(a))/(2*a**(3/2))
```

Mathematica [C] time = 0.264595, size = 130, normalized size = 0.96

$$\frac{\sqrt[4]{-1}\left(\frac{\left(\left(\sqrt{3}-i\right)aB-2iA\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt{a}}\right)}{\sqrt{\sqrt{3}-i}}-\frac{\left(\left(\sqrt{3}+i\right)aB+2iA\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt{a}}\right)}{\sqrt{\sqrt{3}+i}}\right)}{\sqrt{6}a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*(((((-2*I)*A + (-I + Sqrt[3])*a*B)*ArcTan[((1 + I)*x)/
(Sqrt[-I + Sqrt[3]]*Sqrt[a])])/Sqrt[-I + Sqrt[3]] - (((2*I)*A + (
I + Sqrt[3])*a*B)*ArcTanh[((1 + I)*x)/(Sqrt[I + Sqrt[3]]*Sqrt[a])
])/Sqrt[I + Sqrt[3]]))/(Sqrt[6]*a^(3/2))
```

Maple [A] time = 0.026, size = 190, normalized size = 1.4

$$-\frac{B\sqrt{3}}{12}\ln\left(a+x^2+\sqrt{3}\sqrt{a}x\right)\frac{1}{\sqrt{a}}+\frac{A\sqrt{3}}{12}\ln\left(a+x^2+\sqrt{3}\sqrt{a}x\right)a^{-\frac{3}{2}}$$
$$+\frac{B}{2}\arctan\left(1\left(2x+\sqrt{3}\sqrt{a}\right)\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}+\frac{A}{2}\arctan\left(1\left(2x+\sqrt{3}\sqrt{a}\right)\frac{1}{\sqrt{a}}\right)a^{-\frac{3}{2}}$$
$$+\frac{B\sqrt{3}}{12}\ln\left(\sqrt{3}\sqrt{a}x-x^2-a\right)\frac{1}{\sqrt{a}}-\frac{A\sqrt{3}}{12}\ln\left(\sqrt{3}\sqrt{a}x-x^2-a\right)a^{-\frac{3}{2}}$$
$$-\frac{B}{2}\arctan\left(1\left(\sqrt{3}\sqrt{a}-2x\right)\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}-\frac{A}{2}\arctan\left(1\left(\sqrt{3}\sqrt{a}-2x\right)\frac{1}{\sqrt{a}}\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(x^4-a*x^2+a^2),x)
```

```
[Out] -1/12/a^(1/2)*ln(a+x^2+3^(1/2)*a^(1/2)*x)*B*3^(1/2)+1/12/a^(3/2)*
ln(a+x^2+3^(1/2)*a^(1/2)*x)*A*3^(1/2)+1/2/a^(1/2)*arctan((2*x+3^(
1/2)*a^(1/2))/a^(1/2))*B+1/2/a^(3/2)*arctan((2*x+3^(1/2)*a^(1/2))
/a^(1/2))*A+1/12/a^(1/2)*ln(3^(1/2)*a^(1/2)*x-x^2-a)*B*3^(1/2)-1/
12/a^(3/2)*ln(3^(1/2)*a^(1/2)*x-x^2-a)*A*3^(1/2)-1/2/a^(1/2)*arct
an((3^(1/2)*a^(1/2)-2*x)/a^(1/2))*B-1/2/a^(3/2)*arctan((3^(1/2)*a
^(1/2)-2*x)/a^(1/2))*A
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{Bx^2+A}{x^4-ax^2+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(x^4 - a*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(x^4 - a*x^2 + a^2), x)
```

Fricas [A] time = 2.50433, size = 9914, normalized size = 72.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(x^4 - a*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] 1/4*(12*sqrt(1/3)*(1/9)^(1/4)*a^3*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2
*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(1/4)*sqrt((B^4*a^4 - 2*A^2*B^2*
a^2 + A^4)/a^6)*arctan(-3*(1/9)^(1/4)*(sqrt(1/3)*(2*B*a^9 + A*a^8
)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/
a^6)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6) - sqrt(1/3)*(B^3*a
^8 + 3*A*B^2*a^7 + 3*A^2*B*a^6 + 2*A^3*a^5)*sqrt((B^4*a^4 - 2*A^2
*B^2*a^2 + A^4)/a^6))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2
*A^3*B*a + A^4)/a^6)^(1/4)/(2*(B^6*a^6 + 5*A*B^5*a^5 + 5*A^2*B^4*
a^4 - 5*A^4*B^2*a^2 - 5*A^5*B*a - A^6 - 2*(B^4*a^7 + A*B^3*a^6 -
A^3*B*a^4 - A^4*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2
+ 2*A^3*B*a + A^4)/a^6))*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^
2*a^2 + 4*A^3*B*a + 2*A^4 - (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt(
(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(
5*B^4*a^4 + 16*A*B^3*a^3 + 30*A^2*B^2*a^2 + 16*A^3*B*a + 5*A^4 -
4*(B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3
*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)))*sqrt(-((13*B^8*a^11 + 97*A
*B^7*a^10 + 292*A^2*B^6*a^9 + 559*A^3*B^5*a^8 + 670*A^4*B^4*a^7 +
 559*A^5*B^3*a^6 + 292*A^6*B^2*a^5 + 97*A^7*B*a^4 + 13*A^8*a^3)*x
^2*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)
/a^6) - 2*(7*B^10*a^10 + 53*A*B^9*a^9 + 204*A^2*B^8*a^8 + 471*A^3
*B^7*a^7 + 765*A^4*B^6*a^6 + 888*A^5*B^5*a^5 + 765*A^6*B^4*a^4 +
471*A^7*B^3*a^3 + 204*A^8*B^2*a^2 + 53*A^9*B*a + 7*A^10)*x^2 + 3*
(1/9)^(1/4)*((14*B^7*a^12 + 91*A*B^6*a^11 + 294*A^2*B^5*a^10 + 45
5*A^3*B^4*a^9 + 490*A^4*B^3*a^8 + 273*A^5*B^2*a^7 + 98*A^6*B*a^6
+ 13*A^7*a^5)*x*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A
^3*B*a + A^4)/a^6) - (13*B^9*a^11 + 111*A*B^8*a^10 + 384*A^2*B^7*
a^9 + 861*A^3*B^6*a^8 + 1218*A^4*B^5*a^7 + 1239*A^5*B^4*a^6 + 840
*A^6*B^3*a^5 + 399*A^7*B^2*a^4 + 105*A^8*B*a^3 + 14*A^9*a^2)*x)*s
qrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4
- (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3
*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(5*B^4*a^4 + 16*A*B^3*a^3 +
 30*A^2*B^2*a^2 + 16*A^3*B*a + 5*A^4 - 4*(B^2*a^5 + 4*A*B*a^4 + A
^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a +
 A^4)/a^6)))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a
+ A^4)/a^6)^(1/4) - (14*B^8*a^12 + 92*A*B^7*a^11 + 302*A^2*B^6*a^
10 + 548*A^3*B^5*a^9 + 680*A^4*B^4*a^8 + 548*A^5*B^3*a^7 + 302*A^
6*B^2*a^6 + 92*A^7*B*a^5 + 14*A^8*a^4 - (13*B^6*a^13 + 84*A*B^5*a
^12 + 195*A^2*B^4*a^11 + 280*A^3*B^3*a^10 + 195*A^4*B^2*a^9 + 84*
A^5*B*a^8 + 13*A^6*a^7)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a
^2 + 2*A^3*B*a + A^4)/a^6))*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B
^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(14*B^10*a^10 + 106*A*B^9*a^9 + 4
08*A^2*B^8*a^8 + 942*A^3*B^7*a^7 + 1530*A^4*B^6*a^6 + 1776*A^5*B^
5*a^5 + 1530*A^6*B^4*a^4 + 942*A^7*B^3*a^3 + 408*A^8*B^2*a^2 + 10
6*A^9*B*a + 14*A^10 - (13*B^8*a^11 + 97*A*B^7*a^10 + 292*A^2*B^6*
a^9 + 559*A^3*B^5*a^8 + 670*A^4*B^4*a^7 + 559*A^5*B^3*a^6 + 292*A
^6*B^2*a^5 + 97*A^7*B*a^4 + 13*A^8*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a
^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))) - 2*(2*(B^4*a^7 + A*
B^3*a^6 - A^3*B*a^4 - A^4*a^3)*x*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*
A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6) - (B^6*a^6 + 5*A*B^5*a^5 + 5*
A^2*B^4*a^4 - 5*A^4*B^2*a^2 - 5*A^5*B*a - A^6)*x)*sqrt((2*B^4*a^4
 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 - (B^2*a^5 + 4
*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 +
 2*A^3*B*a + A^4)/a^6))/(5*B^4*a^4 + 16*A*B^3*a^3 + 30*A^2*B^2*a^
2 + 16*A^3*B*a + 5*A^4 - 4*(B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((
B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))) +
 3*(1/9)^(1/4)*(B^5*a^7 + A*B^4*a^6 - A^3*B^2*a^4 - A^4*B*a^3 - (
A*B^2*a^7 - A^3*a^5)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2
+ 2*A^3*B*a + A^4)/a^6))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2
+ 2*A^3*B*a + A^4)/a^6)^(1/4))) + 12*sqrt(1/3)*(1/9)^(1/4)*a^3*((
B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(1/
4)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6)*arctan(-3*(1/9)^(1/4
)*(sqrt(1/3)*(2*B*a^9 + A*a^8)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^
2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 +
 A^4)/a^6) - sqrt(1/3)*(B^3*a^8 + 3*A*B^2*a^7 + 3*A^2*B*a^6 + 2*A
^3*a^5)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6))*((B^4*a^4 + 2*
A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(1/4)/(2*(B^6*a
^6 + 5*A*B^5*a^5 + 5*A^2*B^4*a^4 - 5*A^4*B^2*a^2 - 5*A^5*B*a - A^
6 - 2*(B^4*a^7 + A*B^3*a^6 - A^3*B*a^4 - A^4*a^3)*sqrt((B^4*a^4 +
 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))*sqrt((2*B^4
```

*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 - (B^2*a^5
 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a
^2 + 2*A^3*B*a + A^4)/a^6))/(5*B^4*a^4 + 16*A*B^3*a^3 + 30*A^2*B^
2*a^2 + 16*A^3*B*a + 5*A^4 - 4*(B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sq
rt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)
))*sqrt(-((13*B^8*a^11 + 97*A*B^7*a^10 + 292*A^2*B^6*a^9 + 559*A^
3*B^5*a^8 + 670*A^4*B^4*a^7 + 559*A^5*B^3*a^6 + 292*A^6*B^2*a^5 +
 97*A^7*B*a^4 + 13*A^8*a^3)*x^2*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A
^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6) - 2*(7*B^10*a^10 + 53*A*B^9*a^
9 + 204*A^2*B^8*a^8 + 471*A^3*B^7*a^7 + 765*A^4*B^6*a^6 + 888*A^5
*B^5*a^5 + 765*A^6*B^4*a^4 + 471*A^7*B^3*a^3 + 204*A^8*B^2*a^2 +
53*A^9*B*a + 7*A^10)*x^2 - 3*(1/9)^(1/4)*((14*B^7*a^12 + 91*A*B^6
*a^11 + 294*A^2*B^5*a^10 + 455*A^3*B^4*a^9 + 490*A^4*B^3*a^8 + 27
3*A^5*B^2*a^7 + 98*A^6*B*a^6 + 13*A^7*a^5)*x*sqrt((B^4*a^4 + 2*A*
B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6) - (13*B^9*a^11 +
111*A*B^8*a^10 + 384*A^2*B^7*a^9 + 861*A^3*B^6*a^8 + 1218*A^4*B^5
*a^7 + 1239*A^5*B^4*a^6 + 840*A^6*B^3*a^5 + 399*A^7*B^2*a^4 + 105
*A^8*B*a^3 + 14*A^9*a^2)*x)*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2
*B^2*a^2 + 4*A^3*B*a + 2*A^4 - (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sq
rt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)
)/(5*B^4*a^4 + 16*A*B^3*a^3 + 30*A^2*B^2*a^2 + 16*A^3*B*a + 5*A^4
 - 4*(B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3
+ 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)))*((B^4*a^4 + 2*A*B^3*a^3
 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(1/4) - (14*B^8*a^12 + 9
2*A*B^7*a^11 + 302*A^2*B^6*a^10 + 548*A^3*B^5*a^9 + 680*A^4*B^4*a
^8 + 548*A^5*B^3*a^7 + 302*A^6*B^2*a^6 + 92*A^7*B*a^5 + 14*A^8*a^
4 - (13*B^6*a^13 + 84*A*B^5*a^12 + 195*A^2*B^4*a^11 + 280*A^3*B^3
*a^10 + 195*A^4*B^2*a^9 + 84*A^5*B*a^8 + 13*A^6*a^7)*sqrt((B^4*a^
4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))*sqrt((B^
4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(14*
B^10*a^10 + 106*A*B^9*a^9 + 408*A^2*B^8*a^8 + 942*A^3*B^7*a^7 + 1
530*A^4*B^6*a^6 + 1776*A^5*B^5*a^5 + 1530*A^6*B^4*a^4 + 942*A^7*B
^3*a^3 + 408*A^8*B^2*a^2 + 106*A^9*B*a + 14*A^10 - (13*B^8*a^11 +
 97*A*B^7*a^10 + 292*A^2*B^6*a^9 + 559*A^3*B^5*a^8 + 670*A^4*B^4*
a^7 + 559*A^5*B^3*a^6 + 292*A^6*B^2*a^5 + 97*A^7*B*a^4 + 13*A^8*a
^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4
)/a^6))) - 2*(2*(B^4*a^7 + A*B^3*a^6 - A^3*B*a^4 - A^4*a^3)*x*sqr
t((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)
- (B^6*a^6 + 5*A*B^5*a^5 + 5*A^2*B^4*a^4 - 5*A^4*B^2*a^2 - 5*A^5*
B*a - A^6)*x)*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A
^3*B*a + 2*A^4 - (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 +
2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(5*B^4*a^4 +
 16*A*B^3*a^3 + 30*A^2*B^2*a^2 + 16*A^3*B*a + 5*A^4 - 4*(B^2*a^5
+ 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^
2 + 2*A^3*B*a + A^4)/a^6))) - 3*(1/9)^(1/4)*(B^5*a^7 + A*B^4*a^6
- A^3*B^2*a^4 - A^4*B*a^3 - (A*B^2*a^7 - A^3*a^5)*sqrt((B^4*a^4 +
 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))*((B^4*a^4 +
 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(1/4))) + (1
/9)^(1/4)*(B^2*a^2 - 2*a^3*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^
2*a^2 + 2*A^3*B*a + A^4)/a^6) + 4*A*B*a + A^2)*((B^4*a^4 + 2*A*B^
3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(1/4)*log(2*(13*B^8
*a^11 + 97*A*B^7*a^10 + 292*A^2*B^6*a^9 + 559*A^3*B^5*a^8 + 670*A
^4*B^4*a^7 + 559*A^5*B^3*a^6 + 292*A^6*B^2*a^5 + 97*A^7*B*a^4 + 1
3*A^8*a^3)*x^2*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^
3*B*a + A^4)/a^6) - 4*(7*B^10*a^10 + 53*A*B^9*a^9 + 204*A^2*B^8*a
^8 + 471*A^3*B^7*a^7 + 765*A^4*B^6*a^6 + 888*A^5*B^5*a^5 + 765*A^
6*B^4*a^4 + 471*A^7*B^3*a^3 + 204*A^8*B^2*a^2 + 53*A^9*B*a + 7*A^
10)*x^2 + 6*(1/9)^(1/4)*((14*B^7*a^12 + 91*A*B^6*a^11 + 294*A^2*B
^5*a^10 + 455*A^3*B^4*a^9 + 490*A^4*B^3*a^8 + 273*A^5*B^2*a^7 + 9
8*A^6*B*a^6 + 13*A^7*a^5)*x*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B
^2*a^2 + 2*A^3*B*a + A^4)/a^6) - (13*B^9*a^11 + 111*A*B^8*a^10 +
384*A^2*B^7*a^9 + 861*A^3*B^6*a^8 + 1218*A^4*B^5*a^7 + 1239*A^5*B
^4*a^6 + 840*A^6*B^3*a^5 + 399*A^7*B^2*a^4 + 105*A^8*B*a^3 + 14*A
^9*a^2)*x)*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*
B*a + 2*A^4 - (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A
*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(5*B^4*a^4 + 16
*A*B^3*a^3 + 30*A^2*B^2*a^2 + 16*A^3*B*a + 5*A^4 - 4*(B^2*a^5 + 4
*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 +
 2*A^3*B*a + A^4)/a^6)))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2
+ 2*A^3*B*a + A^4)/a^6)^(1/4) - 2*(14*B^8*a^12 + 92*A*B^7*a^11 +
302*A^2*B^6*a^10 + 548*A^3*B^5*a^9 + 680*A^4*B^4*a^8 + 548*A^5*B^
3*a^7 + 302*A^6*B^2*a^6 + 92*A^7*B*a^5 + 14*A^8*a^4 - (13*B^6*a^1

```
3 + 84*A*B^5*a^12 + 195*A^2*B^4*a^11 + 280*A^3*B^3*a^10 + 195*A^4
*B^2*a^9 + 84*A^5*B*a^8 + 13*A^6*a^7)*sqrt((B^4*a^4 + 2*A*B^3*a^3
 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))*sqrt((B^4*a^4 + 2*A*B^3
*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)) - (1/9)^(1/4)*(B^2*
a^2 - 2*a^3*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B
*a + A^4)/a^6) + 4*A*B*a + A^2)*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B
^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(1/4)*log(2*(13*B^8*a^11 + 97*A*B^
7*a^10 + 292*A^2*B^6*a^9 + 559*A^3*B^5*a^8 + 670*A^4*B^4*a^7 + 55
9*A^5*B^3*a^6 + 292*A^6*B^2*a^5 + 97*A^7*B*a^4 + 13*A^8*a^3)*x^2*
sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^
6) - 4*(7*B^10*a^10 + 53*A*B^9*a^9 + 204*A^2*B^8*a^8 + 471*A^3*B^
7*a^7 + 765*A^4*B^6*a^6 + 888*A^5*B^5*a^5 + 765*A^6*B^4*a^4 + 471
*A^7*B^3*a^3 + 204*A^8*B^2*a^2 + 53*A^9*B*a + 7*A^10)*x^2 - 6*(1/
9)^(1/4)*((14*B^7*a^12 + 91*A*B^6*a^11 + 294*A^2*B^5*a^10 + 455*A
^3*B^4*a^9 + 490*A^4*B^3*a^8 + 273*A^5*B^2*a^7 + 98*A^6*B*a^6 + 1
3*A^7*a^5)*x*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*
B*a + A^4)/a^6) - (13*B^9*a^11 + 111*A*B^8*a^10 + 384*A^2*B^7*a^9
 + 861*A^3*B^6*a^8 + 1218*A^4*B^5*a^7 + 1239*A^5*B^4*a^6 + 840*A^
6*B^3*a^5 + 399*A^7*B^2*a^4 + 105*A^8*B*a^3 + 14*A^9*a^2)*x)*sqrt
((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 - (
B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^
2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(5*B^4*a^4 + 16*A*B^3*a^3 + 30
*A^2*B^2*a^2 + 16*A^3*B*a + 5*A^4 - 4*(B^2*a^5 + 4*A*B*a^4 + A^2*
a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^
4)/a^6)))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A
^4)/a^6)^(1/4) - 2*(14*B^8*a^12 + 92*A*B^7*a^11 + 302*A^2*B^6*a^1
0 + 548*A^3*B^5*a^9 + 680*A^4*B^4*a^8 + 548*A^5*B^3*a^7 + 302*A^6
*B^2*a^6 + 92*A^7*B*a^5 + 14*A^8*a^4 - (13*B^6*a^13 + 84*A*B^5*a^
12 + 195*A^2*B^4*a^11 + 280*A^3*B^3*a^10 + 195*A^4*B^2*a^9 + 84*A
^5*B*a^8 + 13*A^6*a^7)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^
2 + 2*A^3*B*a + A^4)/a^6))*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^
2*a^2 + 2*A^3*B*a + A^4)/a^6)))/((B^2*a^2 - 2*a^3*sqrt((B^4*a^4 +
 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6) + 4*A*B*a +
A^2)*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a +
2*A^4 - (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a
^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(5*B^4*a^4 + 16*A*B^3
*a^3 + 30*A^2*B^2*a^2 + 16*A^3*B*a + 5*A^4 - 4*(B^2*a^5 + 4*A*B*a
^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3
*B*a + A^4)/a^6))))
```

Sympy [A] time = 4.17263, size = 172, normalized size = 1.26

$$\operatorname{RootSum}\left(144t^4a^6 + t^2\left(12A^2a^3 + 48ABa^4 + 12B^2a^5\right) + A^4 + 2A^3Ba + 3A^2B^2a^2 + 2AB^3a^3 + B^4a^4, \left(t \mapsto t\log\left(x + \frac{24t^3Aa^5 + \cdots}{\cdots}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(x**4-a*x**2+a**2),x)
```

```
[Out] RootSum(144*_t**4*a**6 + _t**2*(12*A**2*a**3 + 48*A*B*a**4 + 12*B
**2*a**5) + A**4 + 2*A**3*B*a + 3*A**2*B**2*a**2 + 2*A*B**3*a**3
+ B**4*a**4, Lambda(_t, _t*log(x + (24*_t**3*A*a**5 + 48*_t**3*B*
a**6 - 2*_t*A**3*a**2 + 6*_t*A**2*B*a**3 + 12*_t*A*B**2*a**4 + 2*
_t*B**3*a**5)/(-A**4 - A**3*B*a + A*B**3*a**3 + B**4*a**4))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{x^4 - ax^2 + a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(x^4 - a*x^2 + a^2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(x^4 - a*x^2 + a^2), x)
```

3.118 $\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4} \, dx$

Optimal. Leaf size=160

$$-\frac{\left(A-\sqrt{a}B\right)\log\left(-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt{3}a^{3/4}}+\frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt{3}a^{3/4}}-\frac{\left(\sqrt{a}B+A\right)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}}+\frac{\left(\sqrt{a}B+A\right)\tan^{-1}\left(\frac{2x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2a^{3/4}}$$

```
[Out] -((A + Sqrt[a]*B)*ArcTan[Sqrt[3] - (2*x)/a^(1/4)])/(2*a^(3/4)) +
((A + Sqrt[a]*B)*ArcTan[Sqrt[3] + (2*x)/a^(1/4)])/(2*a^(3/4)) - (
(A - Sqrt[a]*B)*Log[Sqrt[a] - Sqrt[3]*a^(1/4)*x + x^2])/(4*Sqrt[3
]*a^(3/4)) + ((A - Sqrt[a]*B)*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*x + x
^2])/(4*Sqrt[3]*a^(3/4))
```

Rubi [A] time = 0.247946, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{\left(A-\sqrt{a}B\right)\log\left(-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt{3}a^{3/4}}+\frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt{3}a^{3/4}}-\frac{\left(\sqrt{a}B+A\right)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}}+\frac{\left(\sqrt{a}B+A\right)\tan^{-1}\left(\frac{2x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2a^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] -((A + Sqrt[a]*B)*ArcTan[Sqrt[3] - (2*x)/a^(1/4)])/(2*a^(3/4)) +
((A + Sqrt[a]*B)*ArcTan[Sqrt[3] + (2*x)/a^(1/4)])/(2*a^(3/4)) - (
(A - Sqrt[a]*B)*Log[Sqrt[a] - Sqrt[3]*a^(1/4)*x + x^2])/(4*Sqrt[3
]*a^(3/4)) + ((A - Sqrt[a]*B)*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*x + x
^2])/(4*Sqrt[3]*a^(3/4))
```

Rubi in Sympy [A] time = 43.2843, size = 168, normalized size = 1.05

$$-\frac{\sqrt{3}\left(A-B\sqrt{a}\right)\log\left(-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{12a^{\frac{3}{4}}}+\frac{\sqrt{3}\left(A-B\sqrt{a}\right)\log\left(\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{12a^{\frac{3}{4}}}-\frac{\left(A+B\sqrt{a}\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\sqrt[4]{a}-\frac{2\sqrt{3}x}{3}\right)}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}}+\frac{\left(A+B\sqrt{a}\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\sqrt[4]{a}+\frac{2\sqrt{3}x}{3}\right)}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(a+x**4-x**2*a**(1/2)),x)
```

```
[Out] -sqrt(3)*(A - B*sqrt(a))*log(-sqrt(3)*a**(1/4)*x + sqrt(a) + x**2
)/(12*a**(3/4)) + sqrt(3)*(A - B*sqrt(a))*log(sqrt(3)*a**(1/4)*x
+ sqrt(a) + x**2)/(12*a**(3/4)) - (A + B*sqrt(a))*atan(sqrt(3)*(a
**(1/4) - 2*sqrt(3)*x/3)/a**(1/4))/(2*a**(3/4)) + (A + B*sqrt(a))
*atan(sqrt(3)*(a**(1/4) + 2*sqrt(3)*x/3)/a**(1/4))/(2*a**(3/4))
```

Mathematica [C] time = 0.241871, size = 138, normalized size = 0.86

$$\frac{\sqrt[4]{-1}\left(\frac{\left(\left(\sqrt{3}-i\right)\sqrt{a}B-2iA\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}-i}}-\frac{\left(\left(\sqrt{3}+i\right)\sqrt{a}B+2iA\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}+i}}\right)}{\sqrt{6}a^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*((((-2*I)*A + (-I + Sqrt[3])*Sqrt[a]*B)*ArcTan[((1 +
I)*x)/(Sqrt[-I + Sqrt[3]]*a^(1/4))])/Sqrt[-I + Sqrt[3]] - (((2*I)
*A + (I + Sqrt[3])*Sqrt[a]*B)*ArcTanh[((1 + I)*x)/(Sqrt[I + Sqrt[
3]]*a^(1/4))])/Sqrt[I + Sqrt[3]]))/(Sqrt[6]*a^(3/4))
```

Maple [A] time = 0.041, size = 198, normalized size = 1.2

$$-\frac{A\sqrt{3}}{12}\ln\left(\sqrt{3}\sqrt[4]{a}x-x^2-\sqrt{a}\right)a^{-\frac{3}{4}}+\frac{B\sqrt{3}}{12}\ln\left(\sqrt{3}\sqrt[4]{a}x-x^2-\sqrt{a}\right)\frac{1}{\sqrt[4]{a}}$$
$$-\frac{A}{2}\arctan\left(1\left(\sqrt{3}\sqrt[4]{a}-2x\right)\frac{1}{\sqrt[4]{a}}\right)a^{-\frac{3}{4}}-\frac{B}{2}\arctan\left(1\left(\sqrt{3}\sqrt[4]{a}-2x\right)\frac{1}{\sqrt[4]{a}}\right)\frac{1}{\sqrt[4]{a}}$$
$$+\frac{A\sqrt{3}}{12}\ln\left(x^2+\sqrt{3}\sqrt[4]{a}x+\sqrt{a}\right)a^{-\frac{3}{4}}-\frac{B\sqrt{3}}{12}\ln\left(x^2+\sqrt{3}\sqrt[4]{a}x+\sqrt{a}\right)\frac{1}{\sqrt[4]{a}}$$
$$+\frac{A}{2}\arctan\left(1\left(2x+\sqrt{3}\sqrt[4]{a}\right)\frac{1}{\sqrt[4]{a}}\right)a^{-\frac{3}{4}}+\frac{B}{2}\arctan\left(1\left(2x+\sqrt{3}\sqrt[4]{a}\right)\frac{1}{\sqrt[4]{a}}\right)\frac{1}{\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(a+x^4-a^(1/2)*x^2),x)
```

```
[Out] -1/12/a^(3/4)*ln(3^(1/2)*a^(1/4)*x-x^2-a^(1/2))*A*3^(1/2)+1/12/a^
(1/4)*ln(3^(1/2)*a^(1/4)*x-x^2-a^(1/2))*B*3^(1/2)-1/2/a^(3/4)*arc
tan((3^(1/2)*a^(1/4)-2*x)/a^(1/4))*A-1/2/a^(1/4)*arctan((3^(1/2)*
a^(1/4)-2*x)/a^(1/4))*B+1/12/a^(3/4)*ln(x^2+3^(1/2)*a^(1/4)*x+a^(
1/2))*A*3^(1/2)-1/12/a^(1/4)*ln(x^2+3^(1/2)*a^(1/4)*x+a^(1/2))*B*
3^(1/2)+1/2/a^(3/4)*arctan((2*x+3^(1/2)*a^(1/4))/a^(1/4))*A+1/2/a
^(1/4)*arctan((2*x+3^(1/2)*a^(1/4))/a^(1/4))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{Bx^2+A}{x^4-\sqrt{a}x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(x^4 - sqrt(a)*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(x^4 - sqrt(a)*x^2 + a), x)
```

Fricas [A] time = 50.2718, size = 5455, normalized size = 34.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(x^4 - sqrt(a)*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/6)*sqrt(-(4*A*B*a + 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2
*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)*log(4*(24*A*
B^25*a^12 + 614*A^3*B^23*a^11 + 4621*A^5*B^21*a^10 + 14496*A^7*B^
19*a^9 + 19031*A^9*B^17*a^8 + 2566*A^11*B^15*a^7 - 18543*A^13*B^1
3*a^6 - 17860*A^15*B^11*a^5 - 5399*A^17*B^9*a^4 + 186*A^19*B^7*a^
3 + 263*A^21*B^5*a^2 + 4*A^23*B^3*a - 3*A^25*B)*sqrt(a)*x + 2*(4*
B^26*a^13 + 296*A^2*B^24*a^12 + 3801*A^4*B^22*a^11 + 18130*A^6*B^
20*a^10 + 37556*A^8*B^18*a^9 + 26715*A^10*B^16*a^8 - 19277*A^12*B
^14*a^7 - 42352*A^14*B^12*a^6 - 22992*A^16*B^10*a^5 - 2939*A^18*B
^8*a^4 + 917*A^20*B^6*a^3 + 156*A^22*B^4*a^2 - 14*A^24*B^2*a - A^
26)*x + 3*sqrt(1/6)*(4*A*B^24*a^13 + 248*A^3*B^22*a^12 + 2617*A^5
*B^20*a^11 + 9824*A^7*B^18*a^10 + 14253*A^9*B^16*a^9 + 2132*A^11*
B^14*a^8 - 14079*A^13*B^12*a^7 - 12596*A^15*B^10*a^6 - 2949*A^17*
B^8*a^5 + 412*A^19*B^6*a^4 + 143*A^21*B^4*a^3 - 8*A^23*B^2*a^2 -
A^25*a - sqrt(1/3)*(8*B^23*a^14 + 548*A^2*B^21*a^13 + 6718*A^4*B^
19*a^12 + 32055*A^6*B^17*a^11 + 74040*A^8*B^15*a^10 + 88634*A^10*
B^13*a^9 + 53146*A^12*B^11*a^8 + 12277*A^14*B^9*a^7 - 1068*A^16*B
^7*a^6 - 646*A^18*B^5*a^5 + 2*A^20*B^3*a^4 + 7*A^22*B*a^3)*sqrt(-
(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (44*A^2*B^23*a^12 + 936*A^4*
B^21*a^11 + 5689*A^6*B^19*a^10 + 13479*A^8*B^17*a^9 + 10330*A^10*
B^15*a^8 - 7330*A^12*B^13*a^7 - 15677*A^14*B^11*a^6 - 7447*A^16*B
^9*a^5 - 402*A^18*B^7*a^4 + 362*A^20*B^5*a^3 + 21*A^22*B^3*a^2 -
5*A^24*B*a - sqrt(1/3)*(92*A*B^22*a^13 + 2212*A^3*B^20*a^12 + 162
07*A^5*B^18*a^11 + 52989*A^7*B^16*a^10 + 87902*A^9*B^14*a^9 + 753
74*A^11*B^12*a^8 + 29941*A^13*B^10*a^7 + 2451*A^15*B^8*a^6 - 1302
*A^17*B^6*a^5 - 166*A^19*B^4*a^4 + 19*A^21*B^2*a^3 + A^23*a^2)*sq
rt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*sqrt(a))*sqrt(-(4*A*B*a +
 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*
a + A^2)*sqrt(a))/a^2)) - 1/2*sqrt(1/6)*sqrt(-(4*A*B*a + 3*sqrt(1
/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*
sqrt(a))/a^2)*log(4*(24*A*B^25*a^12 + 614*A^3*B^23*a^11 + 4621*A^
5*B^21*a^10 + 14496*A^7*B^19*a^9 + 19031*A^9*B^17*a^8 + 2566*A^11
*B^15*a^7 - 18543*A^13*B^13*a^6 - 17860*A^15*B^11*a^5 - 5399*A^17
*B^9*a^4 + 186*A^19*B^7*a^3 + 263*A^21*B^5*a^2 + 4*A^23*B^3*a - 3
*A^25*B)*sqrt(a)*x + 2*(4*B^26*a^13 + 296*A^2*B^24*a^12 + 3801*A^
4*B^22*a^11 + 18130*A^6*B^20*a^10 + 37556*A^8*B^18*a^9 + 26715*A^
10*B^16*a^8 - 19277*A^12*B^14*a^7 - 42352*A^14*B^12*a^6 - 22992*A
^16*B^10*a^5 - 2939*A^18*B^8*a^4 + 917*A^20*B^6*a^3 + 156*A^22*B^
4*a^2 - 14*A^24*B^2*a - A^26)*x - 3*sqrt(1/6)*(4*A*B^24*a^13 + 24
8*A^3*B^22*a^12 + 2617*A^5*B^20*a^11 + 9824*A^7*B^18*a^10 + 14253
*A^9*B^16*a^9 + 2132*A^11*B^14*a^8 - 14079*A^13*B^12*a^7 - 12596*
A^15*B^10*a^6 - 2949*A^17*B^8*a^5 + 412*A^19*B^6*a^4 + 143*A^21*B
^4*a^3 - 8*A^23*B^2*a^2 - A^25*a - sqrt(1/3)*(8*B^23*a^14 + 548*A
^2*B^21*a^13 + 6718*A^4*B^19*a^12 + 32055*A^6*B^17*a^11 + 74040*A
^8*B^15*a^10 + 88634*A^10*B^13*a^9 + 53146*A^12*B^11*a^8 + 12277*
A^14*B^9*a^7 - 1068*A^16*B^7*a^6 - 646*A^18*B^5*a^5 + 2*A^20*B^3*
a^4 + 7*A^22*B*a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (4
4*A^2*B^23*a^12 + 936*A^4*B^21*a^11 + 5689*A^6*B^19*a^10 + 13479*
A^8*B^17*a^9 + 10330*A^10*B^15*a^8 - 7330*A^12*B^13*a^7 - 15677*A
^14*B^11*a^6 - 7447*A^16*B^9*a^5 - 402*A^18*B^7*a^4 + 362*A^20*B^
5*a^3 + 21*A^22*B^3*a^2 - 5*A^24*B*a - sqrt(1/3)*(92*A*B^22*a^13
+ 2212*A^3*B^20*a^12 + 16207*A^5*B^18*a^11 + 52989*A^7*B^16*a^10
+ 87902*A^9*B^14*a^9 + 75374*A^11*B^12*a^8 + 29941*A^13*B^10*a^7
+ 2451*A^15*B^8*a^6 - 1302*A^17*B^6*a^5 - 166*A^19*B^4*a^4 + 19*A
^21*B^2*a^3 + A^23*a^2)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))
*sqrt(a))*sqrt(-(4*A*B*a + 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2
*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)) + 1/2*sqrt(1/6)
*sqrt(-(4*A*B*a - 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a +
A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)*log(4*(24*A*B^25*a^12 + 6
14*A^3*B^23*a^11 + 4621*A^5*B^21*a^10 + 14496*A^7*B^19*a^9 + 1903
1*A^9*B^17*a^8 + 2566*A^11*B^15*a^7 - 18543*A^13*B^13*a^6 - 17860
*A^15*B^11*a^5 - 5399*A^17*B^9*a^4 + 186*A^19*B^7*a^3 + 263*A^21*
B^5*a^2 + 4*A^23*B^3*a - 3*A^25*B)*sqrt(a)*x + 2*(4*B^26*a^13 + 2
96*A^2*B^24*a^12 + 3801*A^4*B^22*a^11 + 18130*A^6*B^20*a^10 + 375
56*A^8*B^18*a^9 + 26715*A^10*B^16*a^8 - 19277*A^12*B^14*a^7 - 423
52*A^14*B^12*a^6 - 22992*A^16*B^10*a^5 - 2939*A^18*B^8*a^4 + 917*
A^20*B^6*a^3 + 156*A^22*B^4*a^2 - 14*A^24*B^2*a - A^26)*x + 3*sqr
```

```
t(1/6)*(4*A*B^24*a^13 + 248*A^3*B^22*a^12 + 2617*A^5*B^20*a^11 +
9824*A^7*B^18*a^10 + 14253*A^9*B^16*a^9 + 2132*A^11*B^14*a^8 - 14
079*A^13*B^12*a^7 - 12596*A^15*B^10*a^6 - 2949*A^17*B^8*a^5 + 412
*A^19*B^6*a^4 + 143*A^21*B^4*a^3 - 8*A^23*B^2*a^2 - A^25*a + sqrt
(1/3)*(8*B^23*a^14 + 548*A^2*B^21*a^13 + 6718*A^4*B^19*a^12 + 320
55*A^6*B^17*a^11 + 74040*A^8*B^15*a^10 + 88634*A^10*B^13*a^9 + 53
146*A^12*B^11*a^8 + 12277*A^14*B^9*a^7 - 1068*A^16*B^7*a^6 - 646*
A^18*B^5*a^5 + 2*A^20*B^3*a^4 + 7*A^22*B*a^3)*sqrt(-(B^4*a^2 - 2*
A^2*B^2*a + A^4)/a^3) + (44*A^2*B^23*a^12 + 936*A^4*B^21*a^11 + 5
689*A^6*B^19*a^10 + 13479*A^8*B^17*a^9 + 10330*A^10*B^15*a^8 - 73
30*A^12*B^13*a^7 - 15677*A^14*B^11*a^6 - 7447*A^16*B^9*a^5 - 402*
A^18*B^7*a^4 + 362*A^20*B^5*a^3 + 21*A^22*B^3*a^2 - 5*A^24*B*a +
sqrt(1/3)*(92*A*B^22*a^13 + 2212*A^3*B^20*a^12 + 16207*A^5*B^18*a
^11 + 52989*A^7*B^16*a^10 + 87902*A^9*B^14*a^9 + 75374*A^11*B^12*
a^8 + 29941*A^13*B^10*a^7 + 2451*A^15*B^8*a^6 - 1302*A^17*B^6*a^5
 - 166*A^19*B^4*a^4 + 19*A^21*B^2*a^3 + A^23*a^2)*sqrt(-(B^4*a^2
- 2*A^2*B^2*a + A^4)/a^3))*sqrt(a))*sqrt(-(4*A*B*a - 3*sqrt(1/3)*
a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt
(a))/a^2)) - 1/2*sqrt(1/6)*sqrt(-(4*A*B*a - 3*sqrt(1/3)*a^2*sqrt(
-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)
*log(4*(24*A*B^25*a^12 + 614*A^3*B^23*a^11 + 4621*A^5*B^21*a^10 +
 14496*A^7*B^19*a^9 + 19031*A^9*B^17*a^8 + 2566*A^11*B^15*a^7 - 1
8543*A^13*B^13*a^6 - 17860*A^15*B^11*a^5 - 5399*A^17*B^9*a^4 + 18
6*A^19*B^7*a^3 + 263*A^21*B^5*a^2 + 4*A^23*B^3*a - 3*A^25*B)*sqrt
(a)*x + 2*(4*B^26*a^13 + 296*A^2*B^24*a^12 + 3801*A^4*B^22*a^11 +
 18130*A^6*B^20*a^10 + 37556*A^8*B^18*a^9 + 26715*A^10*B^16*a^8 -
 19277*A^12*B^14*a^7 - 42352*A^14*B^12*a^6 - 22992*A^16*B^10*a^5
- 2939*A^18*B^8*a^4 + 917*A^20*B^6*a^3 + 156*A^22*B^4*a^2 - 14*A^
24*B^2*a - A^26)*x - 3*sqrt(1/6)*(4*A*B^24*a^13 + 248*A^3*B^22*a^
12 + 2617*A^5*B^20*a^11 + 9824*A^7*B^18*a^10 + 14253*A^9*B^16*a^9
 + 2132*A^11*B^14*a^8 - 14079*A^13*B^12*a^7 - 12596*A^15*B^10*a^6
 - 2949*A^17*B^8*a^5 + 412*A^19*B^6*a^4 + 143*A^21*B^4*a^3 - 8*A^
23*B^2*a^2 - A^25*a + sqrt(1/3)*(8*B^23*a^14 + 548*A^2*B^21*a^13
+ 6718*A^4*B^19*a^12 + 32055*A^6*B^17*a^11 + 74040*A^8*B^15*a^10
+ 88634*A^10*B^13*a^9 + 53146*A^12*B^11*a^8 + 12277*A^14*B^9*a^7
- 1068*A^16*B^7*a^6 - 646*A^18*B^5*a^5 + 2*A^20*B^3*a^4 + 7*A^22*
B*a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (44*A^2*B^23*a^
12 + 936*A^4*B^21*a^11 + 5689*A^6*B^19*a^10 + 13479*A^8*B^17*a^9
+ 10330*A^10*B^15*a^8 - 7330*A^12*B^13*a^7 - 15677*A^14*B^11*a^6
- 7447*A^16*B^9*a^5 - 402*A^18*B^7*a^4 + 362*A^20*B^5*a^3 + 21*A^
22*B^3*a^2 - 5*A^24*B*a + sqrt(1/3)*(92*A*B^22*a^13 + 2212*A^3*B^
20*a^12 + 16207*A^5*B^18*a^11 + 52989*A^7*B^16*a^10 + 87902*A^9*B
^14*a^9 + 75374*A^11*B^12*a^8 + 29941*A^13*B^10*a^7 + 2451*A^15*B
^8*a^6 - 1302*A^17*B^6*a^5 - 166*A^19*B^4*a^4 + 19*A^21*B^2*a^3 +
 A^23*a^2)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*sqrt(a))*sqr
t(-(4*A*B*a - 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)
/a^3) + (B^2*a + A^2)*sqrt(a))/a^2))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(a+x**4-x**2*a**(1/2)),x)

[Out] Exception raised: PolynomialError
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(x^4 - sqrt(a)*x^2 + a),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.119 $\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4} \, dx$

Optimal. Leaf size=414

$$-\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(-x\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(x\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}-\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}-2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}+\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}$$

```
[Out] -((Sqrt[a]*B + A*Sqrt[c])*ArcTan[(Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a
*c]] - 2*Sqrt[c]*x)/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]])/(2*Sqrt
[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]) + ((Sqrt[a]*B +
A*Sqrt[c])*ArcTan[(Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]] + 2*Sqrt[c
]*x)/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]])/(2*Sqrt[a]*Sqrt[c]*Sqr
t[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]) - ((A - (Sqrt[a]*B)/Sqrt[c])*Lo
g[Sqrt[a] - Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x + Sqrt[c]*x^2])
/(4*Sqrt[a]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]) + ((A - (Sqrt[a]
*B)/Sqrt[c])*Log[Sqrt[a] + Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x
+ Sqrt[c]*x^2])/(4*Sqrt[a]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]])
```

Rubi [A] time = 0.891831, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$-\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(-x\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(x\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}-\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}-2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}+\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(a - Sqrt[a*c]*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[a]*B + A*Sqrt[c])*ArcTan[(Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a
*c]] - 2*Sqrt[c]*x)/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]])/(2*Sqrt
[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]) + ((Sqrt[a]*B +
A*Sqrt[c])*ArcTan[(Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]] + 2*Sqrt[c
]*x)/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]])/(2*Sqrt[a]*Sqrt[c]*Sqr
t[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]) - ((A - (Sqrt[a]*B)/Sqrt[c])*Lo
g[Sqrt[a] - Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x + Sqrt[c]*x^2])
/(4*Sqrt[a]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]) + ((A - (Sqrt[a]
*B)/Sqrt[c])*Log[Sqrt[a] + Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x
+ Sqrt[c]*x^2])/(4*Sqrt[a]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]])
```

Rubi in Sympy [A] time = 106.045, size = 386, normalized size = 0.93

$$-\frac{\left(A\sqrt{c}-B\sqrt{a}\right)\log\left(\frac{\sqrt{a}}{\sqrt{c}}+x^2-\frac{x\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}\right)}{4\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\left(A\sqrt{c}-B\sqrt{a}\right)\log\left(\frac{\sqrt{a}}{\sqrt{c}}+x^2+\frac{x\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}\right)}{4\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\left(A\sqrt{c}+B\sqrt{a}\right)\operatorname{atan}\left(\frac{2\sqrt{c}x-\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}+\frac{\left(A\sqrt{c}+B\sqrt{a}\right)\operatorname{atan}\left(\frac{2\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((B*x**2+A)/(a+c*x**4-x**2*(a*c)**(1/2)),x)
```

```
[Out] -(A*sqrt(c) - B*sqrt(a))*log(sqrt(a)/sqrt(c) + x**2 - x*sqrt(2*sq
rt(a)*sqrt(c) + sqrt(a*c))/sqrt(c))/(4*sqrt(a)*sqrt(c)*sqrt(2*sqr
t(a)*sqrt(c) + sqrt(a*c))) + (A*sqrt(c) - B*sqrt(a))*log(sqrt(a)/
sqrt(c) + x**2 + x*sqrt(2*sqrt(a)*sqrt(c) + sqrt(a*c))/sqrt(c))/(
4*sqrt(a)*sqrt(c)*sqrt(2*sqrt(a)*sqrt(c) + sqrt(a*c))) + (A*sqrt(
c) + B*sqrt(a))*atan((2*sqrt(c)*x - sqrt(2*sqrt(a)*sqrt(c) + sqrt
(a*c)))/sqrt(2*sqrt(a)*sqrt(c) - sqrt(a*c)))/(2*sqrt(a)*sqrt(c)*s
qrt(2*sqrt(a)*sqrt(c) - sqrt(a*c))) + (A*sqrt(c) + B*sqrt(a))*ata
n((2*sqrt(c)*x + sqrt(2*sqrt(a)*sqrt(c) + sqrt(a*c)))/sqrt(2*sqrt
(a)*sqrt(c) - sqrt(a*c)))/(2*sqrt(a)*sqrt(c)*sqrt(2*sqrt(a)*sqrt(
c) - sqrt(a*c)))
```

Mathematica [C] time = 0.317388, size = 247, normalized size = 0.6

$$\frac{\frac{\left(\sqrt{3}\sqrt{a}B\sqrt{c}-i\left(B\sqrt{ac}+2Ac\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-\sqrt{ac}-i\sqrt{3}\sqrt{a}\sqrt{c}}}\right)}{\sqrt{-\sqrt{ac}-i\sqrt{3}\sqrt{a}\sqrt{c}}}+\frac{\left(\sqrt{3}\sqrt{a}B\sqrt{c}+i\left(B\sqrt{ac}+2Ac\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-\sqrt{ac}+i\sqrt{3}\sqrt{a}\sqrt{c}}}\right)}{\sqrt{-\sqrt{ac}+i\sqrt{3}\sqrt{a}\sqrt{c}}}}{\sqrt{6}\sqrt{ac}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(A + B*x^2)/(a - Sqrt[a*c]*x^2 + c*x^4),x]
```

```
[Out] (((Sqrt[3]*Sqrt[a]*B*Sqrt[c] - I*(2*A*c + B*Sqrt[a*c]))*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[(-I)*Sqrt[3]*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]]
)/Sqrt[(-I)*Sqrt[3]*Sqrt[a]*Sqrt[c] - Sqrt[a*c]] + ((Sqrt[3]*Sqrt
[a]*B*Sqrt[c] + I*(2*A*c + B*Sqrt[a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[I*Sqrt[3]*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]])/Sqrt[I*Sqrt[3]*S
qrt[a]*Sqrt[c] - Sqrt[a*c]])/(Sqrt[6]*Sqrt[a]*c)
```

Maple [A] time = 0.074, size = 404, normalized size = 1.

$$-\frac{B\sqrt{3}}{12\,a}\ln\left(x^2\sqrt{c}+\sqrt{3}\sqrt[4]{ac}x+\sqrt{a}\right)(ac)^{\frac{3}{4}}c^{-\frac{3}{2}}+\frac{A\sqrt{3}}{12\,c}\ln\left(x^2\sqrt{c}+\sqrt{3}\sqrt[4]{ac}x+\sqrt{a}\right)(ac)^{\frac{3}{4}}a^{-\frac{3}{2}}$$
$$+\frac{A}{2}\arctan\left(1\left(2\,x\sqrt{c}+\sqrt{3}\sqrt[4]{ac}\right)\frac{1}{\sqrt{4\,\sqrt{a}\sqrt{c}-3\,\sqrt{ac}}}\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{4\,\sqrt{a}\sqrt{c}-3\,\sqrt{ac}}}$$
$$+\frac{B}{2}\arctan\left(1\left(2\,x\sqrt{c}+\sqrt{3}\sqrt[4]{ac}\right)\frac{1}{\sqrt{4\,\sqrt{a}\sqrt{c}-3\,\sqrt{ac}}}\right)\frac{1}{\sqrt{c}}\frac{1}{\sqrt{4\,\sqrt{a}\sqrt{c}-3\,\sqrt{ac}}}$$
$$+\frac{B\sqrt{3}}{12\,a}\ln\left(\sqrt{3}\sqrt[4]{ac}x-x^2\sqrt{c}-\sqrt{a}\right)(ac)^{\frac{3}{4}}c^{-\frac{3}{2}}-\frac{A\sqrt{3}}{12\,c}\ln\left(\sqrt{3}\sqrt[4]{ac}x-x^2\sqrt{c}-\sqrt{a}\right)(ac)^{\frac{3}{4}}a^{-\frac{3}{2}}$$
$$-\frac{A}{2}\arctan\left(1\left(\sqrt{3}\sqrt[4]{ac}-2\,x\sqrt{c}\right)\frac{1}{\sqrt{4\,\sqrt{a}\sqrt{c}-3\,\sqrt{ac}}}\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{4\,\sqrt{a}\sqrt{c}-3\,\sqrt{ac}}}$$
$$-\frac{B}{2}\arctan\left(1\left(\sqrt{3}\sqrt[4]{ac}-2\,x\sqrt{c}\right)\frac{1}{\sqrt{4\,\sqrt{a}\sqrt{c}-3\,\sqrt{ac}}}\right)\frac{1}{\sqrt{c}}\frac{1}{\sqrt{4\,\sqrt{a}\sqrt{c}-3\,\sqrt{ac}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x)
```

```
[Out] -1/12/a/c^(3/2)*ln(x^2*c^(1/2)+3^(1/2)*(a*c)^(1/4)*x+a^(1/2))*B*3
^(1/2)*(a*c)^(3/4)+1/12/a^(3/2)/c*ln(x^2*c^(1/2)+3^(1/2)*(a*c)^(1
/4)*x+a^(1/2))*A*3^(1/2)*(a*c)^(3/4)+1/2/a^(1/2)/(4*a^(1/2)*c^(1/
2)-3*(a*c)^(1/2))^(1/2)*arctan((2*x*c^(1/2)+3^(1/2)*(a*c)^(1/4))/
(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2))*A+1/2/c^(1/2)/(4*a^(1/2)
*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arctan((2*x*c^(1/2)+3^(1/2)*(a*c)^(
1/4))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2))*B+1/12/a/c^(3/2)*l
n(3^(1/2)*(a*c)^(1/4)*x-x^2*c^(1/2)-a^(1/2))*B*3^(1/2)*(a*c)^(3/4
)-1/12/a^(3/2)/c*ln(3^(1/2)*(a*c)^(1/4)*x-x^2*c^(1/2)-a^(1/2))*A*
3^(1/2)*(a*c)^(3/4)-1/2/a^(1/2)/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))
^(1/2)*arctan((3^(1/2)*(a*c)^(1/4)-2*x*c^(1/2))/(4*a^(1/2)*c^(1/2
)-3*(a*c)^(1/2))^(1/2))*A-1/2/c^(1/2)/(4*a^(1/2)*c^(1/2)-3*(a*c)^
(1/2))^(1/2)*arctan((3^(1/2)*(a*c)^(1/4)-2*x*c^(1/2))/(4*a^(1/2)*
c^(1/2)-3*(a*c)^(1/2))^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{cx^4 - \sqrt{ac}x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(c*x^4 - sqrt(a*c)*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 - sqrt(a*c)*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(c*x^4 - sqrt(a*c)*x^2 + a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(a+c*x**4-x**2*(a*c)**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(c*x^4 - sqrt(a*c)*x^2 + a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.120 $\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4} \, dx$

Optimal. Leaf size=234

$$-\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}}+\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\tan^{-1}\left(\frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2a^{3/4}c^{3/4}}-\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(-\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}+\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}$$

```
[Out] -((Sqrt[a]*B + A*Sqrt[c])*ArcTan[Sqrt[3] - (2*c^(1/4)*x)/a^(1/4)]
)/(2*a^(3/4)*c^(3/4)) + ((Sqrt[a]*B + A*Sqrt[c])*ArcTan[Sqrt[3] +
 (2*c^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*c^(3/4)) - ((A - (Sqrt[a]*B)/
Sqrt[c])*Log[Sqrt[a] - Sqrt[3]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/
(4*Sqrt[3]*a^(3/4)*c^(1/4)) + ((A - (Sqrt[a]*B)/Sqrt[c])*Log[Sqrt
[a] + Sqrt[3]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[3]*a^(3/4
)*c^(1/4))
```

Rubi [A] time = 0.39161, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156

$$-\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}}+\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\tan^{-1}\left(\frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2a^{3/4}c^{3/4}}-\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(-\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}+\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(a - Sqrt[a]*Sqrt[c]*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[a]*B + A*Sqrt[c])*ArcTan[Sqrt[3] - (2*c^(1/4)*x)/a^(1/4)]
)/(2*a^(3/4)*c^(3/4)) + ((Sqrt[a]*B + A*Sqrt[c])*ArcTan[Sqrt[3] +
 (2*c^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*c^(3/4)) - ((A - (Sqrt[a]*B)/
Sqrt[c])*Log[Sqrt[a] - Sqrt[3]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/
(4*Sqrt[3]*a^(3/4)*c^(1/4)) + ((A - (Sqrt[a]*B)/Sqrt[c])*Log[Sqrt
[a] + Sqrt[3]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[3]*a^(3/4
)*c^(1/4))
```

Rubi in Sympy [A] time = 88.3607, size = 240, normalized size = 1.03

$$-\frac{\sqrt{3}\left(A\sqrt{c}-B\sqrt{a}\right)\log\left(-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}+x^2\right)}{12a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{3}\left(A\sqrt{c}-B\sqrt{a}\right)\log\left(\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}+x^2\right)}{12a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\left(A\sqrt{c}+B\sqrt{a}\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\sqrt[4]{a}-\frac{2\sqrt{3}\sqrt[4]{c}x}{3}\right)}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{\left(A\sqrt{c}+B\sqrt{a}\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\sqrt[4]{a}+\frac{2\sqrt{3}\sqrt[4]{c}x}{3}\right)}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(a+c*x**4-x**2*a**(1/2)*c**(1/2)),x)
```

```
[Out] -sqrt(3)*(A*sqrt(c) - B*sqrt(a))*log(-sqrt(3)*a**(1/4)*x/c**(1/4)
 + sqrt(a)/sqrt(c) + x**2)/(12*a**(3/4)*c**(3/4)) + sqrt(3)*(A*sq
```

```
rt(c) - B*sqrt(a))*log(sqrt(3)*a**(1/4)*x/c**(1/4) + sqrt(a)/sqrt
(c) + x**2)/(12*a**(3/4)*c**(3/4)) - (A*sqrt(c) + B*sqrt(a))*atan
(sqrt(3)*(a**(1/4) - 2*sqrt(3)*c**(1/4)*x/3)/a**(1/4))/(2*a**(3/4
)*c**(3/4)) + (A*sqrt(c) + B*sqrt(a))*atan(sqrt(3)*(a**(1/4) + 2*
sqrt(3)*c**(1/4)*x/3)/a**(1/4))/(2*a**(3/4)*c**(3/4))
```

Mathematica [C] time = 0.309243, size = 163, normalized size = 0.7

$$\frac{\sqrt[4]{-1}\left(\frac{\left(\left(\sqrt{3}-i\right)\sqrt{a}B-2iA\sqrt{c}\right)\tan^{-1}\left(\frac{(1+i)\sqrt[4]{c}x}{\sqrt{\sqrt{3}-i}\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}-i}}-\frac{\left(\left(\sqrt{3}+i\right)\sqrt{a}B+2iA\sqrt{c}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt[4]{c}x}{\sqrt{\sqrt{3}+i}\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}+i}}\right)}{\sqrt{6}a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(A + B*x^2)/(a - Sqrt[a]*Sqrt[c]*x^2 + c*x^4),x]
```

```
[Out] ((-1)^(1/4)*(((((-I + Sqrt[3])*Sqrt[a]*B - (2*I)*A*Sqrt[c])*ArcTan
[((1 + I)*c^(1/4)*x)/(Sqrt[-I + Sqrt[3]]*a^(1/4))])/Sqrt[-I + Sqr
t[3]] - (((I + Sqrt[3])*Sqrt[a]*B + (2*I)*A*Sqrt[c])*ArcTanh[((1
+ I)*c^(1/4)*x)/(Sqrt[I + Sqrt[3]]*a^(1/4))])/Sqrt[I + Sqrt[3]]))
/(Sqrt[6]*a^(3/4)*c^(3/4))
```

Maple [A] time = 0.07, size = 320, normalized size = 1.4

$$\begin{aligned}
&-\frac{A\sqrt{3}}{12}\ln\left(\sqrt[4]{a}\sqrt[4]{c}x\sqrt{3}-x^2\sqrt{c}-\sqrt{a}\right)\frac{1}{\sqrt[4]{c}}a^{-\frac{3}{4}}+\frac{B\sqrt{3}}{12}\ln\left(\sqrt[4]{a}\sqrt[4]{c}x\sqrt{3}-x^2\sqrt{c}-\sqrt{a}\right)c^{-\frac{3}{4}}\frac{1}{\sqrt[4]{a}}\\
&-\frac{A}{2}\arctan\left(1\left(\sqrt{3}\sqrt[4]{c}\sqrt[4]{a}-2x\sqrt{c}\right)\frac{1}{\sqrt{\sqrt{a}\sqrt{c}}}\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{\sqrt{a}\sqrt{c}}}\\
&-\frac{B}{2}\arctan\left(1\left(\sqrt{3}\sqrt[4]{c}\sqrt[4]{a}-2x\sqrt{c}\right)\frac{1}{\sqrt{\sqrt{a}\sqrt{c}}}\right)\frac{1}{\sqrt{c}}\frac{1}{\sqrt{\sqrt{a}\sqrt{c}}}\\
&+\frac{A\sqrt{3}}{12}\ln\left(\sqrt[4]{a}\sqrt[4]{c}x\sqrt{3}+\sqrt{a}+x^2\sqrt{c}\right)\frac{1}{\sqrt[4]{c}}a^{-\frac{3}{4}}-\frac{B\sqrt{3}}{12}\ln\left(\sqrt[4]{a}\sqrt[4]{c}x\sqrt{3}+\sqrt{a}+x^2\sqrt{c}\right)c^{-\frac{3}{4}}\frac{1}{\sqrt[4]{a}}\\
&+\frac{A}{2}\arctan\left(1\left(2x\sqrt{c}+\sqrt{3}\sqrt[4]{c}\sqrt[4]{a}\right)\frac{1}{\sqrt{\sqrt{a}\sqrt{c}}}\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{\sqrt{a}\sqrt{c}}}\\
&+\frac{B}{2}\arctan\left(1\left(2x\sqrt{c}+\sqrt{3}\sqrt[4]{c}\sqrt[4]{a}\right)\frac{1}{\sqrt{\sqrt{a}\sqrt{c}}}\right)\frac{1}{\sqrt{c}}\frac{1}{\sqrt{\sqrt{a}\sqrt{c}}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x)
```

```
[Out] -1/12/c^(1/4)/a^(3/4)*ln(a^(1/4)*c^(1/4)*x*3^(1/2)-x^2*c^(1/2)-a^
(1/2))*A*3^(1/2)+1/12/c^(3/4)/a^(1/4)*ln(a^(1/4)*c^(1/4)*x*3^(1/2
)-x^2*c^(1/2)-a^(1/2))*B*3^(1/2)-1/2/a^(1/2)/(a^(1/2)*c^(1/2))^(1
/2)*arctan((3^(1/2)*c^(1/4)*a^(1/4)-2*x*c^(1/2))/(a^(1/2)*c^(1/2)
)^(1/2))*A-1/2/c^(1/2)/(a^(1/2)*c^(1/2))^(1/2)*arctan((3^(1/2)*c^
(1/4)*a^(1/4)-2*x*c^(1/2))/(a^(1/2)*c^(1/2))^(1/2))*B+1/12/c^(1/4
)/a^(3/4)*ln(a^(1/4)*c^(1/4)*x*3^(1/2)+a^(1/2)+x^2*c^(1/2))*A*3^(
1/2)-1/12/c^(3/4)/a^(1/4)*ln(a^(1/4)*c^(1/4)*x*3^(1/2)+a^(1/2)+x^
2*c^(1/2))*B*3^(1/2)+1/2/a^(1/2)/(a^(1/2)*c^(1/2))^(1/2)*arctan((
2*x*c^(1/2)+3^(1/2)*c^(1/4)*a^(1/4))/(a^(1/2)*c^(1/2))^(1/2))*A+1
/2/c^(1/2)/(a^(1/2)*c^(1/2))^(1/2)*arctan((2*x*c^(1/2)+3^(1/2)*c^
(1/4)*a^(1/4))/(a^(1/2)*c^(1/2))^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{cx^4 - \sqrt{a}\sqrt{c}x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(c*x^4 - sqrt(a)*sqrt(c)*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 - sqrt(a)*sqrt(c)*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(c*x^4 - sqrt(a)*sqrt(c)*x^2 + a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(a+c*x**4-x**2*a**(1/2)*c**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(c*x^4 - sqrt(a)*sqrt(c)*x^2 + a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.121 $\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx$

Optimal. Leaf size=96

$$\sqrt{7+2\sqrt{13}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7-\sqrt{13}\right)\right)-\sqrt{\frac{1}{2}\left(\sqrt{13}-1\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7-\sqrt{13}\right)\right)$$

```
[Out] -(Sqrt[(-1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(1 + Sqrt[13])]
*x], (-7 - Sqrt[13])/6]) + Sqrt[7 + 2*Sqrt[13]]*EllipticF[ArcSin[
Sqrt[2/(1 + Sqrt[13])]*x], (-7 - Sqrt[13])/6]
```

Rubi [A] time = 0.294908, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\sqrt{7+2\sqrt{13}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7-\sqrt{13}\right)\right)-\sqrt{\frac{1}{2}\left(\sqrt{13}-1\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7-\sqrt{13}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - x^2)/Sqrt[3 + x^2 - x^4],x]
```

```
[Out] -(Sqrt[(-1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(1 + Sqrt[13])]
*x], (-7 - Sqrt[13])/6]) + Sqrt[7 + 2*Sqrt[13]]*EllipticF[ArcSin[
Sqrt[2/(1 + Sqrt[13])]*x], (-7 - Sqrt[13])/6]
```

Rubi in Sympy [A] time = 32.4582, size = 114, normalized size = 1.19

$$-\frac{\sqrt{6}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x\sqrt{-1+\sqrt{13}}}{6}\right)\middle|-\frac{7}{6}-\frac{\sqrt{13}}{6}\right)}{\sqrt{1+\sqrt{13}}}-\frac{2\sqrt{6}\left(\frac{\sqrt{13}}{2}+\frac{5}{2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt{6}x\sqrt{-1+\sqrt{13}}}{6}\right)\middle|-\frac{7}{6}-\frac{\sqrt{13}}{6}\right)}{\sqrt{1+\sqrt{13}}\left(-\sqrt{13}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+3)/(-x**4+x**2+3)**(1/2),x)
```

```
[Out] -sqrt(6)*elliptic_e(asin(sqrt(6)*x*sqrt(-1 + sqrt(13))/6), -7/6 -
 sqrt(13)/6)/sqrt(1 + sqrt(13)) - 2*sqrt(6)*(sqrt(13)/2 + 5/2)*el
liptic_f(asin(sqrt(6)*x*sqrt(-1 + sqrt(13))/6), -7/6 - sqrt(13)/6
)/(sqrt(1 + sqrt(13))*(-sqrt(13) + 1))
```

Mathematica [C] time = 0.140548, size = 103, normalized size = 1.07

$$-\frac{i\left(\left(1+\sqrt{13}\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)-\left(\sqrt{13}-5\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)\right)}{\sqrt{2\left(1+\sqrt{13}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(3 - x^2)/Sqrt[3 + x^2 - x^4],x]
```

```
[Out] ((-I)*((1 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(-1 + Sqrt[13])]
*x], (-7 + Sqrt[13])/6] - (-5 + Sqrt[13])*EllipticF[I*ArcSinh[Sqr
```

```
t[2/(-1 + Sqrt[13])]*x], (-7 + Sqrt[13])/6]))/Sqrt[2*(1 + Sqrt[13
])]
```

Maple [B] time = 0.125, size = 200, normalized size = 2.1

$$36\frac{\sqrt{1-\left(-1/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-1/6-1/6\sqrt{13}\right)x^2}\left(\mathit{EllipticF}\left(1/6\,x\sqrt{-6+6\sqrt{13}},i/6\sqrt{3}+i/6\sqrt{39}\right)-\mathit{EllipticE}\left(1/6\right.\right.}{\sqrt{-6+6\sqrt{13}}\sqrt{-x^4+x^2+3}\left(1+\sqrt{13}\right)}$$
$$+18\frac{\sqrt{1-\left(-1/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-1/6-1/6\sqrt{13}\right)x^2}\,\mathit{EllipticF}\left(1/6\,x\sqrt{-6+6\sqrt{13}},i/6\sqrt{3}+i/6\sqrt{39}\right)}{\sqrt{-6+6\sqrt{13}}\sqrt{-x^4+x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+3)/(-x^4+x^2+3)^(1/2),x)
```

```
[Out] 36/(-6+6*13^(1/2))^(1/2)*(1-(-1/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-1
/6-1/6*13^(1/2))*x^2)^(1/2)/(-x^4+x^2+3)^(1/2)/(1+13^(1/2))*(Elli
pticF(1/6*x*(-6+6*13^(1/2))^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2))-E
llipticE(1/6*x*(-6+6*13^(1/2))^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2)
))+18/(-6+6*13^(1/2))^(1/2)*(1-(-1/6+1/6*13^(1/2))*x^2)^(1/2)*(1-
(-1/6-1/6*13^(1/2))*x^2)^(1/2)/(-x^4+x^2+3)^(1/2)*EllipticF(1/6*x
*(-6+6*13^(1/2))^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2-3}{\sqrt{-x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 3)/sqrt(-x^4 + x^2 + 3),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 + x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{x^2-3}{\sqrt{-x^4+x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 3)/sqrt(-x^4 + x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(-(x^2 - 3)/sqrt(-x^4 + x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2}{\sqrt{-x^4+x^2+3}}\,dx-\int\left(-\frac{3}{\sqrt{-x^4+x^2+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+3)/(-x**4+x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 + x**2 + 3), x) - Integral(-3/sqrt(-x**
4 + x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 - 3}{\sqrt{-x^4 + x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 3)/sqrt(-x^4 + x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 + x^2 + 3), x)
```

3.122 $\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}}\,dx$

Optimal. Leaf size=25

$$4F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-E\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)$$

```
[Out] -EllipticE[ArcSin[x/Sqrt[3]], -3] + 4*EllipticF[ArcSin[x/Sqrt[3]]
, -3]
```

Rubi [A] time = 0.0990293, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$4F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-E\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 + 2*x^2 - x^4],x]
```

```
[Out] -EllipticE[ArcSin[x/Sqrt[3]], -3] + 4*EllipticF[ArcSin[x/Sqrt[3]]
, -3]
```

Rubi in Sympy [A] time = 18.821, size = 27, normalized size = 1.08

$$-E\left(\operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)\middle|-3\right)+4F\left(\operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)\middle|-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-x**2+3)/(-x**4+2*x**2+3)**(1/2),x)
```

```
[Out] -elliptic_e(asin(sqrt(3)*x/3), -3) + 4*elliptic_f(asin(sqrt(3)*x/
3), -3)
```

Mathematica [C] time = 0.0434694, size = 19, normalized size = 0.76

$$-i\sqrt{3}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x^2)/Sqrt[3 + 2*x^2 - x^4],x]
```

```
[Out] (-I)*Sqrt[3]*EllipticE[I*ArcSinh[x], -1/3]
```

Maple [B] time = 0.019, size = 113, normalized size = 4.5

$$\frac{\sqrt{3}}{3}\sqrt{-3\,x^2+9}\sqrt{x^2+1}\left(\mathit{EllipticF}\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)-\mathit{EllipticE}\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)\right)\frac{1}{\sqrt{-x^4+2\,x^2+3}}$$
$$+\sqrt{3}\sqrt{-3\,x^2+9}\sqrt{x^2+1}\,\mathit{EllipticF}\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)\frac{1}{\sqrt{-x^4+2\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*(-3*x^2+9)^(1/2)*(x^2+1)^(1/2)/(-x^4+2*x^2+3)^(1/2)*(
EllipticF(1/3*x*3^(1/2),I*3^(1/2))-EllipticE(1/3*x*3^(1/2),I*3^(1
/2)))+3^(1/2)*(-3*x^2+9)^(1/2)*(x^2+1)^(1/2)/(-x^4+2*x^2+3)^(1/2)
*EllipticF(1/3*x*3^(1/2),I*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-3}{\sqrt{-x^4+2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 3)/sqrt(-x^4 + 2*x^2 + 3),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 + 2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^2-3}{\sqrt{-x^4+2x^2+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 3)/sqrt(-x^4 + 2*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(-(x^2 - 3)/sqrt(-x^4 + 2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4+2x^2+3}}\,dx-\int \left(-\frac{3}{\sqrt{-x^4+2x^2+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+3)/(-x**4+2*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 + 2*x**2 + 3), x) - Integral(-3/sqrt(-x
**4 + 2*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-3}{\sqrt{-x^4+2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 3)/sqrt(-x^4 + 2*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 + 2*x^2 + 3), x)
```

3.123 $\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}} \, dx$

Optimal. Leaf size=96

$$\sqrt{9+2\sqrt{21}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right)-\sqrt{\frac{1}{2}\left(\sqrt{21}-3\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right)$$

```
[Out] -(Sqrt[(-3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(3 + Sqrt[21])]
*x], (-5 - Sqrt[21])/2]) + Sqrt[9 + 2*Sqrt[21]]*EllipticF[ArcSin[
Sqrt[2/(3 + Sqrt[21])]*x], (-5 - Sqrt[21])/2]
```

Rubi [A] time = 0.449654, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\sqrt{9+2\sqrt{21}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right)-\sqrt{\frac{1}{2}\left(\sqrt{21}-3\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - x^2)/Sqrt[3 + 3*x^2 - x^4],x]
```

```
[Out] -(Sqrt[(-3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(3 + Sqrt[21])]
*x], (-5 - Sqrt[21])/2]) + Sqrt[9 + 2*Sqrt[21]]*EllipticF[ArcSin[
Sqrt[2/(3 + Sqrt[21])]*x], (-5 - Sqrt[21])/2]
```

Rubi in Sympy [A] time = 32.9355, size = 114, normalized size = 1.19

$$-\frac{\sqrt{6}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x\sqrt{-3+\sqrt{21}}}{6}\right)\middle|-\frac{5}{2}-\frac{\sqrt{21}}{2}\right)}{\sqrt{3+\sqrt{21}}}-\frac{2\sqrt{6}\left(\frac{3}{2}+\frac{\sqrt{21}}{2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt{6}x\sqrt{-3+\sqrt{21}}}{6}\right)\middle|-\frac{5}{2}-\frac{\sqrt{21}}{2}\right)}{\sqrt{3+\sqrt{21}}\left(-\sqrt{21}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+3)/(-x**4+3*x**2+3)**(1/2),x)
```

```
[Out] -sqrt(6)*elliptic_e(asin(sqrt(6)*x*sqrt(-3 + sqrt(21))/6), -5/2 -
 sqrt(21)/2)/sqrt(3 + sqrt(21)) - 2*sqrt(6)*(3/2 + sqrt(21)/2)*el
liptic_f(asin(sqrt(6)*x*sqrt(-3 + sqrt(21))/6), -5/2 - sqrt(21)/2
)/(sqrt(3 + sqrt(21))*(-sqrt(21) + 3))
```

Mathematica [C] time = 0.183024, size = 103, normalized size = 1.07

$$-\frac{i\left(\left(3+\sqrt{21}\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)-\left(\sqrt{21}-3\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)\right)}{\sqrt{2\left(3+\sqrt{21}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(3 - x^2)/Sqrt[3 + 3*x^2 - x^4],x]
```

```
[Out] ((-I)*((3 + Sqrt[21])*EllipticE[I*ArcSinh[Sqrt[2/(-3 + Sqrt[21])]
*x], (-5 + Sqrt[21])/2] - (-3 + Sqrt[21])*EllipticF[I*ArcSinh[Sqr
```

```
t[2/(-3 + Sqrt[21])]*x], (-5 + Sqrt[21])/2]))/Sqrt[2*(3 + Sqrt[21
])]
```

Maple [B] time = 0.121, size = 204, normalized size = 2.1

$$36\frac{\sqrt{1-\left(-1/2+1/6\sqrt{21}\right)x^2}\sqrt{1-\left(-1/2-1/6\sqrt{21}\right)x^2}\left(EllipticF\left(1/6\,x\sqrt{-18+6\sqrt{21}},i/2\sqrt{3}+i/2\sqrt{7}\right)-EllipticE\left(1/6\right.\right.}{\sqrt{-18+6\sqrt{21}}\sqrt{-x^4+3x^2+3}\left(3+\sqrt{21}\right)}$$
$$+18\frac{\sqrt{1-\left(-1/2+1/6\sqrt{21}\right)x^2}\sqrt{1-\left(-1/2-1/6\sqrt{21}\right)x^2}EllipticF\left(1/6\,x\sqrt{-18+6\sqrt{21}},i/2\sqrt{3}+i/2\sqrt{7}\right)}{\sqrt{-18+6\sqrt{21}}\sqrt{-x^4+3x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x)
```

```
[Out] 36/(-18+6*21^(1/2))^(1/2)*(1-(-1/2+1/6*21^(1/2))*x^2)^(1/2)*(1-(-
1/2-1/6*21^(1/2))*x^2)^(1/2)/(-x^4+3*x^2+3)^(1/2)/(3+21^(1/2))*(E
llipticF(1/6*x*(-18+6*21^(1/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1/2)
)-EllipticE(1/6*x*(-18+6*21^(1/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1
/2)))+18/(-18+6*21^(1/2))^(1/2)*(1-(-1/2+1/6*21^(1/2))*x^2)^(1/2)
*(1-(-1/2-1/6*21^(1/2))*x^2)^(1/2)/(-x^4+3*x^2+3)^(1/2)*EllipticF
(1/6*x*(-18+6*21^(1/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-3}{\sqrt{-x^4+3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 3)/sqrt(-x^4 + 3*x^2 + 3),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 + 3*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^2-3}{\sqrt{-x^4+3x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 3)/sqrt(-x^4 + 3*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(-(x^2 - 3)/sqrt(-x^4 + 3*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4+3x^2+3}}\,dx-\int \left(-\frac{3}{\sqrt{-x^4+3x^2+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+3)/(-x**4+3*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 + 3*x**2 + 3), x) - Integral(-3/sqrt(-x
**4 + 3*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-3}{\sqrt{-x^4+3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 3)/sqrt(-x^4 + 3*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 + 3*x^2 + 3), x)
```

3.124 $\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx$

Optimal. Leaf size=92

$$\sqrt{5+2\sqrt{13}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)-\sqrt{\frac{1}{2}\left(1+\sqrt{13}\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)$$

```
[Out] -(Sqrt[(1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(-1 + Sqrt[13])]
*x], (-7 + Sqrt[13])/6]) + Sqrt[5 + 2*Sqrt[13]]*EllipticF[ArcSin[
Sqrt[2/(-1 + Sqrt[13])]*x], (-7 + Sqrt[13])/6]
```

Rubi [A] time = 0.308997, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\sqrt{5+2\sqrt{13}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)-\sqrt{\frac{1}{2}\left(1+\sqrt{13}\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - x^2)/Sqrt[3 - x^2 - x^4],x]
```

```
[Out] -(Sqrt[(1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(-1 + Sqrt[13])]
*x], (-7 + Sqrt[13])/6]) + Sqrt[5 + 2*Sqrt[13]]*EllipticF[ArcSin[
Sqrt[2/(-1 + Sqrt[13])]*x], (-7 + Sqrt[13])/6]
```

Rubi in Sympy [A] time = 32.2979, size = 109, normalized size = 1.18

$$-\frac{\sqrt{6}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x\sqrt{1+\sqrt{13}}}{6}\right)\middle|-\frac{7}{6}+\frac{\sqrt{13}}{6}\right)}{\sqrt{-1+\sqrt{13}}}+\frac{2\sqrt{6}\left(\frac{\sqrt{13}}{2}+\frac{7}{2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt{6}x\sqrt{1+\sqrt{13}}}{6}\right)\middle|-\frac{7}{6}+\frac{\sqrt{13}}{6}\right)}{\sqrt{-1+\sqrt{13}}\left(1+\sqrt{13}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+3)/(-x**4-x**2+3)**(1/2),x)
```

```
[Out] -sqrt(6)*elliptic_e(asin(sqrt(6)*x*sqrt(1 + sqrt(13))/6), -7/6 +
sqrt(13)/6)/sqrt(-1 + sqrt(13)) + 2*sqrt(6)*(sqrt(13)/2 + 7/2)*el
liptic_f(asin(sqrt(6)*x*sqrt(1 + sqrt(13))/6), -7/6 + sqrt(13)/6)
/(sqrt(-1 + sqrt(13))*(1 + sqrt(13)))
```

Mathematica [C] time = 0.154836, size = 107, normalized size = 1.16

$$-\frac{i\left(\left(\sqrt{13}-1\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)|-\frac{7}{6}-\frac{\sqrt{13}}{6}\right)-\left(\sqrt{13}-7\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)|-\frac{7}{6}-\frac{\sqrt{13}}{6}\right)\right)}{\sqrt{2\left(\sqrt{13}-1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(3 - x^2)/Sqrt[3 - x^2 - x^4],x]
```

```
[Out] ((-I)*((-1 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(1 + Sqrt[13])]
*x], -7/6 - Sqrt[13]/6] - (-7 + Sqrt[13])*EllipticF[I*ArcSinh[Sqr
t[2/(1 + Sqrt[13])]*x], -7/6 - Sqrt[13]/6]))/Sqrt[2*(-1 + Sqrt[13
])]
```

Maple [B] time = 0.115, size = 204, normalized size = 2.2

$$36\frac{\sqrt{1-\left(1/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(1/6-1/6\sqrt{13}\right)x^2}\left(EllipticF\left(1/6\,x\sqrt{6+6\sqrt{13}},i/6\sqrt{39}-i/6\sqrt{3}\right)-EllipticE\left(1/6\,x\sqrt{6}\right.\right.}{\sqrt{6+6\sqrt{13}}\sqrt{-x^4-x^2+3}\left(-1+\sqrt{13}\right)}$$
$$+18\frac{\sqrt{1-\left(1/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(1/6-1/6\sqrt{13}\right)x^2}EllipticF\left(1/6\,x\sqrt{6+6\sqrt{13}},i/6\sqrt{39}-i/6\sqrt{3}\right)}{\sqrt{6+6\sqrt{13}}\sqrt{-x^4-x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+3)/(-x^4-x^2+3)^(1/2),x)
```

```
[Out] 36/(6+6*13^(1/2))^(1/2)*(1-(1/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(1/6-
1/6*13^(1/2))*x^2)^(1/2)/(-x^4-x^2+3)^(1/2)/(-1+13^(1/2))*(Ellipt
icF(1/6*x*(6+6*13^(1/2))^(1/2),1/6*I*39^(1/2)-1/6*I*3^(1/2))-Elli
pticE(1/6*x*(6+6*13^(1/2))^(1/2),1/6*I*39^(1/2)-1/6*I*3^(1/2)))+1
8/(6+6*13^(1/2))^(1/2)*(1-(1/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(1/6-1
/6*13^(1/2))*x^2)^(1/2)/(-x^4-x^2+3)^(1/2)*EllipticF(1/6*x*(6+6*1
3^(1/2))^(1/2),1/6*I*39^(1/2)-1/6*I*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-3}{\sqrt{-x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 3)/sqrt(-x^4 - x^2 + 3),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 - x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^2-3}{\sqrt{-x^4-x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 3)/sqrt(-x^4 - x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(-(x^2 - 3)/sqrt(-x^4 - x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4-x^2+3}}\,dx-\int \left(-\frac{3}{\sqrt{-x^4-x^2+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+3)/(-x**4-x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 - x**2 + 3), x) - Integral(-3/sqrt(-x**
4 - x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-3}{\sqrt{-x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 3)/sqrt(-x^4 - x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 - x^2 + 3), x)
```

3.125 $\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx$

Optimal. Leaf size=27

$$2\sqrt{3}F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)-\sqrt{3}E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)$$

```
[Out] -(Sqrt[3]*EllipticE[ArcSin[x], -1/3]) + 2*Sqrt[3]*EllipticF[ArcSi
n[x], -1/3]
```

Rubi [A] time = 0.131619, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$2\sqrt{3}F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)-\sqrt{3}E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - x^2)/Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] -(Sqrt[3]*EllipticE[ArcSin[x], -1/3]) + 2*Sqrt[3]*EllipticF[ArcSi
n[x], -1/3]
```

Rubi in Sympy [A] time = 22.6944, size = 27, normalized size = 1.

$$-\sqrt{3}E\left(\operatorname{asin}(x)\middle|-\frac{1}{3}\right)+2\sqrt{3}F\left(\operatorname{asin}(x)\middle|-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+3)/(-x**4-2*x**2+3)**(1/2),x)
```

```
[Out] -sqrt(3)*elliptic_e(asin(x), -1/3) + 2*sqrt(3)*elliptic_f(asin(x)
, -1/3)
```

Mathematica [C] time = 0.0549004, size = 35, normalized size = 1.3

$$-i\left(2F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)+E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 - x^2)/Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] (-I)*(EllipticE[I*ArcSinh[x/Sqrt[3]], -3] + 2*EllipticF[I*ArcSinh
[x/Sqrt[3]], -3])
```

Maple [B] time = 0.013, size = 95, normalized size = 3.5

$$\left(\mathit{EllipticF}\left(x,\frac{i}{3}\sqrt{3}\right)-\mathit{EllipticE}\left(x,\frac{i}{3}\sqrt{3}\right)\right)\sqrt{-x^2+1}\sqrt{3\,x^2+9}\frac{1}{\sqrt{-x^4-2\,x^2+3}}+\mathit{EllipticF}\left(x,\frac{i}{3}\sqrt{3}\right)\sqrt{-x^2+1}\sqrt{3\,x^2+9}\frac{1}{\sqrt{-x^4-2\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x)
```

```
[Out] (-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*(EllipticF(x,
1/3*I*3^(1/2))-EllipticE(x,1/3*I*3^(1/2)))+(-x^2+1)^(1/2)*(3*x^2+
9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-3}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 3)/sqrt(-x^4 - 2*x^2 + 3),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 - 2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^2-3}{\sqrt{-x^4-2\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 3)/sqrt(-x^4 - 2*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(-(x^2 - 3)/sqrt(-x^4 - 2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4-2x^2+3}}\,dx-\int \left(-\frac{3}{\sqrt{-x^4-2x^2+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+3)/(-x**4-2*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 - 2*x**2 + 3), x) - Integral(-3/sqrt(-x
**4 - 2*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-3}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 3)/sqrt(-x^4 - 2*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 - 2*x^2 + 3), x)
```

3.126 $\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}} \, dx$

Optimal. Leaf size=92

$$\sqrt{3+2\sqrt{21}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right) - \sqrt{\frac{1}{2}\left(3+\sqrt{21}\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)$$

```
[Out] -(Sqrt[(3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(-3 + Sqrt[21])]
*x], (-5 + Sqrt[21])/2]) + Sqrt[3 + 2*Sqrt[21]]*EllipticF[ArcSin[
Sqrt[2/(-3 + Sqrt[21])]*x], (-5 + Sqrt[21])/2]
```

Rubi [A] time = 0.450876, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\sqrt{3+2\sqrt{21}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right) - \sqrt{\frac{1}{2}\left(3+\sqrt{21}\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - x^2)/Sqrt[3 - 3*x^2 - x^4],x]
```

```
[Out] -(Sqrt[(3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(-3 + Sqrt[21])]
*x], (-5 + Sqrt[21])/2]) + Sqrt[3 + 2*Sqrt[21]]*EllipticF[ArcSin[
Sqrt[2/(-3 + Sqrt[21])]*x], (-5 + Sqrt[21])/2]
```

Rubi in Sympy [A] time = 33.6331, size = 109, normalized size = 1.18

$$-\frac{\sqrt{6}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x\sqrt{3+\sqrt{21}}}{6}\right)\middle|-\frac{5}{2}+\frac{\sqrt{21}}{2}\right)}{\sqrt{-3+\sqrt{21}}} + \frac{2\sqrt{6}\left(\frac{\sqrt{21}}{2}+\frac{9}{2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt{6}x\sqrt{3+\sqrt{21}}}{6}\right)\middle|-\frac{5}{2}+\frac{\sqrt{21}}{2}\right)}{\sqrt{-3+\sqrt{21}}\left(3+\sqrt{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+3)/(-x**4-3*x**2+3)**(1/2),x)
```

```
[Out] -sqrt(6)*elliptic_e(asin(sqrt(6)*x*sqrt(3 + sqrt(21))/6), -5/2 +
sqrt(21)/2)/sqrt(-3 + sqrt(21)) + 2*sqrt(6)*(sqrt(21)/2 + 9/2)*el
liptic_f(asin(sqrt(6)*x*sqrt(3 + sqrt(21))/6), -5/2 + sqrt(21)/2)
/(sqrt(-3 + sqrt(21))*(3 + sqrt(21)))
```

Mathematica [C] time = 0.196686, size = 107, normalized size = 1.16

$$-\frac{i\left(\left(\sqrt{21}-3\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)|-\frac{5}{2}-\frac{\sqrt{21}}{2}\right)-\left(\sqrt{21}-9\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)|-\frac{5}{2}-\frac{\sqrt{21}}{2}\right)\right)}{\sqrt{2\left(\sqrt{21}-3\right)}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(3 - x^2)/Sqrt[3 - 3*x^2 - x^4],x]
```

```
[Out] ((-I)*((-3 + Sqrt[21])*EllipticE[I*ArcSinh[Sqrt[2/(3 + Sqrt[21])]
*x], -5/2 - Sqrt[21]/2] - (-9 + Sqrt[21])*EllipticF[I*ArcSinh[Sqr
t[2/(3 + Sqrt[21])]*x], -5/2 - Sqrt[21]/2]))/Sqrt[2*(-3 + Sqrt[21
])]
```

Maple [B] time = 0.117, size = 204, normalized size = 2.2

$$36\,\frac{\sqrt{1-\left(1/2+1/6\,\sqrt{21}\right)x^2}\sqrt{1-\left(1/2-1/6\,\sqrt{21}\right)x^2}\left(EllipticF\left(1/6\,x\sqrt{18+6\,\sqrt{21}},i/2\sqrt{7}-i/2\sqrt{3}\right)-EllipticE\left(1/6\,x\sqrt{1\right.\right.}{\sqrt{18+6\,\sqrt{21}}\sqrt{-x^4-3\,x^2+3}\left(-3+\sqrt{21}\right)}$$

$$+18\,\frac{\sqrt{1-\left(1/2+1/6\,\sqrt{21}\right)x^2}\sqrt{1-\left(1/2-1/6\,\sqrt{21}\right)x^2}EllipticF\left(1/6\,x\sqrt{18+6\,\sqrt{21}},i/2\sqrt{7}-i/2\sqrt{3}\right)}{\sqrt{18+6\,\sqrt{21}}\sqrt{-x^4-3\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x)
```

```
[Out] 36/(18+6*21^(1/2))^(1/2)*(1-(1/2+1/6*21^(1/2))*x^2)^(1/2)*(1-(1/2
-1/6*21^(1/2))*x^2)^(1/2)/(-x^4-3*x^2+3)^(1/2)/(-3+21^(1/2))*(Ell
ipticF(1/6*x*(18+6*21^(1/2))^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2))-E
llipticE(1/6*x*(18+6*21^(1/2))^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2))
)+18/(18+6*21^(1/2))^(1/2)*(1-(1/2+1/6*21^(1/2))*x^2)^(1/2)*(1-(1
/2-1/6*21^(1/2))*x^2)^(1/2)/(-x^4-3*x^2+3)^(1/2)*EllipticF(1/6*x*
(18+6*21^(1/2))^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-3}{\sqrt{-x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 3)/sqrt(-x^4 - 3*x^2 + 3),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 - 3*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^2-3}{\sqrt{-x^4-3\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 3)/sqrt(-x^4 - 3*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(-(x^2 - 3)/sqrt(-x^4 - 3*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4-3x^2+3}}\,dx-\int \left(-\frac{3}{\sqrt{-x^4-3x^2+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+3)/(-x**4-3*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 - 3*x**2 + 3), x) - Integral(-3/sqrt(-x
**4 - 3*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-3}{\sqrt{-x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 3)/sqrt(-x^4 - 3*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 - 3*x^2 + 3), x)
```

3.127 $\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=296

$$\frac{\left(-\sqrt{b^2-4ac}+2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}-\frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2) - (
2*a^(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/Sqrt[a + b*x^2 + c*x^4] + ((b
+ 2*Sqrt[a]*Sqrt[c] - Sqrt[b^2 - 4*a*c])*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1
/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.270245, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{\left(-\sqrt{b^2-4ac}+2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}-\frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2) - (
2*a^(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/Sqrt[a + b*x^2 + c*x^4] + ((b
+ 2*Sqrt[a]*Sqrt[c] - Sqrt[b^2 - 4*a*c])*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1
/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 35.8378, size = 269, normalized size = 0.91

$$-\frac{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}+\frac{\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(2\sqrt{a}\sqrt{c}+b-\sqrt{-4ac+b^2}\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*c*x**2-(-4*a*c+b**2)**(1/2)+b)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -2*a**(1/4)*c**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c
)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x
/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/sqrt(a + b*x**2 + c*x**4
) + 2*sqrt(c)*x*sqrt(a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2
) + sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt
(a) + sqrt(c)*x**2)*(2*sqrt(a)*sqrt(c) + b - sqrt(-4*a*c + b**2))
*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(
c)))/(2*a**(1/4)*c**(1/4)*sqrt(a + b*x**2 + c*x**4))
```

Mathematica [C] time = 0.367432, size = 187, normalized size = 0.63

$$-\frac{2i\sqrt{2}a\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((-2*I)*Sqrt[2]*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
 + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
- 4*a*c])])/Sqrt[a + b*x^2 + c*x^4]
```

Maple [A] time = 0.062, size = 515, normalized size = 1.7

$$-\frac{\sqrt{2}}{4}\sqrt{-4\,ac+b^2}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{\right.\right.$$

$$+\frac{b\sqrt{2}}{4}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\,\frac{b\,(b}{}}\right.$$

$$-ac\sqrt{2}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\,\frac{b}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/
2))/a/c)^(1/2))+1/4*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(
4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2)
)/a/c)^(1/2))-c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*
(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Ellipt
icF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-
4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c
)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,cx^2 + b - \sqrt{b^2 - 4\,ac}}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a),x, algorithm='
```

```
[Out] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 +
a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{2\,cx^2 + b - \sqrt{b^2 - 4\,ac}}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a),x, algorithm='
```

```
[Out] integral((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b + 2cx^2 - \sqrt{-4ac + b^2}}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2-(-4*a*c+b**2)**(1/2)+b)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/sqrt(a + b*x**2 + c
*x**4), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a),x, algorithm='
```

```
[Out] Exception raised: TypeError
```

3.128 $\int (d+ex^2)^4 (a+cx^4)\, dx$

Optimal. Leaf size=106

$$\frac{1}{9}e^2x^9\left(ae^2+6cd^2\right)+\frac{4}{7}dex^7\left(ae^2+cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+cd^2\right)+ad^4x+\frac{4}{3}ad^3ex^3+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}$$

```
[Out] a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*
d*e*(c*d^2 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c
*d*e^3*x^11)/11 + (c*e^4*x^13)/13
```

Rubi [A] time = 0.162781, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{1}{9}e^2x^9\left(ae^2+6cd^2\right)+\frac{4}{7}dex^7\left(ae^2+cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+cd^2\right)+ad^4x+\frac{4}{3}ad^3ex^3+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^4*(a + c*x^4),x]
```

```
[Out] a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*
d*e*(c*d^2 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c
*d*e^3*x^11)/11 + (c*e^4*x^13)/13
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{4ad^3ex^3}{3}+\frac{4cde^3x^{11}}{11}+\frac{ce^4x^{13}}{13}+d^4\int a\,dx+\frac{d^2x^5\left(6ae^2+cd^2\right)}{5}+\frac{4dex^7\left(ae^2+cd^2\right)}{7}+\frac{e^2x^9\left(ae^2+6cd^2\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**4*(c*x**4+a),x)
```

```
[Out] 4*a*d**3*e*x**3/3 + 4*c*d*e**3*x**11/11 + c*e**4*x**13/13 + d**4*
Integral(a, x) + d**2*x**5*(6*a*e**2 + c*d**2)/5 + 4*d*e*x**7*(a*
e**2 + c*d**2)/7 + e**2*x**9*(a*e**2 + 6*c*d**2)/9
```

Mathematica [A] time = 0.0310268, size = 106, normalized size = 1.

$$\frac{1}{9}e^2x^9\left(ae^2+6cd^2\right)+\frac{4}{7}dex^7\left(ae^2+cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+cd^2\right)+ad^4x+\frac{4}{3}ad^3ex^3+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^4*(a + c*x^4),x]
```

```
[Out] a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*
d*e*(c*d^2 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c
*d*e^3*x^11)/11 + (c*e^4*x^13)/13
```

Maple [A] time = 0.002, size = 97, normalized size = 0.9

$$\frac{ce^4x^{13}}{13}+\frac{4\,cde^3x^{11}}{11}+\frac{\left(ae^4+6\,d^2e^2c\right)x^9}{9}+\frac{\left(4\,de^3a+4\,d^3ec\right)x^7}{7}+\frac{\left(6\,d^2e^2a+d^4c\right)x^5}{5}+\frac{4\,ad^3ex^3}{3}+ad^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^4*(c*x^4+a),x)
```

```
[Out] 1/13*c*e^4*x^13+4/11*c*d*e^3*x^11+1/9*(a*e^4+6*c*d^2*e^2)*x^9+1/7
*(4*a*d*e^3+4*c*d^3*e)*x^7+1/5*(6*a*d^2*e^2+c*d^4)*x^5+4/3*a*d^3*
e*x^3+a*d^4*x
```

Maxima [A] time = 0.699931, size = 127, normalized size = 1.2

$$\frac{1}{13}ce^4x^{13} + \frac{4}{11}cde^3x^{11} + \frac{1}{9}\left(6\,cd^2e^2 + ae^4\right)x^9 + \frac{4}{3}ad^3ex^3 + \frac{4}{7}\left(cd^3e + ade^3\right)x^7 + ad^4x + \frac{1}{5}\left(cd^4 + 6\,ad^2e^2\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)*(e*x^2 + d)^4,x, algorithm="maxima")
```

```
[Out] 1/13*c*e^4*x^13 + 4/11*c*d*e^3*x^11 + 1/9*(6*c*d^2*e^2 + a*e^4)*x
^9 + 4/3*a*d^3*e*x^3 + 4/7*(c*d^3*e + a*d*e^3)*x^7 + a*d^4*x + 1/
5*(c*d^4 + 6*a*d^2*e^2)*x^5
```

Fricas [A] time = 0.274835, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}e^4c + \frac{4}{11}x^{11}e^3dc + \frac{2}{3}x^9e^2d^2c + \frac{1}{9}x^9e^4a + \frac{4}{7}x^7ed^3c + \frac{4}{7}x^7e^3da + \frac{1}{5}x^5d^4c + \frac{6}{5}x^5e^2d^2a + \frac{4}{3}x^3ed^3a + xd^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)*(e*x^2 + d)^4,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^4*c + 4/11*x^11*e^3*d*c + 2/3*x^9*e^2*d^2*c + 1/9*x^9
*e^4*a + 4/7*x^7*e*d^3*c + 4/7*x^7*e^3*d*a + 1/5*x^5*d^4*c + 6/5*
x^5*e^2*d^2*a + 4/3*x^3*e*d^3*a + x*d^4*a
```

Sympy [A] time = 0.13508, size = 110, normalized size = 1.04

$$ad^4x + \frac{4ad^3ex^3}{3} + \frac{4cde^3x^{11}}{11} + \frac{ce^4x^{13}}{13} + x^9\left(\frac{ae^4}{9} + \frac{2cd^2e^2}{3}\right) + x^7\left(\frac{4ade^3}{7} + \frac{4cd^3e}{7}\right) + x^5\left(\frac{6ad^2e^2}{5} + \frac{cd^4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**4*(c*x**4+a),x)
```

```
[Out] a*d**4*x + 4*a*d**3*e*x**3/3 + 4*c*d*e**3*x**11/11 + c*e**4*x**13
/13 + x**9*(a*e**4/9 + 2*c*d**2*e**2/3) + x**7*(4*a*d*e**3/7 + 4*
c*d**3*e/7) + x**5*(6*a*d**2*e**2/5 + c*d**4/5)
```

GIAC/XCAS [A] time = 0.267683, size = 127, normalized size = 1.2

$$\frac{1}{13}cx^{13}e^4 + \frac{4}{11}cdx^{11}e^3 + \frac{2}{3}cd^2x^9e^2 + \frac{4}{7}cd^3x^7e + \frac{1}{9}ax^9e^4 + \frac{1}{5}cd^4x^5 + \frac{4}{7}adx^7e^3 + \frac{6}{5}ad^2x^5e^2 + \frac{4}{3}ad^3x^3e + ad^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)*(e*x^2 + d)^4,x, algorithm="giac")
```

```
[Out] 1/13*c*x^13*e^4 + 4/11*c*d*x^11*e^3 + 2/3*c*d^2*x^9*e^2 + 4/7*c*d
^3*x^7*e + 1/9*a*x^9*e^4 + 1/5*c*d^4*x^5 + 4/7*a*d*x^7*e^3 + 6/5*
a*d^2*x^5*e^2 + 4/3*a*d^3*x^3*e + a*d^4*x
```

3.129 $\int \left(d+ex^2\right)^3 \left(a+cx^4\right) \, dx$

Optimal. Leaf size=79

$$\frac{1}{7}ex^7\left(ae^2+3cd^2\right)+\frac{1}{5}dx^5\left(3ae^2+cd^2\right)+ad^3x+ad^2ex^3+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}$$

```
[Out] a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2
 + a*e^2)*x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11
```

Rubi [A] time = 0.112233, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{1}{7}ex^7\left(ae^2+3cd^2\right)+\frac{1}{5}dx^5\left(3ae^2+cd^2\right)+ad^3x+ad^2ex^3+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^3*(a + c*x^4),x]
```

```
[Out] a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2
 + a*e^2)*x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$ad^2ex^3+\frac{cde^2x^9}{3}+\frac{ce^3x^{11}}{11}+d^3\int a\,dx+\frac{dx^5\left(3ae^2+cd^2\right)}{5}+\frac{ex^7\left(ae^2+3cd^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**3*(c*x**4+a),x)
```

```
[Out] a*d**2*e*x**3 + c*d*e**2*x**9/3 + c*e**3*x**11/11 + d**3*Integral
(a, x) + d*x**5*(3*a*e**2 + c*d**2)/5 + e*x**7*(a*e**2 + 3*c*d**2
)/7
```

Mathematica [A] time = 0.02543, size = 79, normalized size = 1.

$$\frac{1}{7}ex^7\left(ae^2+3cd^2\right)+\frac{1}{5}dx^5\left(3ae^2+cd^2\right)+ad^3x+ad^2ex^3+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^3*(a + c*x^4),x]
```

```
[Out] a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2
 + a*e^2)*x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11
```

Maple [A] time = 0.002, size = 72, normalized size = 0.9

$$\frac{ce^3x^{11}}{11}+\frac{cde^2x^9}{3}+\frac{\left(ae^3+3\,cd^2e\right)x^7}{7}+\frac{\left(3\,e^2da+d^3c\right)x^5}{5}+ad^2ex^3+ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((e*x^2+d)^3*(c*x^4+a),x)
```

```
[Out] 1/11*c*e^3*x^11+1/3*c*d*e^2*x^9+1/7*(a*e^3+3*c*d^2*e)*x^7+1/5*(3*
a*d*e^2+c*d^3)*x^5+a*d^2*e*x^3+a*d^3*x
```

Maxima [A] time = 0.733364, size = 96, normalized size = 1.22

$$\frac{1}{11}\,ce^3x^{11} + \frac{1}{3}\,cde^2x^9 + \frac{1}{7}\left(3\,cd^2e + ae^3\right)x^7 + ad^2ex^3 + \frac{1}{5}\left(cd^3 + 3\,ade^2\right)x^5 + ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((c*x^4 + a)*(e*x^2 + d)^3,x, algorithm="maxima")
```

```
[Out] 1/11*c*e^3*x^11 + 1/3*c*d*e^2*x^9 + 1/7*(3*c*d^2*e + a*e^3)*x^7 +
 a*d^2*e*x^3 + 1/5*(c*d^3 + 3*a*d*e^2)*x^5 + a*d^3*x
```

Fricas [A] time = 0.256773, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}e^3c + \frac{1}{3}x^9e^2dc + \frac{3}{7}x^7ed^2c + \frac{1}{7}x^7e^3a + \frac{1}{5}x^5d^3c + \frac{3}{5}x^5e^2da + x^3ed^2a + xd^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((c*x^4 + a)*(e*x^2 + d)^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^3*c + 1/3*x^9*e^2*d*c + 3/7*x^7*e*d^2*c + 1/7*x^7*e^3
*a + 1/5*x^5*d^3*c + 3/5*x^5*e^2*d*a + x^3*e*d^2*a + x*d^3*a
```

Sympy [A] time = 0.115066, size = 78, normalized size = 0.99

$$ad^3x + ad^2ex^3 + \frac{cde^2x^9}{3} + \frac{ce^3x^{11}}{11} + x^7\left(\frac{ae^3}{7} + \frac{3cd^2e}{7}\right) + x^5\left(\frac{3ade^2}{5} + \frac{cd^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((e*x**2+d)**3*(c*x**4+a),x)
```

```
[Out] a*d**3*x + a*d**2*e*x**3 + c*d*e**2*x**9/3 + c*e**3*x**11/11 + x*
*7*(a*e**3/7 + 3*c*d**2*e/7) + x**5*(3*a*d*e**2/5 + c*d**3/5)
```

GIAC/XCAS [A] time = 0.268923, size = 96, normalized size = 1.22

$$\frac{1}{11}\,cx^{11}e^3 + \frac{1}{3}\,cdx^9e^2 + \frac{3}{7}\,cd^2x^7e + \frac{1}{5}\,cd^3x^5 + \frac{1}{7}\,ax^7e^3 + \frac{3}{5}\,adx^5e^2 + ad^2x^3e + ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((c*x^4 + a)*(e*x^2 + d)^3,x, algorithm="giac")
```

```
[Out] 1/11*c*x^11*e^3 + 1/3*c*d*x^9*e^2 + 3/7*c*d^2*x^7*e + 1/5*c*d^3*x
^5 + 1/7*a*x^7*e^3 + 3/5*a*d*x^5*e^2 + a*d^2*x^3*e + a*d^3*x
```

3.130 $\int \left(d+ex^2\right)^2 \left(a+cx^4\right) \, dx$

Optimal. Leaf size=56

$$\frac{1}{5}x^5\left(ae^2+cd^2\right)+ad^2x+\frac{2}{3}adex^3+\frac{2}{7}cdex^7+\frac{1}{9}ce^2x^9$$

```
[Out] a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^
7)/7 + (c*e^2*x^9)/9
```

Rubi [A] time = 0.0685135, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{1}{5}x^5\left(ae^2+cd^2\right)+ad^2x+\frac{2}{3}adex^3+\frac{2}{7}cdex^7+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^2*(a + c*x^4),x]
```

```
[Out] a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^
7)/7 + (c*e^2*x^9)/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2adex^3}{3}+\frac{2cdex^7}{7}+\frac{ce^2x^9}{9}+d^2\int a\,dx+x^5\left(\frac{ae^2}{5}+\frac{cd^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2*(c*x**4+a),x)
```

```
[Out] 2*a*d*e*x**3/3 + 2*c*d*e*x**7/7 + c*e**2*x**9/9 + d**2*Integral(a
, x) + x**5*(a*e**2/5 + c*d**2/5)
```

Mathematica [A] time = 0.0167498, size = 56, normalized size = 1.

$$\frac{1}{5}x^5\left(ae^2+cd^2\right)+ad^2x+\frac{2}{3}adex^3+\frac{2}{7}cdex^7+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^2*(a + c*x^4),x]
```

```
[Out] a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^
7)/7 + (c*e^2*x^9)/9
```

Maple [A] time = 0.002, size = 49, normalized size = 0.9

$$ad^2x+\frac{2\,adex^3}{3}+\frac{\left(ae^2+cd^2\right)x^5}{5}+\frac{2\,cdex^7}{7}+\frac{ce^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^2*(c*x^4+a),x)
```

```
[Out] a*d^2*x+2/3*a*d*e*x^3+1/5*(a*e^2+c*d^2)*x^5+2/7*c*d*e*x^7+1/9*c*e
^2*x^9
```

Maxima [A] time = 0.745005, size = 65, normalized size = 1.16

$$\frac{1}{9}ce^2x^9 + \frac{2}{7}cdex^7 + \frac{2}{3}adex^3 + \frac{1}{5}\left(cd^2 + ae^2\right)x^5 + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)*(e*x^2 + d)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c*e^2*x^9 + 2/7*c*d*e*x^7 + 2/3*a*d*e*x^3 + 1/5*(c*d^2 + a*e^
2)*x^5 + a*d^2*x
```

Fricas [A] time = 0.265453, size = 1, normalized size = 0.02

$$\frac{1}{9}x^9e^2c + \frac{2}{7}x^7edc + \frac{1}{5}x^5d^2c + \frac{1}{5}x^5e^2a + \frac{2}{3}x^3eda + xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)*(e*x^2 + d)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^2*c + 2/7*x^7*e*d*c + 1/5*x^5*d^2*c + 1/5*x^5*e^2*a + 2
/3*x^3*e*d*a + x*d^2*a
```

Sympy [A] time = 0.101235, size = 56, normalized size = 1.

$$ad^2x + \frac{2adex^3}{3} + \frac{2cdex^7}{7} + \frac{ce^2x^9}{9} + x^5\left(\frac{ae^2}{5} + \frac{cd^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**2*(c*x**4+a),x)
```

```
[Out] a*d**2*x + 2*a*d*e*x**3/3 + 2*c*d*e*x**7/7 + c*e**2*x**9/9 + x**5
*(a*e**2/5 + c*d**2/5)
```

GIAC/XCAS [A] time = 0.269131, size = 68, normalized size = 1.21

$$\frac{1}{9}cx^9e^2 + \frac{2}{7}cdx^7e + \frac{1}{5}cd^2x^5 + \frac{1}{5}ax^5e^2 + \frac{2}{3}adx^3e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)*(e*x^2 + d)^2,x, algorithm="giac")
```

```
[Out] 1/9*c*x^9*e^2 + 2/7*c*d*x^7*e + 1/5*c*d^2*x^5 + 1/5*a*x^5*e^2 + 2
/3*a*d*x^3*e + a*d^2*x
```

3.131 $\int \left(d+ex^2\right)\left(a+cx^4\right)\,dx$

Optimal. Leaf size=32

$$adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7$$

```
[Out] a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

Rubi [A] time = 0.0328197, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)*(a + c*x^4),x]

[Out] a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{aex^3}{3}+\frac{cdx^5}{5}+\frac{cex^7}{7}+d\int a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)*(c*x**4+a),x)

[Out] a*e*x**3/3 + c*d*x**5/5 + c*e*x**7/7 + d*Integral(a, x)
```

Mathematica [A] time = 0.00253939, size = 32, normalized size = 1.

$$adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)*(a + c*x^4),x]

[Out] a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

Maple [A] time = 0.001, size = 27, normalized size = 0.8

$$adx+\frac{aex^3}{3}+\frac{cdx^5}{5}+\frac{cex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)*(c*x^4+a),x)
```

```
[Out] a*d*x+1/3*a*e*x^3+1/5*c*d*x^5+1/7*c*e*x^7
```

Maxima [A] time = 0.727102, size = 35, normalized size = 1.09

$$\frac{1}{7}\,cex^7 + \frac{1}{5}\,cdx^5 + \frac{1}{3}\,aex^3 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)*(e*x^2 + d),x, algorithm="maxima")
```

```
[Out] 1/7*c*e*x^7 + 1/5*c*d*x^5 + 1/3*a*e*x^3 + a*d*x
```

Fricas [A] time = 0.261315, size = 1, normalized size = 0.03

$$\frac{1}{7}x^7ec + \frac{1}{5}x^5dc + \frac{1}{3}x^3ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)*(e*x^2 + d),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e*c + 1/5*x^5*d*c + 1/3*x^3*e*a + x*d*a
```

Sympy [A] time = 0.074758, size = 29, normalized size = 0.91

$$adx + \frac{aex^3}{3} + \frac{cdx^5}{5} + \frac{cex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)*(c*x**4+a),x)
```

```
[Out] a*d*x + a*e*x**3/3 + c*d*x**5/5 + c*e*x**7/7
```

GIAC/XCAS [A] time = 0.268922, size = 38, normalized size = 1.19

$$\frac{1}{7}\,cx^7e + \frac{1}{5}\,cdx^5 + \frac{1}{3}\,ax^3e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)*(e*x^2 + d),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7*e + 1/5*c*d*x^5 + 1/3*a*x^3*e + a*d*x
```

3.132 $\int \frac{a+cx^4}{d+ex^2} dx$

Optimal. Leaf size=55

$$\frac{\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}-\frac{cdx}{e^2}+\frac{cx^3}{3e}$$

```
[Out] -((c*d*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))
```

Rubi [A] time = 0.0758465, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}-\frac{cdx}{e^2}+\frac{cx^3}{3e}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^4)/(d + e*x^2),x]
```

```
[Out] -((c*d*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))
```

Rubi in Sympy [A] time = 17.0536, size = 49, normalized size = 0.89

$$-\frac{cdx}{e^2}+\frac{cx^3}{3e}+\frac{\left(ae^2+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+a)/(e*x**2+d),x)
```

```
[Out] -c*d*x/e**2 + c*x**3/(3*e) + (a*e**2 + c*d**2)*atan(sqrt(e)*x/sqr
t(d))/(sqrt(d)*e**(5/2))
```

Mathematica [A] time = 0.0570117, size = 55, normalized size = 1.

$$\frac{\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}-\frac{cdx}{e^2}+\frac{cx^3}{3e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^4)/(d + e*x^2),x]
```

```
[Out] -((c*d*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))
```

Maple [A] time = 0.008, size = 57, normalized size = 1.

$$\frac{cx^3}{3\,e}-\frac{cdx}{e^2}+a\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{cd^2}{e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+a)/(e*x^2+d),x)
```

```
[Out] 1/3*c*x^3/e-c*d*x/e^2+1/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+1/e
^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)/(e*x^2 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.296494, size = 1, normalized size = 0.02

$$\left[\frac{3\left(cd^2+ae^2\right)\log\left(\frac{2\,dex+\left(ex^2-d\right)\sqrt{-de}}{ex^2+d}\right)+2\left(cex^3-3\,cdx\right)\sqrt{-de}}{6\sqrt{-de}e^2},\ \frac{3\left(cd^2+ae^2\right)\arctan\left(\frac{\sqrt{de}x}{d}\right)+\left(cex^3-3\,cdx\right)\sqrt{de}}{3\sqrt{de}e^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)/(e*x^2 + d),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(c*d^2 + a*e^2)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e
*x^2 + d)) + 2*(c*e*x^3 - 3*c*d*x)*sqrt(-d*e))/(sqrt(-d*e)*e^2),
1/3*(3*(c*d^2 + a*e^2)*arctan(sqrt(d*e)*x/d) + (c*e*x^3 - 3*c*d*x
)*sqrt(d*e))/(sqrt(d*e)*e^2)]
```

Sympy [A] time = 1.66288, size = 104, normalized size = 1.89

$$-\frac{cdx}{e^2}+\frac{cx^3}{3e}-\frac{\sqrt{-\frac{1}{de^5}}\left(ae^2+cd^2\right)\log\left(-de^2\sqrt{-\frac{1}{de^5}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{de^5}}\left(ae^2+cd^2\right)\log\left(de^2\sqrt{-\frac{1}{de^5}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+a)/(e*x**2+d),x)
```

```
[Out] -c*d*x/e**2 + c*x**3/(3*e) - sqrt(-1/(d*e**5))*(a*e**2 + c*d**2)*
log(-d*e**2*sqrt(-1/(d*e**5)) + x)/2 + sqrt(-1/(d*e**5))*(a*e**2
+ c*d**2)*log(d*e**2*sqrt(-1/(d*e**5)) + x)/2
```

GIAC/XCAS [A] time = 0.268124, size = 59, normalized size = 1.07

$$\frac{\left(cd^2+ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{\sqrt{d}}+\frac{1}{3}\left(cx^3e^2-3\,cdxe\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)/(e*x^2 + d),x, algorithm="giac")
```

```
[Out] (c*d^2 + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d) + 1/3*
(c*x^3*e^2 - 3*c*d*x*e)*e^(-3)
```

3.133 $\int \frac{a+cx^4}{(d+ex^2)^2} dx$

Optimal. Leaf size=74

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{2d\left(d+ex^2\right)}-\frac{\left(3cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}+\frac{cx}{e^2}$$

```
[Out] (c*x)/e^2 + ((a + (c*d^2)/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 -
 a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rubi [A] time = 0.113144, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{2d\left(d+ex^2\right)}-\frac{\left(3cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((a + (c*d^2)/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 -
 a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rubi in Sympy [A] time = 23.8383, size = 65, normalized size = 0.88

$$\frac{cx}{e^2}+\frac{x\left(\frac{a}{2d}+\frac{cd}{2e^2}\right)}{d+ex^2}+\frac{\left(ae^2-3cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{\frac{3}{2}}e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a/(2*d) + c*d/(2*e**2))/(d + e*x**2) + (a*e**2 - 3*
c*d**2)*atan(sqrt(e)*x/sqrt(d))/(2*d**(3/2)*e**(5/2))
```

Mathematica [A] time = 0.0833572, size = 78, normalized size = 1.05

$$\frac{x\left(ae^2+cd^2\right)}{2de^2\left(d+ex^2\right)}-\frac{\left(3cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2
 - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Maple [A] time = 0.012, size = 82, normalized size = 1.1

$$\frac{cx}{e^2}+\frac{ax}{2\,d\left(ex^2+d\right)}+\frac{cdx}{2\,e^2\left(ex^2+d\right)}+\frac{a}{2\,d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{3\,cd}{2\,e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+a)/(e*x^2+d)^2,x)
```

```
[Out] c*x/e^2+1/2/d*x/(e*x^2+d)*a+1/2/e^2*d*x/(e*x^2+d)*c+1/2/d/(d*e)^(
1/2)*arctan(x*e/(d*e)^(1/2))*a-3/2/e^2*d/(d*e)^(1/2)*arctan(x*e/(
d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)/(e*x^2 + d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.29123, size = 1, normalized size = 0.01

$$\left[-\frac{\left(3\,cd^{3}-ade^{2}+\left(3\,cd^{2}e-ae^{3}\right)x^{2}\right)\log\left(\frac{2\,dex+\left(ex^{2}-d\right)\sqrt{-de}}{ex^{2}+d}\right)-2\left(2\,cdex^{3}+\left(3\,cd^{2}+ae^{2}\right)x\right)\sqrt{-de}}{4\left(de^{3}x^{2}+d^{2}e^{2}\right)\sqrt{-de}},\right.$$
$$\left.-\frac{\left(3\,cd^{3}-ade^{2}+\left(3\,cd^{2}e-ae^{3}\right)x^{2}\right)\arctan\left(\frac{\sqrt{de}x}{d}\right)-\left(2\,cdex^{3}+\left(3\,cd^{2}+ae^{2}\right)x\right)\sqrt{de}}{2\left(de^{3}x^{2}+d^{2}e^{2}\right)\sqrt{de}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)/(e*x^2 + d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((3*c*d^3 - a*d*e^2 + (3*c*d^2*e - a*e^3)*x^2)*log((2*d*e*x
 + (e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d)) - 2*(2*c*d*e*x^3 + (3*c*d
^2 + a*e^2)*x)*sqrt(-d*e))/((d*e^3*x^2 + d^2*e^2)*sqrt(-d*e)), -1
/2*((3*c*d^3 - a*d*e^2 + (3*c*d^2*e - a*e^3)*x^2)*arctan(sqrt(d*e
)*x/d) - (2*c*d*e*x^3 + (3*c*d^2 + a*e^2)*x)*sqrt(d*e))/((d*e^3*x
^2 + d^2*e^2)*sqrt(d*e))]
```

Sympy [A] time = 2.29111, size = 138, normalized size = 1.86

$$\frac{cx}{e^{2}}+\frac{x\left(ae^{2}+cd^{2}\right)}{2d^{2}e^{2}+2de^{3}x^{2}}-\frac{\sqrt{-\frac{1}{d^{3}e^{5}}}\left(ae^{2}-3cd^{2}\right)\log\left(-d^{2}e^{2}\sqrt{-\frac{1}{d^{3}e^{5}}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{d^{3}e^{5}}}\left(ae^{2}-3cd^{2}\right)\log\left(d^{2}e^{2}\sqrt{-\frac{1}{d^{3}e^{5}}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sq
rt(-1/(d**3*e**5))*(a*e**2 - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d*
*3*e**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 - 3*c*d**2)*log(d
```

```
**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4
```

GIAC/XCAS [A] time = 0.270064, size = 84, normalized size = 1.14

$$cxe^{(-2)} - \frac{\left(3\,cd^2 - ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{\left(cd^2x + axe^2\right)e^{(-2)}}{2\,(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)/(e*x^2 + d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-
5/2)/d^(3/2) + 1/2*(c*d^2*x + a*x*e^2)*e^(-2)/((x^2*e + d)*d)
```

3.134 $\int \frac{a+cx^4}{(d+ex^2)^3} dx$

Optimal. Leaf size=93

$$\frac{x\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)}{8\left(d+ex^2\right)}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d\left(d+ex^2\right)^2}+\frac{3\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

```
[Out] ((a + (c*d^2)/e^2)*x)/(4*d*(d + e*x^2)^2) + (((3*a)/d^2 - (5*c)/e
^2)*x)/(8*(d + e*x^2)) + (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sq
rt[d]])/(8*d^(5/2)*e^(5/2))
```

Rubi [A] time = 0.143192, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{x\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)}{8\left(d+ex^2\right)}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d\left(d+ex^2\right)^2}+\frac{3\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] ((a + (c*d^2)/e^2)*x)/(4*d*(d + e*x^2)^2) + (((3*a)/d^2 - (5*c)/e
^2)*x)/(8*(d + e*x^2)) + (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sq
rt[d]])/(8*d^(5/2)*e^(5/2))
```

Rubi in Sympy [A] time = 24.203, size = 85, normalized size = 0.91

$$\frac{x\left(\frac{3a}{8d^2}-\frac{5c}{8e^2}\right)}{d+ex^2}+\frac{x\left(\frac{a}{4d}+\frac{cd}{4e^2}\right)}{\left(d+ex^2\right)^2}+\frac{3\left(ae^2+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{\frac{5}{2}}e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+a)/(e*x**2+d)**3,x)
```

```
[Out] x*(3*a/(8*d**2) - 5*c/(8*e**2))/(d + e*x**2) + x*(a/(4*d) + c*d/(
4*e**2))/(d + e*x**2)**2 + 3*(a*e**2 + c*d**2)*atan(sqrt(e)*x/sqr
t(d))/(8*d**(5/2)*e**(5/2))
```

Mathematica [A] time = 0.098541, size = 92, normalized size = 0.99

$$\frac{ae^2x\left(5d+3ex^2\right)-cd^2x\left(3d+5ex^2\right)}{8d^2e^2\left(d+ex^2\right)^2}+\frac{3\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] (a*e^2*x*(5*d + 3*e*x^2) - c*d^2*x*(3*d + 5*e*x^2))/(8*d^2*e^2*(d
 + e*x^2)^2) + (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8
*d^(5/2)*e^(5/2))
```

Maple [A] time = 0.012, size = 99, normalized size = 1.1

$$\frac{1}{\left(ex^{2}+d\right)^{2}}\left(\frac{\left(3\,ae^{2}-5\,cd^{2}\right)x^{3}}{8\,d^{2}e}+\frac{\left(5\,ae^{2}-3\,cd^{2}\right)x}{8\,e^{2}d}\right)+\frac{3\,a}{8\,d^{2}}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{3\,c}{8\,e^{2}}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+a)/(e*x^2+d)^3,x)
```

```
[Out] (1/8*(3*a*e^2-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-3*c*d^2)/e^2/d*x)/(
e*x^2+d)^2+3/8/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+3/8/e^2/
(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)/(e*x^2 + d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.290071, size = 1, normalized size = 0.01

$$\left[\frac{3\left(cd^{4}+ad^{2}e^{2}+\left(cd^{2}e^{2}+ae^{4}\right)x^{4}+2\left(cd^{3}e+ade^{3}\right)x^{2}\right)\log\left(\frac{2\,dex+\left(ex^{2}-d\right)\sqrt{-de}}{ex^{2}+d}\right)-2\left(\left(5\,cd^{2}e-3\,ae^{3}\right)x^{3}+\left(3\,cd^{3}-5\,ade^{2}\right)\right.}{16\left(d^{2}e^{4}x^{4}+2\,d^{3}e^{3}x^{2}+d^{4}e^{2}\right)\sqrt{-de}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)/(e*x^2 + d)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(c*d^4 + a*d^2*e^2 + (c*d^2*e^2 + a*e^4)*x^4 + 2*(c*d^3*
e + a*d*e^3)*x^2)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2 +
 d)) - 2*((5*c*d^2*e - 3*a*e^3)*x^3 + (3*c*d^3 - 5*a*d*e^2)*x)*sq
rt(-d*e))/((d^2*e^4*x^4 + 2*d^3*e^3*x^2 + d^4*e^2)*sqrt(-d*e)), 1
/8*(3*(c*d^4 + a*d^2*e^2 + (c*d^2*e^2 + a*e^4)*x^4 + 2*(c*d^3*e +
 a*d*e^3)*x^2)*arctan(sqrt(d*e)*x/d) - ((5*c*d^2*e - 3*a*e^3)*x^3
 + (3*c*d^3 - 5*a*d*e^2)*x)*sqrt(d*e))/((d^2*e^4*x^4 + 2*d^3*e^3*
x^2 + d^4*e^2)*sqrt(d*e))]
```

Sympy [A] time = 3.11724, size = 219, normalized size = 2.35

$$-\frac{3\sqrt{-\frac{1}{d^{5}e^{5}}}\left(ae^{2}+cd^{2}\right)\log\left(-\frac{3d^{3}e^{2}\sqrt{-\frac{1}{d^{5}e^{5}}}\left(ae^{2}+cd^{2}\right)}{3ae^{2}+3cd^{2}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{d^{5}e^{5}}}\left(ae^{2}+cd^{2}\right)\log\left(\frac{3d^{3}e^{2}\sqrt{-\frac{1}{d^{5}e^{5}}}\left(ae^{2}+cd^{2}\right)}{3ae^{2}+3cd^{2}}+x\right)}{16}+\frac{x^{3}\left(3ae^{3}-5cd^{2}e\right)+x\left(5ade^{2}-3cd^{3}\right)}{8d^{4}e^{2}+16d^{3}e^{3}x^{2}+8d^{2}e^{4}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+a)/(e*x**2+d)**3,x)
```

```
[Out] -3*sqrt(-1/(d**5*e**5))*(a*e**2 + c*d**2)*log(-3*d**3*e**2*sqrt(-
1/(d**5*e**5))*(a*e**2 + c*d**2)/(3*a*e**2 + 3*c*d**2) + x)/16 +
3*sqrt(-1/(d**5*e**5))*(a*e**2 + c*d**2)*log(3*d**3*e**2*sqrt(-1/
(d**5*e**5))*(a*e**2 + c*d**2)/(3*a*e**2 + 3*c*d**2) + x)/16 + (x
**3*(3*a*e**3 - 5*c*d**2*e) + x*(5*a*d*e**2 - 3*c*d**3))/(8*d**4*
e**2 + 16*d**3*e**3*x**2 + 8*d**2*e**4*x**4)
```

GIAC/XCAS [A] time = 0.271349, size = 104, normalized size = 1.12

$$\frac{3\left(cd^{2}+ae^{2}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{8\,d^{\frac{5}{2}}}-\frac{\left(5\,cd^{2}x^{3}e+3\,cd^{3}x-3\,ax^{3}e^{3}-5\,adxe^{2}\right)e^{\left(-2\right)}}{8\left(x^{2}e+d\right)^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)/(e*x^2 + d)^3,x, algorithm="giac")
```

```
[Out] 3/8*(c*d^2 + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(5/2) -
1/8*(5*c*d^2*x^3*e + 3*c*d^3*x - 3*a*x^3*e^3 - 5*a*d*x*e^2)*e^(-2
)/((x^2*e + d)^2*d^2)
```

3.135 $\int \frac{a+cx^4}{(d+ex^2)^4} dx$

Optimal. Leaf size=126

$$\frac{x\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)}{24\left(d+ex^2\right)^2}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d\left(d+ex^2\right)^3}+\frac{\left(5ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}+\frac{x\left(5ae^2+cd^2\right)}{16d^3e^2\left(d+ex^2\right)}$$

```
[Out] ((a + (c*d^2)/e^2)*x)/(6*d*(d + e*x^2)^3) + (((5*a)/d^2 - (7*c)/e
^2)*x)/(24*(d + e*x^2)^2) + ((c*d^2 + 5*a*e^2)*x)/(16*d^3*e^2*(d
+ e*x^2)) + ((c*d^2 + 5*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d
^(7/2)*e^(5/2))
```

Rubi [A] time = 0.222384, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{x\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)}{24\left(d+ex^2\right)^2}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d\left(d+ex^2\right)^3}+\frac{\left(5ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}+\frac{x\left(5ae^2+cd^2\right)}{16d^3e^2\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] ((a + (c*d^2)/e^2)*x)/(6*d*(d + e*x^2)^3) + (((5*a)/d^2 - (7*c)/e
^2)*x)/(24*(d + e*x^2)^2) + ((c*d^2 + 5*a*e^2)*x)/(16*d^3*e^2*(d
+ e*x^2)) + ((c*d^2 + 5*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d
^(7/2)*e^(5/2))
```

Rubi in Sympy [A] time = 28.812, size = 112, normalized size = 0.89

$$\frac{x\left(\frac{5a}{16d^3}+\frac{c}{16de^2}\right)}{d+ex^2}+\frac{x\left(\frac{5a}{24d^2}-\frac{7c}{24e^2}\right)}{\left(d+ex^2\right)^2}+\frac{x\left(\frac{a}{6d}+\frac{cd}{6e^2}\right)}{\left(d+ex^2\right)^3}+\frac{\left(5ae^2+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{\frac{7}{2}}e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+a)/(e*x**2+d)**4,x)
```

```
[Out] x*(5*a/(16*d**3) + c/(16*d*e**2))/(d + e*x**2) + x*(5*a/(24*d**2)
 - 7*c/(24*e**2))/(d + e*x**2)**2 + x*(a/(6*d) + c*d/(6*e**2))/(d
 + e*x**2)**3 + (5*a*e**2 + c*d**2)*atan(sqrt(e)*x/sqrt(d))/(16*d
**(7/2)*e**(5/2))
```

Mathematica [A] time = 0.139761, size = 113, normalized size = 0.9

$$\frac{\left(5ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}+\frac{x\left(ae^2\left(33d^2+40dex^2+15e^2x^4\right)+cd^2\left(-3d^2-8dex^2+3e^2x^4\right)\right)}{48d^3e^2\left(d+ex^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] (x*(c*d^2*(-3*d^2 - 8*d*e*x^2 + 3*e^2*x^4) + a*e^2*(33*d^2 + 40*d
*e*x^2 + 15*e^2*x^4)))/(48*d^3*e^2*(d + e*x^2)^3) + ((c*d^2 + 5*a
```

```
*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))
```

Maple [A] time = 0.013, size = 122, normalized size = 1.

$$\frac{1}{(ex^2+d)^3}\left(\frac{(5\,ae^2+cd^2)\,x^5}{16\,d^3}+\frac{(5\,ae^2-cd^2)\,x^3}{6\,d^2e}+\frac{(11\,ae^2-cd^2)\,x}{16\,e^2d}\right)+\frac{5\,a}{16\,d^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{c}{16\,e^2d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+a)/(e*x^2+d)^4,x)
```

```
[Out] (1/16*(5*a*e^2+c*d^2)/d^3*x^5+1/6*(5*a*e^2-c*d^2)/d^2/e*x^3+1/16*
(11*a*e^2-c*d^2)/e^2/d*x)/(e*x^2+d)^3+5/16/d^3/(d*e)^(1/2)*arctan
(x*e/(d*e)^(1/2))*a+1/16/d/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)
)*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)/(e*x^2 + d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.287885, size = 1, normalized size = 0.01

$$\left[\frac{3\left(\left(cd^2e^3+5\,ae^5\right)x^6+cd^5+5\,ad^3e^2+3\left(cd^3e^2+5\,ade^4\right)x^4+3\left(cd^4e+5\,ad^2e^3\right)x^2\right)\log\left(\frac{2\,dex+\left(ex^2-d\right)\sqrt{-de}}{ex^2+d}\right)+2\left(3\left(cd^2e^2\right.\right.}{96\left(d^3e^5x^6+3\,d^4e^4x^4+3\,d^5e^3x^2+d^6e^2\right)\sqrt{-de}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)/(e*x^2 + d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(3*((c*d^2*e^3 + 5*a*e^5)*x^6 + c*d^5 + 5*a*d^3*e^2 + 3*(c*
d^3*e^2 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e + 5*a*d^2*e^3)*x^2)*log((2*
d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d)) + 2*(3*(c*d^2*e^2 +
5*a*e^4)*x^5 - 8*(c*d^3*e - 5*a*d*e^3)*x^3 - 3*(c*d^4 - 11*a*d^2*
e^2)*x)*sqrt(-d*e))/((d^3*e^5*x^6 + 3*d^4*e^4*x^4 + 3*d^5*e^3*x^2
 + d^6*e^2)*sqrt(-d*e)), 1/48*(3*((c*d^2*e^3 + 5*a*e^5)*x^6 + c*d
^5 + 5*a*d^3*e^2 + 3*(c*d^3*e^2 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e + 5
*a*d^2*e^3)*x^2)*arctan(sqrt(d*e)*x/d) + (3*(c*d^2*e^2 + 5*a*e^4)
*x^5 - 8*(c*d^3*e - 5*a*d*e^3)*x^3 - 3*(c*d^4 - 11*a*d^2*e^2)*x)*
sqrt(d*e))/((d^3*e^5*x^6 + 3*d^4*e^4*x^4 + 3*d^5*e^3*x^2 + d^6*e^
2)*sqrt(d*e))]
```

Sympy [A] time = 4.03331, size = 204, normalized size = 1.62

$$-\frac{\sqrt{-\frac{1}{d^7e^5}}\left(5ae^2+cd^2\right)\log\left(-d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{d^7e^5}}\left(5ae^2+cd^2\right)\log\left(d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}+\frac{x^5\left(15ae^4+3cd^2e^2\right)+x^3\left(40ade^3-8cd^3e\right)+x\left(33ad^2e^2-3cd^4\right)}{48d^6e^2+144d^5e^3x^2+144d^4e^4x^4+48d^3e^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+a)/(e*x**2+d)**4,x)
```

```
[Out] -sqrt(-1/(d**7*e**5))*(5*a*e**2 + c*d**2)*log(-d**4*e**2*sqrt(-1/
(d**7*e**5)) + x)/32 + sqrt(-1/(d**7*e**5))*(5*a*e**2 + c*d**2)*l
og(d**4*e**2*sqrt(-1/(d**7*e**5)) + x)/32 + (x**5*(15*a*e**4 + 3*
c*d**2*e**2) + x**3*(40*a*d*e**3 - 8*c*d**3*e) + x*(33*a*d**2*e**
2 - 3*c*d**4))/(48*d**6*e**2 + 144*d**5*e**3*x**2 + 144*d**4*e**4
*x**4 + 48*d**3*e**5*x**6)
```

GIAC/XCAS [A] time = 0.27065, size = 135, normalized size = 1.07

$$\frac{\left(cd^2+5\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{16\,d^{\frac{7}{2}}}+\frac{\left(3\,cd^2x^5e^2-8\,cd^3x^3e+15\,ax^5e^4-3\,cd^4x+40\,adx^3e^3+33\,ad^2xe^2\right)e^{(-2)}}{48\,(x^2e+d)^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)/(e*x^2 + d)^4,x, algorithm="giac")
```

```
[Out] 1/16*(c*d^2 + 5*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(7/2)
 + 1/48*(3*c*d^2*x^5*e^2 - 8*c*d^3*x^3*e + 15*a*x^5*e^4 - 3*c*d^4
*x + 40*a*d*x^3*e^3 + 33*a*d^2*x*e^2)*e^(-2)/((x^2*e + d)^3*d^3)
```

3.136 $\int (d+ex^2)^3 (a+cx^4)^2 \, dx$

Optimal. Leaf size=133

$$a^2d^3x+a^2d^2ex^3+\frac{1}{11}cex^{11}\left(2ae^2+3cd^2\right)+\frac{1}{9}cdx^9\left(6ae^2+cd^2\right)+\frac{1}{7}aex^7\left(ae^2+6cd^2\right)+\frac{1}{5}adx^5\left(3ae^2+2cd^2\right)+\frac{3}{13}c^2de^2x^{13}+\frac{1}{15}c^2e^3x^{15}$$

```
[Out] a^2*d^3*x + a^2*d^2*e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*
e*(6*c*d^2 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e
*(3*c*d^2 + 2*a*e^2)*x^11)/11 + (3*c^2*d*e^2*x^13)/13 + (c^2*e^3*
x^15)/15
```

Rubi [A] time = 0.222269, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$a^2d^3x+a^2d^2ex^3+\frac{1}{11}cex^{11}\left(2ae^2+3cd^2\right)+\frac{1}{9}cdx^9\left(6ae^2+cd^2\right)+\frac{1}{7}aex^7\left(ae^2+6cd^2\right)+\frac{1}{5}adx^5\left(3ae^2+2cd^2\right)+\frac{3}{13}c^2de^2x^{13}+\frac{1}{15}c^2e^3x^{15}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^3*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + a^2*d^2*e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*
e*(6*c*d^2 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e
*(3*c*d^2 + 2*a*e^2)*x^11)/11 + (3*c^2*d*e^2*x^13)/13 + (c^2*e^3*
x^15)/15
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2d^2ex^3+\frac{adx^5\left(3ae^2+2cd^2\right)}{5}+\frac{aex^7\left(ae^2+6cd^2\right)}{7}+\frac{3c^2de^2x^{13}}{13}+\frac{c^2e^3x^{15}}{15}+\frac{cdx^9\left(6ae^2+cd^2\right)}{9}+\frac{cex^{11}\left(2ae^2+3cd^2\right)}{11}+d^3\int a^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**3*(c*x**4+a)**2,x)
```

```
[Out] a**2*d**2*e*x**3 + a*d*x**5*(3*a*e**2 + 2*c*d**2)/5 + a*e*x**7*(a
*e**2 + 6*c*d**2)/7 + 3*c**2*d*e**2*x**13/13 + c**2*e**3*x**15/15
 + c*d*x**9*(6*a*e**2 + c*d**2)/9 + c*e*x**11*(2*a*e**2 + 3*c*d**
2)/11 + d**3*Integral(a**2, x)
```

Mathematica [A] time = 0.0396229, size = 133, normalized size = 1.

$$a^2d^3x+a^2d^2ex^3+\frac{1}{11}cex^{11}\left(2ae^2+3cd^2\right)+\frac{1}{9}cdx^9\left(6ae^2+cd^2\right)+\frac{1}{7}aex^7\left(ae^2+6cd^2\right)+\frac{1}{5}adx^5\left(3ae^2+2cd^2\right)+\frac{3}{13}c^2de^2x^{13}+\frac{1}{15}c^2e^3x^{15}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^3*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + a^2*d^2*e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*
e*(6*c*d^2 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e
*(3*c*d^2 + 2*a*e^2)*x^11)/11 + (3*c^2*d*e^2*x^13)/13 + (c^2*e^3*
x^15)/15
```

Maple [A] time = 0.002, size = 130, normalized size = 1.

$$\frac{c^2e^3x^{15}}{15}+\frac{3\,c^2de^2x^{13}}{13}+\frac{\left(2\,e^3ac+3\,d^2ec^2\right)x^{11}}{11}+\frac{\left(6\,acde^2+c^2d^3\right)x^9}{9}+\frac{\left(a^2e^3+6\,acd^2e\right)x^7}{7}+\frac{\left(3\,da^2e^2+2\,d^3ac\right)x^5}{5}+a^2d^2ex^3+a^2d^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^3*(c*x^4+a)^2,x)
```

```
[Out] 1/15*c^2*e^3*x^15+3/13*c^2*d*e^2*x^13+1/11*(2*a*c*e^3+3*c^2*d^2*e
)*x^11+1/9*(6*a*c*d*e^2+c^2*d^3)*x^9+1/7*(a^2*e^3+6*a*c*d^2*e)*x^
7+1/5*(3*a^2*d*e^2+2*a*c*d^3)*x^5+a^2*d^2*e*x^3+a^2*d^3*x
```

Maxima [A] time = 0.737076, size = 174, normalized size = 1.31

$$\frac{1}{15}\,c^2e^3x^{15}+\frac{3}{13}\,c^2de^2x^{13}+\frac{1}{11}\left(3\,c^2d^2e+2\,ace^3\right)x^{11}+\frac{1}{9}\left(c^2d^3+6\,acde^2\right)x^9+a^2d^2ex^3+\frac{1}{7}\left(6\,acd^2e+a^2e^3\right)x^7+a^2d^3x+\frac{1}{5}\left(2\,acd^3+3\,a^2de^2\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2*(e*x^2 + d)^3,x, algorithm="maxima")
```

```
[Out] 1/15*c^2*e^3*x^15 + 3/13*c^2*d*e^2*x^13 + 1/11*(3*c^2*d^2*e + 2*a
*c*e^3)*x^11 + 1/9*(c^2*d^3 + 6*a*c*d*e^2)*x^9 + a^2*d^2*e*x^3 +
1/7*(6*a*c*d^2*e + a^2*e^3)*x^7 + a^2*d^3*x + 1/5*(2*a*c*d^3 + 3*
a^2*d*e^2)*x^5
```

Fricas [A] time = 0.255478, size = 1, normalized size = 0.01

$$\frac{1}{15}x^{15}e^3c^2+\frac{3}{13}x^{13}e^2dc^2+\frac{3}{11}x^{11}ed^2c^2+\frac{2}{11}x^{11}e^3ca+\frac{1}{9}x^9d^3c^2+\frac{2}{3}x^9e^2dca+\frac{6}{7}x^7ed^2ca+\frac{1}{7}x^7e^3a^2+\frac{2}{5}x^5d^3ca+\frac{3}{5}x^5e^2da^2+x^3ed^2a^2+xd^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2*(e*x^2 + d)^3,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*e^3*c^2 + 3/13*x^13*e^2*d*c^2 + 3/11*x^11*e*d^2*c^2 + 2
/11*x^11*e^3*c*a + 1/9*x^9*d^3*c^2 + 2/3*x^9*e^2*d*c*a + 6/7*x^7*
e*d^2*c*a + 1/7*x^7*e^3*a^2 + 2/5*x^5*d^3*c*a + 3/5*x^5*e^2*d*a^2
 + x^3*e*d^2*a^2 + x*d^3*a^2
```

Sympy [A] time = 0.166246, size = 144, normalized size = 1.08

$$a^2d^3x + a^2d^2ex^3 + \frac{3c^2de^2x^{13}}{13} + \frac{c^2e^3x^{15}}{15} + x^{11}\left(\frac{2ace^3}{11} + \frac{3c^2d^2e}{11}\right) + x^9\left(\frac{2acde^2}{3} + \frac{c^2d^3}{9}\right) + x^7\left(\frac{a^2e^3}{7} + \frac{6acd^2e}{7}\right) + x^5\left(\frac{3a^2de^2}{5} + \frac{2acd^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**3*(c*x**4+a)**2,x)
```

```
[Out] a**2*d**3*x + a**2*d**2*e*x**3 + 3*c**2*d*e**2*x**13/13 + c**2*e*
*3*x**15/15 + x**11*(2*a*c*e**3/11 + 3*c**2*d**2*e/11) + x**9*(2*
a*c*d*e**2/3 + c**2*d**3/9) + x**7*(a**2*e**3/7 + 6*a*c*d**2*e/7)
 + x**5*(3*a**2*d*e**2/5 + 2*a*c*d**3/5)
```

GIAC/XCAS [A] time = 0.269175, size = 173, normalized size = 1.3

$$\frac{1}{15}c^2x^{15}e^3 + \frac{3}{13}c^2dx^{13}e^2 + \frac{3}{11}c^2d^2x^{11}e + \frac{1}{9}c^2d^3x^9 + \frac{2}{11}acx^{11}e^3 + \frac{2}{3}acdx^9e^2 + \frac{6}{7}acd^2x^7e + \frac{2}{5}acd^3x^5 + \frac{1}{7}a^2x^7e^3 + \frac{3}{5}a^2dx^5e^2 + a^2d^2x^3e + a^2d^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2*(e*x^2 + d)^3,x, algorithm="giac")
```

```
[Out] 1/15*c^2*x^15*e^3 + 3/13*c^2*d*x^13*e^2 + 3/11*c^2*d^2*x^11*e + 1
/9*c^2*d^3*x^9 + 2/11*a*c*x^11*e^3 + 2/3*a*c*d*x^9*e^2 + 6/7*a*c*
d^2*x^7*e + 2/5*a*c*d^3*x^5 + 1/7*a^2*x^7*e^3 + 3/5*a^2*d*x^5*e^2
 + a^2*d^2*x^3*e + a^2*d^3*x
```

3.137 $\int (d+ex^2)^2 (a+cx^4)^2 \, dx$

Optimal. Leaf size=97

$$a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{9}cx^9\left(2ae^2+cd^2\right)+\frac{1}{5}ax^5\left(ae^2+2cd^2\right)+\frac{4}{7}acdex^7+\frac{2}{11}c^2dex^{11}+\frac{1}{13}c^2e^2x^{13}$$

```
[Out] a^2*d^2*x + (2*a^2*d*e*x^3)/3 + (a*(2*c*d^2 + a*e^2)*x^5)/5 + (4*
a*c*d*e*x^7)/7 + (c*(c*d^2 + 2*a*e^2)*x^9)/9 + (2*c^2*d*e*x^11)/1
1 + (c^2*e^2*x^13)/13
```

Rubi [A] time = 0.143199, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{9}cx^9\left(2ae^2+cd^2\right)+\frac{1}{5}ax^5\left(ae^2+2cd^2\right)+\frac{4}{7}acdex^7+\frac{2}{11}c^2dex^{11}+\frac{1}{13}c^2e^2x^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^2*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a^2*d*e*x^3)/3 + (a*(2*c*d^2 + a*e^2)*x^5)/5 + (4*
a*c*d*e*x^7)/7 + (c*(c*d^2 + 2*a*e^2)*x^9)/9 + (2*c^2*d*e*x^11)/1
1 + (c^2*e^2*x^13)/13
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2a^2dex^3}{3}+\frac{4acdex^7}{7}+\frac{ax^5\left(ae^2+2cd^2\right)}{5}+\frac{2c^2dex^{11}}{11}+\frac{c^2e^2x^{13}}{13}+\frac{cx^9\left(2ae^2+cd^2\right)}{9}+d^2\int a^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2*(c*x**4+a)**2,x)
```

```
[Out] 2*a**2*d*e*x**3/3 + 4*a*c*d*e*x**7/7 + a*x**5*(a*e**2 + 2*c*d**2)
/5 + 2*c**2*d*e*x**11/11 + c**2*e**2*x**13/13 + c*x**9*(2*a*e**2
+ c*d**2)/9 + d**2*Integral(a**2, x)
```

Mathematica [A] time = 0.0298394, size = 97, normalized size = 1.

$$a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{9}cx^9\left(2ae^2+cd^2\right)+\frac{1}{5}ax^5\left(ae^2+2cd^2\right)+\frac{4}{7}acdex^7+\frac{2}{11}c^2dex^{11}+\frac{1}{13}c^2e^2x^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^2*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a^2*d*e*x^3)/3 + (a*(2*c*d^2 + a*e^2)*x^5)/5 + (4*
a*c*d*e*x^7)/7 + (c*(c*d^2 + 2*a*e^2)*x^9)/9 + (2*c^2*d*e*x^11)/1
1 + (c^2*e^2*x^13)/13
```

Maple [A] time = 0.001, size = 90, normalized size = 0.9

$$\frac{c^2e^2x^{13}}{13}+\frac{2\,c^2dex^{11}}{11}+\frac{\left(2\,ace^2+c^2d^2\right)x^9}{9}+\frac{4\,acdex^7}{7}+\frac{\left(a^2e^2+2\,acd^2\right)x^5}{5}+\frac{2\,a^2dex^3}{3}+a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^2*(c*x^4+a)^2,x)
```

```
[Out] 1/13*c^2*e^2*x^13+2/11*c^2*d*e*x^11+1/9*(2*a*c*e^2+c^2*d^2)*x^9+4
/7*a*c*d*e*x^7+1/5*(a^2*e^2+2*a*c*d^2)*x^5+2/3*a^2*d*e*x^3+a^2*d^
2*x
```

Maxima [A] time = 0.740474, size = 120, normalized size = 1.24

$$\frac{1}{13}\,c^2e^2x^{13}+\frac{2}{11}\,c^2dex^{11}+\frac{4}{7}\,acdex^7+\frac{1}{9}\,\left(c^2d^2+2\,ace^2\right)x^9+\frac{2}{3}\,a^2dex^3+\frac{1}{5}\,\left(2\,acd^2+a^2e^2\right)x^5+a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2*(e*x^2 + d)^2,x, algorithm="maxima")
```

```
[Out] 1/13*c^2*e^2*x^13 + 2/11*c^2*d*e*x^11 + 4/7*a*c*d*e*x^7 + 1/9*(c^
2*d^2 + 2*a*c*e^2)*x^9 + 2/3*a^2*d*e*x^3 + 1/5*(2*a*c*d^2 + a^2*e
^2)*x^5 + a^2*d^2*x
```

Fricas [A] time = 0.252813, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}e^2c^2+\frac{2}{11}x^{11}edc^2+\frac{1}{9}x^9d^2c^2+\frac{2}{9}x^9e^2ca+\frac{4}{7}x^7edca+\frac{2}{5}x^5d^2ca+\frac{1}{5}x^5e^2a^2+\frac{2}{3}x^3eda^2+xd^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2*(e*x^2 + d)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^2*c^2 + 2/11*x^11*e*d*c^2 + 1/9*x^9*d^2*c^2 + 2/9*x^9
*e^2*c*a + 4/7*x^7*e*d*c*a + 2/5*x^5*d^2*c*a + 1/5*x^5*e^2*a^2 +
2/3*x^3*e*d*a^2 + x*d^2*a^2
```

Sympy [A] time = 0.138414, size = 104, normalized size = 1.07

$$a^2d^2x+\frac{2a^2dex^3}{3}+\frac{4acdex^7}{7}+\frac{2c^2dex^{11}}{11}+\frac{c^2e^2x^{13}}{13}+x^9\left(\frac{2ace^2}{9}+\frac{c^2d^2}{9}\right)+x^5\left(\frac{a^2e^2}{5}+\frac{2acd^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**2*(c*x**4+a)**2,x)
```

```
[Out] a**2*d**2*x + 2*a**2*d*e*x**3/3 + 4*a*c*d*e*x**7/7 + 2*c**2*d*e*x
**11/11 + c**2*e**2*x**13/13 + x**9*(2*a*c*e**2/9 + c**2*d**2/9)
+ x**5*(a**2*e**2/5 + 2*a*c*d**2/5)
```

GIAC/XCAS [A] time = 0.267689, size = 123, normalized size = 1.27

$$\frac{1}{13}\,c^2x^{13}e^2+\frac{2}{11}\,c^2dx^{11}e+\frac{1}{9}\,c^2d^2x^9+\frac{2}{9}\,acx^9e^2+\frac{4}{7}\,acdx^7e+\frac{2}{5}\,acd^2x^5+\frac{1}{5}\,a^2x^5e^2+\frac{2}{3}\,a^2dx^3e+a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2*(e*x^2 + d)^2,x, algorithm="giac")
```

```
[Out] 1/13*c^2*x^13*e^2 + 2/11*c^2*d*x^11*e + 1/9*c^2*d^2*x^9 + 2/9*a*c
*x^9*e^2 + 4/7*a*c*d*x^7*e + 2/5*a*c*d^2*x^5 + 1/5*a^2*x^5*e^2 +
2/3*a^2*d*x^3*e + a^2*d^2*x
```

3.138 $\int (d+ex^2) (a+cx^4)^2 \, dx$

Optimal. Leaf size=60

$$a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

```
[Out] a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^
2*d*x^9)/9 + (c^2*e*x^11)/11
```

Rubi [A] time = 0.0646378, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)*(a + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^
2*d*x^9)/9 + (c^2*e*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2ex^3}{3}+a^2\int d\,dx+\frac{2acdx^5}{5}+\frac{2acex^7}{7}+\frac{c^2dx^9}{9}+\frac{c^2ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)*(c*x**4+a)**2,x)
```

```
[Out] a**2*e*x**3/3 + a**2*Integral(d, x) + 2*a*c*d*x**5/5 + 2*a*c*e*x*
*7/7 + c**2*d*x**9/9 + c**2*e*x**11/11
```

Mathematica [A] time = 0.00456328, size = 60, normalized size = 1.

$$a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)*(a + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^
2*d*x^9)/9 + (c^2*e*x^11)/11
```

Maple [A] time = 0.001, size = 51, normalized size = 0.9

$$a^2dx+\frac{a^2ex^3}{3}+\frac{2\,acdx^5}{5}+\frac{2\,acex^7}{7}+\frac{c^2dx^9}{9}+\frac{c^2ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)*(c*x^4+a)^2,x)
```

```
[Out] a^2*d*x+1/3*a^2*e*x^3+2/5*a*c*d*x^5+2/7*a*c*e*x^7+1/9*c^2*d*x^9+1
/11*c^2*e*x^11
```

Maxima [A] time = 0.732181, size = 68, normalized size = 1.13

$$\frac{1}{11}c^2ex^{11}+\frac{1}{9}c^2dx^9+\frac{2}{7}acex^7+\frac{2}{5}acdx^5+\frac{1}{3}a^2ex^3+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2*(e*x^2 + d),x, algorithm="maxima")
```

```
[Out] 1/11*c^2*e*x^11 + 1/9*c^2*d*x^9 + 2/7*a*c*e*x^7 + 2/5*a*c*d*x^5 +
 1/3*a^2*e*x^3 + a^2*d*x
```

Fricas [A] time = 0.256993, size = 1, normalized size = 0.02

$$\frac{1}{11}x^{11}ec^2+\frac{1}{9}x^9dc^2+\frac{2}{7}x^7eca+\frac{2}{5}x^5dca+\frac{1}{3}x^3ea^2+xda^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2*(e*x^2 + d),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e*c^2 + 1/9*x^9*d*c^2 + 2/7*x^7*e*c*a + 2/5*x^5*d*c*a +
 1/3*x^3*e*a^2 + x*d*a^2
```

Sympy [A] time = 0.108154, size = 60, normalized size = 1.

$$a^2dx+\frac{a^2ex^3}{3}+\frac{2acdx^5}{5}+\frac{2acex^7}{7}+\frac{c^2dx^9}{9}+\frac{c^2ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)*(c*x**4+a)**2,x)
```

```
[Out] a**2*d*x + a**2*e*x**3/3 + 2*a*c*d*x**5/5 + 2*a*c*e*x**7/7 + c**2
*d*x**9/9 + c**2*e*x**11/11
```

GIAC/XCAS [A] time = 0.268951, size = 72, normalized size = 1.2

$$\frac{1}{11}c^2x^{11}e+\frac{1}{9}c^2dx^9+\frac{2}{7}acx^7e+\frac{2}{5}acdx^5+\frac{1}{3}a^2x^3e+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2*(e*x^2 + d),x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11*e + 1/9*c^2*d*x^9 + 2/7*a*c*x^7*e + 2/5*a*c*d*x^5 +
 1/3*a^2*x^3*e + a^2*d*x
```

3.139 $\int \left(a+cx^4\right)^2 \, dx$

Optimal. Leaf size=25

$$a^2x+\frac{2}{5}acx^5+\frac{c^2x^9}{9}$$

```
[Out] a^2*x + (2*a*c*x^5)/5 + (c^2*x^9)/9
```

Rubi [A] time = 0.0188496, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$a^2x+\frac{2}{5}acx^5+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^4)^2,x]

[Out] a^2*x + (2*a*c*x^5)/5 + (c^2*x^9)/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2acx^5}{5}+\frac{c^2x^9}{9}+\int a^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+a)**2,x)

[Out] 2*a*c*x**5/5 + c**2*x**9/9 + Integral(a**2, x)
```

Mathematica [A] time = 0.00110682, size = 25, normalized size = 1.

$$a^2x+\frac{2}{5}acx^5+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^4)^2,x]

[Out] a^2*x + (2*a*c*x^5)/5 + (c^2*x^9)/9
```

Maple [A] time = 0.001, size = 22, normalized size = 0.9

$$a^2x+\frac{2\,acx^5}{5}+\frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+a)^2,x)
```

```
[Out] a^2*x+2/5*a*c*x^5+1/9*c^2*x^9
```

Maxima [A] time = 0.748751, size = 28, normalized size = 1.12

$$\frac{1}{9}c^2x^9 + \frac{2}{5}acx^5 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

Fricas [A] time = 0.258944, size = 1, normalized size = 0.04

$$\frac{1}{9}x^9c^2 + \frac{2}{5}x^5ca + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^2 + 2/5*x^5*c*a + x*a^2
```

Sympy [A] time = 0.077818, size = 22, normalized size = 0.88

$$a^2x + \frac{2acx^5}{5} + \frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+a)**2,x)
```

```
[Out] a**2*x + 2*a*c*x**5/5 + c**2*x**9/9
```

GIAC/XCAS [A] time = 0.26951, size = 28, normalized size = 1.12

$$\frac{1}{9}c^2x^9 + \frac{2}{5}acx^5 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

3.140 $\int \frac{(a+cx^4)^2}{d+ex^2} dx$

Optimal. Leaf size=108

$$\frac{(ae^2+cd^2)^2 \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}} - \frac{cdx(2ae^2+cd^2)}{e^4} + \frac{cx^3(2ae^2+cd^2)}{3e^3} - \frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e}$$

```
[Out] -((c*d*(c*d^2 + 2*a*e^2)*x)/e^4) + (c*(c*d^2 + 2*a*e^2)*x^3)/(3*e
^3) - (c^2*d*x^5)/(5*e^2) + (c^2*x^7)/(7*e) + ((c*d^2 + a*e^2)^2*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(9/2))
```

Rubi [A] time = 0.147923, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{(ae^2+cd^2)^2 \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}} - \frac{cdx(2ae^2+cd^2)}{e^4} + \frac{cx^3(2ae^2+cd^2)}{3e^3} - \frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^4)^2/(d + e*x^2),x]
```

```
[Out] -((c*d*(c*d^2 + 2*a*e^2)*x)/e^4) + (c*(c*d^2 + 2*a*e^2)*x^3)/(3*e
^3) - (c^2*d*x^5)/(5*e^2) + (c^2*x^7)/(7*e) + ((c*d^2 + a*e^2)^2*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(9/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e} + \frac{cx^3(2ae^2+cd^2)}{3e^3} - \frac{d(2ae^2+cd^2)\int c\,dx}{e^4} + \frac{(ae^2+cd^2)^2 \operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+a)**2/(e*x**2+d),x)
```

```
[Out] -c**2*d*x**5/(5*e**2) + c**2*x**7/(7*e) + c*x**3*(2*a*e**2 + c*d*
*2)/(3*e**3) - d*(2*a*e**2 + c*d**2)*Integral(c, x)/e**4 + (a*e**
2 + c*d**2)**2*atan(sqrt(e)*x/sqrt(d))/(sqrt(d)*e**(9/2))
```

Mathematica [A] time = 0.127111, size = 97, normalized size = 0.9

$$\frac{(ae^2+cd^2)^2 \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}} + \frac{cx\left(70ae^2(ex^2-3d)+c(-105d^3+35d^2ex^2-21de^2x^4+15e^3x^6)\right)}{105e^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^4)^2/(d + e*x^2),x]
```

```
[Out] (c*x*(70*a*e^2*(-3*d + e*x^2) + c*(-105*d^3 + 35*d^2*e*x^2 - 21*d
*e^2*x^4 + 15*e^3*x^6)))/(105*e^4) + ((c*d^2 + a*e^2)^2*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(9/2))
```

Maple [A] time = 0.005, size = 136, normalized size = 1.3

$$\frac{c^2x^7}{7\,e}-\frac{c^2dx^5}{5\,e^2}+\frac{2\,cx^3a}{3\,e}+\frac{c^2x^3d^2}{3\,e^3}-2\,\frac{acdx}{e^2}-\frac{c^2d^3x}{e^4}+a^2\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+2\,\frac{acd^2}{e^2\sqrt{de}}\arctan\left(\frac{ex}{\sqrt{de}}\right)+\frac{c^2d^4}{e^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+a)^2/(e*x^2+d),x)
```

```
[Out] 1/7*c^2*x^7/e-1/5*c^2*d*x^5/e^2+2/3*c/e*x^3*a+1/3*c^2/e^3*x^3*d^2
-2*c/e^2*d*a*x-c^2/e^4*d^3*x+1/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)
)*a^2+2/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*c*d^2+1/e^4/(d*
e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c^2*d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2/(e*x^2 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.319095, size = 1, normalized size = 0.01

$$\left[\frac{105\,\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\log\left(\frac{2\,dex+\left(ex^2-d\right)\sqrt{-de}}{ex^2+d}\right)+2\,\left(15\,c^2e^3x^7-21\,c^2de^2x^5+35\,\left(c^2d^2e+2\,ace^3\right)x^3-105\,\left(c^2d^3+2\,a\right.\right.}{210\,\sqrt{-de}e^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2/(e*x^2 + d),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*log((2*d*e*x + (e
*x^2 - d)*sqrt(-d*e))/(e*x^2 + d)) + 2*(15*c^2*e^3*x^7 - 21*c^2*d
*e^2*x^5 + 35*(c^2*d^2*e + 2*a*c*e^3)*x^3 - 105*(c^2*d^3 + 2*a*c*
d*e^2)*x)*sqrt(-d*e))/(sqrt(-d*e)*e^4), 1/105*(105*(c^2*d^4 + 2*a
*c*d^2*e^2 + a^2*e^4)*arctan(sqrt(d*e)*x/d) + (15*c^2*e^3*x^7 - 2
1*c^2*d*e^2*x^5 + 35*(c^2*d^2*e + 2*a*c*e^3)*x^3 - 105*(c^2*d^3 +
 2*a*c*d*e^2)*x)*sqrt(d*e))/(sqrt(d*e)*e^4)]
```

Sympy [A] time = 2.19622, size = 235, normalized size = 2.18

$$-\frac{c^2dx^5}{5e^2}+\frac{c^2x^7}{7e}-\frac{\sqrt{-\frac{1}{de^9}}\left(ae^2+cd^2\right)^2\log\left(-\frac{de^4\sqrt{-\frac{1}{de^9}}\left(ae^2+cd^2\right)^2}{a^2e^4+2acd^2e^2+c^2d^4}+x\right)}{2}+\frac{\sqrt{-\frac{1}{de^9}}\left(ae^2+cd^2\right)^2\log\left(\frac{de^4\sqrt{-\frac{1}{de^9}}\left(ae^2+cd^2\right)^2}{a^2e^4+2acd^2e^2+c^2d^4}+x\right)}{2}+\frac{x^3\left(2ace^2+c^2d^2\right)}{3e^3}-\frac{x\left(2acde^2+c^2d^3\right)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+a)**2/(e*x**2+d),x)
```

```
[Out] -c**2*d*x**5/(5*e**2) + c**2*x**7/(7*e) - sqrt(-1/(d*e**9))*(a*e*
*2 + c*d**2)**2*log(-d*e**4*sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**
2/(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4) + x)/2 + sqrt(-1/(d*e
**9))*(a*e**2 + c*d**2)**2*log(d*e**4*sqrt(-1/(d*e**9))*(a*e**2 +
 c*d**2)**2/(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4) + x)/2 + x*
*3*(2*a*c*e**2 + c**2*d**2)/(3*e**3) - x*(2*a*c*d*e**2 + c**2*d**
3)/e**4
```

GIAC/XCAS [A] time = 0.268768, size = 142, normalized size = 1.31

$$\frac{\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{\sqrt{d}} + \frac{1}{105}\left(15\,c^2x^7e^6 - 21\,c^2dx^5e^5 + 35\,c^2d^2x^3e^4 - 105\,c^2d^3xe^3 + 70\,acx^3e^6 - 210\,acdxe^5\right)e^{(-7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2/(e*x^2 + d),x, algorithm="giac")
```

```
[Out] (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(
-9/2)/sqrt(d) + 1/105*(15*c^2*x^7*e^6 - 21*c^2*d*x^5*e^5 + 35*c^2
*d^2*x^3*e^4 - 105*c^2*d^3*x*e^3 + 70*a*c*x^3*e^6 - 210*a*c*d*x*e
^5)*e^(-7)
```

3.141 $\int \frac{(a+cx^4)^2}{(d+ex^2)^2} dx$

Optimal. Leaf size=131

$$\frac{x(ae^2+cd^2)^2}{2de^4(d+ex^2)}+\frac{cx(2ae^2+3cd^2)}{e^4}-\frac{(7cd^2-ae^2)(ae^2+cd^2)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}-\frac{2c^2dx^3}{3e^3}+\frac{c^2x^5}{5e^2}$$

```
[Out] (c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (2*c^2*d*x^3)/(3*e^3) + (c^2*x^5)
/(5*e^2) + ((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c*d^
2 - a*e^2)*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2
)*e^(9/2))
```

Rubi [A] time = 0.299731, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{x(ae^2+cd^2)^2}{2de^4(d+ex^2)}+\frac{cx(2ae^2+3cd^2)}{e^4}-\frac{(7cd^2-ae^2)(ae^2+cd^2)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}-\frac{2c^2dx^3}{3e^3}+\frac{c^2x^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^4)^2/(d + e*x^2)^2,x]
```


```
[Out] (c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (2*c^2*d*x^3)/(3*e^3) + (c^2*x^5)
/(5*e^2) + ((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c*d^
2 - a*e^2)*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2
)*e^(9/2))
```

Rubi in Sympy [A] time = 79.1253, size = 146, normalized size = 1.11

$$-\frac{7c^2dx^3}{15e^3}+\frac{c^2x^7}{5e(d+ex^2)}+\frac{2cx(5ae^2+7cd^2)}{5e^4}+\frac{x(5a^2e^4+10acd^2e^2+7c^2d^4)}{10de^4(d+ex^2)}+\frac{(ae^2-7cd^2)(ae^2+cd^2)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{\frac{3}{2}}e^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+a)**2/(e*x**2+d)**2,x)
```

```
[Out] -7*c**2*d*x**3/(15*e**3) + c**2*x**7/(5*e*(d + e*x**2)) + 2*c*x*(
5*a*e**2 + 7*c*d**2)/(5*e**4) + x*(5*a**2*e**4 + 10*a*c*d**2*e**2
 + 7*c**2*d**4)/(10*d*e**4*(d + e*x**2)) + (a*e**2 - 7*c*d**2)*(a
*e**2 + c*d**2)*atan(sqrt(e)*x/sqrt(d))/(2*d**(3/2)*e**(9/2))
```

Mathematica [A] time = 0.189828, size = 134, normalized size = 1.02

$$-\frac{(-a^2e^4+6acd^2e^2+7c^2d^4)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}+\frac{x(ae^2+cd^2)^2}{2de^4(d+ex^2)}+\frac{cx(2ae^2+3cd^2)}{e^4}-\frac{2c^2dx^3}{3e^3}+\frac{c^2x^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^4)^2/(d + e*x^2)^2,x]
```

```
[Out] (c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (2*c^2*d*x^3)/(3*e^3) + (c^2*x^5)
/(5*e^2) + ((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c^2*
d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^
(3/2)*e^(9/2))
```

Maple [A] time = 0.013, size = 170, normalized size = 1.3

$$\frac{c^2x^5}{5\,e^2} - \frac{2\,c^2dx^3}{3\,e^3} + 2\,\frac{xac}{e^2} + 3\,\frac{c^2d^2x}{e^4} + \frac{a^2x}{2\,d\,(ex^2+d)} + \frac{adxc}{e^2\,(ex^2+d)} + \frac{d^3xc^2}{2\,e^4\,(ex^2+d)} + \frac{a^2}{2\,d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}} - 3\,\frac{acd}{e^2\sqrt{de}}\arctan\left(\frac{ex}{\sqrt{de}}\right) - \frac{7\,c^2d^3}{2\,e^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+a)^2/(e*x^2+d)^2,x)
```

```
[Out] 1/5*c^2*x^5/e^2-2/3*c^2*d*x^3/e^3+2*c/e^2*a*x+3*c^2/e^4*d^2*x+1/2
/d*x/(e*x^2+d)*a^2+1/e^2*d*x/(e*x^2+d)*a*c+1/2/e^4*d^3*x/(e*x^2+d
)*c^2+1/2/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a^2-3/e^2*d/(d*e)
^(1/2)*arctan(x*e/(d*e)^(1/2))*a*c-7/2/e^4*d^3/(d*e)^(1/2)*arctan
(x*e/(d*e)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2/(e*x^2 + d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.289905, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(7\,c^2d^5+6\,acd^3e^2-a^2de^4+\left(7\,c^2d^4e+6\,acd^2e^3-a^2e^5\right)x^2\right)\log\left(\frac{2\,dex+(ex^2-d)\sqrt{-de}}{ex^2+d}\right)-2\left(6\,c^2de^3x^7-14\,c^2d^2e^2x^5+1\right.}{60\,(de^5x^2+d^2e^4)\sqrt{-de}}\right.$$

$$-\frac{15\left(7\,c^2d^5+6\,acd^3e^2-a^2de^4+\left(7\,c^2d^4e+6\,acd^2e^3-a^2e^5\right)x^2\right)\arctan\left(\frac{\sqrt{de}x}{d}\right)-\left(6\,c^2de^3x^7-14\,c^2d^2e^2x^5+10\left(7\,c^2d^3e+\right.\right.}{30\,(de^5x^2+d^2e^4)\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2/(e*x^2 + d)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(7*c^2*d^5 + 6*a*c*d^3*e^2 - a^2*d*e^4 + (7*c^2*d^4*e
+ 6*a*c*d^2*e^3 - a^2*e^5)*x^2)*log((2*d*e*x + (e*x^2 - d)*sqrt(-
d*e))/(e*x^2 + d)) - 2*(6*c^2*d*e^3*x^7 - 14*c^2*d^2*e^2*x^5 + 10
*(7*c^2*d^3*e + 6*a*c*d*e^3)*x^3 + 15*(7*c^2*d^4 + 6*a*c*d^2*e^2
+ a^2*e^4)*x)*sqrt(-d*e))/((d*e^5*x^2 + d^2*e^4)*sqrt(-d*e)), -1/
30*(15*(7*c^2*d^5 + 6*a*c*d^3*e^2 - a^2*d*e^4 + (7*c^2*d^4*e + 6*
a*c*d^2*e^3 - a^2*e^5)*x^2)*arctan(sqrt(d*e)*x/d) - (6*c^2*d*e^3*
x^7 - 14*c^2*d^2*e^2*x^5 + 10*(7*c^2*d^3*e + 6*a*c*d*e^3)*x^3 + 1
5*(7*c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(d*e))/((d*e^5*x^2
```

```
+ d^2*e^4)*sqrt(d*e))]
```

Sympy [A] time = 3.76256, size = 314, normalized size = 2.4

$$-\frac{2c^2dx^3}{3e^3}+\frac{c^2x^5}{5e^2}+\frac{x\left(a^2e^4+2acd^2e^2+c^2d^4\right)}{2d^2e^4+2de^5x^2}-\frac{\sqrt{-\frac{1}{d^3e^9}}\left(ae^2-7cd^2\right)\left(ae^2+cd^2\right)\log\left(-\frac{d^2e^4\sqrt{-\frac{1}{d^3e^9}}\left(ae^2-7cd^2\right)\left(ae^2+cd^2\right)}{a^2e^4-6acd^2e^2-7c^2d^4}+x\right)}{4}+\frac{\sqrt{-\frac{1}{d^3e^9}}\left(ae^2-7cd^2\right)\left(ae^2+cd^2\right)\log\left(\frac{d^2e^4\sqrt{-\frac{1}{d^3e^9}}\left(ae^2-7cd^2\right)\left(ae^2+cd^2\right)}{a^2e^4-6acd^2e^2-7c^2d^4}+x\right)}{4}+\frac{x\left(2ace^2+3c^2d^2\right)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+a)**2/(e*x**2+d)**2,x)
```

```
[Out] -2*c**2*d*x**3/(3*e**3) + c**2*x**5/(5*e**2) + x*(a**2*e**4 + 2*a
*c*d**2*e**2 + c**2*d**4)/(2*d**2*e**4 + 2*d*e**5*x**2) - sqrt(-1
/(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)*log(-d**2*e**
4*sqrt(-1/(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)/(a**
2*e**4 - 6*a*c*d**2*e**2 - 7*c**2*d**4) + x)/4 + sqrt(-1/(d**3*e*
*9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)*log(d**2*e**4*sqrt(-1/
(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)/(a**2*e**4 - 6
*a*c*d**2*e**2 - 7*c**2*d**4) + x)/4 + x*(2*a*c*e**2 + 3*c**2*d**
2)/e**4
```

GIAC/XCAS [A] time = 0.270411, size = 173, normalized size = 1.32

$$\frac{1}{15}\left(3\,c^2x^5e^8-10\,c^2dx^3e^7+45\,c^2d^2xe^6+30\,acxe^8\right)e^{(-10)}-\frac{\left(7\,c^2d^4+6\,acd^2e^2-a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{2\,d^{\frac{3}{2}}}+\frac{\left(c^2d^4x+2\,acd^2xe^2+a^2xe^4\right)e^{(-4)}}{2\,(x^2e+d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2/(e*x^2 + d)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*c^2*x^5*e^8 - 10*c^2*d*x^3*e^7 + 45*c^2*d^2*x*e^6 + 30*a*
c*x*e^8)*e^(-10) - 1/2*(7*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*arct
an(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(3/2) + 1/2*(c^2*d^4*x + 2*a*c*d
^2*x*e^2 + a^2*x*e^4)*e^(-4)/((x^2*e + d)*d)
```

3.142 $\int \frac{(a+cx^4)^2}{(d+ex^2)^3} dx$

Optimal. Leaf size=155

$$\frac{(3a^2e^4+6acd^2e^2+35c^2d^4)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}-\frac{x(13cd^2-3ae^2)(ae^2+cd^2)}{8d^2e^4(d+ex^2)}+\frac{x(ae^2+cd^2)^2}{4de^4(d+ex^2)^2}-\frac{3c^2dx}{e^4}+\frac{c^2x^3}{3e^3}$$

```
[Out] (-3*c^2*d*x)/e^4 + (c^2*x^3)/(3*e^3) + ((c*d^2 + a*e^2)^2*x)/(4*d
*e^4*(d + e*x^2)^2) - ((13*c*d^2 - 3*a*e^2)*(c*d^2 + a*e^2)*x)/(8
*d^2*e^4*(d + e*x^2)) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(9/2))
```

Rubi [A] time = 0.398501, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{(3a^2e^4+6acd^2e^2+35c^2d^4)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}-\frac{x(13cd^2-3ae^2)(ae^2+cd^2)}{8d^2e^4(d+ex^2)}+\frac{x(ae^2+cd^2)^2}{4de^4(d+ex^2)^2}-\frac{3c^2dx}{e^4}+\frac{c^2x^3}{3e^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^4)^2/(d + e*x^2)^3,x]
```

```
[Out] (-3*c^2*d*x)/e^4 + (c^2*x^3)/(3*e^3) + ((c*d^2 + a*e^2)^2*x)/(4*d
*e^4*(d + e*x^2)^2) - ((13*c*d^2 - 3*a*e^2)*(c*d^2 + a*e^2)*x)/(8
*d^2*e^4*(d + e*x^2)) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(9/2))
```

Rubi in Sympy [A] time = 85.9228, size = 177, normalized size = 1.14

$$-\frac{7c^2dx}{3e^4}+\frac{c^2x^7}{3e(d+ex^2)^2}+\frac{x(3a^2e^4+6acd^2e^2+7c^2d^4)}{12de^4(d+ex^2)^2}+\frac{x(3a^2e^4-10acd^2e^2-21c^2d^4)}{8d^2e^4(d+ex^2)}+\frac{(3a^2e^4+6acd^2e^2+35c^2d^4)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{\frac{5}{2}}e^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+a)**2/(e*x**2+d)**3,x)
```

```
[Out] -7*c**2*d*x/(3*e**4) + c**2*x**7/(3*e*(d + e*x**2)**2) + x*(3*a**
2*e**4 + 6*a*c*d**2*e**2 + 7*c**2*d**4)/(12*d*e**4*(d + e*x**2)**
2) + x*(3*a**2*e**4 - 10*a*c*d**2*e**2 - 21*c**2*d**4)/(8*d**2*e*
*4*(d + e*x**2)) + (3*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)
*atan(sqrt(e)*x/sqrt(d))/(8*d**(5/2)*e**(9/2))
```

Mathematica [A] time = 0.187493, size = 154, normalized size = 0.99

$$\frac{x\left(3a^2e^4(5d+3ex^2)-6acd^2e^2(3d+5ex^2)-c^2d^2(105d^3+175d^2ex^2+56de^2x^4-8e^3x^6)\right)}{24d^2e^4(d+ex^2)^2}+\frac{(3a^2e^4+6acd^2e^2+35c^2d^4)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^4)^2/(d + e*x^2)^3,x]
```

```
[Out] (x*(3*a^2*e^4*(5*d + 3*e*x^2) - 6*a*c*d^2*e^2*(3*d + 5*e*x^2) - c
^2*d^2*(105*d^3 + 175*d^2*e*x^2 + 56*d*e^2*x^4 - 8*e^3*x^6)))/(24
*d^2*e^4*(d + e*x^2)^2) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^
4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(9/2))
```

Maple [A] time = 0.015, size = 211, normalized size = 1.4

$$\frac{c^2x^3}{3\,e^3}-3\,\frac{c^2dx}{e^4}+\frac{3\,a^2ex^3}{8\,(ex^2+d)^2\,d^2}-\frac{5\,x^3ac}{4\,e\,(ex^2+d)^2}-\frac{13\,d^2x^3c^2}{8\,e^3\,(ex^2+d)^2}+\frac{5\,a^2x}{8\,(ex^2+d)^2\,d}-\frac{3\,adxc}{4\,e^2\,(ex^2+d)^2}-\frac{11\,d^3xc^2}{8\,e^4\,(ex^2+d)^2}+\frac{3\,a^2}{8\,d^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{3\,ac}{4\,e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{35\,c^2d^2}{8\,e^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+a)^2/(e*x^2+d)^3,x)
```

```
[Out] 1/3*c^2*x^3/e^3-3*c^2*d*x/e^4+3/8*e/(e*x^2+d)^2/d^2*x^3*a^2-5/4/e
/(e*x^2+d)^2*x^3*a*c-13/8/e^3/(e*x^2+d)^2*d^2*x^3*c^2+5/8/(e*x^2+
d)^2/d*x*a^2-3/4/e^2/(e*x^2+d)^2*d*x*a*c-11/8/e^4/(e*x^2+d)^2*d^3
*x*c^2+3/8/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a^2+3/4/e^2/(d
*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*c+35/8/e^4*d^2/(d*e)^(1/2)*ar
ctan(x*e/(d*e)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2/(e*x^2 + d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285642, size = 1, normalized size = 0.01

$$\left[\frac{3\left(35\,c^2d^6+6\,acd^4e^2+3\,a^2d^2e^4+\left(35\,c^2d^4e^2+6\,acd^2e^4+3\,a^2e^6\right)x^4+2\left(35\,c^2d^5e+6\,acd^3e^3+3\,a^2de^5\right)x^2\right)\log\left(\frac{2\,dex+(ex^2-}{ex^2+}\right.}{48\left(d^2e^6x^4+2\,d^3e^5\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2/(e*x^2 + d)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(35*c^2*d^6 + 6*a*c*d^4*e^2 + 3*a^2*d^2*e^4 + (35*c^2*d^
4*e^2 + 6*a*c*d^2*e^4 + 3*a^2*e^6)*x^4 + 2*(35*c^2*d^5*e + 6*a*c*
d^3*e^3 + 3*a^2*d*e^5)*x^2)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e)
)/(e*x^2 + d)) + 2*(8*c^2*d^2*e^3*x^7 - 56*c^2*d^3*e^2*x^5 - (175
*c^2*d^4*e + 30*a*c*d^2*e^3 - 9*a^2*e^5)*x^3 - 3*(35*c^2*d^5 + 6*
a*c*d^3*e^2 - 5*a^2*d*e^4)*x)*sqrt(-d*e))/((d^2*e^6*x^4 + 2*d^3*e
^5*x^2 + d^4*e^4)*sqrt(-d*e)), 1/24*(3*(35*c^2*d^6 + 6*a*c*d^4*e^
```

```
2 + 3*a^2*d^2*e^4 + (35*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + 3*a^2*e^6)*
x^4 + 2*(35*c^2*d^5*e + 6*a*c*d^3*e^3 + 3*a^2*d*e^5)*x^2)*arctan(
sqrt(d*e)*x/d) + (8*c^2*d^2*e^3*x^7 - 56*c^2*d^3*e^2*x^5 - (175*c
^2*d^4*e + 30*a*c*d^2*e^3 - 9*a^2*e^5)*x^3 - 3*(35*c^2*d^5 + 6*a*
c*d^3*e^2 - 5*a^2*d*e^4)*x)*sqrt(d*e))/((d^2*e^6*x^4 + 2*d^3*e^5*
x^2 + d^4*e^4)*sqrt(d*e))]
```

Sympy [A] time = 6.49274, size = 257, normalized size = 1.66

$$- \frac{3c^2dx}{e^4} + \frac{c^2x^3}{3e^3} - \frac{\sqrt{-\frac{1}{d^5e^9}}\left(3a^2e^4 + 6acd^2e^2 + 35c^2d^4\right)\log\left(-d^3e^4\sqrt{-\frac{1}{d^5e^9}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{d^5e^9}}\left(3a^2e^4 + 6acd^2e^2 + 35c^2d^4\right)\log\left(d^3e^4\sqrt{-\frac{1}{d^5e^9}} + x\right)}{16} + \frac{x^3\left(3a^2e^5 - 10acd^2e^3 - 13c^2d^4e\right) + x\left(5a^2de^4 - 6acd^3e^2 - 11c^2d^5\right)}{8d^4e^4 + 16d^3e^5x^2 + 8d^2e^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+a)**2/(e*x**2+d)**3,x)
```

```
[Out] -3*c**2*d*x/e**4 + c**2*x**3/(3*e**3) - sqrt(-1/(d**5*e**9))*(3*a
**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(-d**3*e**4*sqrt(-1
/(d**5*e**9)) + x)/16 + sqrt(-1/(d**5*e**9))*(3*a**2*e**4 + 6*a*c
*d**2*e**2 + 35*c**2*d**4)*log(d**3*e**4*sqrt(-1/(d**5*e**9)) + x
)/16 + (x**3*(3*a**2*e**5 - 10*a*c*d**2*e**3 - 13*c**2*d**4*e) +
x*(5*a**2*d*e**4 - 6*a*c*d**3*e**2 - 11*c**2*d**5))/(8*d**4*e**4
+ 16*d**3*e**5*x**2 + 8*d**2*e**6*x**4)
```

GIAC/XCAS [A] time = 0.273537, size = 196, normalized size = 1.26

$$\frac{1}{3}\left(c^2x^3e^6 - 9\,c^2dxe^5\right)e^{(-9)} + \frac{\left(35\,c^2d^4 + 6\,acd^2e^2 + 3\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{8\,d^{\frac{5}{2}}} - \frac{\left(13\,c^2d^4x^3e + 11\,c^2d^5x + 10\,acd^2x^3e^3 + 6\,acd^3xe^2 - 3\,a^2x^3e^5 - 5\,a^2dxe^4\right)e^{(-4)}}{8\left(x^2e + d\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2/(e*x^2 + d)^3,x, algorithm="giac")
```

```
[Out] 1/3*(c^2*x^3*e^6 - 9*c^2*d*x*e^5)*e^(-9) + 1/8*(35*c^2*d^4 + 6*a*
c*d^2*e^2 + 3*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(5/2)
 - 1/8*(13*c^2*d^4*x^3*e + 11*c^2*d^5*x + 10*a*c*d^2*x^3*e^3 + 6*
a*c*d^3*x*e^2 - 3*a^2*x^3*e^5 - 5*a^2*d*x*e^4)*e^(-4)/((x^2*e + d
)^2*d^2)
```

3.143 $\int \frac{(a+cx^4)^2}{(d+ex^2)^4} dx$

Optimal. Leaf size=187

$$-\frac{(-5a^2e^4-2acd^2e^2+35c^2d^4)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}+\frac{x(5a^2e^4+2acd^2e^2+29c^2d^4)}{16d^3e^4(d+ex^2)}-\frac{x(19cd^2-5ae^2)(ae^2+cd^2)}{24d^2e^4(d+ex^2)^2}+\frac{x(ae^2+cd^2)^2}{6de^4(d+ex^2)^3}+\frac{c^2x}{e^4}$$

```
[Out] (c^2*x)/e^4 + ((c*d^2 + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) - ((1
9*c*d^2 - 5*a*e^2)*(c*d^2 + a*e^2)*x)/(24*d^2*e^4*(d + e*x^2)^2)
+ ((29*c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*x)/(16*d^3*e^4*(d + e
*x^2)) - ((35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

Rubi [A] time = 0.498573, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$-\frac{(-5a^2e^4-2acd^2e^2+35c^2d^4)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}+\frac{x(5a^2e^4+2acd^2e^2+29c^2d^4)}{16d^3e^4(d+ex^2)}-\frac{x(19cd^2-5ae^2)(ae^2+cd^2)}{24d^2e^4(d+ex^2)^2}+\frac{x(ae^2+cd^2)^2}{6de^4(d+ex^2)^3}+\frac{c^2x}{e^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (c^2*x)/e^4 + ((c*d^2 + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) - ((1
9*c*d^2 - 5*a*e^2)*(c*d^2 + a*e^2)*x)/(24*d^2*e^4*(d + e*x^2)^2)
+ ((29*c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*x)/(16*d^3*e^4*(d + e
*x^2)) - ((35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

Rubi in Sympy [A] time = 82.6091, size = 207, normalized size = 1.11

$$\frac{c^2x^7}{e(d+ex^2)^3}+\frac{x(a^2e^4+2acd^2e^2+7c^2d^4)}{6de^4(d+ex^2)^3}+\frac{x(5a^2e^4-14acd^2e^2-91c^2d^4)}{24d^2e^4(d+ex^2)^2}+\frac{x(5a^2e^4+2acd^2e^2+77c^2d^4)}{16d^3e^4(d+ex^2)}+\frac{(5a^2e^4+2acd^2e^2-35c^2d^4)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{\frac{7}{2}}e^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+a)**2/(e*x**2+d)**4,x)
```

```
[Out] c**2*x**7/(e*(d + e*x**2)**3) + x*(a**2*e**4 + 2*a*c*d**2*e**2 +
7*c**2*d**4)/(6*d*e**4*(d + e*x**2)**3) + x*(5*a**2*e**4 - 14*a*c
*d**2*e**2 - 91*c**2*d**4)/(24*d**2*e**4*(d + e*x**2)**2) + x*(5*
a**2*e**4 + 2*a*c*d**2*e**2 + 77*c**2*d**4)/(16*d**3*e**4*(d + e*
x**2)) + (5*a**2*e**4 + 2*a*c*d**2*e**2 - 35*c**2*d**4)*atan(sqrt
(e)*x/sqrt(d))/(16*d**(7/2)*e**(9/2))
```

Mathematica [A] time = 0.251571, size = 174, normalized size = 0.93

$$\frac{x\left(a^2e^4\left(33d^2+40dex^2+15e^2x^4\right)-2acd^2e^2\left(3d^2+8dex^2-3e^2x^4\right)+c^2d^3\left(105d^3+280d^2ex^2+231de^2x^4+48e^3x^6\right)\right)}{48d^3e^4\left(d+ex^2\right)^3}-\frac{\left(-5a^2e^4-2acd^2e^2+35c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (x*(-2*a*c*d^2*e^2*(3*d^2 + 8*d*e*x^2 - 3*e^2*x^4) + a^2*e^4*(33*
d^2 + 40*d*e*x^2 + 15*e^2*x^4) + c^2*d^3*(105*d^3 + 280*d^2*e*x^2
 + 231*d*e^2*x^4 + 48*e^3*x^6)))/(48*d^3*e^4*(d + e*x^2)^3) - ((3
5*c^2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(16*d^(7/2)*e^(9/2))
```

Maple [A] time = 0.016, size = 262, normalized size = 1.4

$$\frac{c^2x}{e^4}+\frac{5\,e^2x^5a^2}{16\,(ex^2+d)^3\,d^3}+\frac{acx^5}{8\,(ex^2+d)^3\,d}+\frac{29\,dx^5c^2}{16\,e^2\,(ex^2+d)^3}+\frac{5\,a^2ex^3}{6\,(ex^2+d)^3\,d^2}-\frac{x^3ac}{3\,e\,(ex^2+d)^3}+\frac{17\,d^2x^3c^2}{6\,e^3\,(ex^2+d)^3}+\frac{11\,a^2x}{16\,(ex^2+d)^3\,d}-\frac{adxc}{8\,e^2\,(ex^2+d)^3}+\frac{19\,d^3xc^2}{16\,e^4\,(ex^2+d)^3}+\frac{5\,a^2}{16\,d^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{ac}{8\,e^2d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{35\,c^2d}{16\,e^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+a)^2/(e*x^2+d)^4,x)
```

```
[Out] c^2*x/e^4+5/16*e^2/(e*x^2+d)^3/d^3*x^5*a^2+1/8/(e*x^2+d)^3/d*x^5*
a*c+29/16/e^2/(e*x^2+d)^3*d*x^5*c^2+5/6*e/(e*x^2+d)^3/d^2*x^3*a^2
-1/3/e/(e*x^2+d)^3*x^3*a*c+17/6/e^3/(e*x^2+d)^3*d^2*x^3*c^2+11/16
/(e*x^2+d)^3/d*x*a^2-1/8/e^2/(e*x^2+d)^3*d*x*a*c+19/16/e^4/(e*x^2
+d)^3*d^3*x*c^2+5/16/d^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a^2+
1/8/e^2/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*c-35/16/e^4*d/(d*
e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2/(e*x^2 + d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.296659, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(35\,c^2d^7-2\,acd^5e^2-5\,a^2d^3e^4+\left(35\,c^2d^4e^3-2\,acd^2e^5-5\,a^2e^7\right)x^6+3\left(35\,c^2d^5e^2-2\,acd^3e^4-5\,a^2de^6\right)x^4+3\left(35\,c^2d^6e\right.\right.}{}\right.$$

$$-\frac{3\left(35\,c^2d^7-2\,acd^5e^2-5\,a^2d^3e^4+\left(35\,c^2d^4e^3-2\,acd^2e^5-5\,a^2e^7\right)x^6+3\left(35\,c^2d^5e^2-2\,acd^3e^4-5\,a^2de^6\right)x^4+3\left(35\,c^2d^6e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2/(e*x^2 + d)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(35*c^2*d^7 - 2*a*c*d^5*e^2 - 5*a^2*d^3*e^4 + (35*c^2*d
^4*e^3 - 2*a*c*d^2*e^5 - 5*a^2*e^7)*x^6 + 3*(35*c^2*d^5*e^2 - 2*a
*c*d^3*e^4 - 5*a^2*d*e^6)*x^4 + 3*(35*c^2*d^6*e - 2*a*c*d^4*e^3 -
 5*a^2*d^2*e^5)*x^2)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^
2 + d)) - 2*(48*c^2*d^3*e^3*x^7 + 3*(77*c^2*d^4*e^2 + 2*a*c*d^2*e
^4 + 5*a^2*e^6)*x^5 + 8*(35*c^2*d^5*e - 2*a*c*d^3*e^3 + 5*a^2*d*e
^5)*x^3 + 3*(35*c^2*d^6 - 2*a*c*d^4*e^2 + 11*a^2*d^2*e^4)*x)*sqrt
(-d*e))/((d^3*e^7*x^6 + 3*d^4*e^6*x^4 + 3*d^5*e^5*x^2 + d^6*e^4)*
sqrt(-d*e)), -1/48*(3*(35*c^2*d^7 - 2*a*c*d^5*e^2 - 5*a^2*d^3*e^4
 + (35*c^2*d^4*e^3 - 2*a*c*d^2*e^5 - 5*a^2*e^7)*x^6 + 3*(35*c^2*d
^5*e^2 - 2*a*c*d^3*e^4 - 5*a^2*d*e^6)*x^4 + 3*(35*c^2*d^6*e - 2*a
*c*d^4*e^3 - 5*a^2*d^2*e^5)*x^2)*arctan(sqrt(d*e)*x/d) - (48*c^2*
d^3*e^3*x^7 + 3*(77*c^2*d^4*e^2 + 2*a*c*d^2*e^4 + 5*a^2*e^6)*x^5
+ 8*(35*c^2*d^5*e - 2*a*c*d^3*e^3 + 5*a^2*d*e^5)*x^3 + 3*(35*c^2*
d^6 - 2*a*c*d^4*e^2 + 11*a^2*d^2*e^4)*x)*sqrt(d*e))/((d^3*e^7*x^6
 + 3*d^4*e^6*x^4 + 3*d^5*e^5*x^2 + d^6*e^4)*sqrt(d*e))]
```

Sympy [A] time = 10.5226, size = 292, normalized size = 1.56

$$\frac{c^2x}{e^4}-\frac{\sqrt{-\frac{1}{d^7e^9}}\left(5a^2e^4+2acd^2e^2-35c^2d^4\right)\log\left(-d^4e^4\sqrt{-\frac{1}{d^7e^9}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{d^7e^9}}\left(5a^2e^4+2acd^2e^2-35c^2d^4\right)\log\left(d^4e^4\sqrt{-\frac{1}{d^7e^9}}+x\right)}{32}+\frac{x^5\left(15a^2e^6+6acd^2e^4+87c^2d^4e^2\right)+x^3\left(40a^2de^5-16acd^3e^3+136c^2d^5e\right)+x\left(33a^2d^2e^4-6acd^4e^2+57c^2d^6\right)}{48d^6e^4+144d^5e^5x^2+144d^4e^6x^4+48d^3e^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+a)**2/(e*x**2+d)**4,x)
```

```
[Out] c**2*x/e**4 - sqrt(-1/(d**7*e**9))*(5*a**2*e**4 + 2*a*c*d**2*e**2
 - 35*c**2*d**4)*log(-d**4*e**4*sqrt(-1/(d**7*e**9)) + x)/32 + sq
rt(-1/(d**7*e**9))*(5*a**2*e**4 + 2*a*c*d**2*e**2 - 35*c**2*d**4)
*log(d**4*e**4*sqrt(-1/(d**7*e**9)) + x)/32 + (x**5*(15*a**2*e**6
 + 6*a*c*d**2*e**4 + 87*c**2*d**4*e**2) + x**3*(40*a**2*d*e**5 -
16*a*c*d**3*e**3 + 136*c**2*d**5*e) + x*(33*a**2*d**2*e**4 - 6*a*
c*d**4*e**2 + 57*c**2*d**6))/(48*d**6*e**4 + 144*d**5*e**5*x**2 +
 144*d**4*e**6*x**4 + 48*d**3*e**7*x**6)
```

GIAC/XCAS [A] time = 0.271176, size = 225, normalized size = 1.2

$$c^2xe^{(-4)} - \frac{\left(35\,c^2d^4 - 2\,acd^2e^2 - 5\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{16\,d^{\frac{7}{2}}} + \frac{\left(87\,c^2d^4x^5e^2 + 136\,c^2d^5x^3e + 6\,acd^2x^5e^4 + 57\,c^2d^6x - 16\,acd^3x^3e^3 + 15\,a^2x^5e^6 - 6\,acd^4xe^2 + 40\,a^2dx^3e^5 + 33\,a^2d^2xe^4\right)e^{(-4}}{48\,(x^2e + d)^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2/(e*x^2 + d)^4,x, algorithm="giac")
```

```
[Out] c^2*x*e^(-4) - 1/16*(35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*arct
an(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(7/2) + 1/48*(87*c^2*d^4*x^5*e^2
 + 136*c^2*d^5*x^3*e + 6*a*c*d^2*x^5*e^4 + 57*c^2*d^6*x - 16*a*c*
d^3*x^3*e^3 + 15*a^2*x^5*e^6 - 6*a*c*d^4*x*e^2 + 40*a^2*d*x^3*e^5
 + 33*a^2*d^2*x*e^4)*e^(-4)/((x^2*e + d)^3*d^3)
```

3.144 $\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx$

Optimal. Leaf size=226

$$-\frac{x\left(-35a^2e^4-6acd^2e^2+93c^2d^4\right)}{128d^4e^4\left(d+ex^2\right)}+\frac{\left(35a^2e^4+6acd^2e^2+35c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{128d^{9/2}e^{9/2}}+\frac{x\left(35a^2e^4+6acd^2e^2+163c^2d^4\right)}{192d^3e^4\left(d+ex^2\right)^2}+\frac{x\left(ae^2+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}-\frac{x\left(25cd^2-7ae^2\right)\left(ae^2+cd^2\right)}{48d^2e^4\left(d+ex^2\right)^3}$$

```
[Out] ((c*d^2 + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) - ((25*c*d^2 - 7*a*
e^2)*(c*d^2 + a*e^2)*x)/(48*d^2*e^4*(d + e*x^2)^3) + ((163*c^2*d^
4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*x)/(192*d^3*e^4*(d + e*x^2)^2) -
((93*c^2*d^4 - 6*a*c*d^2*e^2 - 35*a^2*e^4)*x)/(128*d^4*e^4*(d + e
*x^2)) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(128*d^(9/2)*e^(9/2))
```

Rubi [A] time = 0.571897, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$-\frac{x\left(-35a^2e^4-6acd^2e^2+93c^2d^4\right)}{128d^4e^4\left(d+ex^2\right)}+\frac{\left(35a^2e^4+6acd^2e^2+35c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{128d^{9/2}e^{9/2}}+\frac{x\left(35a^2e^4+6acd^2e^2+163c^2d^4\right)}{192d^3e^4\left(d+ex^2\right)^2}+\frac{x\left(ae^2+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}-\frac{x\left(25cd^2-7ae^2\right)\left(ae^2+cd^2\right)}{48d^2e^4\left(d+ex^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((c*d^2 + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) - ((25*c*d^2 - 7*a*
e^2)*(c*d^2 + a*e^2)*x)/(48*d^2*e^4*(d + e*x^2)^3) + ((163*c^2*d^
4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*x)/(192*d^3*e^4*(d + e*x^2)^2) -
((93*c^2*d^4 - 6*a*c*d^2*e^2 - 35*a^2*e^4)*x)/(128*d^4*e^4*(d + e
*x^2)) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(128*d^(9/2)*e^(9/2))
```

Rubi in Sympy [A] time = 92.5557, size = 255, normalized size = 1.13

$$-\frac{c^2x^7}{e\left(d+ex^2\right)^4}+\frac{x\left(a^2e^4+2acd^2e^2-7c^2d^4\right)}{8de^4\left(d+ex^2\right)^4}+\frac{x\left(7a^2e^4-18acd^2e^2+119c^2d^4\right)}{48d^2e^4\left(d+ex^2\right)^3}+\frac{x\left(35a^2e^4+6acd^2e^2-413c^2d^4\right)}{192d^3e^4\left(d+ex^2\right)^2}+\frac{x\left(35a^2e^4+6acd^2e^2+35c^2d^4\right)}{128d^4e^4\left(d+ex^2\right)}+\frac{\left(35a^2e^4+6acd^2e^2+35c^2d^4\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{128d^{\frac{9}{2}}e^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+a)**2/(e*x**2+d)**5,x)
```

```
[Out] -c**2*x**7/(e*(d + e*x**2)**4) + x*(a**2*e**4 + 2*a*c*d**2*e**2 -
 7*c**2*d**4)/(8*d*e**4*(d + e*x**2)**4) + x*(7*a**2*e**4 - 18*a*
c*d**2*e**2 + 119*c**2*d**4)/(48*d**2*e**4*(d + e*x**2)**3) + x*(
35*a**2*e**4 + 6*a*c*d**2*e**2 - 413*c**2*d**4)/(192*d**3*e**4*(d
 + e*x**2)**2) + x*(35*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4
)/(128*d**4*e**4*(d + e*x**2)) + (35*a**2*e**4 + 6*a*c*d**2*e**2
```

```
+ 35*c**2*d**4)*atan(sqrt(e)*x/sqrt(d))/(128*d**(9/2)*e**(9/2))
```

Mathematica [A] time = 0.330338, size = 200, normalized size = 0.88

$$\frac{3\left(35a^2e^4+6acd^2e^2+35c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\frac{\sqrt{d}\sqrt{e}x\left(a^2e^4\left(279d^3+511d^2ex^2+385de^2x^4+105e^3x^6\right)-6acd^2e^2\left(3d^3+11d^2ex^2-11de^2x^4-3e^3x^6\right)-c^2\right.}{(d+ex^2)^4}}{384d^{9/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((Sqrt[d]*Sqrt[e]*x*(-6*a*c*d^2*e^2*(3*d^3 + 11*d^2*e*x^2 - 11*d*
e^2*x^4 - 3*e^3*x^6) + a^2*e^4*(279*d^3 + 511*d^2*e*x^2 + 385*d*e
^2*x^4 + 105*e^3*x^6) - c^2*d^4*(105*d^3 + 385*d^2*e*x^2 + 511*d*
e^2*x^4 + 279*e^3*x^6)))/(d + e*x^2)^4 + 3*(35*c^2*d^4 + 6*a*c*d^
2*e^2 + 35*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(384*d^(9/2)*e^(
9/2))
```

Maple [A] time = 0.016, size = 231, normalized size = 1.

$$\frac{1}{(ex^2+d)^4}\left(\frac{\left(35\,a^2e^4+6\,acd^2e^2-93\,c^2d^4\right)x^7}{128\,d^4e}+\frac{\left(385\,a^2e^4+66\,acd^2e^2-511\,c^2d^4\right)x^5}{384\,d^3e^2}+\frac{\left(511\,a^2e^4-66\,acd^2e^2-385\,c^2d^4\right)x}{384\,d^2e^3}\right.$$
$$+\frac{35\,a^2}{128\,d^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{3\,ac}{64\,d^2e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{35\,c^2}{128\,e^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+a)^2/(e*x^2+d)^5,x)
```

```
[Out] (1/128*(35*a^2*e^4+6*a*c*d^2*e^2-93*c^2*d^4)/d^4/e*x^7+1/384*(385
*a^2*e^4+66*a*c*d^2*e^2-511*c^2*d^4)/d^3/e^2*x^5+1/384*(511*a^2*e
^4-66*a*c*d^2*e^2-385*c^2*d^4)/d^2/e^3*x^3+1/128*(93*a^2*e^4-6*a*
c*d^2*e^2-35*c^2*d^4)/d/e^4*x)/(e*x^2+d)^4+35/128/d^4/(d*e)^(1/2)
*arctan(x*e/(d*e)^(1/2))*a^2+3/64/d^2/e^2/(d*e)^(1/2)*arctan(x*e/
(d*e)^(1/2))*a*c+35/128/e^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c
^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2/(e*x^2 + d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281606, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2/(e*x^2 + d)^5,x, algorithm="fricas")
```

```
[Out] [1/768*(3*(35*c^2*d^8 + 6*a*c*d^6*e^2 + 35*a^2*d^4*e^4 + (35*c^2*
d^4*e^4 + 6*a*c*d^2*e^6 + 35*a^2*e^8)*x^8 + 4*(35*c^2*d^5*e^3 + 6
*a*c*d^3*e^5 + 35*a^2*d*e^7)*x^6 + 6*(35*c^2*d^6*e^2 + 6*a*c*d^4*
e^4 + 35*a^2*d^2*e^6)*x^4 + 4*(35*c^2*d^7*e + 6*a*c*d^5*e^3 + 35*
a^2*d^3*e^5)*x^2)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2 +
 d)) - 2*(3*(93*c^2*d^4*e^3 - 6*a*c*d^2*e^5 - 35*a^2*e^7)*x^7 + (
511*c^2*d^5*e^2 - 66*a*c*d^3*e^4 - 385*a^2*d*e^6)*x^5 + (385*c^2*
d^6*e + 66*a*c*d^4*e^3 - 511*a^2*d^2*e^5)*x^3 + 3*(35*c^2*d^7 + 6
*a*c*d^5*e^2 - 93*a^2*d^3*e^4)*x)*sqrt(-d*e))/((d^4*e^8*x^8 + 4*d
^5*e^7*x^6 + 6*d^6*e^6*x^4 + 4*d^7*e^5*x^2 + d^8*e^4)*sqrt(-d*e))
, 1/384*(3*(35*c^2*d^8 + 6*a*c*d^6*e^2 + 35*a^2*d^4*e^4 + (35*c^2
*d^4*e^4 + 6*a*c*d^2*e^6 + 35*a^2*e^8)*x^8 + 4*(35*c^2*d^5*e^3 +
6*a*c*d^3*e^5 + 35*a^2*d*e^7)*x^6 + 6*(35*c^2*d^6*e^2 + 6*a*c*d^4
*e^4 + 35*a^2*d^2*e^6)*x^4 + 4*(35*c^2*d^7*e + 6*a*c*d^5*e^3 + 35
*a^2*d^3*e^5)*x^2)*arctan(sqrt(d*e)*x/d) - (3*(93*c^2*d^4*e^3 - 6
*a*c*d^2*e^5 - 35*a^2*e^7)*x^7 + (511*c^2*d^5*e^2 - 66*a*c*d^3*e^
4 - 385*a^2*d*e^6)*x^5 + (385*c^2*d^6*e + 66*a*c*d^4*e^3 - 511*a^
2*d^2*e^5)*x^3 + 3*(35*c^2*d^7 + 6*a*c*d^5*e^2 - 93*a^2*d^3*e^4)*
x)*sqrt(d*e))/((d^4*e^8*x^8 + 4*d^5*e^7*x^6 + 6*d^6*e^6*x^4 + 4*d
^7*e^5*x^2 + d^8*e^4)*sqrt(d*e))]
```

Sympy [A] time = 17.5187, size = 335, normalized size = 1.48

$$-\frac{\sqrt{-\frac{1}{d^9e^9}}\left(35a^2e^4+6acd^2e^2+35c^2d^4\right)\log\left(-d^5e^4\sqrt{-\frac{1}{d^9e^9}}+x\right)}{256}+\frac{\sqrt{-\frac{1}{d^9e^9}}\left(35a^2e^4+6acd^2e^2+35c^2d^4\right)\log\left(d^5e^4\sqrt{-\frac{1}{d^9e^9}}+x\right)}{256}+\frac{x^7\left(105a^2e^7+18acd^2e^5-279c^2d^4e^3\right)+x^5\left(385a^2de^6+66acd^3e^4-511c^2d^5e^2\right)+x^3\left(511a^2d^2e^5-66acd^4e^3-385c^2d^6e\right)+}{384d^8e^4+1536d^7e^5x^2+2304d^6e^6x^4+1536d^5e^7x^6+384d^4e^8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+a)**2/(e*x**2+d)**5,x)
```

```
[Out] -sqrt(-1/(d**9*e**9))*(35*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d
**4)*log(-d**5*e**4*sqrt(-1/(d**9*e**9)) + x)/256 + sqrt(-1/(d**9
*e**9))*(35*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(d**5*
e**4*sqrt(-1/(d**9*e**9)) + x)/256 + (x**7*(105*a**2*e**7 + 18*a*
c*d**2*e**5 - 279*c**2*d**4*e**3) + x**5*(385*a**2*d*e**6 + 66*a*
c*d**3*e**4 - 511*c**2*d**5*e**2) + x**3*(511*a**2*d**2*e**5 - 66
*a*c*d**4*e**3 - 385*c**2*d**6*e) + x*(279*a**2*d**3*e**4 - 18*a*
c*d**5*e**2 - 105*c**2*d**7))/(384*d**8*e**4 + 1536*d**7*e**5*x**
2 + 2304*d**6*e**6*x**4 + 1536*d**5*e**7*x**6 + 384*d**4*e**8*x**
8)
```

GIAC/XCAS [A] time = 0.275597, size = 267, normalized size = 1.18

$$\frac{\left(35\,c^2d^4+6\,acd^2e^2+35\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{128\,d^{\frac{9}{2}}}-\frac{\left(279\,c^2d^4x^7e^3+511\,c^2d^5x^5e^2-18\,acd^2x^7e^5+385\,c^2d^6x^3e-66\,acd^3x^5e^4+105\,c^2d^7x-105\,a^2x^7e^7+66\,acd^4x^3e^3-385\,a\right.}{384\,(x^2e+d)^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^2/(e*x^2 + d)^5,x, algorithm="giac")
```

```
[Out] 1/128*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*arctan(x*e^(1/2)/
sqrt(d))*e^(-9/2)/d^(9/2) - 1/384*(279*c^2*d^4*x^7*e^3 + 511*c^2*
d^5*x^5*e^2 - 18*a*c*d^2*x^7*e^5 + 385*c^2*d^6*x^3*e - 66*a*c*d^3
*x^5*e^4 + 105*c^2*d^7*x - 105*a^2*x^7*e^7 + 66*a*c*d^4*x^3*e^3 -
 385*a^2*d*x^5*e^6 + 18*a*c*d^5*x*e^2 - 511*a^2*d^2*x^3*e^5 - 279
*a^2*d^3*x*e^4)*e^(-4)/((x^2*e + d)^4*d^4)
```

3.145 $\int \frac{(d+ex^2)^4}{a+cx^4} dx$

Optimal. Leaf size=437

$$-\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\sqrt{c}de\left(cd^2-ae^2\right)+c^2d^4\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{9/4}}+\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\sqrt{c}de\left(cd^2-ae^2\right)+c^2d^4\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{9/4}}-\frac{\left(a^2e^4-6acd^2e^2+4\sqrt{a}\sqrt{c}de\left(cd^2-ae^2\right)+c^2d^4\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{9/4}}+\frac{\left(a^2e^4-6acd^2e^2+4\sqrt{a}\sqrt{c}de\left(cd^2-ae^2\right)+c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}c^{9/4}}+\frac{e^2x\left(6cd^2-ae^2\right)}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}$$

```
[Out] (e^2*(6*c*d^2 - a*e^2)*x)/c^2 + (4*d*e^3*x^3)/(3*c) + (e^4*x^5)/(
5*c) - ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*
e*(c*d^2 - a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sq
rt[2]*a^(3/4)*c^(9/4)) + ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*
Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*ArcTan[1 + (Sqrt[2]*c^(1/4)*
x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(9/4)) - ((c^2*d^4 - 6*a*c*d^2*
e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
c^(9/4)) + ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c
]*d*e*(c*d^2 - a*e^2))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(9/4))
```

Rubi [A] time = 0.876325, antiderivative size = 437, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$-\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\sqrt{c}de\left(cd^2-ae^2\right)+c^2d^4\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{9/4}}+\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\sqrt{c}de\left(cd^2-ae^2\right)+c^2d^4\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{9/4}}-\frac{\left(a^2e^4-6acd^2e^2+4\sqrt{a}\sqrt{c}de\left(cd^2-ae^2\right)+c^2d^4\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{9/4}}+\frac{\left(a^2e^4-6acd^2e^2+4\sqrt{a}\sqrt{c}de\left(cd^2-ae^2\right)+c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}c^{9/4}}+\frac{e^2x\left(6cd^2-ae^2\right)}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^4/(a + c*x^4),x]
```

```
[Out] (e^2*(6*c*d^2 - a*e^2)*x)/c^2 + (4*d*e^3*x^3)/(3*c) + (e^4*x^5)/(
5*c) - ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*
e*(c*d^2 - a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sq
rt[2]*a^(3/4)*c^(9/4)) + ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*
Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*ArcTan[1 + (Sqrt[2]*c^(1/4)*
x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(9/4)) - ((c^2*d^4 - 6*a*c*d^2*
e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
```

```
c^(9/4)) + ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c
]*d*e*(c*d^2 - a*e^2))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(9/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-e^{2}\left(ae^{2}-6cd^{2}\right)\int\frac{1}{c^{2}}\,dx+\frac{4de^{3}x^{3}}{3c}+\frac{e^{4}x^{5}}{5c}-\frac{\sqrt{2}\left(-4\sqrt{a}\sqrt{c}de\left(ae^{2}-cd^{2}\right)+a^{2}e^{4}-6acd^{2}e^{2}+c^{2}d^{4}\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}c^{\frac{9}{4}}}+\frac{\sqrt{2}\left(-4\sqrt{a}\sqrt{c}de\left(ae^{2}-cd^{2}\right)+a^{2}e^{4}-6acd^{2}e^{2}+c^{2}d^{4}\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}c^{\frac{9}{4}}}-\frac{\sqrt{2}\left(4\sqrt{a}\sqrt{c}de\left(ae^{2}-cd^{2}\right)+a^{2}e^{4}-6acd^{2}e^{2}+c^{2}d^{4}\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^{2}\right)}{8a^{\frac{3}{4}}c^{\frac{9}{4}}}+\frac{\sqrt{2}\left(4\sqrt{a}\sqrt{c}de\left(ae^{2}-cd^{2}\right)+a^{2}e^{4}-6acd^{2}e^{2}+c^{2}d^{4}\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^{2}\right)}{8a^{\frac{3}{4}}c^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**4/(c*x**4+a),x)
```

```
[Out] -e**2*(a*e**2 - 6*c*d**2)*Integral(c**(-2), x) + 4*d*e**3*x**3/(3
*c) + e**4*x**5/(5*c) - sqrt(2)*(-4*sqrt(a)*sqrt(c)*d*e*(a*e**2 -
 c*d**2) + a**2*e**4 - 6*a*c*d**2*e**2 + c**2*d**4)*atan(1 - sqrt
(2)*c**(1/4)*x/a**(1/4))/(4*a**(3/4)*c**(9/4)) + sqrt(2)*(-4*sqrt
(a)*sqrt(c)*d*e*(a*e**2 - c*d**2) + a**2*e**4 - 6*a*c*d**2*e**2 +
 c**2*d**4)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/(4*a**(3/4)*c**
(9/4)) - sqrt(2)*(4*sqrt(a)*sqrt(c)*d*e*(a*e**2 - c*d**2) + a**2*
e**4 - 6*a*c*d**2*e**2 + c**2*d**4)*log(-sqrt(2)*a**(1/4)*c**(3/4
)*x + sqrt(a)*sqrt(c) + c*x**2)/(8*a**(3/4)*c**(9/4)) + sqrt(2)*(
4*sqrt(a)*sqrt(c)*d*e*(a*e**2 - c*d**2) + a**2*e**4 - 6*a*c*d**2*
e**2 + c**2*d**4)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(
c) + c*x**2)/(8*a**(3/4)*c**(9/4))
```

Mathematica [A] time = 0.729257, size = 444, normalized size = 1.02

$$160a^{3/4}c^{5/4}de^{3}x^{3}+24a^{3/4}c^{5/4}e^{4}x^{5}-120a^{3/4}\sqrt[4]{c}e^{2}x\left(ae^{2}-6cd^{2}\right)-15\sqrt{2}\left(4a^{3/2}\sqrt{c}de^{3}+a^{2}e^{4}-4\sqrt{a}c^{3/2}d^{3}e-6acd^{2}e^{2}+c^{2}d\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^4/(a + c*x^4),x]
```

```
[Out] (-120*a^(3/4)*c^(1/4)*e^2*(-6*c*d^2 + a*e^2)*x + 160*a^(3/4)*c^(5
/4)*d*e^3*x^3 + 24*a^(3/4)*c^(5/4)*e^4*x^5 - 30*Sqrt[2]*(c^2*d^4
+ 4*Sqrt[a]*c^(3/2)*d^3*e - 6*a*c*d^2*e^2 - 4*a^(3/2)*Sqrt[c]*d*e
^3 + a^2*e^4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 30*Sqrt[2
]*(c^2*d^4 + 4*Sqrt[a]*c^(3/2)*d^3*e - 6*a*c*d^2*e^2 - 4*a^(3/2)*
Sqrt[c]*d*e^3 + a^2*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
- 15*Sqrt[2]*(c^2*d^4 - 4*Sqrt[a]*c^(3/2)*d^3*e - 6*a*c*d^2*e^2 +
 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2] + 15*Sqrt[2]*(c^2*d^4 - 4*Sqrt[a]*c^(3/
2)*d^3*e - 6*a*c*d^2*e^2 + 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(120*a^(3/4)
*c^(9/4))
```

Maple [B] time = 0.014, size = 741, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^4/(c*x^4+a),x)
```

```
[Out] 1/5*e^4*x^5/c+4/3*d*e^3*x^3/c-e^4/c^2*a*x+6*e^2/c*d^2*x+1/8/c^2*(
1/c*a)^(1/4)*a*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1
/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*e^4-3/4/c*(1/c*
a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(
x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^2*e^2+1/8*(1/c*a)^(
1/4)/a*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^
2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^4+1/4/c^2*(1/c*a)^(1/
4)*a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*e^4-3/2/c*(1/c*a)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^2*e^2+1/4*(1/c*
a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^4+1/4/c^2*
(1/c*a)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*e^4-3/2
/c*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^2*e^
2+1/4*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d
^4-1/2/c^2/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+
(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*a*d*e
^3+1/2/c/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1
/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^3*e-1
/c^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*a*d*
e^3+1/c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d
^3*e-1/c^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1
)*a*d*e^3+1/c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*
x+1)*d^3*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^4/(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.59068, size = 3885, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^4/(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] 1/60*(12*c*e^4*x^5 + 80*c*d*e^3*x^3 + 15*c^2*sqrt(-(8*c^3*d^7*e -
 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqrt(-
(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c
^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*
a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c
^4))*log((c^8*d^16 - 24*a*c^7*d^14*e^2 - 36*a^2*c^6*d^12*e^4 + 88
*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*a^5*c^3*d^6*e^10 - 3
6*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x + (a*c^8*d^1
2 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e
^6 + 239*a^5*c^4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 + 4
```

```
*(a^3*c^8*d^3*e - a^4*c^7*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*
e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4
*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*
c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(-(8*c^3*d^7*e - 56*a*c^2*
d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqrt(-(c^8*d^16
- 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^
6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^
4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))) - 15
*c^2*sqrt(-(8*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8
*a^3*d*e^7 + a*c^4*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*
c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 397
6*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a
^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d^16 - 24*a*c^7*d^14*e^2 -
 36*a^2*c^6*d^12*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8
+ 88*a^5*c^3*d^6*e^10 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 +
 a^8*e^16)*x - (a*c^8*d^12 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^
8*e^4 - 476*a^4*c^5*d^6*e^6 + 239*a^5*c^4*d^4*e^8 - 34*a^6*c^3*d^
2*e^10 + a^7*c^2*e^12 + 4*(a^3*c^8*d^3*e - a^4*c^7*d*e^3)*sqrt(-(
c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^
5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a
^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(
-(8*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7
 + a*c^4*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e
^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*
d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(
a^3*c^9)))/(a*c^4))) + 15*c^2*sqrt(-(8*c^3*d^7*e - 56*a*c^2*d^5*e
^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*sqrt(-(c^8*d^16 - 56*
a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6
470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^1
2 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d
^16 - 24*a*c^7*d^14*e^2 - 36*a^2*c^6*d^12*e^4 + 88*a^3*c^5*d^10*e
^6 + 198*a^4*c^4*d^8*e^8 + 88*a^5*c^3*d^6*e^10 - 36*a^6*c^2*d^4*e
^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x + (a*c^8*d^12 - 34*a^2*c^7*
d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e^6 + 239*a^5*c^
4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 - 4*(a^3*c^8*d^3*e
 - a^4*c^7*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c
^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976
*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^
8*e^16)/(a^3*c^9)))*sqrt(-(8*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^
2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*sqrt(-(c^8*d^16 - 56*a*c^7*d^14
*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^
4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7
*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))) - 15*c^2*sqrt(-(8*c
^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*
c^4*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 -
3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e
^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c
^9)))/(a*c^4))*log((c^8*d^16 - 24*a*c^7*d^14*e^2 - 36*a^2*c^6*d^1
2*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*a^5*c^3*d^
6*e^10 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x -
(a*c^8*d^12 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4
*c^5*d^6*e^6 + 239*a^5*c^4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^
2*e^12 - 4*(a^3*c^8*d^3*e - a^4*c^7*d*e^3)*sqrt(-(c^8*d^16 - 56*a
*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 64
70*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12
 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(-(8*c^3*d^7*e -
 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*sqrt(-
(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c
^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*
a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c
^4))) + 60*(6*c*d^2*e^2 - a*e^4)*x)/c^2
```

Sympy [A] time = 12.3877, size = 500, normalized size = 1.14

$$\text{RootSum}\Bigg(256t^4a^3c^9 + t^2\left(-256a^5c^5de^7 + 1792a^4c^6d^3e^5 - 1792a^3c^7d^5e^3 + 256a^2c^8d^7e\right) + a^8e^{16} + 8a^7cd^2e^{14} + 28a^6c^2d^4e^{12} +$$

$$+ \frac{4de^3x^3}{3c} + \frac{e^4x^5}{5c} - \frac{x\left(ae^4 - 6cd^2e^2\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**4/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**9 + _t**2*(-256*a**5*c**5*d*e**7 + 1792
*a**4*c**6*d**3*e**5 - 1792*a**3*c**7*d**5*e**3 + 256*a**2*c**8*d
**7*e) + a**8*e**16 + 8*a**7*c*d**2*e**14 + 28*a**6*c**2*d**4*e**
12 + 56*a**5*c**3*d**6*e**10 + 70*a**4*c**4*d**8*e**8 + 56*a**3*c
**5*d**10*e**6 + 28*a**2*c**6*d**12*e**4 + 8*a*c**7*d**14*e**2 +
c**8*d**16, Lambda(_t, _t*log(x + (256*_t**3*a**4*c**7*d*e**3 - 2
56*_t**3*a**3*c**8*d**3*e + 4*_t*a**7*c**2*e**12 - 264*_t*a**6*c*
*3*d**2*e**10 + 1980*_t*a**5*c**4*d**4*e**8 - 3696*_t*a**4*c**5*d
**6*e**6 + 1980*_t*a**3*c**6*d**8*e**4 - 264*_t*a**2*c**7*d**10*e
**2 + 4*_t*a*c**8*d**12)/(a**8*e**16 - 24*a**7*c*d**2*e**14 - 36*
a**6*c**2*d**4*e**12 + 88*a**5*c**3*d**6*e**10 + 198*a**4*c**4*d*
*8*e**8 + 88*a**3*c**5*d**10*e**6 - 36*a**2*c**6*d**12*e**4 - 24*
a*c**7*d**14*e**2 + c**8*d**16)))) + 4*d*e**3*x**3/(3*c) + e**4*x
**5/(5*c) - x*(a*e**4 - 6*c*d**2*e**2)/c**2
```

GIAC/XCAS [A] time = 0.27947, size = 672, normalized size = 1.54

$$\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^4 - 6\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e^2 + 4\left(ac^3\right)^{\frac{3}{4}}cd^3e + \left(ac^3\right)^{\frac{1}{4}}a^2ce^4 - 4\left(ac^3\right)^{\frac{3}{4}}ade^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^4} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^4 - 6\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e^2 + 4\left(ac^3\right)^{\frac{3}{4}}cd^3e + \left(ac^3\right)^{\frac{1}{4}}a^2ce^4 - 4\left(ac^3\right)^{\frac{3}{4}}ade^3\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^4} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^4 - 6\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e^2 - 4\left(ac^3\right)^{\frac{3}{4}}cd^3e + \left(ac^3\right)^{\frac{1}{4}}a^2ce^4 + 4\left(ac^3\right)^{\frac{3}{4}}ade^3\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8ac^4} - \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^4 - 6\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e^2 - 4\left(ac^3\right)^{\frac{3}{4}}cd^3e + \left(ac^3\right)^{\frac{1}{4}}a^2ce^4 + 4\left(ac^3\right)^{\frac{3}{4}}ade^3\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8ac^4} + \frac{3c^4x^5e^4 + 20c^4dx^3e^3 + 90c^4d^2xe^2 - 15ac^3xe^4}{15c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^4/(c*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^
2 + 4*(a*c^3)^(3/4)*c*d^3*e + (a*c^3)^(1/4)*a^2*c*e^4 - 4*(a*c^3)
^(3/4)*a*d*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(a*c^4) + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^
3)^(1/4)*a*c^2*d^2*e^2 + 4*(a*c^3)^(3/4)*c*d^3*e + (a*c^3)^(1/4)*
a^2*c*e^4 - 4*(a*c^3)^(3/4)*a*d*e^3)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/8*sqrt(2)*((a*c^3)^(1
/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 4*(a*c^3)^(3/4)*c*d
^3*e + (a*c^3)^(1/4)*a^2*c*e^4 + 4*(a*c^3)^(3/4)*a*d*e^3)*ln(x^2
+ sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4) - 1/8*sqrt(2)*((a*c^
3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 4*(a*c^3)^(3/4
)*c*d^3*e + (a*c^3)^(1/4)*a^2*c*e^4 + 4*(a*c^3)^(3/4)*a*d*e^3)*ln
(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4) + 1/15*(3*c^4*x
^5*e^4 + 20*c^4*d*x^3*e^3 + 90*c^4*d^2*x*e^2 - 15*a*c^3*x*e^4)/c^
5
```

3.146 $\int \frac{(d+ex^2)^3}{a+cx^4}\,dx$

Optimal. Leaf size=370

$$-\frac{\left(\sqrt{c}d\left(cd^2-3ae^2\right)-\sqrt{a}e\left(3cd^2-ae^2\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{7/4}}+\frac{\left(\sqrt{c}d\left(cd^2-3ae^2\right)-\sqrt{a}e\left(3cd^2-ae^2\right)\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{7/4}}-\frac{\left(\sqrt{c}d\left(cd^2-3ae^2\right)+\sqrt{a}e\left(3cd^2-ae^2\right)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{7/4}}+\frac{\left(\sqrt{c}d\left(cd^2-3ae^2\right)+\sqrt{a}e\left(3cd^2-ae^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}c^{7/4}}+\frac{3de^2x}{c}+\frac{e^3x^3}{3c}$$

```
[Out] (3*d*e^2*x)/c + (e^3*x^3)/(3*c) - ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) +
 Sqrt[a]*e*(3*c*d^2 - a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/(2*Sqrt[2]*a^(3/4)*c^(7/4)) + ((Sqrt[c]*d*(c*d^2 - 3*a*e^2)
 + Sqrt[a]*e*(3*c*d^2 - a*e^2))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^
(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(7/4)) - ((Sqrt[c]*d*(c*d^2 - 3*a*e^
2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4)) + ((Sqrt[c]*
d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7
/4))
```

Rubi [A] time = 0.909209, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368

$$-\frac{\left(\sqrt{c}d\left(cd^2-3ae^2\right)-\sqrt{a}e\left(3cd^2-ae^2\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{7/4}}+\frac{\left(\sqrt{c}d\left(cd^2-3ae^2\right)-\sqrt{a}e\left(3cd^2-ae^2\right)\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{7/4}}-\frac{\left(\sqrt{c}d\left(cd^2-3ae^2\right)+\sqrt{a}e\left(3cd^2-ae^2\right)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{7/4}}+\frac{\left(\sqrt{c}d\left(cd^2-3ae^2\right)+\sqrt{a}e\left(3cd^2-ae^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}c^{7/4}}+\frac{3de^2x}{c}+\frac{e^3x^3}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^3/(a + c*x^4),x]
```

```
[Out] (3*d*e^2*x)/c + (e^3*x^3)/(3*c) - ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) +
 Sqrt[a]*e*(3*c*d^2 - a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/(2*Sqrt[2]*a^(3/4)*c^(7/4)) + ((Sqrt[c]*d*(c*d^2 - 3*a*e^2)
 + Sqrt[a]*e*(3*c*d^2 - a*e^2))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^
(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(7/4)) - ((Sqrt[c]*d*(c*d^2 - 3*a*e^
2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4)) + ((Sqrt[c]*
d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7
/4))
```

Rubi in Sympy [A] time = 88.3478, size = 345, normalized size = 0.93

$$\frac{3de^2x}{c}+\frac{e^3x^3}{3c}-\frac{\sqrt{2}\left(\sqrt{a}e\left(ae^2-3cd^2\right)-\sqrt{c}d\left(3ae^2-cd^2\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}c^{\frac{7}{4}}}$$
$$+\frac{\sqrt{2}\left(\sqrt{a}e\left(ae^2-3cd^2\right)-\sqrt{c}d\left(3ae^2-cd^2\right)\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}c^{\frac{7}{4}}}$$
$$+\frac{\sqrt{2}\left(\sqrt{a}e\left(ae^2-3cd^2\right)+\sqrt{c}d\left(3ae^2-cd^2\right)\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}c^{\frac{7}{4}}}$$
$$-\frac{\sqrt{2}\left(\sqrt{a}e\left(ae^2-3cd^2\right)+\sqrt{c}d\left(3ae^2-cd^2\right)\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}c^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**3/(c*x**4+a),x)
```

```
[Out] 3*d*e**2*x/c + e**3*x**3/(3*c) - sqrt(2)*(sqrt(a)*e*(a*e**2 - 3*c
*d**2) - sqrt(c)*d*(3*a*e**2 - c*d**2))*log(-sqrt(2)*a**(1/4)*c**
(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(8*a**(3/4)*c**(7/4)) + sqrt(
2)*(sqrt(a)*e*(a*e**2 - 3*c*d**2) - sqrt(c)*d*(3*a*e**2 - c*d**2)
)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(8*
a**(3/4)*c**(7/4)) + sqrt(2)*(sqrt(a)*e*(a*e**2 - 3*c*d**2) + sqr
t(c)*d*(3*a*e**2 - c*d**2))*atan(1 - sqrt(2)*c**(1/4)*x/a**(1/4))
/(4*a**(3/4)*c**(7/4)) - sqrt(2)*(sqrt(a)*e*(a*e**2 - 3*c*d**2) +
 sqrt(c)*d*(3*a*e**2 - c*d**2))*atan(1 + sqrt(2)*c**(1/4)*x/a**(1
/4))/(4*a**(3/4)*c**(7/4))
```

Mathematica [A] time = 0.51865, size = 360, normalized size = 0.97

$$\frac{-3\sqrt{2}\left(a^{3/2}e^3-3\sqrt{a}cd^2e-3a\sqrt{c}de^2+c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+3\sqrt{2}\left(a^{3/2}e^3-3\sqrt{a}cd^2e-3a\sqrt{c}de^2+c^{3/2}d\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^3/(a + c*x^4),x]
```

```
[Out] (72*a^(3/4)*c^(3/4)*d*e^2*x + 8*a^(3/4)*c^(3/4)*e^3*x^3 + 6*Sqrt[
2]*(-(c^(3/2)*d^3) - 3*Sqrt[a]*c*d^2*e + 3*a*Sqrt[c]*d*e^2 + a^(3
/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*Sqrt[2]*(c^(
3/2)*d^3 + 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 - a^(3/2)*e^3)*A
rcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*(c^(3/2)*d^3 -
 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2]*(c^(3/2)*
d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(24*a^(3/4)*c^(
7/4))
```

Maple [A] time = 0.006, size = 572, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^3/(c*x^4+a),x)
```

```
[Out] 1/3*e^3*x^3/c+3*d*e^2*x/c-3/4/c*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(1/c*a)^(1/4)*x-1)*d*e^2+1/4*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2
^(1/2)/(1/c*a)^(1/4)*x-1)*d^3-3/8/c*(1/c*a)^(1/4)*2^(1/2)*ln((x^2
+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1
/2)+(1/c*a)^(1/2)))*d*e^2+1/8*(1/c*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/
c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+
(1/c*a)^(1/2)))*d^3-3/4/c*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
/c*a)^(1/4)*x+1)*d*e^2+1/4*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)
/(1/c*a)^(1/4)*x+1)*d^3-1/8/c^2/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/
c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+
(1/c*a)^(1/2)))*a*e^3+3/8/c/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)
^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c
*a)^(1/2)))*d^2*e-1/4/c^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
/c*a)^(1/4)*x-1)*a*e^3+3/4/c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(1/c*a)^(1/4)*x-1)*d^2*e-1/4/c^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(1/c*a)^(1/4)*x+1)*a*e^3+3/4/c/(1/c*a)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^2*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.21967, size = 2880, normalized size = 7.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] 1/12*(4*e^3*x^3 + 36*d*e^2*x - 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^
3*e^3 + 6*a^2*d*e^5 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 +
 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8
- 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))/(a*c^3))*log(-(c^6*d^
12 - 12*a*c^5*d^10*e^2 - 27*a^2*c^4*d^8*e^4 + 27*a^4*c^2*d^4*e^8
+ 12*a^5*c*d^2*e^10 - a^6*e^12)*x + (a*c^6*d^9 - 18*a^2*c^5*d^7*e
^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^5*c^2*d*e^8 +
(3*a^3*c^6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e
^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*
e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*
e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c
^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4
*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))/(a*c^3))
) + 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + a*c^3
*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*
a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e
^12)/(a^3*c^7)))/(a*c^3))*log(-(c^6*d^12 - 12*a*c^5*d^10*e^2 - 27
*a^2*c^4*d^8*e^4 + 27*a^4*c^2*d^4*e^8 + 12*a^5*c*d^2*e^10 - a^6*e
^12)*x - (a*c^6*d^9 - 18*a^2*c^5*d^7*e^2 + 60*a^3*c^4*d^5*e^4 - 4
6*a^4*c^3*d^3*e^6 + 3*a^5*c^2*d*e^8 + (3*a^3*c^6*d^2*e - a^4*c^5*
e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 -
452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a
^6*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*
d*e^5 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d
^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2
*e^10 + a^6*e^12)/(a^3*c^7)))/(a*c^3))) - 3*c*sqrt(-(6*c^2*d^5*e
- 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5
```

```
*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c
^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))/(a*c^3))*l
og(-(c^6*d^12 - 12*a*c^5*d^10*e^2 - 27*a^2*c^4*d^8*e^4 + 27*a^4*c
^2*d^4*e^8 + 12*a^5*c*d^2*e^10 - a^6*e^12)*x + (a*c^6*d^9 - 18*a^
2*c^5*d^7*e^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^5*c
^2*d*e^8 - (3*a^3*c^6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a
*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a
^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))*sqrt(-
(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - a*c^3*sqrt(-(c^6*d^
12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^
6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)
))/(a*c^3))) + 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*
e^5 - a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8
*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e
^10 + a^6*e^12)/(a^3*c^7)))/(a*c^3))*log(-(c^6*d^12 - 12*a*c^5*d^
10*e^2 - 27*a^2*c^4*d^8*e^4 + 27*a^4*c^2*d^4*e^8 + 12*a^5*c*d^2*e
^10 - a^6*e^12)*x - (a*c^6*d^9 - 18*a^2*c^5*d^7*e^2 + 60*a^3*c^4*
d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^5*c^2*d*e^8 - (3*a^3*c^6*d^2*e
 - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4
*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d
^2*e^10 + a^6*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e
^3 + 6*a^2*d*e^5 - a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 25
5*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 3
0*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))/(a*c^3))))/c
```

Sympy [A] time = 7.94747, size = 350, normalized size = 0.95

$$\mathrm{RootSum}\left(256t^4a^3c^7 + t^2\left(192a^4c^4de^5 - 640a^3c^5d^3e^3 + 192a^2c^6d^5e\right) + a^6e^{12} + 6a^5cd^2e^{10} + 15a^4c^2d^4e^8 + 20a^3c^3d^6e^6 + 15a^2c\right.$$

$$+ \frac{3de^2x}{c} + \frac{e^3x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**3/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**7 + _t**2*(192*a**4*c**4*d*e**5 - 640*a
**3*c**5*d**3*e**3 + 192*a**2*c**6*d**5*e) + a**6*e**12 + 6*a**5*
c*d**2*e**10 + 15*a**4*c**2*d**4*e**8 + 20*a**3*c**3*d**6*e**6 +
15*a**2*c**4*d**8*e**4 + 6*a*c**5*d**10*e**2 + c**6*d**12, Lambda
(_t, _t*log(x + (-64*_t**3*a**4*c**5*e**3 + 192*_t**3*a**3*c**6*d
**2*e - 36*_t*a**5*c**2*d*e**8 + 336*_t*a**4*c**3*d**3*e**6 - 504
*_t*a**3*c**4*d**5*e**4 + 144*_t*a**2*c**5*d**7*e**2 - 4*_t*a*c**
6*d**9)/(a**6*e**12 - 12*a**5*c*d**2*e**10 - 27*a**4*c**2*d**4*e*
*8 + 27*a**2*c**4*d**8*e**4 + 12*a*c**5*d**10*e**2 - c**6*d**12))
)) + 3*d*e**2*x/c + e**3*x**3/(3*c)
```

GIAC/XCAS [A] time = 0.28042, size = 547, normalized size = 1.48

$$\frac{c^2x^3e^3 + 9\,c^2dxe^2}{3\,c^3} + \frac{\sqrt{2}\Big((ac^3)^{\frac{1}{4}}c^3d^3 - 3\,(ac^3)^{\frac{1}{4}}ac^2de^2 + 3\,(ac^3)^{\frac{3}{4}}cd^2e - (ac^3)^{\frac{3}{4}}ae^3\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^4} + \frac{\sqrt{2}\Big((ac^3)^{\frac{1}{4}}c^3d^3 - 3\,(ac^3)^{\frac{1}{4}}ac^2de^2 + 3\,(ac^3)^{\frac{3}{4}}cd^2e - (ac^3)^{\frac{3}{4}}ae^3\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^4} + \frac{\sqrt{2}\Big((ac^3)^{\frac{1}{4}}c^3d^3 - 3\,(ac^3)^{\frac{1}{4}}ac^2de^2 - 3\,(ac^3)^{\frac{3}{4}}cd^2e + (ac^3)^{\frac{3}{4}}ae^3\Big)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac^4} - \frac{\sqrt{2}\Big((ac^3)^{\frac{1}{4}}c^3d^3 - 3\,(ac^3)^{\frac{1}{4}}ac^2de^2 - 3\,(ac^3)^{\frac{3}{4}}cd^2e + (ac^3)^{\frac{3}{4}}ae^3\Big)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/(c*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/3*(c^2*x^3*e^3 + 9*c^2*d*x*e^2)/c^3 + 1/4*sqrt(2)*((a*c^3)^(1/4
)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e
 - (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(
1/4))/(a/c)^(1/4))/(a*c^4) + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 -
 3*(a*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^
(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)
^(1/4))/(a*c^4) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^
(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3
)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4) - 1/8*sqrt(
2)*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^
3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*ln(x^2 - sqrt(2)*x*(a/c)^
(1/4) + sqrt(a/c))/(a*c^4)
```

3.147 $\int \frac{(d+ex^2)^2}{a+cx^4}\,dx$

Optimal. Leaf size=297

$$-\frac{\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}+\frac{\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}-\frac{\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}}+\frac{\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}c^{5/4}}+\frac{e^2x}{c}$$

```
[Out] (e^2*x)/c - ((c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 - (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^
2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)
/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(5/4)) - ((c*d^2 - 2*Sqrt[a]*Sqrt
[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c
]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2 - 2*Sqrt[a]*Sqrt[c]
*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4))
```

Rubi [A] time = 0.531393, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$-\frac{\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}+\frac{\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}-\frac{\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}}+\frac{\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}c^{5/4}}+\frac{e^2x}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^2/(a + c*x^4),x]
```

```
[Out] (e^2*x)/c - ((c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 - (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^
2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)
/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(5/4)) - ((c*d^2 - 2*Sqrt[a]*Sqrt
[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c
]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2 - 2*Sqrt[a]*Sqrt[c]
*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2 \int \frac{1}{c}\,dx + \frac{\sqrt{2}\left(-2\sqrt{a}\sqrt{c}de + ae^2 - cd^2\right)\operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}c^{\frac{5}{4}}} - \frac{\sqrt{2}\left(-2\sqrt{a}\sqrt{c}de + ae^2 - cd^2\right)\operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}c^{\frac{5}{4}}} + \frac{\sqrt{2}\left(2\sqrt{a}\sqrt{c}de + ae^2 - cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x + \sqrt{a}\sqrt{c} + cx^2\right)}{8a^{\frac{3}{4}}c^{\frac{5}{4}}} - \frac{\sqrt{2}\left(2\sqrt{a}\sqrt{c}de + ae^2 - cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x + \sqrt{a}\sqrt{c} + cx^2\right)}{8a^{\frac{3}{4}}c^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2/(c*x**4+a),x)
```

```
[Out] e**2*Integral(1/c, x) + sqrt(2)*(-2*sqrt(a)*sqrt(c)*d*e + a*e**2
- c*d**2)*atan(1 - sqrt(2)*c**(1/4)*x/a**(1/4))/(4*a**(3/4)*c**(5
/4)) - sqrt(2)*(-2*sqrt(a)*sqrt(c)*d*e + a*e**2 - c*d**2)*atan(1
+ sqrt(2)*c**(1/4)*x/a**(1/4))/(4*a**(3/4)*c**(5/4)) + sqrt(2)*(2
*sqrt(a)*sqrt(c)*d*e + a*e**2 - c*d**2)*log(-sqrt(2)*a**(1/4)*c**
(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(8*a**(3/4)*c**(5/4)) - sqrt(
2)*(2*sqrt(a)*sqrt(c)*d*e + a*e**2 - c*d**2)*log(sqrt(2)*a**(1/4)
*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(8*a**(3/4)*c**(5/4))
```

Mathematica [A] time = 0.485712, size = 269, normalized size = 0.91

$$\frac{8a^{3/4}\sqrt[4]{c}e^2x + \sqrt{2}\left(2\sqrt{a}\sqrt{c}de + ae^2 - cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2\right) + \sqrt{2}\left(-2\sqrt{a}\sqrt{c}de - ae^2 + cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \right.}{8a^{3/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^2/(a + c*x^4),x]
```

```
[Out] (8*a^(3/4)*c^(1/4)*e^2*x - 2*Sqrt[2]*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d
*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(
c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)] + Sqrt[2]*(-(c*d^2) + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2
]*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(8*a^(3/4)*c^(5/4))
```

Maple [A] time = 0.005, size = 412, normalized size = 1.4

$$\frac{e^2x}{c} - \frac{\sqrt{2}e^2}{4c}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right) + \frac{\sqrt{2}d^2}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)$$
$$-\frac{\sqrt{2}e^2}{8c}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}d^2}{8a}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$-\frac{\sqrt{2}e^2}{4c}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right) + \frac{\sqrt{2}d^2}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)$$
$$+\frac{de\sqrt{2}}{4c}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$+\frac{de\sqrt{2}}{2c}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}} + \frac{de\sqrt{2}}{2c}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^2/(c*x^4+a),x)
```

```
[Out] e^2*x/c-1/4/c*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*
x-1)*e^2+1/4*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)
*x-1)*d^2-1/8/c*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(
1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*
e^2+1/8*(1/c*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(
1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^2-1/
4/c*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*e^2+1
/4*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^2+
1/4/c*d*e/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(
1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/2/c*
d*e/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/2/c
*d*e/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.460831, size = 1998, normalized size = 6.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*e^2*x + c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 + a*c^2*sqrt(-(c^4*
d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 +
a^4*e^8)/(a^3*c^5)))/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10
*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x + (a*c^4*d^6 - 7*
a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2*a^3*c^4*d*e*s
qrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*
d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*d*e^3 + a*c
^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^
3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))) - c*sqrt(-(4*c*d^3*e
 - 4*a*d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c
^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))*log
((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^
6 + a^4*e^8)*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e
^4 - a^4*c*e^6 + 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2
+ 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sq
rt(-(4*c*d^3*e - 4*a*d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*
e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5))
)/(a*c^2))) + c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d
^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a
^4*e^8)/(a^3*c^5)))/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*
a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x + (a*c^4*d^6 - 7*a
^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 - 2*a^3*c^4*d*e*sq
rt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d
^2*e^6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*d*e^3 - a*c^
2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3
*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))) - c*sqrt(-(4*c*d^3*e
- 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^
2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))*log(
(c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6
 + a^4*e^8)*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^
4 - a^4*c*e^6 - 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 +
 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sqr
t(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e
^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))
/(a*c^2))))/c
```

Sympy [A] time = 5.08583, size = 238, normalized size = 0.8

$$\text{RootSum}\left(256t^4a^3c^5 + t^2\left(-128a^3c^3de^3 + 128a^2c^4d^3e\right) + a^4e^8 + 4a^3cd^2e^6 + 6a^2c^2d^4e^4 + 4ac^3d^6e^2 + c^4d^8, \left(t \mapsto t\log\left(x + \frac{}{}\right.\right.\right.$$

$$+ \frac{e^2x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**2/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**5 + _t**2*(-128*a**3*c**3*d*e**3 + 128*
a**2*c**4*d**3*e) + a**4*e**8 + 4*a**3*c*d**2*e**6 + 6*a**2*c**2*
d**4*e**4 + 4*a*c**3*d**6*e**2 + c**4*d**8, Lambda(_t, _t*log(x +
 (-128*_t**3*a**3*c**4*d*e - 4*_t*a**4*c*e**6 + 60*_t*a**3*c**2*d
**2*e**4 - 60*_t*a**2*c**3*d**4*e**2 + 4*_t*a*c**4*d**6)/(a**4*e*
*8 - 4*a**3*c*d**2*e**6 - 10*a**2*c**2*d**4*e**4 - 4*a*c**3*d**6*
e**2 + c**4*d**8)))) + e**2*x/c
```

GIAC/XCAS [A] time = 0.279545, size = 454, normalized size = 1.53

$$\frac{xe^2}{c} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2 - \left(ac^3\right)^{\frac{1}{4}}ace^2 + 2\,\left(ac^3\right)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} - \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2 - \left(ac^3\right)^{\frac{1}{4}}ace^2 - 2\,\left(ac^3\right)^{\frac{3}{4}}de\right)\ln\left(x^2 - \sqrt{2}x\,\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac^3} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}ac^4d^2 - \left(ac^3\right)^{\frac{1}{4}}a^2c^3e^2 + 2\,\left(ac^3\right)^{\frac{3}{4}}ac^2de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,a^2c^5} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}ac^4d^2 - \left(ac^3\right)^{\frac{1}{4}}a^2c^3e^2 - 2\,\left(ac^3\right)^{\frac{3}{4}}ac^2de\right)\ln\left(x^2 + \sqrt{2}x\,\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,a^2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/(c*x^4 + a),x, algorithm="giac")
```

```
[Out] x*e^2/c + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*
e^2 + 2*(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^
2 - (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*ln(x^2 - sqrt(2)
*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) + 1/4*sqrt(2)*((a*c^3)^(1/4)*
a*c^4*d^2 - (a*c^3)^(1/4)*a^2*c^3*e^2 + 2*(a*c^3)^(3/4)*a*c^2*d*e
)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^
2*c^5) + 1/8*sqrt(2)*((a*c^3)^(1/4)*a*c^4*d^2 - (a*c^3)^(1/4)*a^2
*c^3*e^2 - 2*(a*c^3)^(3/4)*a*c^2*d*e)*ln(x^2 + sqrt(2)*x*(a/c)^(1
/4) + sqrt(a/c))/(a^2*c^5)
```

3.148 $\int \frac{d+ex^2}{a+cx^4}\,dx$

Optimal. Leaf size=247

$$-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}c^{3/4}}$$

```
[Out] -((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
)/(2*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((
Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d - Sqrt[a]
*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sq
rt[2]*a^(3/4)*c^(3/4))
```

Rubi [A] time = 0.286568, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(a + c*x^4),x]
```

```
[Out] -((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
)/(2*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((
Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d - Sqrt[a]
*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sq
rt[2]*a^(3/4)*c^(3/4))
```

Rubi in Sympy [A] time = 59.6804, size = 230, normalized size = 0.93

$$\frac{\sqrt{2}\left(\sqrt{a}e-\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}e-\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}e+\sqrt{c}d\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}e+\sqrt{c}d\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(c*x**4+a),x)
```

```
[Out] sqrt(2)*(sqrt(a)*e - sqrt(c)*d)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x
+ sqrt(a)*sqrt(c) + c*x**2)/(8*a**(3/4)*c**(3/4)) - sqrt(2)*(sqrt
(a)*e - sqrt(c)*d)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt
(c) + c*x**2)/(8*a**(3/4)*c**(3/4)) - sqrt(2)*(sqrt(a)*e + sqrt(c
```

```
)*d)*atan(1 - sqrt(2)*c**(1/4)*x/a**(1/4))/(4*a**(3/4)*c**(3/4))
+ sqrt(2)*(sqrt(a)*e + sqrt(c)*d)*atan(1 + sqrt(2)*c**(1/4)*x/a**
(1/4))/(4*a**(3/4)*c**(3/4))
```

Mathematica [A] time = 0.113403, size = 183, normalized size = 0.74

$$\frac{-\left(\sqrt{c}d-\sqrt{a}e\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)\right)-2\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2}{4\sqrt{2}a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/(a + c*x^4),x]
```

```
[Out] (-2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4
)] + 2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(
1/4)] - (Sqrt[c]*d - Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^
(1/4)*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x
+ Sqrt[c]*x^2]))/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

Maple [A] time = 0.004, size = 260, normalized size = 1.1

$$\frac{d\sqrt{2}}{8a}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{d\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{d\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)+\frac{e\sqrt{2}}{8c}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{e\sqrt{2}}{4c}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{e\sqrt{2}}{4c}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/8*d*(1/c*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/
c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/4*d*(1
/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/4*d*(1/
c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)+1/8*e/c/(1
/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)
)/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/4*e/c/(1/c*a)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/4*e/c/(1/c*a)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.28496, size = 1035, normalized size = 4.19

$$-\frac{1}{4}\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+2\,de}{ac}}\log\left(-\left(c^2d^4-a^2e^4\right)x+\left(a^3c^2e\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+ac^2d^3-a^2cde^2\right)\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+2\,de}{ac}}\right)$$
$$+\frac{1}{4}\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+2\,de}{ac}}\log\left(-\left(c^2d^4-a^2e^4\right)x-\left(a^3c^2e\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+ac^2d^3-a^2cde^2\right)\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+2\,de}{ac}}\right)$$
$$+\frac{1}{4}\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-2\,de}{ac}}\log\left(-\left(c^2d^4-a^2e^4\right)x+\left(a^3c^2e\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-ac^2d^3+a^2cde^2\right)\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-2\,de}{ac}}\right)$$
$$-\frac{1}{4}\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-2\,de}{ac}}\log\left(-\left(c^2d^4-a^2e^4\right)x-\left(a^3c^2e\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-ac^2d^3+a^2cde^2\right)\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-2\,de}{ac}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^
3)) + 2*d*e)/(a*c))*log(-(c^2*d^4 - a^2*e^4)*x + (a^3*c^2*e*sqrt(
-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + a*c^2*d^3 - a^2
*c*d*e^2)*sqrt(-(a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a
^3*c^3)) + 2*d*e)/(a*c))) + 1/4*sqrt(-(a*c*sqrt(-(c^2*d^4 - 2*a*c
*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))*log(-(c^2*d^4 - a^
2*e^4)*x - (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(
a^3*c^3)) + a*c^2*d^3 - a^2*c*d*e^2)*sqrt(-(a*c*sqrt(-(c^2*d^4 -
2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))) + 1/4*sqrt((
a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) - 2*d*e)
/(a*c))*log(-(c^2*d^4 - a^2*e^4)*x + (a^3*c^2*e*sqrt(-(c^2*d^4 -
2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) - a*c^2*d^3 + a^2*c*d*e^2)*sq
rt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) - 2*
d*e)/(a*c))) - 1/4*sqrt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2
*e^4)/(a^3*c^3)) - 2*d*e)/(a*c))*log(-(c^2*d^4 - a^2*e^4)*x - (a^
3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) - a*
c^2*d^3 + a^2*c*d*e^2)*sqrt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 +
 a^2*e^4)/(a^3*c^3)) - 2*d*e)/(a*c)))
```

Sympy [A] time = 1.90984, size = 109, normalized size = 0.44

$$\operatorname{RootSum}\left(256t^4a^3c^3 + 64t^2a^2c^2de + a^2e^4 + 2acd^2e^2 + c^2d^4, \left(t \mapsto t\log\left(x + \frac{64t^3a^3c^2e + 12ta^2cde^2 - 4tac^2d^3}{a^2e^4 - c^2d^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**3 + 64*_t**2*a**2*c**2*d*e + a**2*e**4
+ 2*a*c*d**2*e**2 + c**2*d**4, Lambda(_t, _t*log(x + (64*_t**3*a*
*3*c**2*e + 12*_t*a**2*c*d*e**2 - 4*_t*a*c**2*d**3)/(a**2*e**4 -
c**2*d**4))))
```

GIAC/XCAS [A] time = 0.27852, size = 331, normalized size = 1.34

$$\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d + \left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d + \left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d - \left(ac^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac^3} - \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d - \left(ac^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(c*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sq
rt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/4*sqrt
(2)*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/8*sqrt(2)*((a*
c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4
) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d - (a*c^
3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)
```

3.149 $\int \frac{1}{a+cx^4}\,dx$

Optimal. Leaf size=185

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

```
[Out] -ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/
4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*
c^(1/4)) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4))
```

Rubi [A] time = 0.193999, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^4)^(-1),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/
4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*
c^(1/4)) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4))
```

Rubi in Sympy [A] time = 46.2574, size = 172, normalized size = 0.93

$$-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{8a^{\frac{3}{4}}\sqrt[4]{c}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{8a^{\frac{3}{4}}\sqrt[4]{c}}-\frac{\sqrt{2}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\sqrt[4]{c}}+\frac{\sqrt{2}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\sqrt[4]{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+a),x)
```

```
[Out] -sqrt(2)*log(-sqrt(2)*a**(1/4)*c**(1/4)*x + sqrt(a) + sqrt(c)*x**
2)/(8*a**(3/4)*c**(1/4)) + sqrt(2)*log(sqrt(2)*a**(1/4)*c**(1/4)*
x + sqrt(a) + sqrt(c)*x**2)/(8*a**(3/4)*c**(1/4)) - sqrt(2)*atan(
1 - sqrt(2)*c**(1/4)*x/a**(1/4))/(4*a**(3/4)*c**(1/4)) + sqrt(2)*
atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/(4*a**(3/4)*c**(1/4))
```

Mathematica [A] time = 0.0336104, size = 134, normalized size = 0.72

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[
2]*c^(1/4)*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
+ Sqrt[c]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c
]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4))
```

Maple [A] time = 0.003, size = 128, normalized size = 0.7

$$\frac{\sqrt{2}}{8a}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+a),x)
```

```
[Out] 1/8*(1/c*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*
a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/4*(1/c*a
)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/4*(1/c*a)^(
1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.299014, size = 142, normalized size = 0.77

$$-\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\arctan\left(\frac{a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}}{x+\sqrt{a^2\sqrt{-\frac{1}{a^3c}}+x^2}}\right)+\frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)-\frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] -(-1/(a^3*c))^(1/4)*arctan(a*(-1/(a^3*c))^(1/4)/(x + sqrt(a^2*sqr
t(-1/(a^3*c)) + x^2))) + 1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/(a^3*c)
)^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4) +
 x)
```

Sympy [A] time = 0.349526, size = 20, normalized size = 0.11

$$\text{RootSum}\left(256t^4a^3c + 1, \left(t \mapsto t\log\left(4ta + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))
```

GIAC/XCAS [A] time = 0.273287, size = 242, normalized size = 1.31

$$\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac}+\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac}+\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\ln\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}-\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\ln\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)
^(1/4))/(a/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqr
t(2)*(a*c^3)^(1/4)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a
*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) +
sqrt(a/c))/(a*c)
```

3.150 $\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=336

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2+cd^2\right)}$$

```
[Out] (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) -
 (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(
3/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.518059, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) -
 (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(
3/4)*(c*d^2 + a*e^2))
```

Rubi in Sympy [A] time = 91.131, size = 304, normalized size = 0.9

$$\frac{e^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2+cd^2\right)}+\frac{\sqrt{2}\sqrt[4]{c}\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\left(ae^2+cd^2\right)}-\frac{\sqrt{2}\sqrt[4]{c}\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\left(ae^2+cd^2\right)}-\frac{\sqrt{2}\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}\left(ae^2+cd^2\right)}+\frac{\sqrt{2}\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] e**(3/2)*atan(sqrt(e)*x/sqrt(d))/(sqrt(d)*(a*e**2 + c*d**2)) + sq
rt(2)*c**(1/4)*(sqrt(a)*e - sqrt(c)*d)*atan(1 - sqrt(2)*c**(1/4)*
x/a**(1/4))/(4*a**(3/4)*(a*e**2 + c*d**2)) - sqrt(2)*c**(1/4)*(sq
rt(a)*e - sqrt(c)*d)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/(4*a**
(3/4)*(a*e**2 + c*d**2)) - sqrt(2)*c**(1/4)*(sqrt(a)*e + sqrt(c)*
d)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(
8*a**(3/4)*(a*e**2 + c*d**2)) + sqrt(2)*c**(1/4)*(sqrt(a)*e + sqr
t(c)*d)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**
2)/(8*a**(3/4)*(a*e**2 + c*d**2))
```

Mathematica [A] time = 0.30676, size = 234, normalized size = 0.7

$$\frac{8a^{3/4}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\sqrt{2}\sqrt[4]{c}\sqrt{d}\left(-\left(\sqrt{a}e+\sqrt{c}d\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)\right)+(2\cdots}{8a^{3/4}\sqrt{d}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (8*a^(3/4)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*c^(1/4)*
Sqrt[d]*((-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)
*x)/a^(1/4)] + 2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*x)/a^(1/4)] - (Sqrt[c]*d + Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(3/4)*Sqrt[d]*(c*d^2 + a*e^2))
```

Maple [A] time = 0.01, size = 363, normalized size = 1.1

$$\begin{aligned}
&\frac{cd\sqrt{2}}{(8\,ae^2+8\,cd^2)\,a}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\\
&+\frac{cd\sqrt{2}}{(4\,ae^2+4\,cd^2)\,a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{cd\sqrt{2}}{(4\,ae^2+4\,cd^2)\,a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\\
&-\frac{e\sqrt{2}}{8\,ae^2+8\,cd^2}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}\\
&-\frac{e\sqrt{2}}{4\,ae^2+4\,cd^2}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}\\
&-\frac{e\sqrt{2}}{4\,ae^2+4\,cd^2}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{e^2}{ae^2+cd^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/8*c/(a*e^2+c*d^2)*d*(1/c*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/c*a)^(1/
4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^
(1/2)))+1/4*c/(a*e^2+c*d^2)*d*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1
```

```
/2)/(1/c*a)^(1/4)*x+1)+1/4*c/(a*e^2+c*d^2)*d*(1/c*a)^(1/4)/a*2^(1
/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)-1/8/(a*e^2+c*d^2)*e/(1/c*a)
^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^
2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))-1/4/(a*e^2+c*d^2)*e/(1/
c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)-1/4/(a*e^2+c
*d^2)*e/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)+e
^2/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.9165, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="fricas")

[Out] [-1/4*((c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e
^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*
c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6
 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*
d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*
a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c
*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^
6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2
*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^
3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e
^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*d^
2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4
)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*
a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))
)/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^
2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^
3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c
^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6
+ a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e
^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 +
4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8
)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*s
qrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3
*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*
e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4
 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^
2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)
*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a
^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))
*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c
^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^
6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d
^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e
 - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c
^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*
c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d
^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a*c^2*d^3 - a^2*
```

```
c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*
d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e
^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d
*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a
*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^
5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + a^3*e^4))) - 2*e*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e
/d) - d)/(e*x^2 + d)))/(c*d^2 + a*e^2), 1/4*(4*e*sqrt(e/d)*arctan
(e*x/(d*sqrt(e/d))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4
+ 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a
^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 +
4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e
^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 + (a^3
*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*
d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2
*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^
4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 +
 a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4
+ 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3
*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^
2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a
^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*
e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c
^2*d^2 - a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e +
 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^
2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4
*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*
d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/
(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^
2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c
*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*
e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 +
 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^
8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c
*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2
*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^
3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e
^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^
3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8
 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*
e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2
)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(
c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d
^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a
*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e
^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 +
4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8
)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(
-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3
*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^
2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))))/(c*d^2 + a*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)/(c*x**4+a),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.284994, size = 458, normalized size = 1.36

$$\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}+\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{3}{2}}}{(cd^2+ae^2)\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] 1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2
*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(
2)*a^2*c^2*e^2) + 1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)
*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d + (a
*c^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(
2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*c^2*d +
(a*c^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqr
t(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + arctan(x*e^(1/2)/sqrt(d))
*e^(3/2)/((c*d^2 + a*e^2)*sqrt(d))
```

3.151 $\int \frac{1}{(d+ex^2)^2(a+cx^4)}\,dx$

Optimal. Leaf size=453

$$-\frac{c^{3/4}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{c^{3/4}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{c^{3/4}\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{c^{3/4}\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{e^2x}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}+\frac{2c\sqrt{d}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(ae^2+cd^2\right)^2}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(ae^2+cd^2\right)}$$

```
[Out] (e^2*x)/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (2*c*Sqrt[d]*e^(3/2)*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (e^(3/2)*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 + a*e^2)) - (c^(3/4)*(c*d
^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*(c*d
^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*(c*d
^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2)
+ (c^(3/4)*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d
^2 + a*e^2)^2)
```

Rubi [A] time = 0.729074, antiderivative size = 453, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$

$$-\frac{c^{3/4}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{c^{3/4}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{c^{3/4}\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{c^{3/4}\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{e^2x}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}+\frac{2c\sqrt{d}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(ae^2+cd^2\right)^2}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)^2*(a + c*x^4)),x]
```

```
[Out] (e^2*x)/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (2*c*Sqrt[d]*e^(3/2)*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (e^(3/2)*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 + a*e^2)) - (c^(3/4)*(c*d
^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
```

```
)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*(c*d
^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*(c*d
^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2)
+ (c^(3/4)*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d
^2 + a*e^2)^2)
```

Rubi in Sympy [A] time = 133.508, size = 425, normalized size = 0.94

$$\frac{2c\sqrt{d}e^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(ae^2+cd^2\right)^2}+\frac{e^2x}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}+\frac{e^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{\frac{3}{2}}\left(ae^2+cd^2\right)}+\frac{\sqrt{2}c^{\frac{3}{4}}\left(-2\sqrt{a}\sqrt{c}de+ae^2-cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}\left(ae^2+cd^2\right)^2}-\frac{\sqrt{2}c^{\frac{3}{4}}\left(-2\sqrt{a}\sqrt{c}de+ae^2-cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}\left(ae^2+cd^2\right)^2}+\frac{\sqrt{2}c^{\frac{3}{4}}\left(2\sqrt{a}\sqrt{c}de+ae^2-cd^2\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\left(ae^2+cd^2\right)^2}-\frac{\sqrt{2}c^{\frac{3}{4}}\left(2\sqrt{a}\sqrt{c}de+ae^2-cd^2\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\left(ae^2+cd^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)**2/(c*x**4+a),x)
```

```
[Out] 2*c*sqrt(d)*e**(3/2)*atan(sqrt(e)*x/sqrt(d))/(a*e**2 + c*d**2)**2
 + e**2*x/(2*d*(d + e*x**2)*(a*e**2 + c*d**2)) + e**(3/2)*atan(sq
rt(e)*x/sqrt(d))/(2*d**(3/2)*(a*e**2 + c*d**2)) + sqrt(2)*c**(3/4
)*(-2*sqrt(a)*sqrt(c)*d*e + a*e**2 - c*d**2)*log(-sqrt(2)*a**(1/4
)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(8*a**(3/4)*(a*e**2 + c*
d**2)**2) - sqrt(2)*c**(3/4)*(-2*sqrt(a)*sqrt(c)*d*e + a*e**2 - c
*d**2)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2
)/(8*a**(3/4)*(a*e**2 + c*d**2)**2) + sqrt(2)*c**(3/4)*(2*sqrt(a)
*sqrt(c)*d*e + a*e**2 - c*d**2)*atan(1 - sqrt(2)*c**(1/4)*x/a**(1
/4))/(4*a**(3/4)*(a*e**2 + c*d**2)**2) - sqrt(2)*c**(3/4)*(2*sqrt
(a)*sqrt(c)*d*e + a*e**2 - c*d**2)*atan(1 + sqrt(2)*c**(1/4)*x/a*
*(1/4))/(4*a**(3/4)*(a*e**2 + c*d**2)**2)
```

Mathematica [A] time = 1.08497, size = 362, normalized size = 0.8

$$\frac{\frac{\sqrt{2}c^{3/4}\left(-2\sqrt{a}\sqrt{c}de+ae^2-cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{3/4}}+\frac{\sqrt{2}c^{3/4}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{3/4}}+\frac{2\sqrt{2}c^{3/4}\left(2\sqrt{a}\sqrt{c}de+ae^2-cd^2\right)}{a^{3/4}}}{8\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)^2*(a + c*x^4)),x]
```

```
[Out] ((4*e^2*(c*d^2 + a*e^2)*x)/(d*(d + e*x^2)) + (4*e^(3/2)*(5*c*d^2
+ a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + (2*Sqrt[2]*c^(3/4
)*(-(c*d^2) + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[2]*
c^(1/4)*x)/a^(1/4)])/a^(3/4) - (2*Sqrt[2]*c^(3/4)*(-(c*d^2) + 2*S
qrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4
```

```
)])/a^(3/4) + (Sqrt[2]*c^(3/4)*(-(c*d^2) - 2*Sqrt[a]*Sqrt[c]*d*e
+ a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/
a^(3/4) + (Sqrt[2]*c^(3/4)*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4)
)/(8*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.015, size = 650, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)^2/(c*x^4+a),x)
```

```
[Out] -1/4/(a*e^2+c*d^2)^2*c*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*
a)^(1/4)*x+1)*e^2+1/4/(a*e^2+c*d^2)^2*c^2*(1/c*a)^(1/4)/a*2^(1/2)
*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^2-1/4/(a*e^2+c*d^2)^2*c*(1/c
*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*e^2+1/4/(a*e^
2+c*d^2)^2*c^2*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/
4)*x-1)*d^2-1/8/(a*e^2+c*d^2)^2*c*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(
1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2
)+(1/c*a)^(1/2)))*e^2+1/8/(a*e^2+c*d^2)^2*c^2*(1/c*a)^(1/4)/a*2^(
1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^
(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^2-1/4/(a*e^2+c*d^2)^2*c*d*e/(1/
c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))
/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))-1/2/(a*e^2+c*d^2)^2
*c*d*e/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)-1/
2/(a*e^2+c*d^2)^2*c*d*e/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c
*a)^(1/4)*x-1)+1/2*e^4/(a*e^2+c*d^2)^2/d*x/(e*x^2+d)*a+1/2*e^2/(a
*e^2+c*d^2)^2*d*x/(e*x^2+d)*c+1/2*e^4/(a*e^2+c*d^2)^2/d/(d*e)^(1/
2)*arctan(x*e/(d*e)^(1/2))*a+5/2*e^2/(a*e^2+c*d^2)^2*d/(d*e)^(1/2
)*arctan(x*e/(d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 19.115, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)^2),x, algorithm="fricas")
```

```
[Out] [1/4*((c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c
*d^3*e^3 + a^2*d*e^5)*x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 + (a
*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^
6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e
^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*
d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4
*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d
```

```
^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2
*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2
*e^2 + a^2*c^2*e^4)*x + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^
2*d^2*e^4 - a^4*c*e^6 + 2*(a^3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6*
a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9)*sqrt(-(c^7*d^8 - 1
2*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c
^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4
 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10
 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*
c^3*d^3*e - 4*a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^
3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*
c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e
^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56
*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28
*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 +
 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^
8))) - (c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*
c*d^3*e^3 + a^2*d*e^5)*x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 + (
a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e
^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*
e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7
*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^
4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*
d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^
2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^
2*e^2 + a^2*c^2*e^4)*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c
^2*d^2*e^4 - a^4*c*e^6 + 2*(a^3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6
*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9)*sqrt(-(c^7*d^8 -
12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*
c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4
 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10
 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4
*c^3*d^3*e - 4*a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a
^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a
*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*
e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 5
6*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 2
8*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8
+ 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e
^8))) + (c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a
*c*d^3*e^3 + a^2*d*e^5)*x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 -
(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*
e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4
*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^
7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c
^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c
*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c
^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d
^2*e^2 + a^2*c^2*e^4)*x + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*
c^2*d^2*e^4 - a^4*c*e^6 - 2*(a^3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3 +
6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9)*sqrt(-(c^7*d^8 -
 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4
*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^
4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^1
0 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((
4*c^3*d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*
a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*
a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3
*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 +
56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 +
28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8
 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*
e^8))) - (c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*
a*c*d^3*e^3 + a^2*d*e^5)*x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 -
 (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2
*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^
4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c
^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*
c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*
c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*
c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*
d^2*e^2 + a^2*c^2*e^4)*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3
```

```
*c^2*d^2*e^4 - a^4*c*e^6 - 2*(a^3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3 +
 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9)*sqrt(-(c^7*d^8
- 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^
4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e
^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^
10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt(
(4*c^3*d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6
*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12
*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^
3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 +
 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 +
 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^
8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5
*e^8))) + (5*c*d^3*e + a*d*e^3 + (5*c*d^2*e^2 + a*e^4)*x^2)*sqrt(
-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) + 2*(c*d^2*
e^2 + a*e^4)*x)/(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5
*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x^2), 1/4*(2*(5*c*d^3*e + a*d*e^3
 + (5*c*d^2*e^2 + a*e^4)*x^2)*sqrt(e/d)*arctan(e*x/(d*sqrt(e/d)))
 + (c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^
3*e^3 + a^2*d*e^5)*x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 + (a*c^
4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 +
 a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4
- 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^1
4*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^
8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*
e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^
4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*e^
2 + a^2*c^2*e^4)*x + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d
^2*e^4 - a^4*c*e^6 + 2*(a^3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6*a^5
*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9)*sqrt(-(c^7*d^8 - 12*a
*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*
e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 5
6*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 2
8*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^3
*d^3*e - 4*a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c
^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6
*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)
/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^
6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^
9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*
a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))
) - (c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d
^3*e^3 + a^2*d*e^5)*x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 + (a*c
^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6
+ a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4
 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^
14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d
^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2
*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d
^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*e
^2 + a^2*c^2*e^4)*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*
d^2*e^4 - a^4*c*e^6 + 2*(a^3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6*a^
5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9)*sqrt(-(c^7*d^8 - 12*
a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3
*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 +
56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 +
28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^
3*d^3*e - 4*a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*
c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^
6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8
)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a
^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a
^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4
*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)
)) + (c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*
d^3*e^3 + a^2*d*e^5)*x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 - (a*
c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6
 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^
4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d
^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*
d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^
2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*
```

```
d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*
e^2 + a^2*c^2*e^4)*x + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2
*d^2*e^4 - a^4*c*e^6 - 2*(a^3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6*a
^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9)*sqrt(-(c^7*d^8 - 12
*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^
3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 +
 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 +
 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c
^3*d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3
*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c
^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^
8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*
a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*
a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 +
4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8
))) - (c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c
*d^3*e^3 + a^2*d*e^5)*x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 - (a
*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^
6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e
^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*
d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4
*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d
^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2
*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2
*e^2 + a^2*c^2*e^4)*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^
2*d^2*e^4 - a^4*c*e^6 - 2*(a^3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6*
a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9)*sqrt(-(c^7*d^8 - 1
2*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c
^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4
+ 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10
+ 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*
c^3*d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^
3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*
c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e
^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56
*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28
*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 +
 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^
8))) + 2*(c*d^2*e^2 + a*e^4)*x)/(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^
2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)**2/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.28403, size = 698, normalized size = 1.54

$$\frac{\left(5\,cd^2e^2+ae^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,(c^2d^5+2\,acd^3e^2+a^2de^4)\sqrt{d}}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2-2\,\left(ac^3\right)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^4+2\,\sqrt{2}a^2c^2d^2e^2+\sqrt{2}a^3ce^4\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2-2\,\left(ac^3\right)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^4+2\,\sqrt{2}a^2c^2d^2e^2+\sqrt{2}a^3ce^4\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2+2\,\left(ac^3\right)^{\frac{3}{4}}de\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^4+2\,\sqrt{2}a^2c^2d^2e^2+\sqrt{2}a^3ce^4\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2+2\,\left(ac^3\right)^{\frac{3}{4}}de\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^4+2\,\sqrt{2}a^2c^2d^2e^2+\sqrt{2}a^3ce^4\right)}+\frac{xe^2}{2\,(cd^3+ade^2)(x^2e+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)^2),x, algorithm="giac")
```

```
[Out] 1/2*(5*c*d^2*e^2 + a*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^
2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d)) + 1/2*((a*c^3)^(1/4)*
c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*arctan(1/2
*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*
d^4 + 2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c*e^4) + 1/2*((a*c^
3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(
2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c*e^4) + 1
/4*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/
4)*d*e)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^
3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c*e^4) - 1/4*((a*
c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)
*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 +
 2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c*e^4) + 1/2*x*e^2/((c*d
^3 + a*d*e^2)*(x^2*e + d))
```

3.152 $\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx$

Optimal. Leaf size=363

$$-\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)\left(ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{7/4}}+\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)\left(ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{7/4}}-\frac{3\left(\sqrt{a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{7/4}}+\frac{3\left(\sqrt{a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}c^{7/4}}+\frac{x\left(3ex^2\left(ae^2+cd^2\right)+d\left(cd^2-3ae^2\right)\right)}{4ac\left(a+cx^4\right)}-\frac{e^3x^3}{c\left(a+cx^4\right)}$$

```
[Out] -((e^3*x^3)/(c*(a + c*x^4))) + (x*(d*(c*d^2 - 3*a*e^2) + 3*e*(c*d
^2 + a*e^2)*x^2))/(4*a*c*(a + c*x^4)) - (3*(Sqrt[c]*d + Sqrt[a]*e
)*(c*d^2 + a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqr
t[2]*a^(7/4)*c^(7/4)) + (3*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 + a*e^2
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(
7/4)) - (3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(
7/4)) + (3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(
7/4))
```

Rubi [A] time = 0.732895, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$-\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)\left(ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{7/4}}+\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)\left(ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{7/4}}-\frac{3\left(\sqrt{a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{7/4}}+\frac{3\left(\sqrt{a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}c^{7/4}}+\frac{x\left(3ex^2\left(ae^2+cd^2\right)+d\left(cd^2-3ae^2\right)\right)}{4ac\left(a+cx^4\right)}-\frac{e^3x^3}{c\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^3/(a + c*x^4)^2,x]
```

```
[Out] -((e^3*x^3)/(c*(a + c*x^4))) + (x*(d*(c*d^2 - 3*a*e^2) + 3*e*(c*d
^2 + a*e^2)*x^2))/(4*a*c*(a + c*x^4)) - (3*(Sqrt[c]*d + Sqrt[a]*e
)*(c*d^2 + a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqr
t[2]*a^(7/4)*c^(7/4)) + (3*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 + a*e^2
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(
7/4)) - (3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(
7/4)) + (3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(
7/4))
```

Rubi in Sympy [A] time = 98.9976, size = 321, normalized size = 0.88

$$-\frac{x\left(d\left(3ae^2-cd^2\right)+ex^2\left(ae^2-3cd^2\right)\right)}{4ac\left(a+cx^4\right)}+\frac{3\sqrt{2}\left(\sqrt{a}e-\sqrt{c}d\right)\left(ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{32a^{\frac{7}{4}}c^{\frac{7}{4}}}-\frac{3\sqrt{2}\left(\sqrt{a}e-\sqrt{c}d\right)\left(ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{32a^{\frac{7}{4}}c^{\frac{7}{4}}}-\frac{3\sqrt{2}\left(\sqrt{a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}c^{\frac{7}{4}}}+\frac{3\sqrt{2}\left(\sqrt{a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}c^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**3/(c*x**4+a)**2,x)
```

```
[Out] -x*(d*(3*a*e**2 - c*d**2) + e*x**2*(a*e**2 - 3*c*d**2))/(4*a*c*(a
 + c*x**4)) + 3*sqrt(2)*(sqrt(a)*e - sqrt(c)*d)*(a*e**2 + c*d**2)
*log(-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(32
*a**(7/4)*c**(7/4)) - 3*sqrt(2)*(sqrt(a)*e - sqrt(c)*d)*(a*e**2 +
 c*d**2)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x*
*2)/(32*a**(7/4)*c**(7/4)) - 3*sqrt(2)*(sqrt(a)*e + sqrt(c)*d)*(a
*e**2 + c*d**2)*atan(1 - sqrt(2)*c**(1/4)*x/a**(1/4))/(16*a**(7/4
)*c**(7/4)) + 3*sqrt(2)*(sqrt(a)*e + sqrt(c)*d)*(a*e**2 + c*d**2)
*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/(16*a**(7/4)*c**(7/4))
```

Mathematica [A] time = 0.488309, size = 371, normalized size = 1.02

$$-\frac{8a^{3/4}c^{3/4}\left(ae^2x\left(3d+ex^2\right)-cd^2x\left(d+3ex^2\right)\right)}{a+cx^4}+3\sqrt{2}\left(a^{3/2}e^3+\sqrt{a}cd^2e-a\sqrt{c}de^2-c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+3\sqrt{2}\left(-c\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^3/(a + c*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*c^(3/4)*(a*e^2*x*(3*d + e*x^2) - c*d^2*x*(d + 3*e*x^
2)))/(a + c*x^4) - 6*Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + a*S
qrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4
)] + 6*Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 +
 a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]
*(-(c^(3/2)*d^3) + Sqrt[a]*c*d^2*e - a*Sqrt[c]*d*e^2 + a^(3/2)*e^
3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqr
t[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 - a^(3/2)*e
^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(32*a
^(7/4)*c^(7/4))
```

Maple [B] time = 0.014, size = 624, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^3/(c*x^4+a)^2,x)
```

```
[Out] (-1/4*e*(a*e^2-3*c*d^2)/a/c*x^3-1/4*d*(3*a*e^2-c*d^2)/a/c*x)/(c*x
^4+a)+3/16*d*(1/c*a)^(1/4)/a/c*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/
4)*x+1)*e^2+3/16*d^3*(1/c*a)^(1/4)/a^2*2^(1/2)*arctan(2^(1/2)/(1/
c*a)^(1/4)*x+1)+3/16*d*(1/c*a)^(1/4)/a/c*2^(1/2)*arctan(2^(1/2)/(
1/c*a)^(1/4)*x-1)*e^2+3/16*d^3*(1/c*a)^(1/4)/a^2*2^(1/2)*arctan(2
^(1/2)/(1/c*a)^(1/4)*x-1)+3/32*d*(1/c*a)^(1/4)/a/c*2^(1/2)*ln((x^
2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(
1/2)+(1/c*a)^(1/2)))*e^2+3/32*d^3*(1/c*a)^(1/4)/a^2*2^(1/2)*ln((x
^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^
(1/2)+(1/c*a)^(1/2)))+3/32*e^3/c^2/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-
(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/
2)+(1/c*a)^(1/2)))+3/32*e/a/c/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*
a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1
/c*a)^(1/2)))*d^2+3/16*e^3/c^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(1/c*a)^(1/4)*x+1)+3/16*e/a/c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(1/c*a)^(1/4)*x+1)*d^2+3/16*e^3/c^2/(1/c*a)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(1/c*a)^(1/4)*x-1)+3/16*e/a/c/(1/c*a)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/(c*x^4 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.404384, size = 2857, normalized size = 7.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/(c*x^4 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(3*c*d^2*e - a*e^3)*x^3 - 3*(a*c^2*x^4 + a^2*c)*sqrt(-(2*
c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12
 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c
^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*
log(-27*(c^5*d^10 + 3*a*c^4*d^8*e^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c
^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e^10)*x + 27*(a^2*c^5*d^7 + a^
3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^6 + a^6*c^5*e*sqrt(
-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e
^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))*s
qrt(-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(
c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6
 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^
3*c^3))) + 3*(a*c^2*x^4 + a^2*c)*sqrt(-(2*c^2*d^5*e + 4*a*c*d^3*e
^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a
^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^
2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*log(-27*(c^5*d^10 + 3*a
*c^4*d^8*e^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^
2*e^8 - a^5*e^10)*x - 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3
*d^3*e^4 - a^5*c^2*d*e^6 + a^6*c^5*e*sqrt(-(c^6*d^12 + 2*a*c^5*d^
10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 +
2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))*sqrt(-(2*c^2*d^5*e + 4*a
*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^10
*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*
a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))) - 3*(a*c^2*x^4
 + a^2*c)*sqrt(-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*
c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*
```

```
c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7
*c^7)))/(a^3*c^3))*log(-27*(c^5*d^10 + 3*a*c^4*d^8*e^2 + 2*a^2*c^
3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e^10)*x + 2
7*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^
6 - a^6*c^5*e*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^
4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*
e^12)/(a^7*c^7)))*sqrt(-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^
5 - a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4
- 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^
12)/(a^7*c^7)))/(a^3*c^3))) + 3*(a*c^2*x^4 + a^2*c)*sqrt(-(2*c^2*
d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*sqrt(-(c^6*d^12 + 2
*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d
^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*log(
-27*(c^5*d^10 + 3*a*c^4*d^8*e^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d
^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e^10)*x - 27*(a^2*c^5*d^7 + a^3*c^
4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^6 - a^6*c^5*e*sqrt(-(c^
6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 -
 a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))*sqrt(
-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*sqrt(-(c^6*
d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a
^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^
3))) + 4*(c*d^3 - 3*a*d*e^2)*x)/(a*c^2*x^4 + a^2*c)
```

Sympy [A] time = 12.6453, size = 352, normalized size = 0.97

$$\operatorname{RootSum}\left(65536t^4a^7c^7 + t^2\left(9216a^6c^4de^5 + 18432a^5c^5d^3e^3 + 9216a^4c^6d^5e\right) + 81a^6e^{12} + 486a^5cd^2e^{10} + 1215a^4c^2d^4e^8 + 1620\right.$$
$$-\frac{x^3\left(ae^3 - 3cd^2e\right) + x\left(3ade^2 - cd^3\right)}{4a^2c + 4ac^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**3/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**7 + _t**2*(9216*a**6*c**4*d*e**5 + 18
432*a**5*c**5*d**3*e**3 + 9216*a**4*c**6*d**5*e) + 81*a**6*e**12
+ 486*a**5*c*d**2*e**10 + 1215*a**4*c**2*d**4*e**8 + 1620*a**3*c*
*3*d**6*e**6 + 1215*a**2*c**4*d**8*e**4 + 486*a*c**5*d**10*e**2 +
 81*c**6*d**12, Lambda(_t, _t*log(x + (4096*_t**3*a**6*c**5*e + 4
32*_t*a**5*c**2*d*e**6 + 720*_t*a**4*c**3*d**3*e**4 + 144*_t*a**3
*c**4*d**5*e**2 - 144*_t*a**2*c**5*d**7)/(27*a**5*e**10 + 81*a**4
*c*d**2*e**8 + 54*a**3*c**2*d**4*e**6 - 54*a**2*c**3*d**6*e**4 -
81*a*c**4*d**8*e**2 - 27*c**5*d**10)))) - (x**3*(a*e**3 - 3*c*d**
2*e) + x*(3*a*d*e**2 - c*d**3))/(4*a**2*c + 4*a*c**2*x**4)
```

GIAC/XCAS [A] time = 0.282376, size = 574, normalized size = 1.58

$$\frac{3\,cd^2x^3e + cd^3x - ax^3e^3 - 3\,adxe^2}{4\,(cx^4 + a)ac} + \frac{3\,\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^3 + \left(ac^3\right)^{\frac{1}{4}}ac^2de^2 + \left(ac^3\right)^{\frac{3}{4}}cd^2e + \left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^4} + \frac{3\,\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^3 + \left(ac^3\right)^{\frac{1}{4}}ac^2de^2 + \left(ac^3\right)^{\frac{3}{4}}cd^2e + \left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^4} + \frac{3\,\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^3 + \left(ac^3\right)^{\frac{1}{4}}ac^2de^2 - \left(ac^3\right)^{\frac{3}{4}}cd^2e - \left(ac^3\right)^{\frac{3}{4}}ae^3\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c^4} - \frac{3\,\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^3 + \left(ac^3\right)^{\frac{1}{4}}ac^2de^2 - \left(ac^3\right)^{\frac{3}{4}}cd^2e - \left(ac^3\right)^{\frac{3}{4}}ae^3\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/(c*x^4 + a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(3*c*d^2*x^3*e + c*d^3*x - a*x^3*e^3 - 3*a*d*x*e^2)/((c*x^4 +
 a)*a*c) + 3/16*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*
c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^4) + 3
/16*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 +
(a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^4) + 3/32*sqrt(2)*
((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/4
)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) +
 sqrt(a/c))/(a^2*c^4) - 3/32*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*
c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*
e^3)*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^4)
```

3.153 $\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx$

Optimal. Leaf size=349

$$-\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}}+\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}}-\frac{\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}}+\frac{\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}c^{5/4}}+\frac{x\left(ae^2+3cd^2+6cdex^2\right)}{12ac\left(a+cx^4\right)}-\frac{e^2x}{3c\left(a+cx^4\right)}$$

```
[Out] -(e^2*x)/(3*c*(a + c*x^4)) + (x*(3*c*d^2 + a*e^2 + 6*c*d*e*x^2))/
(12*a*c*(a + c*x^4)) - ((3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(5
/4)) + ((3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 + (Sqr
t[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(5/4)) - ((3*c*d^2
 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(5/4)) + ((3*c*d^2
 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(5/4))
```

Rubi [A] time = 0.583859, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$-\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}}+\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}}-\frac{\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}}+\frac{\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}c^{5/4}}+\frac{x\left(ae^2+3cd^2+6cdex^2\right)}{12ac\left(a+cx^4\right)}-\frac{e^2x}{3c\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^2/(a + c*x^4)^2,x]
```

```
[Out] -(e^2*x)/(3*c*(a + c*x^4)) + (x*(3*c*d^2 + a*e^2 + 6*c*d*e*x^2))/
(12*a*c*(a + c*x^4)) - ((3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(5
/4)) + ((3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 + (Sqr
t[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(5/4)) - ((3*c*d^2
 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(5/4)) + ((3*c*d^2
 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(5/4))
```

Rubi in Sympy [A] time = 94.8331, size = 318, normalized size = 0.91

$$-\frac{x\left(ae^2-cd^2-2cdex^2\right)}{4ac\left(a+cx^4\right)}-\frac{\sqrt{2}\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{32a^{\frac{7}{4}}c^{\frac{5}{4}}}+\frac{\sqrt{2}\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{32a^{\frac{7}{4}}c^{\frac{5}{4}}}-\frac{\sqrt{2}\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}c^{\frac{5}{4}}}+\frac{\sqrt{2}\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}c^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2/(c*x**4+a)**2,x)
```

```
[Out] -x*(a*e**2 - c*d**2 - 2*c*d*e*x**2)/(4*a*c*(a + c*x**4)) - sqrt(2
)*(-2*sqrt(a)*sqrt(c)*d*e + a*e**2 + 3*c*d**2)*log(-sqrt(2)*a**(1
/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(32*a**(7/4)*c**(5/4))
 + sqrt(2)*(-2*sqrt(a)*sqrt(c)*d*e + a*e**2 + 3*c*d**2)*log(sqrt(
2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(32*a**(7/4)*c
**(5/4)) - sqrt(2)*(2*sqrt(a)*sqrt(c)*d*e + a*e**2 + 3*c*d**2)*at
an(1 - sqrt(2)*c**(1/4)*x/a**(1/4))/(16*a**(7/4)*c**(5/4)) + sqrt
(2)*(2*sqrt(a)*sqrt(c)*d*e + a*e**2 + 3*c*d**2)*atan(1 + sqrt(2)*
c**(1/4)*x/a**(1/4))/(16*a**(7/4)*c**(5/4))
```

Mathematica [A] time = 0.29115, size = 295, normalized size = 0.85

$$-\frac{8a^{3/4}\sqrt[4]{c}\left(ae^2x-cdx\left(d+2ex^2\right)\right)}{a+cx^4}-\sqrt{2}\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+\sqrt{2}\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^2/(a + c*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*c^(1/4)*(a*e^2*x - c*d*x*(d + 2*e*x^2)))/(a + c*x^4)
 - 2*Sqrt[2]*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 -
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(3*c*d^2 + 2*Sqrt[a]*Sq
rt[c]*d*e + a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt
[2]*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*(3*c*d^2 - 2*Sqrt[a
]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(32*a^(7/4)*c^(5/4))
```

Maple [A] time = 0.012, size = 464, normalized size = 1.3

$$\frac{1}{cx^4+a}\left(\frac{dex^3}{2a}-\frac{(ae^2-cd^2)x}{4ac}\right)+\frac{\sqrt{2}e^2}{16ac}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)$$
$$+\frac{3\sqrt{2}d^2}{16a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)$$
$$+\frac{\sqrt{2}e^2}{32ac}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$+\frac{3\sqrt{2}d^2}{32a^2}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}e^2}{16ac}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{3\sqrt{2}d^2}{16a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)$$
$$+\frac{de\sqrt{2}}{16ac}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$+\frac{de\sqrt{2}}{8ac}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{de\sqrt{2}}{8ac}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^2/(c*x^4+a)^2,x)
```

```
[Out] (1/2*d*e/a*x^3-1/4*(a*e^2-c*d^2)/a/c*x)/(c*x^4+a)+1/16*(1/c*a)^(1
/4)/a/c*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*e^2+3/16*(1/c*a
)^(1/4)/a^2*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^2+1/32*(1
/c*a)^(1/4)/a/c*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(
1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*e^2+3/32*(1/c*
a)^(1/4)/a^2*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2
))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^2+1/16*(1/c*a)^
(1/4)/a/c*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*e^2+3/16*(1/c
*a)^(1/4)/a^2*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^2+1/16/
c*d*e/a/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/
c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/8/c*d*
e/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/8/c
*d*e/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/(c*x^4 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.50171, size = 2155, normalized size = 6.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/(c*x^4 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*c*d*e*x^3 + (a*c^2*x^4 + a^2*c)*sqrt(-(a^3*c^2*sqrt(-(81*
c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6
 + a^4*e^8)/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))*log((
81*c^4*d^8 + 108*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^
2*e^6 + a^4*e^8)*x + (2*a^6*c^4*d*e*sqrt(-(81*c^4*d^8 + 36*a*c^3*
d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^
5)) + 27*a^2*c^4*d^6 + 15*a^3*c^3*d^4*e^2 + 5*a^4*c^2*d^2*e^4 + a
^5*c*e^6)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 2
2*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 12*c*
d^3*e + 4*a*d*e^3)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)*sqrt(-(a^3*c
^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*
a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^
3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4
+ 12*a^3*c*d^2*e^6 + a^4*e^8)*x - (2*a^6*c^4*d*e*sqrt(-(81*c^4*d^
8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4
*e^8)/(a^7*c^5)) + 27*a^2*c^4*d^6 + 15*a^3*c^3*d^4*e^2 + 5*a^4*c^
2*d^2*e^4 + a^5*c*e^6)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^
3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*
c^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)
*sqrt((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*
d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*
a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^2 + 38*a^2*
c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x + (2*a^6*c^4*d*e*sqrt
(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d
^2*e^6 + a^4*e^8)/(a^7*c^5)) - 27*a^2*c^4*d^6 - 15*a^3*c^3*d^4*e^
2 - 5*a^4*c^2*d^2*e^4 - a^5*c*e^6)*sqrt((a^3*c^2*sqrt(-(81*c^4*d^
8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4
*e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))) + (a*c^2*x
^4 + a^2*c)*sqrt((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 +
22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c
*d^3*e - 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^
2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x - (2*a^6*c
^4*d*e*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4
+ 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 27*a^2*c^4*d^6 - 15*a^3
*c^3*d^4*e^2 - 5*a^4*c^2*d^2*e^4 - a^5*c*e^6)*sqrt((a^3*c^2*sqrt(
-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^
2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c^2)))
 + 4*(c*d^2 - a*e^2)*x)/(a*c^2*x^4 + a^2*c)
```

Sympy [A] time = 7.22033, size = 275, normalized size = 0.79

$$\operatorname{RootSum}\left(65536t^4a^7c^5 + t^2\left(2048a^5c^3de^3 + 6144a^4c^4d^3e\right) + a^4e^8 + 20a^3cd^2e^6 + 118a^2c^2d^4e^4 + 180ac^3d^6e^2 + 81c^4d^8, \left(t \mapsto t\right.\right.$$

$$+ \frac{2cdex^3 + x\left(-ae^2 + cd^2\right)}{4a^2c + 4ac^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**2/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**5 + _t**2*(2048*a**5*c**3*d*e**3 + 61
44*a**4*c**4*d**3*e) + a**4*e**8 + 20*a**3*c*d**2*e**6 + 118*a**2
*c**2*d**4*e**4 + 180*a*c**3*d**6*e**2 + 81*c**4*d**8, Lambda(_t,
 _t*log(x + (-8192*_t**3*a**6*c**4*d*e + 16*_t*a**5*c*e**6 - 48*_
t*a**4*c**2*d**2*e**4 - 144*_t*a**3*c**3*d**4*e**2 + 432*_t*a**2*
c**4*d**6)/(a**4*e**8 + 12*a**3*c*d**2*e**6 + 38*a**2*c**2*d**4*e
**4 + 108*a*c**3*d**6*e**2 + 81*c**4*d**8)))) + (2*c*d*e*x**3 + x
*(-a*e**2 + c*d**2))/(4*a**2*c + 4*a*c**2*x**4)
```

GIAC/XCAS [A] time = 0.281599, size = 497, normalized size = 1.42

$$\frac{2\,cdx^3e + cd^2x - axe^2}{4\,(cx^4+a)ac} + \frac{\sqrt{2}\Big(3\,\left(ac^3\right)^{\frac{1}{4}}c^2d^2 + \left(ac^3\right)^{\frac{1}{4}}ace^2 + 2\,\left(ac^3\right)^{\frac{3}{4}}de\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} - \frac{\sqrt{2}\Big(3\,\left(ac^3\right)^{\frac{1}{4}}c^2d^2 + \left(ac^3\right)^{\frac{1}{4}}ace^2 - 2\,\left(ac^3\right)^{\frac{3}{4}}de\Big)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3} + \frac{\sqrt{2}\Big(3\,\left(ac^3\right)^{\frac{1}{4}}ac^4d^2 + \left(ac^3\right)^{\frac{1}{4}}a^2c^3e^2 + 2\,\left(ac^3\right)^{\frac{3}{4}}ac^2de\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^3c^5} + \frac{\sqrt{2}\Big(3\,\left(ac^3\right)^{\frac{1}{4}}ac^4d^2 + \left(ac^3\right)^{\frac{1}{4}}a^2c^3e^2 - 2\,\left(ac^3\right)^{\frac{3}{4}}ac^2de\Big)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/(c*x^4 + a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*c*d*x^3*e + c*d^2*x - a*x*e^2)/((c*x^4 + a)*a*c) + 1/16*sq
rt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3
)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)
^(1/4))/(a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^
3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*ln(x^2 - sqrt(2)*x*(a/c)^
(1/4) + sqrt(a/c))/(a^2*c^3) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)*a*c^
4*d^2 + (a*c^3)^(1/4)*a^2*c^3*e^2 + 2*(a*c^3)^(3/4)*a*c^2*d*e)*ar
ctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^
5) + 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*a*c^4*d^2 + (a*c^3)^(1/4)*a^2*
c^3*e^2 - 2*(a*c^3)^(3/4)*a*c^2*d*e)*ln(x^2 + sqrt(2)*x*(a/c)^(1/
4) + sqrt(a/c))/(a^3*c^5)
```

3.154 $\int \frac{d+ex^2}{(a+cx^4)^2}\,dx$

Optimal. Leaf size=275

$$-\frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}+\frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}-\frac{\left(\sqrt{a}e+3\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}+\frac{\left(\sqrt{a}e+3\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}c^{3/4}}+\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

```
[Out] (x*(d + e*x^2))/(4*a*(a + c*x^4)) - ((3*Sqrt[c]*d + Sqrt[a]*e)*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)
) + ((3*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - ((3*Sqrt[c]*d - Sqrt[a]*e)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2
]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4
))
```

Rubi [A] time = 0.38873, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412

$$-\frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}+\frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}-\frac{\left(\sqrt{a}e+3\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}+\frac{\left(\sqrt{a}e+3\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}c^{3/4}}+\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(a + c*x^4)^2,x]
```


```
[Out] (x*(d + e*x^2))/(4*a*(a + c*x^4)) - ((3*Sqrt[c]*d + Sqrt[a]*e)*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)
) + ((3*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - ((3*Sqrt[c]*d - Sqrt[a]*e)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2
]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4
))
```

Rubi in Sympy [A] time = 75.1655, size = 255, normalized size = 0.93

$$\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}+\frac{\sqrt{2}\left(\sqrt{a}e-3\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{32a^{\frac{7}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}e-3\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{32a^{\frac{7}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}e+3\sqrt{c}d\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}e+3\sqrt{c}d\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] x*(d + e*x**2)/(4*a*(a + c*x**4)) + sqrt(2)*(sqrt(a)*e - 3*sqrt(c
)*d)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)
/(32*a**(7/4)*c**(3/4)) - sqrt(2)*(sqrt(a)*e - 3*sqrt(c)*d)*log(s
qrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(32*a**(7/
4)*c**(3/4)) - sqrt(2)*(sqrt(a)*e + 3*sqrt(c)*d)*atan(1 - sqrt(2)
*c**(1/4)*x/a**(1/4))/(16*a**(7/4)*c**(3/4)) + sqrt(2)*(sqrt(a)*e
 + 3*sqrt(c)*d)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/(16*a**(7/4
)*c**(3/4))
```

Mathematica [A] time = 0.605513, size = 267, normalized size = 0.97

$$\frac{\frac{\sqrt{2}\left(a^{3/4}e-3\sqrt[4]{a}\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}}+\frac{\sqrt{2}\left(3\sqrt[4]{a}\sqrt{c}d-a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}}-\frac{2\sqrt{2}\sqrt[4]{a}\left(\sqrt{a}e+3\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}}+\cdots}{32a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/(a + c*x^4)^2,x]
```

```
[Out] ((8*a*x*(d + e*x^2))/(a + c*x^4) - (2*Sqrt[2]*a^(1/4)*(3*Sqrt[c]*
d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) +
 (2*Sqrt[2]*a^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]
*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (Sqrt[2]*(-3*a^(1/4)*Sqrt[c]*d +
a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
)/c^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[c]*d - a^(3/4)*e)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(32*a^2)
```

Maple [A] time = 0.007, size = 303, normalized size = 1.1

$$\frac{dx}{4a(cx^4+a)}+\frac{3d\sqrt{2}}{32a^2}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{3d\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{3d\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)+\frac{ex^3}{4a(cx^4+a)}+\frac{e\sqrt{2}}{32ac}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{e\sqrt{2}}{16ac}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{e\sqrt{2}}{16ac}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*d*x/a/(c*x^4+a)+3/32*d/a^2*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c
*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(
1/c*a)^(1/2)))+3/16*d/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
/c*a)^(1/4)*x+1)+3/16*d/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(1/c*a)^(1/4)*x-1)+1/4*e*x^3/a/(c*x^4+a)+1/32*e/a/c/(1/c*a)^(1/4)
*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c
*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/16*e/a/c/(1/c*a)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/16*e/a/c/(1/c*a)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(c*x^4 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.290185, size = 1179, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(c*x^4 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*e*x^3 - (a*c*x^4 + a^2)*sqrt(-(a^3*c*sqrt(-(81*c^2*d^4 -
18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))*log(-(81*c
^2*d^4 - a^2*e^4)*x + (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e
^2 + a^2*e^4)/(a^7*c^3)) + 27*a^2*c^2*d^3 - 3*a^3*c*d*e^2)*sqrt(-
(a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) +
 6*d*e)/(a^3*c))) + (a*c*x^4 + a^2)*sqrt(-(a^3*c*sqrt(-(81*c^2*d^
4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))*log(-(
81*c^2*d^4 - a^2*e^4)*x - (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d
^2*e^2 + a^2*e^4)/(a^7*c^3)) + 27*a^2*c^2*d^3 - 3*a^3*c*d*e^2)*sq
rt(-(a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3
)) + 6*d*e)/(a^3*c))) + (a*c*x^4 + a^2)*sqrt((a^3*c*sqrt(-(81*c^2
*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))*log
(-(81*c^2*d^4 - a^2*e^4)*x + (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*
c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 27*a^2*c^2*d^3 + 3*a^3*c*d*e^2)
*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c
^3)) - 6*d*e)/(a^3*c))) - (a*c*x^4 + a^2)*sqrt((a^3*c*sqrt(-(81*c
^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))*l
og(-(81*c^2*d^4 - a^2*e^4)*x - (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*
a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 27*a^2*c^2*d^3 + 3*a^3*c*d*e^
2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7
*c^3)) - 6*d*e)/(a^3*c))) + 4*d*x)/(a*c*x^4 + a^2)
```

Sympy [A] time = 3.8241, size = 136, normalized size = 0.49

$$\operatorname{RootSum}\left(65536t^4a^7c^3 + 3072t^2a^4c^2de + a^2e^4 + 18acd^2e^2 + 81c^2d^4, \left(t \mapsto t\log\left(x + \frac{4096t^3a^6c^2e + 144ta^3cde^2 - 432ta^2c^2d}{a^2e^4 - 81c^2d^4}\right.\right.\right.$$
$$+ \frac{dx + ex^3}{4a^2 + 4acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**3 + 3072*_t**2*a**4*c**2*d*e + a**2*e
**4 + 18*a*c*d**2*e**2 + 81*c**2*d**4, Lambda(_t, _t*log(x + (409
6*_t**3*a**6*c**2*e + 144*_t*a**3*c*d*e**2 - 432*_t*a**2*c**2*d**
3)/(a**2*e**4 - 81*c**2*d**4)))) + (d*x + e*x**3)/(4*a**2 + 4*a*c
*x**4)
```

GIAC/XCAS [A] time = 0.277716, size = 369, normalized size = 1.34

$$\frac{x^3e + dx}{4\,(cx^4 + a)a} + \frac{\sqrt{2}\left(3\,\left(ac^3\right)^{\frac{1}{4}}c^2d + \left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{\sqrt{2}\left(3\,\left(ac^3\right)^{\frac{1}{4}}c^2d + \left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{\sqrt{2}\left(3\,\left(ac^3\right)^{\frac{1}{4}}c^2d - \left(ac^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c^3} - \frac{\sqrt{2}\left(3\,\left(ac^3\right)^{\frac{1}{4}}c^2d - \left(ac^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(c*x^4 + a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(x^3*e + d*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)
*c^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)
^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)*c^
2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1
/4))/(a/c)^(1/4))/(a^2*c^3) + 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d
 - (a*c^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)
*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)
```

3.155 $\int \frac{1}{(a+cx^4)^2}\,dx$

Optimal. Leaf size=202

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{x}{4a\left(a+cx^4\right)}$$

```
[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(8*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) +
(3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sq
rt[2]*a^(7/4)*c^(1/4))
```

Rubi [A] time = 0.240471, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{x}{4a\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^4)^(-2),x]
```

```
[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(8*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) +
(3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sq
rt[2]*a^(7/4)*c^(1/4))
```

Rubi in Sympy [A] time = 57.1851, size = 190, normalized size = 0.94

$$\frac{x}{4a\left(a+cx^4\right)}-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{32a^{\frac{7}{4}}\sqrt[4]{c}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{32a^{\frac{7}{4}}\sqrt[4]{c}}-\frac{3\sqrt{2}\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}\sqrt[4]{c}}+\frac{3\sqrt{2}\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}\sqrt[4]{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+a)**2,x)
```

```
[Out] x/(4*a*(a + c*x**4)) - 3*sqrt(2)*log(-sqrt(2)*a**(1/4)*c**(1/4)*x
 + sqrt(a) + sqrt(c)*x**2)/(32*a**(7/4)*c**(1/4)) + 3*sqrt(2)*log
(sqrt(2)*a**(1/4)*c**(1/4)*x + sqrt(a) + sqrt(c)*x**2)/(32*a**(7/
4)*c**(1/4)) - 3*sqrt(2)*atan(1 - sqrt(2)*c**(1/4)*x/a**(1/4))/(1
6*a**(7/4)*c**(1/4)) + 3*sqrt(2)*atan(1 + sqrt(2)*c**(1/4)*x/a**(
```

```
1/4))/(16*a**(7/4)*c**(1/4))
```

Mathematica [A] time = 0.259062, size = 183, normalized size = 0.91

$$\frac{\frac{8a^{3/4}x}{a+cx^4}-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{c}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^4)^(-2),x]
```

```
[Out] ((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*
x)/a^(1/4)])/c^(1/4) - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(32*a^(7/4))
```

Maple [A] time = 0.006, size = 143, normalized size = 0.7

$$\frac{x}{4\,a\,(cx^4+a)}+\frac{3\,\sqrt{2}}{32\,a^2}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{3\,\sqrt{2}}{16\,a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{3\,\sqrt{2}}{16\,a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+a)^2,x)
```

```
[Out] 1/4*x/a/(c*x^4+a)+3/32/a^2*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^
(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*
a)^(1/2)))+3/16/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^
(1/4)*x+1)+3/16/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^
(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^(-2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.293152, size = 213, normalized size = 1.05

$$-\frac{12\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\arctan\left(\frac{a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}}{x+\sqrt{a^4\sqrt{-\frac{1}{a^7c}}+x^2}}\right)-3\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)+3\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}}{16\left(acx^4+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^(-2),x, algorithm="fricas")
```

```
[Out] -1/16*(12*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*arctan(a^2*(-1/(a^7*
c))^(1/4)/(x + sqrt(a^4*sqrt(-1/(a^7*c)) + x^2))) - 3*(a*c*x^4 +
a^2)*(-1/(a^7*c))^(1/4)*log(a^2*(-1/(a^7*c))^(1/4) + x) + 3*(a*c*
x^4 + a^2)*(-1/(a^7*c))^(1/4)*log(-a^2*(-1/(a^7*c))^(1/4) + x) -
4*x)/(a*c*x^4 + a^2)
```

Sympy [A] time = 1.87491, size = 39, normalized size = 0.19

$$\frac{x}{4a^2 + 4acx^4} + \operatorname{RootSum}\left(65536t^4a^7c + 81, \left(t \mapsto t\log\left(\frac{16ta^2}{3} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+a)**2,x)
```

```
[Out] x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda
(_t, _t*log(16*_t*a**2/3 + x)))
```

GIAC/XCAS [A] time = 0.270863, size = 262, normalized size = 1.3

$$\frac{x}{4\,(cx^4 + a)a} + \frac{3\,\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c} + \frac{3\,\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c} + \frac{3\,\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\ln\left(x^2 + \sqrt{2}x\,\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c} - \frac{3\,\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\ln\left(x^2 - \sqrt{2}x\,\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^(-2),x, algorithm="giac")
```

```
[Out] 1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt
(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(a^2*c) + 3/32*sqrt(2)*(a*c^3)^(1/4)*ln(x^2 + sqrt(2
)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c) - 3/32*sqrt(2)*(a*c^3)^(1/4)
*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c)
```

3.156 $\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=689

$$-\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}+\frac{cx\left(d-ex^2\right)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2+cd^2\right)^2}$$

```
[Out] (c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)
*e^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*
d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d - Sqrt[a]*e)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d
^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (S
qrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) -
 (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2
) - (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)
) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e
^2))
```

Rubi [A] time = 1.14594, antiderivative size = 689, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$

$$-\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}+\frac{cx\left(d-ex^2\right)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)
*e^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*
d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d - Sqrt[a]*e)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d
^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (S
qrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) -
 (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2
) - (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)
) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e
^2))
```

Rubi in Sympy [A] time = 174.876, size = 636, normalized size = 0.92

$$\frac{e^{\frac{7}{2}}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^{2}+cd^{2}\right)^{2}}+\frac{cx\left(d-ex^{2}\right)}{4a\left(a+cx^{4}\right)\left(ae^{2}+cd^{2}\right)}+\frac{\sqrt{2}\sqrt[4]{c}e^{2}\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\left(ae^{2}+cd^{2}\right)^{2}}$$
$$-\frac{\sqrt{2}\sqrt[4]{c}e^{2}\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\left(ae^{2}+cd^{2}\right)^{2}}$$
$$-\frac{\sqrt{2}\sqrt[4]{c}e^{2}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^{2}\right)}{8a^{\frac{3}{4}}\left(ae^{2}+cd^{2}\right)^{2}}$$
$$+\frac{\sqrt{2}\sqrt[4]{c}e^{2}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^{2}\right)}{8a^{\frac{3}{4}}\left(ae^{2}+cd^{2}\right)^{2}}$$
$$+\frac{\sqrt{2}\sqrt[4]{c}\left(\sqrt{a}e-3\sqrt{c}d\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}\left(ae^{2}+cd^{2}\right)}-\frac{\sqrt{2}\sqrt[4]{c}\left(\sqrt{a}e-3\sqrt{c}d\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}\left(ae^{2}+cd^{2}\right)}$$
$$-\frac{\sqrt{2}\sqrt[4]{c}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^{2}\right)}{32a^{\frac{7}{4}}\left(ae^{2}+cd^{2}\right)}$$
$$+\frac{\sqrt{2}\sqrt[4]{c}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^{2}\right)}{32a^{\frac{7}{4}}\left(ae^{2}+cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] e**(7/2)*atan(sqrt(e)*x/sqrt(d))/(sqrt(d)*(a*e**2 + c*d**2)**2) +
 c*x*(d - e*x**2)/(4*a*(a + c*x**4)*(a*e**2 + c*d**2)) + sqrt(2)*
c**(1/4)*e**2*(sqrt(a)*e - sqrt(c)*d)*atan(1 - sqrt(2)*c**(1/4)*x
/a**(1/4))/(4*a**(3/4)*(a*e**2 + c*d**2)**2) - sqrt(2)*c**(1/4)*e
**2*(sqrt(a)*e - sqrt(c)*d)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))
/(4*a**(3/4)*(a*e**2 + c*d**2)**2) - sqrt(2)*c**(1/4)*e**2*(sqrt(
a)*e + sqrt(c)*d)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt
(c) + c*x**2)/(8*a**(3/4)*(a*e**2 + c*d**2)**2) + sqrt(2)*c**(1/4
)*e**2*(sqrt(a)*e + sqrt(c)*d)*log(sqrt(2)*a**(1/4)*c**(3/4)*x +
sqrt(a)*sqrt(c) + c*x**2)/(8*a**(3/4)*(a*e**2 + c*d**2)**2) + sqr
t(2)*c**(1/4)*(sqrt(a)*e - 3*sqrt(c)*d)*atan(1 - sqrt(2)*c**(1/4)
*x/a**(1/4))/(16*a**(7/4)*(a*e**2 + c*d**2)) - sqrt(2)*c**(1/4)*(
sqrt(a)*e - 3*sqrt(c)*d)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/(1
6*a**(7/4)*(a*e**2 + c*d**2)) - sqrt(2)*c**(1/4)*(sqrt(a)*e + 3*s
qrt(c)*d)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*
x**2)/(32*a**(7/4)*(a*e**2 + c*d**2)) + sqrt(2)*c**(1/4)*(sqrt(a)
*e + 3*sqrt(c)*d)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(
c) + c*x**2)/(32*a**(7/4)*(a*e**2 + c*d**2))
```

Mathematica [A] time = 0.535228, size = 429, normalized size = 0.62

$$-\frac{\sqrt{2}\sqrt[4]{c}\left(5a^{3/2}e^{3}+\sqrt{a}cd^{2}e+7a\sqrt{c}de^{2}+3c^{3/2}d^{3}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^{2}\right)}{a^{7/4}}+\frac{\sqrt{2}\sqrt[4]{c}\left(5a^{3/2}e^{3}+\sqrt{a}cd^{2}e+7a\sqrt{c}de^{2}+3c^{3/2}d^{3}\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^{2}\right)}{a^{7/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*c*(c*d^2 + a*e^2)*x*(d - e*x^2))/(a*(a + c*x^4)) + (32*e^(7/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[2]*c^(1/4)*(-3*c
^(3/2)*d^3 + Sqrt[a]*c*d^2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)
```

```
*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^
(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e - 7*a*Sqrt[c]*d*e^2 + 5*a
^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) - (S
qrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqrt[c]*d*e
^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*
d^2*e + 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.021, size = 873, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*x^3*e^3-1/4*c^2/(a*e^2+c*d^2)^2/
(c*x^4+a)*e/a*x^3*d^2+1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*x*d*e^2+1/4
*c^2/(a*e^2+c*d^2)^2/(c*x^4+a)*d^3/a*x+7/16*c/(a*e^2+c*d^2)^2/a*(
1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d*e^2+3/16
*c^2/(a*e^2+c*d^2)^2/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
c*a)^(1/4)*x+1)*d^3+7/16*c/(a*e^2+c*d^2)^2/a*(1/c*a)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d*e^2+3/16*c^2/(a*e^2+c*d^2)^
2/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^3
+7/32*c/(a*e^2+c*d^2)^2/a*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(
1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a
)^(1/2)))*d*e^2+3/32*c^2/(a*e^2+c*d^2)^2/a^2*(1/c*a)^(1/4)*2^(1/2
)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/
4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^3-5/32/(a*e^2+c*d^2)^2/(1/c*a)^(1/
4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1
/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*e^3-1/32*c/(a*e^2+c*d^2)^2/
a/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(
1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^2*e-5/16/(a*
e^2+c*d^2)^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x
+1)*e^3-1/16*c/(a*e^2+c*d^2)^2/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(1/c*a)^(1/4)*x+1)*d^2*e-5/16/(a*e^2+c*d^2)^2/(1/c*a)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*e^3-1/16*c/(a*e^2+c*d^2
)^2/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^2
*e+e^4/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 22.1855, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(c^2*d^2*e + a*c*e^3)*x^3 + (a^2*c^2*d^4 + 2*a^3*c*d^2*
e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*
sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*
d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a
^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^
8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2
*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 +
28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8
+ 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14
 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*
e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^
6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^
8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^
4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a
^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^1
0*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^
2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4
*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*
a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 7
0*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12
+ 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d
^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^
5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 7
38*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 -
 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a
^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*
c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^
13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 +
 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^
8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*
a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^
3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5
*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 73
8*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 -
529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^
7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c
^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^1
3*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 +
4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8
))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 +
750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^
3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a
^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6
*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt
(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748
*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 +
625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d
^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^
3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16
)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*
c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6
 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^
5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5
*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^
2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*
e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*
e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*
d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) + (a^2*c^2*d^4 + 2*a^3*c*d
^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^
4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c
^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6
+ a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5
*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*
c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2
 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e
^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e
^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d
^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4
*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c
*e^8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5
*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 - (a^6*c^5*d^10*e +
9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*
```

a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10
*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*
d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 +
8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6
+ 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^
12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^
2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6
*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12
+ 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^
6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)
/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^
10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28
*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^
8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7
*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 +
2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2
*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*
a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 +
738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6
- 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/
(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^1
0*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*
a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8
+ 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*
e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4
+ 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186
*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 17
5*a^6*c*d*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*
d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*s
qrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2
748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10
+ 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^
6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12
*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e
^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a
^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*
e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2
*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*
a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14
*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d
^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d
^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c
^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - 8*(a*c*e^3*x^4 + a^2*
e^3)*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) -
4*(c^2*d^3 + a*c*d*e^2)*x)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*
e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4), -1/16*(4*
(c^2*d^2*e + a*c*e^3)*x^3 - 16*(a*c*e^3*x^4 + a^2*e^3)*sqrt(e/d)*
arctan(e*x/(d*sqrt(e/d))) + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*
e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^
3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^
4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sq
rt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 27
48*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10
+ 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6
*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*
c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^
16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^
6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 13
76*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27
*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^
5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8
*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^
9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a
^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 195
0*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^
14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4
*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c
*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 7
0*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*
e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d
^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c
^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^1

6 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e
^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4
*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3
*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - (a^
2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^
2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70
*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e
^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^
10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^
3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16
 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^
6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*
e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*
d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(8
1*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^
2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*
e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8
 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*
a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*
d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d
^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*
e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 +
56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10
 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((
6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 +
4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8
)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4
+ 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e
^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9
*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a
^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^1
5*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 +
4*a^6*c*d^2*e^6 + a^7*e^8))) + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a
^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6
*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4
*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)
*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 +
 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^
10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*
c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^
12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15
*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4
*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 +
 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x +
(27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198
*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*
d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2
*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 238
3*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 -
1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7
*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*
c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^1
4*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3
+ 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d
^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^
6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^
4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*
d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^1
0*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*
d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*
c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) -
(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2
*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 +
 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^
4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6
*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4
*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d
^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10
*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d
^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c
^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(
-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3

```
*c^2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d
^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*
e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 +
34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c
^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^
4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6
*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4
 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e
^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqr
t((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8
 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*
e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e
^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^
2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*
a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 5
6*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 +
a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4
 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - 4*(c^2*d^3 + a*c*d*e^2)*x)/(a^2
*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2
*e^2 + a^3*c*e^4)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.288334, size = 814, normalized size = 1.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c
^3)^(3/4)*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sq
rt(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/8*(3*(a*c^3)^(1/
4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/4)*c*d^2*e
- 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^
(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*
e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a
*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)
*a*e^3)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*
c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16
*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)
^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*ln(x^2 - sqrt(2)*x*(a/c)^
(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e
^2 + sqrt(2)*a^4*c^2*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(7/2)/((c
^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d)) - 1/4*(c*x^3*e - c*d*x
)/((c*x^4 + a)*(a*c*d^2 + a^2*e^2))
```

3.157 $\int \frac{1}{(d+ex^2)^2(a+cx^4)^2}\,dx$

Optimal. Leaf size=864

$$\frac{xe^4}{2d\left(cd^2+ae^2\right)^2\left(ex^2+d\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}\left(cd^2+ae^2\right)^2}+\frac{4c\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{\left(cd^2+ae^2\right)^3}-\frac{c^{3/4}\left(3cd^2-4\sqrt{a}\sqrt{c}ed-ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^2}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3}+\frac{c^{3/4}\left(3cd^2-4\sqrt{a}\sqrt{c}ed-ae^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^2}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3}-\frac{c^{3/4}\left(3cd^2+4\sqrt{a}\sqrt{c}ed-ae^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^2}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3}+\frac{c^{3/4}\left(3cd^2+4\sqrt{a}\sqrt{c}ed-ae^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^2}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3}-\frac{c^{3/4}\left(3cd^2-2\sqrt{a}\sqrt{c}ed-3ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)^2}+\frac{c^{3/4}\left(3cd^2-2\sqrt{a}\sqrt{c}ed-3ae^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)^2}-\frac{c^{3/4}\left(3cd^2+2\sqrt{a}\sqrt{c}ed-3ae^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)^2}+\frac{c^{3/4}\left(3cd^2+2\sqrt{a}\sqrt{c}ed-3ae^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)^2}+\frac{cx\left(cd^2-2cex^2d-ae^2\right)}{4a\left(cd^2+ae^2\right)^2\left(cx^4+a\right)}$$

```
[Out] (e^4*x)/(2*d*(c*d^2 + a*e^2)^2*(d + e*x^2)) + (c*x*(c*d^2 - a*e^2
 - 2*c*d*e*x^2))/(4*a*(c*d^2 + a*e^2)^2*(a + c*x^4)) + (4*c*Sqrt[
d]*e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^3 + (e^(7
/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 + a*e^2)^2) -
(c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 -
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)
^3) - (c^(3/4)*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*ArcTan
[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*
e^2)^2) + (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*
ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^
2 + a*e^2)^3) + (c^(3/4)*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e
^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(
c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(3*c*d^2 + 4*Sqrt[a]*Sqrt[c]*d*e
 - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) - (c^(3/4)*(3*c*d^2 + 2*Sq
rt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) + (c^(
3/4)*e^2*(3*c*d^2 + 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d
^2 + a*e^2)^3) + (c^(3/4)*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*
e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*
Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 1.6853, antiderivative size = 864, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$\frac{xe^4}{2d(cd^2+ae^2)^2(ex^2+d)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}(cd^2+ae^2)^2}+\frac{4c\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{(cd^2+ae^2)^3}-\frac{c^{3/4}\left(3cd^2-4\sqrt{a}\sqrt{c}ed-ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^2}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^3}+\frac{c^{3/4}\left(3cd^2-4\sqrt{a}\sqrt{c}ed-ae^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^2}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^3}-\frac{c^{3/4}\left(3cd^2+4\sqrt{a}\sqrt{c}ed-ae^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^2}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^3}+\frac{c^{3/4}\left(3cd^2+4\sqrt{a}\sqrt{c}ed-ae^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^2}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^3}-\frac{c^{3/4}\left(3cd^2-2\sqrt{a}\sqrt{c}ed-3ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^2+ae^2)^2}+\frac{c^{3/4}\left(3cd^2-2\sqrt{a}\sqrt{c}ed-3ae^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}(cd^2+ae^2)^2}-\frac{c^{3/4}\left(3cd^2+2\sqrt{a}\sqrt{c}ed-3ae^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}(cd^2+ae^2)^2}+\frac{c^{3/4}\left(3cd^2+2\sqrt{a}\sqrt{c}ed-3ae^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}(cd^2+ae^2)^2}+\frac{cx\left(cd^2-2cex^2d-ae^2\right)}{4a(cd^2+ae^2)^2(cx^4+a)}$$

Antiderivative was successfully verified.

```
[In]   Int[1/((d + e*x^2)^2*(a + c*x^4)^2),x]
```

```
[Out] (e^4*x)/(2*d*(c*d^2 + a*e^2)^2*(d + e*x^2)) + (c*x*(c*d^2 - a*e^2
 - 2*c*d*e*x^2))/(4*a*(c*d^2 + a*e^2)^2*(a + c*x^4)) + (4*c*Sqrt[
d]*e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^3 + (e^(7
/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 + a*e^2)^2) -
(c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 -
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)
^3) - (c^(3/4)*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*ArcTan
[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*
e^2)^2) + (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*
ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^
2 + a*e^2)^3) + (c^(3/4)*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e
^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(
c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(3*c*d^2 + 4*Sqrt[a]*Sqrt[c]*d*e
 - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) - (c^(3/4)*(3*c*d^2 + 2*Sq
rt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) + (c^(
3/4)*e^2*(3*c*d^2 + 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d
^2 + a*e^2)^3) + (c^(3/4)*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*
e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*
Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2)
```

Rubi in Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)**2/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.1605, size = 540, normalized size = 0.62

$$\frac{-\frac{\sqrt{2}c^{3/4}\left(18a^{3/2}\sqrt{c}de^3-7a^2e^4+2\sqrt{a}c^{3/2}d^3e+12acd^2e^2+3c^2d^4\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{7/4}}+\frac{\sqrt{2}c^{3/4}\left(18a^{3/2}\sqrt{c}de^3-7a^2e^4+2\sqrt{a}c^{3/2}d^3e+12acd^2e^2+3c^2d^4\right.}{a^{7/4}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)^2*(a + c*x^4)^2),x]
```

```
[Out] ((16*e^4*(c*d^2 + a*e^2)*x)/(d*(d + e*x^2)) + (8*c*(c*d^2 + a*e^2
)*x*(-(a*e^2) + c*d*(d - 2*e*x^2)))/(a*(a + c*x^4)) + (16*e^(7/2)
*(9*c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + (2*Sqrt
[2]*c^(3/4)*(-3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e - 12*a*c*d^2*e^
2 + 18*a^(3/2)*Sqrt[c]*d*e^3 + 7*a^2*e^4)*ArcTan[1 - (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(3/4)*(-3*c^2*d^4 + 2*Sq
rt[a]*c^(3/2)*d^3*e - 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 +
 7*a^2*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) - (S
qrt[2]*c^(3/4)*(3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e + 12*a*c*d^2*
e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 - 7*a^2*e^4)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(3/4)*(3*
c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e + 12*a*c*d^2*e^2 + 18*a^(3/2)*S
qrt[c]*d*e^3 - 7*a^2*e^4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^3)
```

Maple [A] time = 0.026, size = 1169, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)^2/(c*x^4+a)^2,x)
```

```
[Out] -1/2/(a*e^2+c*d^2)^3*c^2/(c*x^4+a)*d*e^3*x^3-1/2/(a*e^2+c*d^2)^3*
c^3/(c*x^4+a)*d^3*e/a*x^3-1/4/(a*e^2+c*d^2)^3*c/(c*x^4+a)*a*x*e^4
+1/4/(a*e^2+c*d^2)^3*c^3/(c*x^4+a)/a*x*d^4-7/16/(a*e^2+c*d^2)^3*c
*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*e^4+3/4/
(a*e^2+c*d^2)^3*c^2/a*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a
)^(1/4)*x+1)*d^2*e^2+3/16/(a*e^2+c*d^2)^3*c^3/a^2*(1/c*a)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^4-7/16/(a*e^2+c*d^2)^3
*c*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*e^4+3/
4/(a*e^2+c*d^2)^3*c^2/a*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c
*a)^(1/4)*x-1)*d^2*e^2+3/16/(a*e^2+c*d^2)^3*c^3/a^2*(1/c*a)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^4-7/32/(a*e^2+c*d^2)
^3*c*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a
)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*e^4+3/8/(a*
e^2+c*d^2)^3*c^2/a*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*
2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)
))*d^2*e^2+3/32/(a*e^2+c*d^2)^3*c^3/a^2*(1/c*a)^(1/4)*2^(1/2)*ln(
(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*
2^(1/2)+(1/c*a)^(1/2)))*d^4-9/16/(a*e^2+c*d^2)^3*c/(1/c*a)^(1/4)*
2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*
a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d*e^3-1/16/(a*e^2+c*d^2)^3*c^2
/a/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^
(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^3*e-9/8/(a*
e^2+c*d^2)^3*c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)
*x+1)*d*e^3-1/8/(a*e^2+c*d^2)^3*c^2/a/(1/c*a)^(1/4)*2^(1/2)*arcta
```

```
n(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^3*e-9/8/(a*e^2+c*d^2)^3*c/(1/c*a)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d*e^3-1/8/(a*e^2+
c*d^2)^3*c^2/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)
*x-1)*d^3*e+1/2*e^6/(a*e^2+c*d^2)^3/d*x/(e*x^2+d)*a+1/2*e^4/(a*e^
2+c*d^2)^3*d*x/(e*x^2+d)*c+1/2*e^6/(a*e^2+c*d^2)^3/d/(d*e)^(1/2)*
arctan(x*e/(d*e)^(1/2))*a+9/2*e^4/(a*e^2+c*d^2)^3*d/(d*e)^(1/2)*a
rctan(x*e/(d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 174.494, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)^2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(c^3*d^4*e^2 - a^2*c*e^6)*x^5 + 4*(c^3*d^5*e + 2*a*c^2*
d^3*e^3 + a^2*c*d*e^5)*x^3 - (a^2*c^3*d^8 + 3*a^3*c^2*d^6*e^2 + 3
*a^4*c*d^4*e^4 + a^5*d^2*e^6 + (a*c^4*d^7*e + 3*a^2*c^3*d^5*e^3 +
 3*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*x^6 + (a*c^4*d^8 + 3*a^2*c^3*d^
6*e^2 + 3*a^3*c^2*d^4*e^4 + a^4*c*d^2*e^6)*x^4 + (a^2*c^3*d^7*e +
 3*a^3*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5 + a^5*d*e^7)*x^2)*sqrt((12*c
^5*d^7*e + 156*a*c^4*d^5*e^3 + 404*a^2*c^3*d^3*e^5 - 252*a^3*c^2*
d*e^7 + (a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 +
 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9
*e^12)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*
d^12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*
a^5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^
14 + 2401*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 6
6*a^9*c^10*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e
^8 + 792*a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c
^5*d^10*e^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66
*a^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))/(a^3*c^6*d
^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^
6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12))*log((81*c^
7*d^10 + 1053*a*c^6*d^8*e^2 + 3602*a^2*c^5*d^6*e^4 - 2958*a^3*c^4
*d^4*e^6 - 23667*a^4*c^3*d^2*e^8 + 2401*a^5*c^2*e^10)*x + (27*a^2
*c^7*d^12 + 312*a^3*c^6*d^10*e^2 + 843*a^4*c^5*d^8*e^4 - 1592*a^5
*c^4*d^6*e^6 - 5967*a^6*c^3*d^4*e^8 + 4032*a^7*c^2*d^2*e^10 - 343
*a^8*c*e^12 + 2*(a^6*c^7*d^15*e + 15*a^7*c^6*d^13*e^3 + 69*a^8*c^
5*d^11*e^5 + 155*a^9*c^4*d^9*e^7 + 195*a^10*c^3*d^7*e^9 + 141*a^1
1*c^2*d^5*e^11 + 55*a^12*c*d^3*e^13 + 9*a^13*d*e^15)*sqrt(-(81*c^
11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^12*e^4 - 4776*a^3
*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^5*c^6*d^6*e^10 +
245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14 + 2401*a^8*c^3*e
^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^10*d^20*e^4
 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8 + 792*a^12*c^7*d
^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5*d^10*e^14 + 495*
a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a^17*c^2*d^4*e^20
+ 12*a^18*c*d^2*e^22 + a^19*e^24)))*sqrt((12*c^5*d^7*e + 156*a*c^
4*d^5*e^3 + 404*a^2*c^3*d^3*e^5 - 252*a^3*c^2*d*e^7 + (a^3*c^6*d^
12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6
 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*sqrt(-(81*c^
```

11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^12*e^4 - 4776*a^3
*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^5*c^6*d^6*e^10 +
245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14 + 2401*a^8*c^3*e
^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^10*d^20*e^4
 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8 + 792*a^12*c^7*d
^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5*d^10*e^14 + 495*
a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a^17*c^2*d^4*e^20
+ 12*a^18*c*d^2*e^22 + a^19*e^24)))/(a^3*c^6*d^12 + 6*a^4*c^5*d^1
0*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*
e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12))) + (a^2*c^3*d^8 + 3*a^3*c^2*d
^6*e^2 + 3*a^4*c*d^4*e^4 + a^5*d^2*e^6 + (a*c^4*d^7*e + 3*a^2*c^3
*d^5*e^3 + 3*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*x^6 + (a*c^4*d^8 + 3*
a^2*c^3*d^6*e^2 + 3*a^3*c^2*d^4*e^4 + a^4*c*d^2*e^6)*x^4 + (a^2*c
^3*d^7*e + 3*a^3*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5 + a^5*d*e^7)*x^2)*
sqrt((12*c^5*d^7*e + 156*a*c^4*d^5*e^3 + 404*a^2*c^3*d^3*e^5 - 25
2*a^3*c^2*d*e^7 + (a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4
*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*
e^10 + a^9*e^12)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 516
4*a^2*c^9*d^12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^
8 - 22856*a^5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*
c^4*d^2*e^14 + 2401*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^
22*e^2 + 66*a^9*c^10*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*
c^8*d^16*e^8 + 792*a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 +
792*a^14*c^5*d^10*e^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6
*e^18 + 66*a^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))/
(a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*
c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12))*
log((81*c^7*d^10 + 1053*a*c^6*d^8*e^2 + 3602*a^2*c^5*d^6*e^4 - 29
58*a^3*c^4*d^4*e^6 - 23667*a^4*c^3*d^2*e^8 + 2401*a^5*c^2*e^10)*x
 - (27*a^2*c^7*d^12 + 312*a^3*c^6*d^10*e^2 + 843*a^4*c^5*d^8*e^4
- 1592*a^5*c^4*d^6*e^6 - 5967*a^6*c^3*d^4*e^8 + 4032*a^7*c^2*d^2*
e^10 - 343*a^8*c*e^12 + 2*(a^6*c^7*d^15*e + 15*a^7*c^6*d^13*e^3 +
 69*a^8*c^5*d^11*e^5 + 155*a^9*c^4*d^9*e^7 + 195*a^10*c^3*d^7*e^9
 + 141*a^11*c^2*d^5*e^11 + 55*a^12*c*d^3*e^13 + 9*a^13*d*e^15)*sq
rt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^12*e^4
- 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^5*c^6*d
^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14 + 2401
*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^1
0*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8 + 792*
a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5*d^10*e
^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a^17*c^2
*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))*sqrt((12*c^5*d^7*e
+ 156*a*c^4*d^5*e^3 + 404*a^2*c^3*d^3*e^5 - 252*a^3*c^2*d*e^7 + (
a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c
^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*sq
rt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^12*e^4
- 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^5*c^6*d
^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14 + 2401
*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^1
0*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8 + 792*
a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5*d^10*e
^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a^17*c^2
*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))/(a^3*c^6*d^12 + 6*a
^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^
7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12))) - (a^2*c^3*d^8 + 3
*a^3*c^2*d^6*e^2 + 3*a^4*c*d^4*e^4 + a^5*d^2*e^6 + (a*c^4*d^7*e +
 3*a^2*c^3*d^5*e^3 + 3*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*x^6 + (a*c^
4*d^8 + 3*a^2*c^3*d^6*e^2 + 3*a^3*c^2*d^4*e^4 + a^4*c*d^2*e^6)*x^
4 + (a^2*c^3*d^7*e + 3*a^3*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5 + a^5*d*
e^7)*x^2)*sqrt((12*c^5*d^7*e + 156*a*c^4*d^5*e^3 + 404*a^2*c^3*d^
3*e^5 - 252*a^3*c^2*d*e^7 - (a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 +
15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*
a^8*c*d^2*e^10 + a^9*e^12)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14
*e^2 + 5164*a^2*c^9*d^12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*
c^7*d^8*e^8 - 22856*a^5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 -
48216*a^7*c^4*d^2*e^14 + 2401*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a
^8*c^11*d^22*e^2 + 66*a^9*c^10*d^20*e^4 + 220*a^10*c^9*d^18*e^6 +
 495*a^11*c^8*d^16*e^8 + 792*a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^
12*e^12 + 792*a^14*c^5*d^10*e^14 + 495*a^15*c^4*d^8*e^16 + 220*a^
16*c^3*d^6*e^18 + 66*a^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^1
9*e^24)))/(a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4
 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a

^9*e^12))*log((81*c^7*d^10 + 1053*a*c^6*d^8*e^2 + 3602*a^2*c^5*d^6*e^4 - 2958*a^3*c^4*d^4*e^6 - 23667*a^4*c^3*d^2*e^8 + 2401*a^5*c^2*e^10)*x + (27*a^2*c^7*d^12 + 312*a^3*c^6*d^10*e^2 + 843*a^4*c^5*d^8*e^4 - 1592*a^5*c^4*d^6*e^6 - 5967*a^6*c^3*d^4*e^8 + 4032*a^7*c^2*d^2*e^10 - 343*a^8*c*e^12 - 2*(a^6*c^7*d^15*e + 15*a^7*c^6*d^13*e^3 + 69*a^8*c^5*d^11*e^5 + 155*a^9*c^4*d^9*e^7 + 195*a^10*c^3*d^7*e^9 + 141*a^11*c^2*d^5*e^11 + 55*a^12*c*d^3*e^13 + 9*a^13*d*e^15)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14 + 2401*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^10*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8 + 792*a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5*d^10*e^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))*sqrt((12*c^5*d^7*e + 156*a*c^4*d^5*e^3 + 404*a^2*c^3*d^3*e^5 - 252*a^3*c^2*d*e^7 - (a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14 + 2401*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^10*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8 + 792*a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5*d^10*e^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))/(a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12))) + (a^2*c^3*d^8 + 3*a^3*c^2*d^6*e^2 + 3*a^4*c*d^4*e^4 + a^5*d^2*e^6 + (a*c^4*d^7*e + 3*a^2*c^3*d^5*e^3 + 3*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*x^6 + (a*c^4*d^8 + 3*a^2*c^3*d^6*e^2 + 3*a^3*c^2*d^4*e^4 + a^4*c*d^2*e^6)*x^4 + (a^2*c^3*d^7*e + 3*a^3*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5 + a^5*d*e^7)*x^2)*sqrt((12*c^5*d^7*e + 156*a*c^4*d^5*e^3 + 404*a^2*c^3*d^3*e^5 - 252*a^3*c^2*d*e^7 - (a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14 + 2401*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^10*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8 + 792*a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5*d^10*e^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))/(a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12))*log((81*c^7*d^10 + 1053*a*c^6*d^8*e^2 + 3602*a^2*c^5*d^6*e^4 - 2958*a^3*c^4*d^4*e^6 - 23667*a^4*c^3*d^2*e^8 + 2401*a^5*c^2*e^10)*x - (27*a^2*c^7*d^12 + 312*a^3*c^6*d^10*e^2 + 843*a^4*c^5*d^8*e^4 - 1592*a^5*c^4*d^6*e^6 - 5967*a^6*c^3*d^4*e^8 + 4032*a^7*c^2*d^2*e^10 - 343*a^8*c*e^12 - 2*(a^6*c^7*d^15*e + 15*a^7*c^6*d^13*e^3 + 69*a^8*c^5*d^11*e^5 + 155*a^9*c^4*d^9*e^7 + 195*a^10*c^3*d^7*e^9 + 141*a^11*c^2*d^5*e^11 + 55*a^12*c*d^3*e^13 + 9*a^13*d*e^15)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14 + 2401*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^10*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8 + 792*a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5*d^10*e^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))*sqrt((12*c^5*d^7*e + 156*a*c^4*d^5*e^3 + 404*a^2*c^3*d^3*e^5 - 252*a^3*c^2*d*e^7 - (a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14 + 2401*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^10*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8 + 792*a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5*d^10*e^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))/(a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6

```
*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12))
) - 4*(9*a^2*c*d^3*e^3 + a^3*d*e^5 + (9*a*c^2*d^2*e^4 + a^2*c*e^6
)*x^6 + (9*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x^4 + (9*a^2*c*d^2*e^4 +
a^3*e^6)*x^2)*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(e*x^
2 + d)) - 4*(c^3*d^6 + a^2*c*d^2*e^4 + 2*a^3*e^6)*x)/(a^2*c^3*d^8
 + 3*a^3*c^2*d^6*e^2 + 3*a^4*c*d^4*e^4 + a^5*d^2*e^6 + (a*c^4*d^7
*e + 3*a^2*c^3*d^5*e^3 + 3*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*x^6 + (
a*c^4*d^8 + 3*a^2*c^3*d^6*e^2 + 3*a^3*c^2*d^4*e^4 + a^4*c*d^2*e^6
)*x^4 + (a^2*c^3*d^7*e + 3*a^3*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5 + a^
5*d*e^7)*x^2), -1/16*(8*(c^3*d^4*e^2 - a^2*c*e^6)*x^5 + 4*(c^3*d^
5*e + 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x^3 - 8*(9*a^2*c*d^3*e^3 + a
^3*d*e^5 + (9*a*c^2*d^2*e^4 + a^2*c*e^6)*x^6 + (9*a*c^2*d^3*e^3 +
 a^2*c*d*e^5)*x^4 + (9*a^2*c*d^2*e^4 + a^3*e^6)*x^2)*sqrt(e/d)*ar
ctan(e*x/(d*sqrt(e/d))) - (a^2*c^3*d^8 + 3*a^3*c^2*d^6*e^2 + 3*a^
4*c*d^4*e^4 + a^5*d^2*e^6 + (a*c^4*d^7*e + 3*a^2*c^3*d^5*e^3 + 3*
a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*x^6 + (a*c^4*d^8 + 3*a^2*c^3*d^6*e
^2 + 3*a^3*c^2*d^4*e^4 + a^4*c*d^2*e^6)*x^4 + (a^2*c^3*d^7*e + 3*
a^3*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5 + a^5*d*e^7)*x^2)*sqrt((12*c^5*
d^7*e + 156*a*c^4*d^5*e^3 + 404*a^2*c^3*d^3*e^5 - 252*a^3*c^2*d*e
^7 + (a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20
*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^
12)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^1
2*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^5
*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14
+ 2401*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a
^9*c^10*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8
+ 792*a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5*
d^10*e^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a^
17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))/(a^3*c^6*d^12
 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 +
 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12))*log((81*c^7*d
^10 + 1053*a*c^6*d^8*e^2 + 3602*a^2*c^5*d^6*e^4 - 2958*a^3*c^4*d^
4*e^6 - 23667*a^4*c^3*d^2*e^8 + 2401*a^5*c^2*e^10)*x + (27*a^2*c^
7*d^12 + 312*a^3*c^6*d^10*e^2 + 843*a^4*c^5*d^8*e^4 - 1592*a^5*c^
4*d^6*e^6 - 5967*a^6*c^3*d^4*e^8 + 4032*a^7*c^2*d^2*e^10 - 343*a^
8*c*e^12 + 2*(a^6*c^7*d^15*e + 15*a^7*c^6*d^13*e^3 + 69*a^8*c^5*d
^11*e^5 + 155*a^9*c^4*d^9*e^7 + 195*a^10*c^3*d^7*e^9 + 141*a^11*c
^2*d^5*e^11 + 55*a^12*c*d^3*e^13 + 9*a^13*d*e^15)*sqrt(-(81*c^11*
d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^12*e^4 - 4776*a^3*c^
8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^5*c^6*d^6*e^10 + 245
004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14 + 2401*a^8*c^3*e^16
)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^10*d^20*e^4 +
220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8 + 792*a^12*c^7*d^14
*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5*d^10*e^14 + 495*a^1
5*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a^17*c^2*d^4*e^20 + 1
2*a^18*c*d^2*e^22 + a^19*e^24)))*sqrt((12*c^5*d^7*e + 156*a*c^4*d
^5*e^3 + 404*a^2*c^3*d^3*e^5 - 252*a^3*c^2*d*e^7 + (a^3*c^6*d^12
+ 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 +
15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*sqrt(-(81*c^11*
d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^12*e^4 - 4776*a^3*c^
8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^5*c^6*d^6*e^10 + 245
004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14 + 2401*a^8*c^3*e^16
)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^10*d^20*e^4 +
220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8 + 792*a^12*c^7*d^14
*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5*d^10*e^14 + 495*a^1
5*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a^17*c^2*d^4*e^20 + 1
2*a^18*c*d^2*e^22 + a^19*e^24)))/(a^3*c^6*d^12 + 6*a^4*c^5*d^10*e
^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8
 + 6*a^8*c*d^2*e^10 + a^9*e^12))) + (a^2*c^3*d^8 + 3*a^3*c^2*d^6*
e^2 + 3*a^4*c*d^4*e^4 + a^5*d^2*e^6 + (a*c^4*d^7*e + 3*a^2*c^3*d^
5*e^3 + 3*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*x^6 + (a*c^4*d^8 + 3*a^2
*c^3*d^6*e^2 + 3*a^3*c^2*d^4*e^4 + a^4*c*d^2*e^6)*x^4 + (a^2*c^3*
d^7*e + 3*a^3*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5 + a^5*d*e^7)*x^2)*sqr
t((12*c^5*d^7*e + 156*a*c^4*d^5*e^3 + 404*a^2*c^3*d^3*e^5 - 252*a
^3*c^2*d*e^7 + (a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^
8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^1
0 + a^9*e^12)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a
^2*c^9*d^12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 -
 22856*a^5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4
*d^2*e^14 + 2401*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*
e^2 + 66*a^9*c^10*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8
*d^16*e^8 + 792*a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792
```

```
*a^14*c^5*d^10*e^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^
18 + 66*a^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))/(a^
3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3
*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12))*log
((81*c^7*d^10 + 1053*a*c^6*d^8*e^2 + 3602*a^2*c^5*d^6*e^4 - 2958*
a^3*c^4*d^4*e^6 - 23667*a^4*c^3*d^2*e^8 + 2401*a^5*c^2*e^10)*x -
(27*a^2*c^7*d^12 + 312*a^3*c^6*d^10*e^2 + 843*a^4*c^5*d^8*e^4 - 1
592*a^5*c^4*d^6*e^6 - 5967*a^6*c^3*d^4*e^8 + 4032*a^7*c^2*d^2*e^1
0 - 343*a^8*c*e^12 + 2*(a^6*c^7*d^15*e + 15*a^7*c^6*d^13*e^3 + 69
*a^8*c^5*d^11*e^5 + 155*a^9*c^4*d^9*e^7 + 195*a^10*c^3*d^7*e^9 +
141*a^11*c^2*d^5*e^11 + 55*a^12*c*d^3*e^13 + 9*a^13*d*e^15)*sqrt(
-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^12*e^4 - 4
776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^5*c^6*d^6*
e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14 + 2401*a^
8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^10*d
^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8 + 792*a^1
2*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5*d^10*e^14
 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a^17*c^2*d^
4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))*sqrt((12*c^5*d^7*e + 1
56*a*c^4*d^5*e^3 + 404*a^2*c^3*d^3*e^5 - 252*a^3*c^2*d*e^7 + (a^3
*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*
d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*sqrt(
-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^12*e^4 - 4
776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^5*c^6*d^6*
e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14 + 2401*a^
8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^10*d
^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8 + 792*a^1
2*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5*d^10*e^14
 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a^17*c^2*d^
4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))/(a^3*c^6*d^12 + 6*a^4*
c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c
^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12))) - (a^2*c^3*d^8 + 3*a^
3*c^2*d^6*e^2 + 3*a^4*c*d^4*e^4 + a^5*d^2*e^6 + (a*c^4*d^7*e + 3*
a^2*c^3*d^5*e^3 + 3*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*x^6 + (a*c^4*d
^8 + 3*a^2*c^3*d^6*e^2 + 3*a^3*c^2*d^4*e^4 + a^4*c*d^2*e^6)*x^4 +
 (a^2*c^3*d^7*e + 3*a^3*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5 + a^5*d*e^7
)*x^2)*sqrt((12*c^5*d^7*e + 156*a*c^4*d^5*e^3 + 404*a^2*c^3*d^3*e
^5 - 252*a^3*c^2*d*e^7 - (a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*
a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8
*c*d^2*e^10 + a^9*e^12)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^
2 + 5164*a^2*c^9*d^12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7
*d^8*e^8 - 22856*a^5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 482
16*a^7*c^4*d^2*e^14 + 2401*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*
c^11*d^22*e^2 + 66*a^9*c^10*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 49
5*a^11*c^8*d^16*e^8 + 792*a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*
e^12 + 792*a^14*c^5*d^10*e^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*
c^3*d^6*e^18 + 66*a^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e
^24)))/(a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 +
20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*
e^12))*log((81*c^7*d^10 + 1053*a*c^6*d^8*e^2 + 3602*a^2*c^5*d^6*e
^4 - 2958*a^3*c^4*d^4*e^6 - 23667*a^4*c^3*d^2*e^8 + 2401*a^5*c^2*
e^10)*x + (27*a^2*c^7*d^12 + 312*a^3*c^6*d^10*e^2 + 843*a^4*c^5*d
^8*e^4 - 1592*a^5*c^4*d^6*e^6 - 5967*a^6*c^3*d^4*e^8 + 4032*a^7*c
^2*d^2*e^10 - 343*a^8*c*e^12 - 2*(a^6*c^7*d^15*e + 15*a^7*c^6*d^1
3*e^3 + 69*a^8*c^5*d^11*e^5 + 155*a^9*c^4*d^9*e^7 + 195*a^10*c^3*
d^7*e^9 + 141*a^11*c^2*d^5*e^11 + 55*a^12*c*d^3*e^13 + 9*a^13*d*e
^15)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^
12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^
5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14
 + 2401*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*
a^9*c^10*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8
 + 792*a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5
*d^10*e^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a
^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))*sqrt((12*c^5
*d^7*e + 156*a*c^4*d^5*e^3 + 404*a^2*c^3*d^3*e^5 - 252*a^3*c^2*d*
e^7 - (a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 2
0*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e
^12)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^
12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^
5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14
 + 2401*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*
a^9*c^10*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8
```

```
 + 792*a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5
*d^10*e^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a
^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))/(a^3*c^6*d^1
2 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6
+ 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12))) + (a^2*c^3*
d^8 + 3*a^3*c^2*d^6*e^2 + 3*a^4*c*d^4*e^4 + a^5*d^2*e^6 + (a*c^4*
d^7*e + 3*a^2*c^3*d^5*e^3 + 3*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*x^6
+ (a*c^4*d^8 + 3*a^2*c^3*d^6*e^2 + 3*a^3*c^2*d^4*e^4 + a^4*c*d^2*
e^6)*x^4 + (a^2*c^3*d^7*e + 3*a^3*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5 +
 a^5*d*e^7)*x^2)*sqrt((12*c^5*d^7*e + 156*a*c^4*d^5*e^3 + 404*a^2
*c^3*d^3*e^5 - 252*a^3*c^2*d*e^7 - (a^3*c^6*d^12 + 6*a^4*c^5*d^10
*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e
^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*sqrt(-(81*c^11*d^16 + 1224*a*c^
10*d^14*e^2 + 5164*a^2*c^9*d^12*e^4 - 4776*a^3*c^8*d^10*e^6 - 651
30*a^4*c^7*d^8*e^8 - 22856*a^5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*
e^12 - 48216*a^7*c^4*d^2*e^14 + 2401*a^8*c^3*e^16)/(a^7*c^12*d^24
 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^10*d^20*e^4 + 220*a^10*c^9*d^1
8*e^6 + 495*a^11*c^8*d^16*e^8 + 792*a^12*c^7*d^14*e^10 + 924*a^13
*c^6*d^12*e^12 + 792*a^14*c^5*d^10*e^14 + 495*a^15*c^4*d^8*e^16 +
 220*a^16*c^3*d^6*e^18 + 66*a^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^2
2 + a^19*e^24)))/(a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*
d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e
^10 + a^9*e^12))*log((81*c^7*d^10 + 1053*a*c^6*d^8*e^2 + 3602*a^2
*c^5*d^6*e^4 - 2958*a^3*c^4*d^4*e^6 - 23667*a^4*c^3*d^2*e^8 + 240
1*a^5*c^2*e^10)*x - (27*a^2*c^7*d^12 + 312*a^3*c^6*d^10*e^2 + 843
*a^4*c^5*d^8*e^4 - 1592*a^5*c^4*d^6*e^6 - 5967*a^6*c^3*d^4*e^8 +
4032*a^7*c^2*d^2*e^10 - 343*a^8*c*e^12 - 2*(a^6*c^7*d^15*e + 15*a
^7*c^6*d^13*e^3 + 69*a^8*c^5*d^11*e^5 + 155*a^9*c^4*d^9*e^7 + 195
*a^10*c^3*d^7*e^9 + 141*a^11*c^2*d^5*e^11 + 55*a^12*c*d^3*e^13 +
9*a^13*d*e^15)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*
a^2*c^9*d^12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8
- 22856*a^5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^
4*d^2*e^14 + 2401*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22
*e^2 + 66*a^9*c^10*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^
8*d^16*e^8 + 792*a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 79
2*a^14*c^5*d^10*e^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e
^18 + 66*a^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))*sq
rt((12*c^5*d^7*e + 156*a*c^4*d^5*e^3 + 404*a^2*c^3*d^3*e^5 - 252*
a^3*c^2*d*e^7 - (a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d
^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^
10 + a^9*e^12)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*
a^2*c^9*d^12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8
- 22856*a^5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^
4*d^2*e^14 + 2401*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22
*e^2 + 66*a^9*c^10*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^
8*d^16*e^8 + 792*a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 79
2*a^14*c^5*d^10*e^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e
^18 + 66*a^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))/(a
^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^
3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12))) -
 4*(c^3*d^6 + a^2*c*d^2*e^4 + 2*a^3*e^6)*x)/(a^2*c^3*d^8 + 3*a^3*
c^2*d^6*e^2 + 3*a^4*c*d^4*e^4 + a^5*d^2*e^6 + (a*c^4*d^7*e + 3*a^
2*c^3*d^5*e^3 + 3*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*x^6 + (a*c^4*d^8
 + 3*a^2*c^3*d^6*e^2 + 3*a^3*c^2*d^4*e^4 + a^4*c*d^2*e^6)*x^4 + (
a^2*c^3*d^7*e + 3*a^3*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5 + a^5*d*e^7)*
x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)**2/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.285917, size = 1, normalized size = 0.

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)^2),x, algorithm="giac")
```

```
[Out] Done
```

3.158 $\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=326

$$\frac{3ex\sqrt{a+cx^4}\,(5cd^2-ae^2)}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{3\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,(5cd^2-ae^2)\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{5\sqrt{c}d(cd^2-ae^2)}{\sqrt{a}}-3ae^3+15cd^2e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10c^{7/4}\sqrt{a+cx^4}}+\frac{de^2x\sqrt{a+cx^4}}{c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}$$

```
[Out] (d*e^2*x*Sqrt[a + c*x^4])/c + (e^3*x^3*Sqrt[a + c*x^4])/(5*c) + (
3*e*(5*c*d^2 - a*e^2)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sq
rt[c]*x^2)) - (3*a^(1/4)*e*(5*c*d^2 - a*e^2)*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[a + c*x^4]) + (a^(1
/4)*(15*c*d^2*e - 3*a*e^3 + (5*Sqrt[c]*d*(c*d^2 - a*e^2))/Sqrt[a]
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(10*c^(7/4)*S
qrt[a + c*x^4])
```

Rubi [A] time = 0.545271, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{3ex\sqrt{a+cx^4}\,(5cd^2-ae^2)}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{3\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,(5cd^2-ae^2)\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{5\sqrt{c}d(cd^2-ae^2)}{\sqrt{a}}-3ae^3+15cd^2e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10c^{7/4}\sqrt{a+cx^4}}+\frac{de^2x\sqrt{a+cx^4}}{c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^3/Sqrt[a + c*x^4],x]
```


```
[Out] (d*e^2*x*Sqrt[a + c*x^4])/c + (e^3*x^3*Sqrt[a + c*x^4])/(5*c) + (
3*e*(5*c*d^2 - a*e^2)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sq
rt[c]*x^2)) - (3*a^(1/4)*e*(5*c*d^2 - a*e^2)*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[a + c*x^4]) + (a^(1
/4)*(15*c*d^2*e - 3*a*e^3 + (5*Sqrt[c]*d*(c*d^2 - a*e^2))/Sqrt[a]
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(10*c^(7/4)*S
qrt[a + c*x^4])
```

Rubi in Sympy [A] time = 69.7671, size = 299, normalized size = 0.92

$$\frac{3\sqrt[4]{a}e\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(ae^2-5cd^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{7}{4}}\sqrt{a+cx^4}} + \frac{de^2x\sqrt{a+cx^4}}{c} + \frac{e^3x^3\sqrt{a+cx^4}}{5c} - \frac{3ex\sqrt{a+cx^4}\left(ae^2-5cd^2\right)}{5c^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(3\sqrt{a}e\left(ae^2-5cd^2\right)+5\sqrt{c}d\left(ae^2-cd^2\right)\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{10\sqrt[4]{a}c^{\frac{7}{4}}\sqrt{a+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] 3*a**(1/4)*e*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt
(a) + sqrt(c)*x**2)*(a*e**2 - 5*c*d**2)*elliptic_e(2*atan(c**(1/4
)*x/a**(1/4)), 1/2)/(5*c**(7/4)*sqrt(a + c*x**4)) + d*e**2*x*sqrt
(a + c*x**4)/c + e**3*x**3*sqrt(a + c*x**4)/(5*c) - 3*e*x*sqrt(a
+ c*x**4)*(a*e**2 - 5*c*d**2)/(5*c**(3/2)*(sqrt(a) + sqrt(c)*x**2
)) - sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sq
rt(c)*x**2)*(3*sqrt(a)*e*(a*e**2 - 5*c*d**2) + 5*sqrt(c)*d*(a*e**
2 - c*d**2))*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2)/(10*a**
(1/4)*c**(7/4)*sqrt(a + c*x**4))
```

Mathematica [C] time = 0.55683, size = 235, normalized size = 0.72

$$\frac{\sqrt{\frac{cx^4}{a}+1}\left(3a^{3/2}e^3-15\sqrt{a}cd^2e+5ia\sqrt{c}de^2-5ic^{3/2}d^3\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-3\sqrt{a}e\sqrt{\frac{cx^4}{a}+1}\left(ae^2-5cd^2\right)E\left(i\sinh\right.}{5c^{3/2}\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^3/Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[c]*e^2*x*(5*d + e*x^2)*(a + c*x^4
) - 3*Sqrt[a]*e*(-5*c*d^2 + a*e^2)*Sqrt[1 + (c*x^4)/a]*EllipticE[
I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + ((-5*I)*c^(3/2)*d^3
 - 15*Sqrt[a]*c*d^2*e + (5*I)*a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*Sq
rt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x
], -1])/(5*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^(3/2)*Sqrt[a + c*x^4])
```

Maple [C] time = 0.017, size = 388, normalized size = 1.2

$$d^{3}\sqrt{1-ix^{2}\sqrt{c}\,\frac{1}{\sqrt{a}}}\sqrt{1+ix^{2}\sqrt{c}\,\frac{1}{\sqrt{a}}}\,EllipticF\left(x\sqrt{i\sqrt{c}\,\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^{4}+a}}+e^{3}\left(\frac{x^{3}}{5c}\sqrt{cx^{4}+a}\right.$$
$$-\frac{3i}{5}a^{\frac{3}{2}}\sqrt{1-ix^{2}\sqrt{c}\,\frac{1}{\sqrt{a}}}\sqrt{1+ix^{2}\sqrt{c}\,\frac{1}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{i\sqrt{c}\,\frac{1}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{i\sqrt{c}\,\frac{1}{\sqrt{a}}},i\right)\right)c^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\left.\frac{1}{\sqrt{cx^{4}+a}}\right)$$
$$+3\,id^{2}e\sqrt{a}\sqrt{1-ix^{2}\sqrt{c}\,\frac{1}{\sqrt{a}}}\sqrt{1+ix^{2}\sqrt{c}\,\frac{1}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{i\sqrt{c}\,\frac{1}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{i\sqrt{c}\,\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^{4}+a}}\frac{1}{\sqrt{c}}$$
$$+3e^{2}d\left(1/3\,\frac{x\sqrt{cx^{4}+a}}{c}-1/3\,\frac{a}{c\sqrt{cx^{4}+a}}\sqrt{1-\frac{i\sqrt{c}x^{2}}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}x^{2}}{\sqrt{a}}}\,EllipticF\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\frac{1}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^3/(c*x^4+a)^(1/2),x)
```

```
[Out] d^3/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/
2)*c^(1/2))^(1/2),I)+e^3*(1/5/c*x^3*(c*x^4+a)^(1/2)-3/5*I/c^(3/2)
*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2
)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I
/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),
I)))+3*I*d^2*e*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(
1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c
^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a
^(1/2)*c^(1/2))^(1/2),I))+3*e^2*d*(1/3*x*(c*x^4+a)^(1/2)/c-1/3/c*
a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*c^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^{2}+d\right)^{3}}{\sqrt{cx^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/sqrt(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^{3}x^{6}+3\,de^{2}x^{4}+3\,d^{2}ex^{2}+d^{3}}{\sqrt{cx^{4}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/sqrt(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)/sqrt(c*x^4 +
 a), x)
```

Sympy [A] time = 7.89018, size = 173, normalized size = 0.53

$$\frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{3d^2ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}+\frac{3de^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}+\frac{e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi
)/a)/(4*sqrt(a)*gamma(5/4)) + 3*d**2*e*x**3*gamma(3/4)*hyper((1/2
, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
+ 3*d*e**2*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_p
olar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + e**3*x**7*gamma(7/4)*hyper
((1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(
11/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^3}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/sqrt(c*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + a), x)
```

3.159 $\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=264

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(6\sqrt{a}\sqrt{c}de-ae^2+3cd^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}c^{5/4}\sqrt{a+cx^4}}-\frac{2\sqrt[4]{a}de\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{2dex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{3c}$$

```
[Out] (e^2*x*Sqrt[a + c*x^4])/(3*c) + (2*d*e*x*Sqrt[a + c*x^4])/(Sqrt[c
]*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1/4)*d*e*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + ((3*c*d^
2 + 6*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], 1/2])/(6*a^(1/4)*c^(5/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.301531, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(6\sqrt{a}\sqrt{c}de-ae^2+3cd^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}c^{5/4}\sqrt{a+cx^4}}-\frac{2\sqrt[4]{a}de\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{2dex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^2/Sqrt[a + c*x^4],x]
```

```
[Out] (e^2*x*Sqrt[a + c*x^4])/(3*c) + (2*d*e*x*Sqrt[a + c*x^4])/(Sqrt[c
]*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1/4)*d*e*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + ((3*c*d^
2 + 6*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], 1/2])/(6*a^(1/4)*c^(5/4)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 40.9805, size = 243, normalized size = 0.92

$$-\frac{2\sqrt[4]{a}de\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{c^{\frac{3}{4}}\sqrt{a+cx^4}}+\frac{e^2x\sqrt{a+cx^4}}{3c}+\frac{2dex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-6\sqrt{a}\sqrt{c}de+ae^2-3cd^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{6\sqrt[4]{a}c^{\frac{5}{4}}\sqrt{a+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] -2*a**(1/4)*d*e*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(s
qrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)), 1/
2)/(c**(3/4)*sqrt(a + c*x**4)) + e**2*x*sqrt(a + c*x**4)/(3*c) +
2*d*e*x*sqrt(a + c*x**4)/(sqrt(c)*(sqrt(a) + sqrt(c)*x**2)) - sqr
t((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x*
*2)*(-6*sqrt(a)*sqrt(c)*d*e + a*e**2 - 3*c*d**2)*elliptic_f(2*ata
n(c**(1/4)*x/a**(1/4)), 1/2)/(6*a**(1/4)*c**(5/4)*sqrt(a + c*x**4
))
```

Mathematica [C] time = 0.391357, size = 195, normalized size = 0.74

$$\frac{i\sqrt{\frac{cx^4}{a}+1}\left(6i\sqrt{a}\sqrt{c}de+ae^2-3cd^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+6\sqrt{a}\sqrt{c}de\sqrt{\frac{cx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+e^2x\sqrt{\cdots}}{3c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^2/Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[(I*Sqrt[c])/Sqrt[a]]*e^2*x*(a + c*x^4) + 6*Sqrt[a]*Sqrt[c]*
d*e*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt
[a]]*x], -1] + I*(-3*c*d^2 + (6*I)*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*S
qrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*
x], -1])/(3*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*Sqrt[a + c*x^4])
```

Maple [C] time = 0.01, size = 266, normalized size = 1.

$$d^2\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$
$$+e^2\left(\frac{x}{3c}\sqrt{cx^4+a}-\frac{a}{3c}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\right)$$
$$+2ide\sqrt{a}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^2/(c*x^4+a)^(1/2),x)
```

```
[Out] d^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/
2)*c^(1/2))^(1/2),I)+e^2*(1/3*x*(c*x^4+a)^(1/2)/c-1/3/c*a/(I/a^(1
/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))
^(1/2),I))+2*I*d*e*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)
*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/
2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*
(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^2+d\right)^2}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/sqrt(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^4 + 2\,dex^2 + d^2}{\sqrt{cx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/sqrt(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/sqrt(c*x^4 + a), x)
```

Sympy [A] time = 5.6902, size = 124, normalized size = 0.47

$$\frac{d^2x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)} + \frac{dex^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)} + \frac{e^2x^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi
)/a)/(4*sqrt(a)*gamma(5/4)) + d*e*x**3*gamma(3/4)*hyper((1/2, 3/4
), (7/4,), c*x**4*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + e**
2*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi
)/a)/(4*sqrt(a)*gamma(9/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2}{\sqrt{cx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/sqrt(c*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

3.160 $\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=226

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{ex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[Out] (e*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4
)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sq
rt[a + c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.174628, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{ex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4
)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sq
rt[a + c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 20.2763, size = 204, normalized size = 0.9

$$-\frac{\sqrt[4]{a}e\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{c^{\frac{3}{4}}\sqrt{a+cx^4}}+\frac{ex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}e+\sqrt{c}d\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{\frac{3}{4}}\sqrt{a+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] -a**(1/4)*e*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(
a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)), 1/2)/(
c**(3/4)*sqrt(a + c*x**4)) + e*x*sqrt(a + c*x**4)/(sqrt(c)*(sqrt(
```

```
a) + sqrt(c)*x**2)) + sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)*
*2)*(sqrt(a) + sqrt(c)*x**2)*(sqrt(a)*e + sqrt(c)*d)*elliptic_f(2
*atan(c**(1/4)*x/a**(1/4)), 1/2)/(2*a**(1/4)*c**(3/4)*sqrt(a + c*
x**4))
```

Mathematica [C] time = 0.111229, size = 131, normalized size = 0.58

$$\frac{\sqrt{\frac{cx^4}{a}+1}\left(\left(-\sqrt{a}e-i\sqrt{c}d\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{a}eE\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)\right)}{\sqrt{c}\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[1 + (c*x^4)/a]*(Sqrt[a]*e*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[
c])/Sqrt[a]]*x], -1] + ((-I)*Sqrt[c]*d - Sqrt[a]*e)*EllipticF[I*A
rcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/(Sqrt[(I*Sqrt[c])/Sqrt
[a]]*Sqrt[c]*Sqrt[a + c*x^4])
```

Maple [C] time = 0.006, size = 169, normalized size = 0.8

$$d\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$
$$+ie\sqrt{a}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(c*x^4+a)^(1/2),x)
```

```
[Out] d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*c^(1/2))^(1/2),I)+I*e*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(
1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)
^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-Elliptic
E(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2+d}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(c*x^4 + a), x)
```

Sympy [A] time = 3.68965, size = 78, normalized size = 0.35

$$\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a
)/(4*sqrt(a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7
/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(c*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + a), x)
```

3.161 $\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=323

$$\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{ae}{d}+\frac{cd}{e}}}{\sqrt{a+cx^4}}\right)}{2d\sqrt{\frac{ae}{d}+\frac{cd}{e}}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\Pi\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\sqrt{a+cx^4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)}$$

```
[Out] ArcTan[(Sqrt[(c*d)/e + (a*e)/d]*x)/Sqrt[a + c*x^4]]/(2*d*Sqrt[(c*
d)/e + (a*e)/d]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + c*x^4])
- (((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[a]*((Sqrt[c]*d)
/Sqrt[a] - e)^2)/(4*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)],
1/2])/(4*a^(1/4)*c^(1/4)*d*((Sqrt[c]*d)/Sqrt[a] - e)*Sqrt[a + c*x
^4])
```

Rubi [A] time = 0.4085, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{ae}{d}+\frac{cd}{e}}}{\sqrt{a+cx^4}}\right)}{2d\sqrt{\frac{ae}{d}+\frac{cd}{e}}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}d\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[(c*d)/e + (a*e)/d]*x)/Sqrt[a + c*x^4]]/(2*d*Sqrt[(c*
d)/e + (a*e)/d]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + c*x^4])
- (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d -
Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4
)], 1/2])/(4*c^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 21.5877, size = 275, normalized size = 0.85

$$\frac{\operatorname{atan}\left(\frac{x\sqrt{\frac{ae}{d}+\frac{cd}{e}}}{\sqrt{a+cx^4}}\right)}{2d\sqrt{\frac{ae}{d}+\frac{cd}{e}}}-\frac{\sqrt[4]{c}\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e-\sqrt{c}d\right)}+\frac{\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}e+\sqrt{c}d\right)\Pi\left(-\frac{\sqrt{a}\left(e-\frac{\sqrt{c}d}{\sqrt{a}}\right)^2}{4\sqrt{c}de};2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\sqrt{a+cx^4}\left(\sqrt{a}e-\sqrt{c}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] atan(x*sqrt(a*e/d + c*d/e)/sqrt(a + c*x**4))/(2*d*sqrt(a*e/d + c*
d/e)) - c**(1/4)*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(
sqrt(a) + sqrt(c)*x**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1
/2)/(2*a**(1/4)*sqrt(a + c*x**4)*(sqrt(a)*e - sqrt(c)*d)) + sqrt(
(a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2
)*(sqrt(a)*e + sqrt(c)*d)*elliptic_pi(-sqrt(a)*(e - sqrt(c)*d/sqr
t(a))**2/(4*sqrt(c)*d*e), 2*atan(c**(1/4)*x/a**(1/4)), 1/2)/(4*a*
*(1/4)*c**(1/4)*d*sqrt(a + c*x**4)*(sqrt(a)*e - sqrt(c)*d))
```

Mathematica [C] time = 0.0833063, size = 95, normalized size = 0.29

$$-\frac{i\sqrt{\frac{cx^4}{a}+1}\,\Pi\left(-\frac{i\sqrt{a}e}{\sqrt{c}d};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d)
, I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/
Sqrt[a]]*d*Sqrt[a + c*x^4])
```

Maple [C] time = 0.041, size = 107, normalized size = 0.3

$$\frac{1}{d}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticPi\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},\frac{ie}{d}\sqrt{a}\frac{1}{\sqrt{c}},1\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}}\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1
/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/
2)/(I/a^(1/2)*c^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^4+a}(ex^2+d)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a+cx^4}\,(d+ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

3.162 $\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=570

$$\frac{e^2x\sqrt{a+cx^4}}{2d(d+ex^2)(ae^2+cd^2)}-\frac{\sqrt{c}ex\sqrt{a+cx^4}}{2d(\sqrt{a}+\sqrt{c}x^2)(ae^2+cd^2)}+\frac{\sqrt[4]{a}\sqrt[4]{c}e(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2d\sqrt{a+cx^4}(ae^2+cd^2)}-\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)(ae^2+3cd^2)\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{c}d^2\sqrt{a+cx^4}(\sqrt{c}d-\sqrt{a}e)(ae^2+cd^2)}+\frac{(ae^2+3cd^2)\tan^{-1}\left(\frac{x\sqrt{\frac{ae}{d}+\frac{cd}{e}}}{\sqrt{a+cx^4}}\right)}{4d^3e\left(\frac{ae}{d}+\frac{cd}{e}\right)^{3/2}}-\frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+cx^4}(\sqrt{a}e-\sqrt{c}d)}$$

```
[Out] -(Sqrt[c]*e*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(Sqrt[a] + Sq
rt[c]*x^2)) + (e^2*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(d + e
*x^2)) + ((3*c*d^2 + a*e^2)*ArcTan[(Sqrt[(c*d)/e + (a*e)/d]*x)/Sq
rt[a + c*x^4]])/(4*d^3*e*((c*d)/e + (a*e)/d)^(3/2)) + (a^(1/4)*c^
(1/4)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*d*(c
*d^2 + a*e^2)*Sqrt[a + c*x^4]) - (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*(-(Sqrt[c]*d) + Sqrt[a]*e
)*Sqrt[a + c*x^4]) - (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(3*c*d^2
+ a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt
[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*c^(1/4)*d^2*(Sq
rt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.918835, antiderivative size = 713, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{e^2x\sqrt{a+cx^4}}{2d(d+ex^2)(ae^2+cd^2)}-\frac{\sqrt{c}ex\sqrt{a+cx^4}}{2d(\sqrt{a}+\sqrt{c}x^2)(ae^2+cd^2)}+\frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(ae^2+3cd^2)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}d\sqrt{a+cx^4}(\sqrt{c}d-\sqrt{a}e)(ae^2+cd^2)}-\frac{\sqrt[4]{a}\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4d\sqrt{a+cx^4}(ae^2+cd^2)}+\frac{\sqrt[4]{a}\sqrt[4]{c}e(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2d\sqrt{a+cx^4}(ae^2+cd^2)}-\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)(ae^2+3cd^2)\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{c}d^2\sqrt{a+cx^4}(\sqrt{c}d-\sqrt{a}e)(ae^2+cd^2)}+\frac{(ae^2+3cd^2)\tan^{-1}\left(\frac{x\sqrt{\frac{ae}{d}+\frac{cd}{e}}}{\sqrt{a+cx^4}}\right)}{4d^3e\left(\frac{ae}{d}+\frac{cd}{e}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] -(Sqrt[c]*e*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(Sqrt[a] + Sq
rt[c]*x^2)) + (e^2*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(d + e
*x^2)) + ((3*c*d^2 + a*e^2)*ArcTan[(Sqrt[(c*d)/e + (a*e)/d]*x)/Sq
rt[a + c*x^4]])/(4*d^3*e*((c*d)/e + (a*e)/d)^(3/2)) + (a^(1/4)*c^
(1/4)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*d*(c
*d^2 + a*e^2)*Sqrt[a + c*x^4]) - (a^(1/4)*c^(1/4)*((Sqrt[c]*d)/Sq
rt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*d
*(c*d^2 + a*e^2)*Sqrt[a + c*x^4]) + (c^(1/4)*(3*c*d^2 + a*e^2)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*d*(Sqr
t[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqrt[a + c*x^4]) - (a^(1/4)*(
(Sqrt[c]*d)/Sqrt[a] + e)*(3*c*d^2 + a*e^2)*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c
]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(8*c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a
*e^2)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 82.3448, size = 619, normalized size = 1.09

$$\frac{\sqrt[4]{a}\sqrt[4]{c}e\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2d\sqrt{a+cx^4}\left(ae^2+cd^2\right)}-\frac{\sqrt{c}ex\sqrt{a+cx^4}}{2d\left(\sqrt{a}+\sqrt{c}x^2\right)\left(ae^2+cd^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}+\frac{\left(ae^2+3cd^2\right)\operatorname{atan}\left(\frac{x\sqrt{\frac{ae}{d}+\frac{cd}{e}}}{\sqrt{a+cx^4}}\right)}{4d^3e\left(\frac{ae}{d}+\frac{cd}{e}\right)^{\frac{3}{2}}}-\frac{\sqrt[4]{c}\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}e+\sqrt{c}d\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4\sqrt[4]{a}d\sqrt{a+cx^4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(ae^2+3cd^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4\sqrt[4]{a}d\sqrt{a+cx^4}\left(\sqrt{a}e-\sqrt{c}d\right)\left(ae^2+cd^2\right)}+\frac{\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}e+\sqrt{c}d\right)\left(ae^2+3cd^2\right)\Pi\left(-\frac{\sqrt{a}\left(e-\frac{\sqrt{c}d}{\sqrt{a}}\right)^2}{4\sqrt{c}de};2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{c}d^2\sqrt{a+cx^4}\left(\sqrt{a}e-\sqrt{c}d\right)\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] a**(1/4)*c**(1/4)*e*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2
)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4))
, 1/2)/(2*d*sqrt(a + c*x**4)*(a*e**2 + c*d**2)) - sqrt(c)*e*x*sqr
t(a + c*x**4)/(2*d*(sqrt(a) + sqrt(c)*x**2)*(a*e**2 + c*d**2)) +
e**2*x*sqrt(a + c*x**4)/(2*d*(d + e*x**2)*(a*e**2 + c*d**2)) + (a
*e**2 + 3*c*d**2)*atan(x*sqrt(a*e/d + c*d/e)/sqrt(a + c*x**4))/(4
*d**3*e*(a*e/d + c*d/e)**(3/2)) - c**(1/4)*sqrt((a + c*x**4)/(sqr
t(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(sqrt(a)*e + sq
rt(c)*d)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2)/(4*a**(1/4)
*d*sqrt(a + c*x**4)*(a*e**2 + c*d**2)) - c**(1/4)*sqrt((a + c*x**
4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(a*e**2
+ 3*c*d**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2)/(4*a**(1
/4)*d*sqrt(a + c*x**4)*(sqrt(a)*e - sqrt(c)*d)*(a*e**2 + c*d**2))
 + sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt
(c)*x**2)*(sqrt(a)*e + sqrt(c)*d)*(a*e**2 + 3*c*d**2)*elliptic_pi
(-sqrt(a)*(e - sqrt(c)*d/sqrt(a))**2/(4*sqrt(c)*d*e), 2*atan(c**(
1/4)*x/a**(1/4)), 1/2)/(8*a**(1/4)*c**(1/4)*d**2*sqrt(a + c*x**4)
```

```
*(sqrt(a)*e - sqrt(c)*d)*(a*e**2 + c*d**2))
```

Mathematica [C] time = 1.00079, size = 522, normalized size = 0.92

$$-3icd^3\sqrt{\frac{cx^4}{a}+1}\left(-\frac{i\sqrt{a}e}{\sqrt{c}d};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-3icd^2ex^2\sqrt{\frac{cx^4}{a}+1}\left(-\frac{i\sqrt{a}e}{\sqrt{c}d};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-iae^3x^2\sqrt{\frac{cx^4}{a}+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] (a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*e^2*x + Sqrt[(I*Sqrt[c])/Sqrt[a]]*
c*d*e^2*x^5 - Sqrt[a]*Sqrt[c]*d*e*(d + e*x^2)*Sqrt[1 + (c*x^4)/a]
*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[c]*
d*(I*Sqrt[c]*d + Sqrt[a]*e)*(d + e*x^2)*Sqrt[1 + (c*x^4)/a]*Ellip
ticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - (3*I)*c*d^3*Sq
rt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcS
inh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - I*a*d*e^2*Sqrt[1 + (c*x^4
)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*S
qrt[c])/Sqrt[a]]*x], -1] - (3*I)*c*d^2*e*x^2*Sqrt[1 + (c*x^4)/a]*
EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c
])/Sqrt[a]]*x], -1] - I*a*e^3*x^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[
((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]
*x], -1])/(2*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d^2*(c*d^2 + a*e^2)*(d + e
*x^2)*Sqrt[a + c*x^4])
```

Maple [C] time = 0.032, size = 556, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/2*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)-1/2*c/(a*e^2+
c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*
(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^
(1/2)*c^(1/2))^(1/2),I)-1/2*I*e*c^(1/2)/(a*e^2+c*d^2)/d*a^(1/2)/(
I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(
1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^
(1/2))^(1/2),I)+1/2*I*e*c^(1/2)/(a*e^2+c*d^2)/d*a^(1/2)/(I/a^(1/2
)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(
1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(
1/2),I)+1/2/(a*e^2+c*d^2)/d^2*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/
a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4
+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2
)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a+3/2
/(a*e^2+c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2
)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticP
i(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c
^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+a}(ex^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^4}\,(d+ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+a}(ex^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

3.163 $\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=213

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{5\sqrt{c}d(ae^2+cd^2)}{\sqrt{a}}-3e\left(ae^2+5cd^2\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\left(ae^2+5cd^2\right)E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}-\frac{de^2x\sqrt{a-cx^4}}{c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

```
[Out] -((d*e^2*x*Sqrt[a - c*x^4])/c) - (e^3*x^3*Sqrt[a - c*x^4])/(5*c)
+ (3*a^(3/4)*e*(5*c*d^2 + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[Ar
cSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4]) + (a^
(3/4)*((5*Sqrt[c]*d*(c*d^2 + a*e^2))/Sqrt[a] - 3*e*(5*c*d^2 + a*e
^2))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -
1])/(5*c^(7/4)*Sqrt[a - c*x^4])
```

Rubi [A] time = 0.579759, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{5\sqrt{c}d(ae^2+cd^2)}{\sqrt{a}}-3e\left(ae^2+5cd^2\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\left(ae^2+5cd^2\right)E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}-\frac{de^2x\sqrt{a-cx^4}}{c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^3/Sqrt[a - c*x^4],x]
```

```
[Out] -((d*e^2*x*Sqrt[a - c*x^4])/c) - (e^3*x^3*Sqrt[a - c*x^4])/(5*c)
+ (3*a^(3/4)*e*(5*c*d^2 + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[Ar
cSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4]) + (a^
(3/4)*((5*Sqrt[c]*d*(c*d^2 + a*e^2))/Sqrt[a] - 3*e*(5*c*d^2 + a*e
^2))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -
1])/(5*c^(7/4)*Sqrt[a - c*x^4])
```

Rubi in Sympy [A] time = 81.8445, size = 194, normalized size = 0.91

$$\frac{3a^{\frac{3}{4}}e\sqrt{1-\frac{cx^4}{a}}\left(ae^2+5cd^2\right)E\left(\operatorname{asin}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{\frac{7}{4}}\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}e\left(ae^2+5cd^2\right)-5\sqrt{c}d\left(ae^2+cd^2\right)\right)F\left(\operatorname{asin}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{\frac{7}{4}}\sqrt{a-cx^4}}-\frac{de^2x\sqrt{a-cx^4}}{c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**3/(-c*x**4+a)**(1/2),x)
```

```
[Out] 3*a**(3/4)*e*sqrt(1 - c*x**4/a)*(a*e**2 + 5*c*d**2)*elliptic_e(as
in(c**(1/4)*x/a**(1/4)), -1)/(5*c**(7/4)*sqrt(a - c*x**4)) - a**(
```

```
1/4)*sqrt(1 - c*x**4/a)*(3*sqrt(a)*e*(a*e**2 + 5*c*d**2) - 5*sqrt
(c)*d*(a*e**2 + c*d**2))*elliptic_f(asin(c**(1/4)*x/a**(1/4)), -1
)/(5*c**(7/4)*sqrt(a - c*x**4)) - d*e**2*x*sqrt(a - c*x**4)/c - e
**3*x**3*sqrt(a - c*x**4)/(5*c)
```

Mathematica [C] time = 0.811455, size = 232, normalized size = 1.09

$$\frac{i\sqrt{1-\frac{cx^4}{a}}\left(3a^{3/2}e^3+15\sqrt{a}cd^2e-5a\sqrt{c}de^2-5c^{3/2}d^3\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-3i\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\left(ae^2+5cd^2\right)E\left(i\sin\right.}{5c^{3/2}\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^3/Sqrt[a - c*x^4],x]
```

```
[Out] (Sqrt[a]*(-(Sqrt[c]/Sqrt[a]))^(3/2)*e^2*x*(5*d + e*x^2)*(a - c*x^
4) - (3*I)*Sqrt[a]*e*(5*c*d^2 + a*e^2)*Sqrt[1 - (c*x^4)/a]*Ellipt
icE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*(-5*c^(3/2)*d^
3 + 15*Sqrt[a]*c*d^2*e - 5*a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*Sqrt[
1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -
1])/(5*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^(3/2)*Sqrt[a - c*x^4])
```

Maple [B] time = 0.028, size = 360, normalized size = 1.7

$$d^3\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{1\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{1\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-cx^4+a}}+e^3\left(-\frac{x^3}{5c}\sqrt{-cx^4+a}\right.$$
$$\left.-\frac{3}{5}a^{\frac{3}{2}}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{1\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{1\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)c^{-\frac{3}{2}}\frac{1}{\sqrt{1\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-cx^4+a}}\right)$$
$$-3\frac{d^2e\sqrt{a}}{\sqrt{-cx^4+a}\sqrt{c}}\sqrt{1-\frac{x^2\sqrt{c}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{c}}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}$$
$$+3e^2d\left(-1/3\frac{x\sqrt{-cx^4+a}}{c}+1/3\frac{a}{c\sqrt{-cx^4+a}}\sqrt{1-\frac{x^2\sqrt{c}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{c}}{\sqrt{a}}}EllipticF\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\frac{1}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^3/(-c*x^4+a)^(1/2),x)
```

```
[Out] d^3/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1
/2)*c^(1/2))^(1/2),I)+e^3*(-1/5/c*x^3*(-c*x^4+a)^(1/2)-3/5/c^(3/2
)*a^(3/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/
2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*(EllipticF(x*
(1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2
),I)))-3*d^2*e*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(
1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/
c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(1/
a^(1/2)*c^(1/2))^(1/2),I))+3*e^2*d*(-1/3/c*x*(-c*x^4+a)^(1/2)+1/3
/c*a/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(
1/2)*c^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^3}{\sqrt{-cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/sqrt(-c*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}{\sqrt{-cx^4+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/sqrt(-c*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)/sqrt(-c*x^4
+ a), x)
```

Sympy [A] time = 8.70571, size = 180, normalized size = 0.85

$$\frac{d^3x\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{3d^2ex^3\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}+\frac{3de^2x^5\,\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}+\frac{e^3x^7\,\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**3/(-c*x**4+a)**(1/2),x)
```

```
[Out] d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*
pi)/a)/(4*sqrt(a)*gamma(5/4)) + 3*d**2*e*x**3*gamma(3/4)*hyper((1
/2, 3/4), (7/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/
4)) + 3*d*e**2*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*e
xp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + e**3*x**7*gamma(7/4)
*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a
)*gamma(11/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^3}{\sqrt{-cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/sqrt(-c*x^4 + a),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + a), x)
```

3.164 $\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=162

$$\frac{2a^{3/4}de\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3c^{5/4}\sqrt{a-cx^4}}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

```
[Out] -(e^2*x*Sqrt[a - c*x^4])/(3*c) + (2*a^(3/4)*d*e*Sqrt[1 - (c*x^4)/
a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a -
c*x^4]) + (a^(1/4)*(3*c*d^2 - 6*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Sqrt
[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(3*c^
(5/4)*Sqrt[a - c*x^4])
```

Rubi [A] time = 0.339464, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$\frac{2a^{3/4}de\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3c^{5/4}\sqrt{a-cx^4}}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^2/Sqrt[a - c*x^4],x]
```

```
[Out] -(e^2*x*Sqrt[a - c*x^4])/(3*c) + (2*a^(3/4)*d*e*Sqrt[1 - (c*x^4)/
a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a -
c*x^4]) + (a^(1/4)*(3*c*d^2 - 6*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Sqrt
[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(3*c^
(5/4)*Sqrt[a - c*x^4])
```

Rubi in Sympy [A] time = 56.9213, size = 150, normalized size = 0.93

$$\frac{2a^{\frac{3}{4}}de\sqrt{1-\frac{cx^4}{a}}E\left(\operatorname{asin}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{\frac{3}{4}}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)F\left(\operatorname{asin}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3c^{\frac{5}{4}}\sqrt{a-cx^4}}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2/(-c*x**4+a)**(1/2),x)
```

```
[Out] 2*a**(3/4)*d*e*sqrt(1 - c*x**4/a)*elliptic_e(asin(c**(1/4)*x/a**(
1/4)), -1)/(c**(3/4)*sqrt(a - c*x**4)) + a**(1/4)*sqrt(1 - c*x**4
/a)*(-6*sqrt(a)*sqrt(c)*d*e + a*e**2 + 3*c*d**2)*elliptic_f(asin(
c**(1/4)*x/a**(1/4)), -1)/(3*c**(5/4)*sqrt(a - c*x**4)) - e**2*x*
sqrt(a - c*x**4)/(3*c)
```

Mathematica [C] time = 0.439196, size = 192, normalized size = 1.19

$$\frac{-i\sqrt{1-\frac{cx^4}{a}}\left(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-6i\sqrt{a}\sqrt{c}de\sqrt{1-\frac{cx^4}{a}}E\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+e^{}}{3c\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^2/Sqrt[a - c*x^4],x]
```

```
[Out] (Sqrt[-(Sqrt[c]/Sqrt[a])]*e^2*x*(-a + c*x^4) - (6*I)*Sqrt[a]*Sqrt
[c]*d*e*Sqrt[1 - (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sq
rt[a])]*x], -1] - I*(3*c*d^2 - 6*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Sqr
t[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x],
 -1])/(3*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*Sqrt[a - c*x^4])
```

Maple [A] time = 0.01, size = 246, normalized size = 1.5

$$d^2\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{1\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{1\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-cx^4+a}}$$
$$+e^2\left(-\frac{x}{3c}\sqrt{-cx^4+a}\right.$$
$$\left.+\frac{a}{3c}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{1\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{1\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-cx^4+a}}\right)$$
$$-2\frac{de\sqrt{a}}{\sqrt{-cx^4+a}\sqrt{c}}\sqrt{1-\frac{x^2\sqrt{c}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{c}}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^2/(-c*x^4+a)^(1/2),x)
```

```
[Out] d^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1
/2)*c^(1/2))^(1/2),I)+e^2*(-1/3/c*x*(-c*x^4+a)^(1/2)+1/3/c*a/(1/a
^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2
)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1
/2))^(1/2),I))-2*d*e*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/
2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^
(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE
(x*(1/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex^2+d)^2}{\sqrt{-cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/sqrt(-c*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^4 + 2\,dex^2 + d^2}{\sqrt{-cx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/sqrt(-c*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/sqrt(-c*x^4 + a), x)
```

Sympy [A] time = 6.25555, size = 129, normalized size = 0.8

$$\frac{d^2x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)} + \frac{dex^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)} + \frac{e^2x^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**2/(-c*x**4+a)**(1/2),x)
```

```
[Out] d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*
pi)/a)/(4*sqrt(a)*gamma(5/4)) + d*e*x**3*gamma(3/4)*hyper((1/2, 3
/4), (7/4,), c*x**4*exp_polar(2*I*pi)/a)/(2*sqrt(a)*gamma(7/4)) +
 e**2*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(
2*I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2}{\sqrt{-cx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/sqrt(-c*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + a), x)
```

3.165 $\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=124

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}$$

```
[Out] (a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/
4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[
a] - e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)]
, -1])/(c^(3/4)*Sqrt[a - c*x^4])
```

Rubi [A] time = 0.220254, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/Sqrt[a - c*x^4],x]
```

```
[Out] (a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/
4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[
a] - e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)]
, -1])/(c^(3/4)*Sqrt[a - c*x^4])
```

Rubi in Sympy [A] time = 36.2828, size = 112, normalized size = 0.9

$$\frac{a^{\frac{3}{4}}e\sqrt{1-\frac{cx^4}{a}}E\left(\operatorname{asin}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{\frac{3}{4}}\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}e-\sqrt{c}d\right)F\left(\operatorname{asin}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{\frac{3}{4}}\sqrt{a-cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

```
[Out] a**(3/4)*e*sqrt(1 - c*x**4/a)*elliptic_e(asin(c**(1/4)*x/a**(1/4)
), -1)/(c**(3/4)*sqrt(a - c*x**4)) - a**(1/4)*sqrt(1 - c*x**4/a)*
(sqrt(a)*e - sqrt(c)*d)*elliptic_f(asin(c**(1/4)*x/a**(1/4)), -1)
/(c**(3/4)*sqrt(a - c*x**4))
```

Mathematica [C] time = 0.126764, size = 127, normalized size = 1.02

$$\frac{i\sqrt{1-\frac{cx^4}{a}}\left(\left(\sqrt{c}d-\sqrt{a}e\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{a}eE\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)\right)}{\sqrt{a}\left(-\frac{\sqrt{c}}{\sqrt{a}}\right)^{3/2}\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/Sqrt[a - c*x^4],x]
```

```
[Out] (I*Sqrt[1 - (c*x^4)/a]*(Sqrt[a]*e*EllipticE[I*ArcSinh[Sqrt[-(Sqrt
[c]/Sqrt[a])]*x], -1] + (Sqrt[c]*d - Sqrt[a]*e)*EllipticF[I*ArcSi
nh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]))/(Sqrt[a]*(-(Sqrt[c]/Sqrt[a]
))^(3/2)*Sqrt[a - c*x^4])
```

Maple [A] time = 0.006, size = 154, normalized size = 1.2

$$d\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{1\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{1\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-cx^4+a}}$$
$$-e\sqrt{a}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{1\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{1\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{1\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-cx^4+a}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(-c*x^4+a)^(1/2),x)
```

```
[Out] d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/
a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2
)*c^(1/2))^(1/2),I)-e*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1
/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)
^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-Elliptic
E(x*(1/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{-cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(-c*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ex^2+d}{\sqrt{-cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(-c*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

Sympy [A] time = 4.11344, size = 82, normalized size = 0.66

$$\frac{dx\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

```
[Out] d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)
/a)/(4*sqrt(a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4),
(7/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{-cx^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(-c*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

3.166 $\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}$$

```
[Out] (a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)
), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[a - c*x^4])
```

Rubi [A] time = 0.121227, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

```
[Out] (a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)
), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[a - c*x^4])
```

Rubi in Sympy [A] time = 15.3858, size = 63, normalized size = 0.88

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\operatorname{asin}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

```
[Out] a**(1/4)*sqrt(1 - c*x**4/a)*elliptic_pi(-sqrt(a)*e/(sqrt(c)*d), a
sin(c**(1/4)*x/a**(1/4)), -1)/(c**(1/4)*d*sqrt(a - c*x**4))
```

Mathematica [C] time = 0.086924, size = 91, normalized size = 1.26

$$-\frac{i\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{d\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)),
I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[
a])]*d*Sqrt[a - c*x^4])
```

Maple [A] time = 0.028, size = 97, normalized size = 1.4

$$\frac{1}{d}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\,EllipticPi\left(x\sqrt{1\sqrt{c}\frac{1}{\sqrt{a}}},-\frac{e}{d}\sqrt{a}\frac{1}{\sqrt{c}},1\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}}\frac{1}{\sqrt{1\sqrt{c}\frac{1}{\sqrt{a}}}}\right)\frac{1}{\sqrt{1\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x)
```

```
[Out] 1/d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(
1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/
2)/(1/a^(1/2)*c^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-cx^4+a}(ex^2+d)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)),x, algorithm="giac")

[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

3.167 $\int \frac{1}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=299

$$-\frac{a^{3/4}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\sqrt{a-cx^4}\left(cd^2-ae^2\right)}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{c}d^2\sqrt{a-cx^4}\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}-\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\sqrt{a-cx^4}\left(\sqrt{a}e+\sqrt{c}d\right)}$$

```
[Out] -(e^2*x*Sqrt[a - c*x^4])/(2*d*(c*d^2 - a*e^2)*(d + e*x^2)) - (a^(
3/4)*c^(1/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a
^(1/4)], -1])/(2*d*(c*d^2 - a*e^2)*Sqrt[a - c*x^4]) - (a^(1/4)*c^
(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)],
-1])/(2*d*(Sqrt[c]*d + Sqrt[a]*e)*Sqrt[a - c*x^4]) + (a^(1/4)*(3*
c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt
[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*c^(1/4)*d^2*(c*d^2
- a*e^2)*Sqrt[a - c*x^4])
```

Rubi [A] time = 0.517639, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$

$$-\frac{a^{3/4}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\sqrt{a-cx^4}\left(cd^2-ae^2\right)}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{c}d^2\sqrt{a-cx^4}\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}-\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\sqrt{a-cx^4}\left(\sqrt{a}e+\sqrt{c}d\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)^2*Sqrt[a - c*x^4]),x]
```

```
[Out] -(e^2*x*Sqrt[a - c*x^4])/(2*d*(c*d^2 - a*e^2)*(d + e*x^2)) - (a^(
3/4)*c^(1/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a
^(1/4)], -1])/(2*d*(c*d^2 - a*e^2)*Sqrt[a - c*x^4]) - (a^(1/4)*c^
(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)],
-1])/(2*d*(Sqrt[c]*d + Sqrt[a]*e)*Sqrt[a - c*x^4]) + (a^(1/4)*(3*
c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt
[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*c^(1/4)*d^2*(c*d^2
- a*e^2)*Sqrt[a - c*x^4])
```

Rubi in Sympy [A] time = 69.3863, size = 265, normalized size = 0.89

$$\frac{a^{\frac{3}{4}}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}E\left(\operatorname{asin}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\sqrt{a-cx^4}\left(ae^2-cd^2\right)}-\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}e-\sqrt{c}d\right)F\left(\operatorname{asin}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\sqrt{a-cx^4}\left(ae^2-cd^2\right)}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(ae^2-3cd^2\right)\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\operatorname{asin}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{c}d^2\sqrt{a-cx^4}\left(ae^2-cd^2\right)}+\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(ae^2-cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)**2/(-c*x**4+a)**(1/2),x)
```

```
[Out] a**(3/4)*c**(1/4)*e*sqrt(1 - c*x**4/a)*elliptic_e(asin(c**(1/4)*x
/a**(1/4)), -1)/(2*d*sqrt(a - c*x**4)*(a*e**2 - c*d**2)) - a**(1/
4)*c**(1/4)*sqrt(1 - c*x**4/a)*(sqrt(a)*e - sqrt(c)*d)*elliptic_f
(asin(c**(1/4)*x/a**(1/4)), -1)/(2*d*sqrt(a - c*x**4)*(a*e**2 - c
*d**2)) + a**(1/4)*sqrt(1 - c*x**4/a)*(a*e**2 - 3*c*d**2)*ellipti
c_pi(-sqrt(a)*e/(sqrt(c)*d), asin(c**(1/4)*x/a**(1/4)), -1)/(2*c*
*(1/4)*d**2*sqrt(a - c*x**4)*(a*e**2 - c*d**2)) + e**2*x*sqrt(a -
 c*x**4)/(2*d*(d + e*x**2)*(a*e**2 - c*d**2))
```

Mathematica [C] time = 1.56114, size = 508, normalized size = 1.7

$$-3icd^3\sqrt{1-\frac{cx^4}{a}}\left(-\frac{\sqrt{a}e}{\sqrt{c}d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-3icd^2ex^2\sqrt{1-\frac{cx^4}{a}}\left(-\frac{\sqrt{a}e}{\sqrt{c}d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+iae^3x^2\sqrt{1-\frac{cx^4}{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)^2*Sqrt[a - c*x^4]),x]
```

```
[Out] (-(a*Sqrt[-(Sqrt[c]/Sqrt[a])]*d*e^2*x) + Sqrt[-(Sqrt[c]/Sqrt[a])]
*c*d*e^2*x^5 + I*Sqrt[a]*Sqrt[c]*d*e*(d + e*x^2)*Sqrt[1 - (c*x^4)
/a]*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*Sqrt
[c]*d*(-(Sqrt[c]*d) + Sqrt[a]*e)*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*
EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - (3*I)*c*d^
3*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*Ar
cSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*a*d*e^2*Sqrt[1 - (c*x^
4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqr
t[c]/Sqrt[a])]*x], -1] - (3*I)*c*d^2*e*x^2*Sqrt[1 - (c*x^4)/a]*El
lipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqr
t[a])]*x], -1] + I*a*e^3*x^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sq
rt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]
)/(2*Sqrt[-(Sqrt[c]/Sqrt[a])]*d^2*(c*d^2 - a*e^2)*(d + e*x^2)*Sqr
t[a - c*x^4])
```

Maple [B] time = 0.033, size = 523, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x)
```

```
[Out] 1/2*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)+1/2*c/(a*e^2
-c*d^2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)
*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/
a^(1/2)*c^(1/2))^(1/2),I)-1/2*e*c^(1/2)/(a*e^2-c*d^2)/d*a^(1/2)/(
1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(
1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c
^(1/2))^(1/2),I)+1/2*e*c^(1/2)/(a*e^2-c*d^2)/d*a^(1/2)/(1/a^(1/2)
*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1
/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(
1/2),I)+1/2/(a*e^2-c*d^2)/d^2*e^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/
a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^
4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^
(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*a-3/2
/(a*e^2-c*d^2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2
)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Elliptic
Pi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c
^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)**2/(-c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2), x)
```

3.168 $\int \frac{d+ex^2}{\sqrt{-a+cx^4}} dx$

Optimal. Leaf size=126

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{cx^4-a}}+\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{cx^4-a}}$$

```
[Out] (a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/
4)], -1])/(c^(3/4)*Sqrt[-a + c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt
[a] - e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)
], -1])/(c^(3/4)*Sqrt[-a + c*x^4])
```

Rubi [A] time = 0.214027, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{cx^4-a}}+\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/
4)], -1])/(c^(3/4)*Sqrt[-a + c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt
[a] - e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)
], -1])/(c^(3/4)*Sqrt[-a + c*x^4])
```

Rubi in Sympy [A] time = 35.8939, size = 112, normalized size = 0.89

$$\frac{a^{\frac{3}{4}}e\sqrt{1-\frac{cx^4}{a}}E\left(\operatorname{asin}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{\frac{3}{4}}\sqrt{-a+cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}e-\sqrt{c}d\right)F\left(\operatorname{asin}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{\frac{3}{4}}\sqrt{-a+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(c*x**4-a)**(1/2),x)
```

```
[Out] a**(3/4)*e*sqrt(1 - c*x**4/a)*elliptic_e(asin(c**(1/4)*x/a**(1/4)
), -1)/(c**(3/4)*sqrt(-a + c*x**4)) - a**(1/4)*sqrt(1 - c*x**4/a)
*(sqrt(a)*e - sqrt(c)*d)*elliptic_f(asin(c**(1/4)*x/a**(1/4)), -1
)/(c**(3/4)*sqrt(-a + c*x**4))
```

Mathematica [C] time = 0.130873, size = 128, normalized size = 1.02

$$\frac{i\sqrt{1-\frac{cx^4}{a}}\left(\left(\sqrt{c}d-\sqrt{a}e\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{a}eE\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)\right)}{\sqrt{a}\left(-\frac{\sqrt{c}}{\sqrt{a}}\right)^{3/2}\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (I*Sqrt[1 - (c*x^4)/a]*(Sqrt[a]*e*EllipticE[I*ArcSinh[Sqrt[-(Sqrt
[c]/Sqrt[a])]*x], -1] + (Sqrt[c]*d - Sqrt[a]*e)*EllipticF[I*ArcSi
nh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]))/(Sqrt[a]*(-(Sqrt[c]/Sqrt[a]
))^(3/2)*Sqrt[-a + c*x^4])
```

Maple [A] time = 0.011, size = 160, normalized size = 1.3

$$d\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4-a}}$$
$$+e\sqrt{a}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4-a}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(c*x^4-a)^(1/2),x)
```

```
[Out] d/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1
/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/
2)*c^(1/2))^(1/2),I)+e*a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^
(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a
)^(1/2)/c^(1/2)*(EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),I)-Ellipt
icE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{cx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(c*x^4 - a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ex^2+d}{\sqrt{cx^4-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(c*x^4 - a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(c*x^4 - a), x)
```

Sympy [A] time = 4.14346, size = 73, normalized size = 0.58

$$-\frac{idx\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}-\frac{iex^3\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(c*x**4-a)**(1/2),x)
```

```
[Out] -I*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*sqrt(a)*
gamma(5/4)) - I*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**
4/a)/(4*sqrt(a)*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{cx^4 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(c*x^4 - a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 - a), x)
```

3.169 $\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=73

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}d\sqrt{cx^4-a}}$$

```
[Out] (a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)
), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[-a + c*x^4])
```

Rubi [A] time = 0.120486, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}d\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)*Sqrt[-a + c*x^4]),x]
```

```
[Out] (a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)
), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[-a + c*x^4])
```

Rubi in Sympy [A] time = 15.5972, size = 63, normalized size = 0.86

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\operatorname{asin}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}d\sqrt{-a+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)/(c*x**4-a)**(1/2),x)
```

```
[Out] a**(1/4)*sqrt(1 - c*x**4/a)*elliptic_pi(-sqrt(a)*e/(sqrt(c)*d), a
sin(c**(1/4)*x/a**(1/4)), -1)/(c**(1/4)*d*sqrt(-a + c*x**4))
```

Mathematica [C] time = 0.0704225, size = 92, normalized size = 1.26

$$-\frac{i\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{d\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)*Sqrt[-a + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)),
I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[
a])]*d*Sqrt[-a + c*x^4])
```

Maple [A] time = 0.023, size = 99, normalized size = 1.4

$$\frac{1}{d}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\,\mathit{EllipticPi}\left(x\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}},\frac{e}{d}\sqrt{a}\frac{1}{\sqrt{c}},1\sqrt{1\sqrt{c}\frac{1}{\sqrt{a}}}\frac{1}{\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}}}\right)\frac{1}{\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)/(c*x^4-a)^(1/2),x)
```

```
[Out] 1/d/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticPi(x*(-1/a^
(1/2)*c^(1/2))^(1/2),e*a^(1/2)/d/c^(1/2),(1/a^(1/2)*c^(1/2))^(1/2
)/(-1/a^(1/2)*c^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4-a}(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 - a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 - a)*(e*x^2 + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^4-a}(ex^2+d)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 - a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(c*x^4 - a)*(e*x^2 + d)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a+cx^4}\,(d+ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)/(c*x**4-a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a + c*x**4)*(d + e*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4-a}(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 - a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 - a)*(e*x^2 + d)), x)
```

3.170 $\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=54

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{cx^4-a}}$$

```
[Out] (a^(3/4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)
], -1])/(c^(1/4)*Sqrt[-a + c*x^4])
```

Rubi [A] time = 0.135692, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a] + Sqrt[c]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (a^(3/4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)
], -1])/(c^(1/4)*Sqrt[-a + c*x^4])
```

Rubi in Sympy [A] time = 25.5978, size = 48, normalized size = 0.89

$$\frac{a^{\frac{3}{4}}\sqrt{1-\frac{cx^4}{a}}E\left(\operatorname{asin}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{-a+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**(1/2)+x**2*c**(1/2))/(c*x**4-a)**(1/2),x)
```

```
[Out] a**(3/4)*sqrt(1 - c*x**4/a)*elliptic_e(asin(c**(1/4)*x/a**(1/4)),
 -1)/(c**(1/4)*sqrt(-a + c*x**4))
```

Mathematica [C] time = 0.0976819, size = 78, normalized size = 1.44

$$\frac{i\sqrt{c}\sqrt{1-\frac{cx^4}{a}}E\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{\left(-\frac{\sqrt{c}}{\sqrt{a}}\right)^{3/2}\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a] + Sqrt[c]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (I*Sqrt[c]*Sqrt[1 - (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]
/Sqrt[a])]*x], -1])/((-(Sqrt[c]/Sqrt[a]))^(3/2)*Sqrt[-a + c*x^4])
```

Maple [B] time = 0.067, size = 158, normalized size = 2.9

$$1\sqrt{a}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4-a}}$$
$$+1\sqrt{a}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\mathit{EllipticF}\left(x\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-\mathit{EllipticE}\left(x\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x)
```

```
[Out] a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2
)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1
/a^(1/2)*c^(1/2))^(1/2),I)+a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+
1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x
^4-a)^(1/2)*(EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(
x*(-1/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{cx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a),x, algorithm="maxima")
```

```
[Out] integrate((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{cx^4-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a),x, algorithm="fricas")
```

```
[Out] integral((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a), x)
```

Sympy [A] time = 4.29037, size = 70, normalized size = 1.3

$$-\frac{ix\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\,\Gamma\left(\frac{5}{4}\right)}-\frac{i\sqrt{c}x^3\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**(1/2)+x**2*c**(1/2))/(c*x**4-a)**(1/2),x)
```

```
[Out] -I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*gamma(5/4)
) - I*sqrt(c)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)
```

```
/(4*sqrt(a)*gamma(7/4))
```

GIAC/XCAS [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.171 $\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=52

$$\frac{\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\sqrt[4]{\frac{c}{a}}x\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\sqrt{cx^4-a}}$$

```
[Out] (Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c/a)^(1/4)*x], -1])/((c/a)
^(1/4)*Sqrt[-a + c*x^4])
```

Rubi [A] time = 0.134496, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$\frac{\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\sqrt[4]{\frac{c}{a}}x\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[c/a]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c/a)^(1/4)*x], -1])/((c/a)
^(1/4)*Sqrt[-a + c*x^4])
```

Rubi in Sympy [A] time = 18.4757, size = 41, normalized size = 0.79

$$\frac{\sqrt{1-\frac{cx^4}{a}}E\left(\operatorname{asin}\left(x\sqrt[4]{\frac{c}{a}}\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\sqrt{-a+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x**2*(c/a)**(1/2))/(c*x**4-a)**(1/2),x)
```

```
[Out] sqrt(1 - c*x**4/a)*elliptic_e(asin(x*(c/a)**(1/4)), -1)/((c/a)**(
1/4)*sqrt(-a + c*x**4))
```

Mathematica [C] time = 0.149101, size = 142, normalized size = 2.73

$$\frac{i\sqrt{1-\frac{cx^4}{a}}\left(\left(\sqrt{c}-\sqrt{a}\sqrt{\frac{c}{a}}\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{a}\sqrt{\frac{c}{a}}E\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)\right)}{\sqrt{a}\left(-\frac{\sqrt{c}}{\sqrt{a}}\right)^{3/2}\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[c/a]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (I*Sqrt[1 - (c*x^4)/a]*(Sqrt[a]*Sqrt[c/a]*EllipticE[I*ArcSinh[Sqr
t[-(Sqrt[c]/Sqrt[a])]*x], -1] + (Sqrt[c] - Sqrt[a]*Sqrt[c/a])*Ell
ipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]))/(Sqrt[a]*(-(S
qrt[c]/Sqrt[a]))^(3/2)*Sqrt[-a + c*x^4])
```

Maple [B] time = 0.042, size = 165, normalized size = 3.2

$$1\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4-a}}$$
$$+1\sqrt{\frac{c}{a}}\sqrt{a}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{-1\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4-a}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x)
```

```
[Out] 1/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1
/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/
2)*c^(1/2))^(1/2),I)+(c/a)^(1/2)*a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/
2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2
)/(c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2)
,I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a),x, algorithm="maxima")
```

```
[Out] integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a),x, algorithm="fricas")
```

```
[Out] integral((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)
```

Sympy [A] time = 4.39726, size = 76, normalized size = 1.46

$$-\frac{ix^3\sqrt{\frac{c}{a}}\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}-\frac{ix\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x**2*(c/a)**(1/2))/(c*x**4-a)**(1/2),x)
```

```
[Out] -I*x**3*sqrt(c/a)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/
(4*sqrt(a)*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,),
 c*x**4/a)/(4*sqrt(a)*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{\frac{c}{a}} + 1}{\sqrt{cx^4 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a),x, algorithm="giac")
```

```
[Out] integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)
```

3.172 $\int \frac{d+ex^2}{\sqrt{-a-cx^4}} dx$

Optimal. Leaf size=236

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{-a-cx^4}} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{-a-cx^4}} - \frac{ex\sqrt{-a-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[Out] -((e*x*Sqrt[-a - c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2))) - (a^
(1/4)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4
)*Sqrt[-a - c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[-a - c*
x^4])
```

Rubi [A] time = 0.175595, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{-a-cx^4}} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{-a-cx^4}} - \frac{ex\sqrt{-a-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/Sqrt[-a - c*x^4],x]
```

```
[Out] -((e*x*Sqrt[-a - c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2))) - (a^
(1/4)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4
)*Sqrt[-a - c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[-a - c*
x^4])
```

Rubi in Sympy [A] time = 21.5477, size = 209, normalized size = 0.89

$$-\frac{\sqrt[4]{a}e\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{c^{\frac{3}{4}}\sqrt{-a-cx^4}} - \frac{ex\sqrt{-a-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}e+\sqrt{c}d\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{\frac{3}{4}}\sqrt{-a-cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(-c*x**4-a)**(1/2),x)
```

```
[Out] -a**(1/4)*e*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(
a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)), 1/2)/(
```

```
c**(3/4)*sqrt(-a - c*x**4)) - e*x*sqrt(-a - c*x**4)/(sqrt(c)*(sqr
t(a) + sqrt(c)*x**2)) + sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2
)**2)*(sqrt(a) + sqrt(c)*x**2)*(sqrt(a)*e + sqrt(c)*d)*elliptic_f
(2*atan(c**(1/4)*x/a**(1/4)), 1/2)/(2*a**(1/4)*c**(3/4)*sqrt(-a -
 c*x**4))
```

Mathematica [C] time = 0.128828, size = 134, normalized size = 0.57

$$\frac{\sqrt{\frac{cx^4}{a}+1}\left(\left(-\sqrt{a}e-i\sqrt{c}d\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{a}eE\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)\right)}{\sqrt{c}\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{-a-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/Sqrt[-a - c*x^4],x]
```

```
[Out] (Sqrt[1 + (c*x^4)/a]*(Sqrt[a]*e*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[
c])/Sqrt[a]]*x], -1] + ((-I)*Sqrt[c]*d - Sqrt[a]*e)*EllipticF[I*A
rcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/(Sqrt[(I*Sqrt[c])/Sqrt
[a]]*Sqrt[c]*Sqrt[-a - c*x^4])
```

Maple [C] time = 0.014, size = 175, normalized size = 0.7

$$d\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-cx^4-a}}$$
$$-ie\sqrt{a}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-cx^4-a}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(-c*x^4-a)^(1/2),x)
```

```
[Out] d/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-I
/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4-a)^(1/2)*EllipticF(x*(-I/a^(1
/2)*c^(1/2))^(1/2),I)-I*e*a^(1/2)/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I
/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x
^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(-I/a^(1/2)*c^(1/2))^(1/2),I)-El
lipticE(x*(-I/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{-cx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(-c*x^4 - a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2+d}{\sqrt{-cx^4-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(-c*x^4 - a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(-c*x^4 - a), x)
```

Sympy [A] time = 4.01527, size = 83, normalized size = 0.35

$$-\frac{idx\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}-\frac{iex^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(-c*x**4-a)**(1/2),x)
```

```
[Out] -I*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi
)/a)/(4*sqrt(a)*gamma(5/4)) - I*e*x**3*gamma(3/4)*hyper((1/2, 3/4
), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{-cx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(-c*x^4 - a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 - a), x)
```

3.173 $\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx$

Optimal. Leaf size=336

$$\frac{\tan^{-1}\left(\frac{x\sqrt{-\frac{ae}{d}-\frac{cd}{e}}}{\sqrt{-a-cx^4}}\right)}{2d\sqrt{-\frac{ae}{d}-\frac{cd}{e}}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{-a-cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\Pi\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\sqrt{-a-cx^4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)}$$

```
[Out] ArcTan[(Sqrt[-((c*d)/e) - (a*e)/d]*x)/Sqrt[-a - c*x^4]]/(2*d*Sqrt
[-((c*d)/e) - (a*e)/d]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a -
c*x^4]) - (((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[a]*((Sq
rt[c]*d)/Sqrt[a] - e)^2)/(4*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*((Sqrt[c]*d)/Sqrt[a] - e)*Sqrt
[-a - c*x^4])
```

Rubi [A] time = 0.397728, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{\tan^{-1}\left(\frac{x\sqrt{-\frac{ae}{d}-\frac{cd}{e}}}{\sqrt{-a-cx^4}}\right)}{2d\sqrt{-\frac{ae}{d}-\frac{cd}{e}}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{-a-cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}d\sqrt{-a-cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)*Sqrt[-a - c*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[-((c*d)/e) - (a*e)/d]*x)/Sqrt[-a - c*x^4]]/(2*d*Sqrt
[-((c*d)/e) - (a*e)/d]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a -
c*x^4]) - (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt
[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x
)/a^(1/4)], 1/2])/(4*c^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a -
c*x^4])
```

Rubi in Sympy [A] time = 22.4853, size = 284, normalized size = 0.85

$$\frac{\operatorname{atan}\left(\frac{x\sqrt{-\frac{ae}{d}-\frac{cd}{e}}}{\sqrt{-a-cx^4}}\right)}{2d\sqrt{-\frac{ae}{d}-\frac{cd}{e}}}-\frac{\sqrt[4]{c}\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{-a-cx^4}\left(\sqrt{a}e-\sqrt{c}d\right)}+\frac{\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}e+\sqrt{c}d\right)\Pi\left(-\frac{\sqrt{a}\left(e-\frac{\sqrt{c}d}{\sqrt{a}}\right)^2}{4\sqrt{c}de};2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\sqrt{-a-cx^4}\left(\sqrt{a}e-\sqrt{c}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)/(-c*x**4-a)**(1/2),x)
```

```
[Out] atan(x*sqrt(-a*e/d - c*d/e)/sqrt(-a - c*x**4))/(2*d*sqrt(-a*e/d -
 c*d/e)) - c**(1/4)*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2
)*(sqrt(a) + sqrt(c)*x**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4))
, 1/2)/(2*a**(1/4)*sqrt(-a - c*x**4)*(sqrt(a)*e - sqrt(c)*d)) + s
qrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*
x**2)*(sqrt(a)*e + sqrt(c)*d)*elliptic_pi(-sqrt(a)*(e - sqrt(c)*d
/sqrt(a))**2/(4*sqrt(c)*d*e), 2*atan(c**(1/4)*x/a**(1/4)), 1/2)/(
4*a**(1/4)*c**(1/4)*d*sqrt(-a - c*x**4)*(sqrt(a)*e - sqrt(c)*d))
```

Mathematica [C] time = 0.0874549, size = 98, normalized size = 0.29

$$-\frac{i\sqrt{\frac{cx^4}{a}+1}\,\Pi\left(-\frac{i\sqrt{a}e}{\sqrt{c}d};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{-a-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)*Sqrt[-a - c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d)
, I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/
Sqrt[a]]*d*Sqrt[-a - c*x^4])
```

Maple [C] time = 0.022, size = 110, normalized size = 0.3

$$\frac{1}{d}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticPi\left(x\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}},\frac{-ie}{d}\sqrt{a}\frac{1}{\sqrt{c}},1\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}\frac{1}{\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}}}\right)\frac{1}{\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-cx^4-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x)
```

```
[Out] 1/d/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
-I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4-a)^(1/2)*EllipticPi(x*(-I/a
^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)*e/d,(I/a^(1/2)*c^(1/2))^
(1/2)/(-I/a^(1/2)*c^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4-a}(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 - a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 - a)*(e*x^2 + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-cx^4 - a}(ex^2 + d)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 - a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(-c*x^4 - a)*(e*x^2 + d)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a - cx^4}\,(d + ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)/(-c*x**4-a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a - c*x**4)*(d + e*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 - a}(ex^2 + d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 - a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 - a)*(e*x^2 + d)), x)
```

3.174 $\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx$

Optimal. Leaf size=40

$$\frac{\left(-\frac{2b}{\sqrt{5}a};\sin^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

```
[Out] EllipticPi[(-2*b)/(Sqrt[5]*a), ArcSin[(5^(1/4)*x)/Sqrt[2]], -1]/(
Sqrt[2]*5^(1/4)*a)
```

Rubi [A] time = 0.174949, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{\left(-\frac{2b}{\sqrt{5}a};\sin^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*Sqrt[4 - 5*x^4]),x]
```

```
[Out] EllipticPi[(-2*b)/(Sqrt[5]*a), ArcSin[(5^(1/4)*x)/Sqrt[2]], -1]/(
Sqrt[2]*5^(1/4)*a)
```

Rubi in Sympy [A] time = 23.7051, size = 42, normalized size = 1.05

$$\frac{\sqrt{2}\cdot 5^{\frac{3}{4}}\left(-\frac{2\sqrt{5}b}{5a};\operatorname{asin}\left(\frac{\sqrt{2}\sqrt[4]{5}x}{2}\right)\middle|-1\right)}{10a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(-5*x**4+4)**(1/2),x)
```

```
[Out] sqrt(2)*5**(3/4)*elliptic_pi(-2*sqrt(5)*b/(5*a), asin(sqrt(2)*5**
(1/4)*x/2), -1)/(10*a)
```

Mathematica [A] time = 0.0628098, size = 43, normalized size = 1.08

$$-\frac{\left(-\frac{2b}{\sqrt{5}a};-\sin^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*Sqrt[4 - 5*x^4]),x]
```

```
[Out] -(EllipticPi[(-2*b)/(Sqrt[5]*a), -ArcSin[(5^(1/4)*x)/Sqrt[2]], -1
]/(Sqrt[2]*5^(1/4)*a))
```

Maple [B] time = 0.072, size = 79, normalized size = 2.

$$\frac{\sqrt{2}5^{\frac{3}{4}}}{5a}\sqrt{1-\frac{\sqrt{5}x^{2}}{2}}\sqrt{1+\frac{\sqrt{5}x^{2}}{2}}EllipticPi\left(\frac{\sqrt[4]{5}x\sqrt{2}}{2},-\frac{2\sqrt{5}b}{5a},\frac{\sqrt{-\frac{\sqrt{5}}{2}}\sqrt{2}5^{\frac{3}{4}}}{5}\right)\frac{1}{\sqrt{-5x^{4}+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x)
```

```
[Out] 1/5/a*2^(1/2)*5^(3/4)*(1-1/2*5^(1/2)*x^2)^(1/2)*(1+1/2*5^(1/2)*x^
2)^(1/2)/(-5*x^4+4)^(1/2)*EllipticPi(1/2*5^(1/4)*x*2^(1/2),-2/5*b
/a*5^(1/2),1/5*(-1/2*5^(1/2))^(1/2)*2^(1/2)*5^(3/4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-5x^{4}+4}(bx^{2}+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-5x^{4}+4}(bx^{2}+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^{2})\sqrt{-5x^{4}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(-5*x**4+4)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(-5*x**4 + 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-5x^{4}+4}(bx^{2}+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)),x, algorithm="giac")

[Out] integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)), x)
```

3.175 $\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx$

Optimal. Leaf size=284

$$\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{5a}{b}+\frac{4b}{a}}}{\sqrt{5x^4+4}}\right)}{2a\sqrt{\frac{5a}{b}+\frac{4b}{a}}}+\frac{\sqrt[4]{5}\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{5x^4+4}\left(\sqrt{5}a-2b\right)}-\frac{\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}x^2+2\right)^2}}\left(\sqrt{5}a+2b\right)\Pi\left(-\frac{\left(\sqrt{5}a-2b\right)^2}{8\sqrt{5}ab};2\tan^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{4\sqrt{2}\sqrt[4]{5}a\sqrt{5x^4+4}\left(\sqrt{5}a-2b\right)}$$

```
[Out] ArcTan[(Sqrt[(5*a)/b + (4*b)/a]*x)/Sqrt[4 + 5*x^4]]/(2*a*Sqrt[(5*
a)/b + (4*b)/a]) + (5^(1/4)*(2 + Sqrt[5]*x^2)*Sqrt[(4 + 5*x^4)/(2
 + Sqrt[5]*x^2)^2]*EllipticF[2*ArcTan[(5^(1/4)*x)/Sqrt[2]], 1/2])
/(2*Sqrt[2]*(Sqrt[5]*a - 2*b)*Sqrt[4 + 5*x^4]) - ((Sqrt[5]*a + 2*
b)*(2 + Sqrt[5]*x^2)*Sqrt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^2]*Ellipt
icPi[-(Sqrt[5]*a - 2*b)^2/(8*Sqrt[5]*a*b), 2*ArcTan[(5^(1/4)*x)/S
qrt[2]], 1/2])/(4*Sqrt[2]*5^(1/4)*a*(Sqrt[5]*a - 2*b)*Sqrt[4 + 5*
x^4])
```

Rubi [A] time = 0.348459, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{5a}{b}+\frac{4b}{a}}}{\sqrt{5x^4+4}}\right)}{2a\sqrt{\frac{5a}{b}+\frac{4b}{a}}}+\frac{\sqrt[4]{5}\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{5x^4+4}\left(\sqrt{5}a-2b\right)}-\frac{\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}x^2+2\right)^2}}\left(\sqrt{5}a+2b\right)\Pi\left(-\frac{\left(\sqrt{5}a-2b\right)^2}{8\sqrt{5}ab};2\tan^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{4\sqrt{2}\sqrt[4]{5}a\sqrt{5x^4+4}\left(\sqrt{5}a-2b\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*Sqrt[4 + 5*x^4]),x]
```


```
[Out] ArcTan[(Sqrt[(5*a)/b + (4*b)/a]*x)/Sqrt[4 + 5*x^4]]/(2*a*Sqrt[(5*
a)/b + (4*b)/a]) + (5^(1/4)*(2 + Sqrt[5]*x^2)*Sqrt[(4 + 5*x^4)/(2
 + Sqrt[5]*x^2)^2]*EllipticF[2*ArcTan[(5^(1/4)*x)/Sqrt[2]], 1/2])
/(2*Sqrt[2]*(Sqrt[5]*a - 2*b)*Sqrt[4 + 5*x^4]) - ((Sqrt[5]*a + 2*
b)*(2 + Sqrt[5]*x^2)*Sqrt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^2]*Ellipt
icPi[-(Sqrt[5]*a - 2*b)^2/(8*Sqrt[5]*a*b), 2*ArcTan[(5^(1/4)*x)/S
qrt[2]], 1/2])/(4*Sqrt[2]*5^(1/4)*a*(Sqrt[5]*a - 2*b)*Sqrt[4 + 5*
x^4])
```

Rubi in Sympy [A] time = 13.3489, size = 253, normalized size = 0.89

$$-\frac{\sqrt{2}\sqrt[4]{5}\sqrt{\frac{5x^4+4}{\left(\frac{\sqrt{5}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt[4]{5}x}{2}\right)\middle|\frac{1}{2}\right)}{4\sqrt{5x^4+4}\left(-\sqrt{5}a+2b\right)}+\frac{\sqrt{2}\cdot 5^{\frac{3}{4}}\sqrt{\frac{5x^4+4}{\left(\frac{\sqrt{5}x^2}{2}+1\right)^2}}\left(\sqrt{5}a+2b\right)\left(\frac{\sqrt{5}x^2}{2}+1\right)\Pi\left(-\frac{\sqrt{5}\left(-\frac{\sqrt{5}a}{2}+b\right)^2}{10ab};2\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt[4]{5}x}{2}\right)\middle|\frac{1}{2}\right)}{40a\sqrt{5x^4+4}\left(-\sqrt{5}a+2b\right)}+\frac{\mathrm{atan}\left(\frac{x\sqrt{\frac{5a}{b}+\frac{4b}{a}}}{\sqrt{5x^4+4}}\right)}{2a\sqrt{\frac{5a}{b}+\frac{4b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(5*x**4+4)**(1/2),x)
```

```
[Out] -sqrt(2)*5**(1/4)*sqrt((5*x**4 + 4)/(sqrt(5)*x**2/2 + 1)**2)*(sqr
t(5)*x**2/2 + 1)*elliptic_f(2*atan(sqrt(2)*5**(1/4)*x/2), 1/2)/(4
*sqrt(5*x**4 + 4)*(-sqrt(5)*a + 2*b)) + sqrt(2)*5**(3/4)*sqrt((5*
x**4 + 4)/(sqrt(5)*x**2/2 + 1)**2)*(sqrt(5)*a + 2*b)*(sqrt(5)*x**
2/2 + 1)*elliptic_pi(-sqrt(5)*(-sqrt(5)*a/2 + b)**2/(10*a*b), 2*a
tan(sqrt(2)*5**(1/4)*x/2), 1/2)/(40*a*sqrt(5*x**4 + 4)*(-sqrt(5)*
a + 2*b)) + atan(x*sqrt(5*a/b + 4*b/a)/sqrt(5*x**4 + 4))/(2*a*sqr
t(5*a/b + 4*b/a))
```

Mathematica [C] time = 0.0633166, size = 50, normalized size = 0.18

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\Pi\left(-\frac{2ib}{\sqrt{5}a};i\sinh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{5}x\right)\middle|-1\right)}{\sqrt[4]{5}a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*Sqrt[4 + 5*x^4]),x]
```

```
[Out] ((-1/2 - I/2)*EllipticPi[((-2*I)*b)/(Sqrt[5]*a), I*ArcSinh[(1/2 +
 I/2)*5^(1/4)*x], -1])/(5^(1/4)*a)
```

Maple [C] time = 0.141, size = 86, normalized size = 0.3

$$\frac{1}{a\sqrt{\frac{i}{2}\sqrt{5}}}\sqrt{1-\frac{i}{2}x^2\sqrt{5}}\sqrt{1+\frac{i}{2}x^2\sqrt{5}}EllipticPi\left(\sqrt{\frac{i}{2}\sqrt{5}}x,\frac{\frac{2i}{5}\sqrt{5}b}{a},\frac{\sqrt{-\frac{i}{2}\sqrt{5}}}{\sqrt{\frac{i}{2}\sqrt{5}}}\right)\frac{1}{\sqrt{5x^4+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(5*x^4+4)^(1/2),x)
```

```
[Out] 1/a/(1/2*I*5^(1/2))^(1/2)*(1-1/2*I*x^2*5^(1/2))^(1/2)*(1+1/2*I*x^
2*5^(1/2))^(1/2)/(5*x^4+4)^(1/2)*EllipticPi((1/2*I*5^(1/2))^(1/2)
*x,2/5*I*5^(1/2)*b/a,(-1/2*I*5^(1/2))^(1/2)/(1/2*I*5^(1/2))^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5x^4+4}(bx^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{5\,x^4+4}(bx^2+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)),x, algorithm="fricas")

[Out] integral(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)\sqrt{5x^4+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(5*x**4+4)**(1/2),x)

[Out] Integral(1/((a + b*x**2)*sqrt(5*x**4 + 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\,x^4+4}(bx^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)),x, algorithm="giac")

[Out] integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)), x)
```

3.176 $\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx$

Optimal. Leaf size=40

$$\frac{\Pi\left(-\frac{2b}{a\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}a\sqrt[4]{d}}$$

```
[Out] EllipticPi[(-2*b)/(a*Sqrt[d]), ArcSin[(d^(1/4)*x)/Sqrt[2]], -1]/(
Sqrt[2]*a*d^(1/4))
```

Rubi [A] time = 0.059881, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{\Pi\left(-\frac{2b}{a\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}a\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[4 - d*x^4]),x]
```

```
[Out] EllipticPi[(-2*b)/(a*Sqrt[d]), ArcSin[(d^(1/4)*x)/Sqrt[2]], -1]/(
Sqrt[2]*a*d^(1/4))
```

Rubi in Sympy [A] time = 8.25028, size = 41, normalized size = 1.02

$$\frac{\sqrt{2}\,\Pi\left(-\frac{2b}{a\sqrt{d}};\operatorname{asin}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{2}\right)\middle|-1\right)}{2a\sqrt[4]{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(b*x**2+a)/(-d*x**4+4)**(1/2),x)
```

```
[Out] sqrt(2)*elliptic_pi(-2*b/(a*sqrt(d)), asin(sqrt(2)*d**(1/4)*x/2),
 -1)/(2*a*d**(1/4))
```

Mathematica [C] time = 0.0678159, size = 59, normalized size = 1.48

$$-\frac{i\,\Pi\left(-\frac{2b}{a\sqrt{d}};i\sinh^{-1}\left(\frac{\sqrt{-\sqrt{d}}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}a\sqrt{-\sqrt{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[4 - d*x^4]),x]
```

```
[Out] ((-I)*EllipticPi[(-2*b)/(a*Sqrt[d]), I*ArcSinh[(Sqrt[-Sqrt[d]]*x)
/Sqrt[2]], -1])/(Sqrt[2]*a*Sqrt[-Sqrt[d]])
```

Maple [B] time = 0.033, size = 78, normalized size = 2.

$$\frac{\sqrt{2}}{a}\sqrt{1-\frac{x^2}{2}\sqrt{d}}\sqrt{1+\frac{x^2}{2}\sqrt{d}}EllipticPi\left(\frac{\sqrt{2}x}{2}\sqrt[4]{d},-2\frac{b}{a\sqrt{d}},\sqrt{2}\sqrt{-\frac{1}{2}\sqrt{d}}\frac{1}{\sqrt[4]{d}}\right)\frac{1}{\sqrt[4]{d}}\frac{1}{\sqrt{-dx^4+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x)
```

```
[Out] 1/a*2^(1/2)/d^(1/4)*(1-1/2*d^(1/2)*x^2)^(1/2)*(1+1/2*d^(1/2)*x^2)
^(1/2)/(-d*x^4+4)^(1/2)*EllipticPi(1/2*d^(1/4)*x*2^(1/2),-2*b/a/d
^(1/2),(-1/2*d^(1/2))^(1/2)*2^(1/2)/d^(1/4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-dx^4+4}(bx^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-dx^4+4}(bx^2+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)\sqrt{-dx^4+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/(b*x**2+a)/(-d*x**4+4)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(-d*x**4 + 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-dx^4+4}(bx^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)),x, algorithm="giac")

[Out] integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)), x)
```

3.177 $\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx$

Optimal. Leaf size=287

$$\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{ad}{b}+\frac{4b}{a}}}{\sqrt{dx^4+4}}\right)}{2a\sqrt{\frac{ad}{b}+\frac{4b}{a}}}-\frac{\sqrt[4]{d}\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{dx^4+4}\left(2b-a\sqrt{d}\right)}+\frac{\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}\left(a\sqrt{d}+2b\right)\Pi\left(-\frac{\left(2b-a\sqrt{d}\right)^2}{8ab\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{4\sqrt{2}a\sqrt[4]{d}\sqrt{dx^4+4}\left(2b-a\sqrt{d}\right)}$$

```
[Out] ArcTan[(Sqrt[(4*b)/a + (a*d)/b]*x)/Sqrt[4 + d*x^4]]/(2*a*Sqrt[(4*
b)/a + (a*d)/b]) - (d^(1/4)*(2 + Sqrt[d]*x^2)*Sqrt[(4 + d*x^4)/(2
 + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/Sqrt[2]], 1/2])
/(2*Sqrt[2]*(2*b - a*Sqrt[d])*Sqrt[4 + d*x^4]) + ((2*b + a*Sqrt[d
])*(2 + Sqrt[d]*x^2)*Sqrt[(4 + d*x^4)/(2 + Sqrt[d]*x^2)^2]*Ellipt
icPi[-(2*b - a*Sqrt[d])^2/(8*a*b*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/S
qrt[2]], 1/2])/(4*Sqrt[2]*a*(2*b - a*Sqrt[d])*d^(1/4)*Sqrt[4 + d*
x^4])
```

Rubi [A] time = 0.289906, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{ad}{b}+\frac{4b}{a}}}{\sqrt{dx^4+4}}\right)}{2a\sqrt{\frac{ad}{b}+\frac{4b}{a}}}-\frac{\sqrt[4]{d}\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{dx^4+4}\left(2b-a\sqrt{d}\right)}+\frac{\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}\left(a\sqrt{d}+2b\right)\Pi\left(-\frac{\left(2b-a\sqrt{d}\right)^2}{8ab\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{4\sqrt{2}a\sqrt[4]{d}\sqrt{dx^4+4}\left(2b-a\sqrt{d}\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*Sqrt[4 + d*x^4]),x]
```


```
[Out] ArcTan[(Sqrt[(4*b)/a + (a*d)/b]*x)/Sqrt[4 + d*x^4]]/(2*a*Sqrt[(4*
b)/a + (a*d)/b]) - (d^(1/4)*(2 + Sqrt[d]*x^2)*Sqrt[(4 + d*x^4)/(2
 + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/Sqrt[2]], 1/2])
/(2*Sqrt[2]*(2*b - a*Sqrt[d])*Sqrt[4 + d*x^4]) + ((2*b + a*Sqrt[d
])*(2 + Sqrt[d]*x^2)*Sqrt[(4 + d*x^4)/(2 + Sqrt[d]*x^2)^2]*Ellipt
icPi[-(2*b - a*Sqrt[d])^2/(8*a*b*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/S
qrt[2]], 1/2])/(4*Sqrt[2]*a*(2*b - a*Sqrt[d])*d^(1/4)*Sqrt[4 + d*
x^4])
```

Rubi in Sympy [A] time = 18.6951, size = 253, normalized size = 0.88

$$-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{\frac{dx^4+4}{\left(\frac{\sqrt{d}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{d}x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{2}\right)\middle|\frac{1}{2}\right)}{4\left(-a\sqrt{d}+2b\right)\sqrt{dx^4+4}}+\frac{\operatorname{atan}\left(\frac{x\sqrt{\frac{ad}{b}+\frac{4b}{a}}}{\sqrt{dx^4+4}}\right)}{2a\sqrt{\frac{ad}{b}+\frac{4b}{a}}}+\frac{\sqrt{2}\sqrt{\frac{dx^4+4}{\left(\frac{\sqrt{d}x^2}{2}+1\right)^2}}\left(a\sqrt{d}+2b\right)\left(\frac{\sqrt{d}x^2}{2}+1\right)\Pi\left(-\frac{\left(-\frac{a\sqrt{d}}{2}+b\right)^2}{2ab\sqrt{d}};2\operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{2}\right)\middle|\frac{1}{2}\right)}{8a\sqrt[4]{d}\left(-a\sqrt{d}+2b\right)\sqrt{dx^4+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**4+4)**(1/2),x)
```

```
[Out] -sqrt(2)*d**(1/4)*sqrt((d*x**4 + 4)/(sqrt(d)*x**2/2 + 1)**2)*(sqr
t(d)*x**2/2 + 1)*elliptic_f(2*atan(sqrt(2)*d**(1/4)*x/2), 1/2)/(4
*(-a*sqrt(d) + 2*b)*sqrt(d*x**4 + 4)) + atan(x*sqrt(a*d/b + 4*b/a
)/sqrt(d*x**4 + 4))/(2*a*sqrt(a*d/b + 4*b/a)) + sqrt(2)*sqrt((d*x
**4 + 4)/(sqrt(d)*x**2/2 + 1)**2)*(a*sqrt(d) + 2*b)*(sqrt(d)*x**2
/2 + 1)*elliptic_pi(-(-a*sqrt(d)/2 + b)**2/(2*a*b*sqrt(d)), 2*ata
n(sqrt(2)*d**(1/4)*x/2), 1/2)/(8*a*d**(1/4)*(-a*sqrt(d) + 2*b)*sq
rt(d*x**4 + 4))
```

Mathematica [C] time = 0.0634357, size = 65, normalized size = 0.23

$$-\frac{i\Pi\left(-\frac{2ib}{a\sqrt{d}};i\sinh^{-1}\left(\frac{\sqrt{i\sqrt{d}}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}a\sqrt{i\sqrt{d}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*Sqrt[4 + d*x^4]),x]
```

```
[Out] ((-I)*EllipticPi[((-2*I)*b)/(a*Sqrt[d]), I*ArcSinh[(Sqrt[I*Sqrt[d
]]*x)/Sqrt[2]], -1])/(Sqrt[2]*a*Sqrt[I*Sqrt[d]])
```

Maple [C] time = 0.033, size = 86, normalized size = 0.3

$$\frac{1}{a}\sqrt{1-\frac{i}{2}\sqrt{d}x^2}\sqrt{1+\frac{i}{2}\sqrt{d}x^2}EllipticPi\left(\sqrt{\frac{i}{2}\sqrt{d}}x,\frac{2\,ib}{a}\frac{1}{\sqrt{d}},1\sqrt{-\frac{i}{2}\sqrt{d}}\frac{1}{\sqrt{\frac{i}{2}\sqrt{d}}}\right)\frac{1}{\sqrt{\frac{i}{2}\sqrt{d}}}\frac{1}{\sqrt{dx^4+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^4+4)^(1/2),x)
```

```
[Out] 1/a/(1/2*I*d^(1/2))^(1/2)*(1-1/2*I*d^(1/2)*x^2)^(1/2)*(1+1/2*I*d^
(1/2)*x^2)^(1/2)/(d*x^4+4)^(1/2)*EllipticPi((1/2*I*d^(1/2))^(1/2)
*x,2*I/d^(1/2)*b/a,(-1/2*I*d^(1/2))^(1/2)/(1/2*I*d^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^4+4}(bx^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{dx^4+4}(bx^2+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)\sqrt{dx^4+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**4+4)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(d*x**4 + 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^4+4}(bx^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)), x)
```

3.178 $\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=112

$$\frac{a\sqrt{1-x^2}\sqrt{\frac{a(x^2+1)}{a+bx^2}}\,\Pi\left(\frac{b}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}x}{\sqrt{bx^2+a}}\right)\middle|-\frac{a-b}{a+b}\right)}{\sqrt{x^2+1}\sqrt{a+b}\sqrt{\frac{a(1-x^2)}{a+bx^2}}}$$

```
[Out] (a*Sqrt[1 - x^2]*Sqrt[(a*(1 + x^2))/(a + b*x^2)]*EllipticPi[b/(a
+ b), ArcSin[(Sqrt[a + b]*x)/Sqrt[a + b*x^2]], -((a - b)/(a + b))
])/(Sqrt[a + b]*Sqrt[1 + x^2]*Sqrt[(a*(1 - x^2))/(a + b*x^2)])
```

Rubi [F] time = 0.0327163, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[Sqrt[a + b*x^2]/Sqrt[1 - x^4],x]

[Out] Defer[Int][Sqrt[a + b*x^2]/Sqrt[1 - x^4], x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)/(-x**4+1)**(1/2),x)

[Out] Integral(sqrt(a + b*x**2)/sqrt(-x**4 + 1), x)
```

Mathematica [A] time = 0.063988, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[a + b*x^2]/Sqrt[1 - x^4],x]

[Out] Integrate[Sqrt[a + b*x^2]/Sqrt[1 - x^4], x]
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int 1\sqrt{bx^2+a}\frac{1}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x)

[Out] int((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{-x^4 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1),x, algorithm="maxima")

[Out] integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2 + a}}{\sqrt{-x^4 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1),x, algorithm="fricas")

[Out] integral(sqrt(b*x^2 + a)/sqrt(-x^4 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2}}{\sqrt{-(x-1)(x+1)(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x**2+a)**(1/2)/(-x**4+1)**(1/2),x)

[Out] Integral(sqrt(a + b*x**2)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{-x^4 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1),x, algorithm="giac")

[Out] integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1), x)
```

3.179 $\int (c+ex^2)^q (a+bx^4)^p \, dx$

Optimal. Leaf size=22

$$\text{Int}\left((a+bx^4)^p (c+ex^2)^q, x\right)$$

```
[Out] Unintegrable[(c + e*x^2)^q*(a + b*x^4)^p, x]
```

Rubi [A] time = 0.0230109, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\text{Int}\left((c+ex^2)^q (a+bx^4)^p, x\right)$$

Verification is Not applicable to the result.

```
[In] Int[(c + e*x^2)^q*(a + b*x^4)^p,x]
```

```
[Out] Defer[Int][(c + e*x^2)^q*(a + b*x^4)^p, x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a+bx^4)^p (c+ex^2)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((e*x**2+c)**q*(b*x**4+a)**p,x)
```

```
[Out] Integral((a + b*x**4)**p*(c + e*x**2)**q, x)
```

Mathematica [A] time = 0.0805122, size = 0, normalized size = 0.

$$\int (c+ex^2)^q (a+bx^4)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + e*x^2)^q*(a + b*x^4)^p,x]
```

```
[Out] Integrate[(c + e*x^2)^q*(a + b*x^4)^p, x]
```

Maple [A] time = 0.117, size = 0, normalized size = 0.

$$\int (ex^2+c)^q (bx^4+a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^q*(b*x^4+a)^p,x)
```

```
[Out] int((e*x^2+c)^q*(b*x^4+a)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^p (ex^2 + c)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^p*(e*x^2 + c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^p*(e*x^2 + c)^q, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((bx^4 + a)^p (ex^2 + c)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^p*(e*x^2 + c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^p*(e*x^2 + c)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+c)**q*(b*x**4+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^p (ex^2 + c)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^p*(e*x^2 + c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p*(e*x^2 + c)^q, x)
```

3.180 $\int \left(c+ex^2\right)^3 \left(a+bx^4\right)^p \, dx$

Optimal. Leaf size=204

$$c^3x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right) - \frac{ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(ae^2-bc^2(4p+7)\right){}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)}{b(4p+7)} + \frac{3}{5}ce^2x^5\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right) + \frac{e^3x^3\left(a+bx^4\right)^{p+1}}{b(4p+7)}$$

```
[Out] (e^3*x^3*(a + b*x^4)^(1 + p))/(b*(7 + 4*p)) + (c^3*x*(a + b*x^4)^
p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^
p - (e*(a*e^2 - b*c^2*(7 + 4*p))*x^3*(a + b*x^4)^p*Hypergeometric
2F1[3/4, -p, 7/4, -((b*x^4)/a)])/(b*(7 + 4*p)*(1 + (b*x^4)/a)^p)
+ (3*c*e^2*x^5*(a + b*x^4)^p*Hypergeometric2F1[5/4, -p, 9/4, -((b
*x^4)/a)])/(5*(1 + (b*x^4)/a)^p)
```

Rubi [A] time = 0.406175, antiderivative size = 196, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$c^3x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right) + ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(c^2-\frac{ae^2}{4bp+7b}\right){}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right) + \frac{3}{5}ce^2x^5\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right) + \frac{e^3x^3\left(a+bx^4\right)^{p+1}}{b(4p+7)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + e*x^2)^3*(a + b*x^4)^p,x]
```

```
[Out] (e^3*x^3*(a + b*x^4)^(1 + p))/(b*(7 + 4*p)) + (c^3*x*(a + b*x^4)^
p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^
p + (e*(c^2 - (a*e^2)/(7*b + 4*b*p))*x^3*(a + b*x^4)^p*Hypergeome
tric2F1[3/4, -p, 7/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (3*c*e^2
*x^5*(a + b*x^4)^p*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)])
/(5*(1 + (b*x^4)/a)^p)
```

Rubi in Sympy [A] time = 33.9907, size = 168, normalized size = 0.82

$$c^3x\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p{}_2F_1\left(\begin{matrix}-p,\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|-\frac{bx^4}{a}\right) + c^2ex^3\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p{}_2F_1\left(\begin{matrix}-p,\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|-\frac{bx^4}{a}\right) + \frac{3ce^2x^5\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p{}_2F_1\left(\begin{matrix}-p,\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|-\frac{bx^4}{a}\right)}{5} + \frac{e^3x^7\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p{}_2F_1\left(\begin{matrix}-p,\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|-\frac{bx^4}{a}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+c)**3*(b*x**4+a)**p,x)
```

```
[Out] c**3*x*(1 + b*x**4/a)**(-p)*(a + b*x**4)**p*hyper((-p, 1/4), (5/4
,), -b*x**4/a) + c**2*e*x**3*(1 + b*x**4/a)**(-p)*(a + b*x**4)**p
*hyper((-p, 3/4), (7/4,), -b*x**4/a) + 3*c*e**2*x**5*(1 + b*x**4/
a)**(-p)*(a + b*x**4)**p*hyper((-p, 5/4), (9/4,), -b*x**4/a)/5 +
```

```
e**3*x**7*(1 + b*x**4/a)**(-p)*(a + b*x**4)**p*hyper((-p, 7/4), (
11/4,), -b*x**4/a)/7
```

Mathematica [A] time = 0.0934814, size = 136, normalized size = 0.67

$$\frac{1}{35}x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(35c^3\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+ex^2\left(35c^2\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)+ex^2\left(21c\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right)+5ex^2\,{}_2F_1\left(\frac{7}{4},-p;\frac{11}{4};-\frac{bx^4}{a}\right)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + e*x^2)^3*(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*(35*c^3*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4
)/a)] + e*x^2*(35*c^2*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a
)] + e*x^2*(21*c*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)] +
5*e*x^2*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^4)/a)]))))/(35*(1
 + (b*x^4)/a)^p)
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)^3\left(bx^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+c)^3*(b*x^4+a)^p,x)
```

```
[Out] int((e*x^2+c)^3*(b*x^4+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)^3\left(bx^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + c)^3*(b*x^4 + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + c)^3*(b*x^4 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^6+3\,ce^2x^4+3\,c^2ex^2+c^3\right)\left(bx^4+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + c)^3*(b*x^4 + a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*c*e^2*x^4 + 3*c^2*e*x^2 + c^3)*(b*x^4 + a)^
p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+c)**3*(b*x**4+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)^3 \left(bx^4+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + c)^3*(b*x^4 + a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + c)^3*(b*x^4 + a)^p, x)
```

3.181 $\int \left(c+ex^2\right)^2 \left(a+bx^4\right)^p \, dx$

Optimal. Leaf size=150

$$-\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(ae^2-bc^2(4p+5)\right)\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)}{b(4p+5)}+\frac{2}{3}cex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)+\frac{e^2x\left(a+bx^4\right)^{p+1}}{b(4p+5)}$$

```
[Out] (e^2*x*(a + b*x^4)^(1 + p))/(b*(5 + 4*p)) - ((a*e^2 - b*c^2*(5 +
4*p))*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a
)])/(b*(5 + 4*p)*(1 + (b*x^4)/a)^p) + (2*c*e*x^3*(a + b*x^4)^p*Hy
pergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)])/(3*(1 + (b*x^4)/a)^p
)
```

Rubi [A] time = 0.269505, antiderivative size = 142, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(c^2-\frac{ae^2}{4bp+5b}\right)\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+\frac{2}{3}cex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)+\frac{e^2x\left(a+bx^4\right)^{p+1}}{b(4p+5)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + e*x^2)^2*(a + b*x^4)^p,x]
```

```
[Out] (e^2*x*(a + b*x^4)^(1 + p))/(b*(5 + 4*p)) + ((c^2 - (a*e^2)/(5*b
+ 4*b*p))*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^
4)/a)])/(1 + (b*x^4)/a)^p + (2*c*e*x^3*(a + b*x^4)^p*Hypergeometr
ic2F1[3/4, -p, 7/4, -((b*x^4)/a)])/(3*(1 + (b*x^4)/a)^p)
```

Rubi in Sympy [A] time = 25.4957, size = 124, normalized size = 0.83

$$c^2x\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p\,{}_2F_1\left(\begin{matrix}-p,\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|-\frac{bx^4}{a}\right)+\frac{2cex^3\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p\,{}_2F_1\left(\begin{matrix}-p,\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|-\frac{bx^4}{a}\right)}{3}+\frac{e^2x^5\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p\,{}_2F_1\left(\begin{matrix}-p,\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|-\frac{bx^4}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+c)**2*(b*x**4+a)**p,x)
```

```
[Out] c**2*x*(1 + b*x**4/a)**(-p)*(a + b*x**4)**p*hyper((-p, 1/4), (5/4
,), -b*x**4/a) + 2*c*e*x**3*(1 + b*x**4/a)**(-p)*(a + b*x**4)**p*
hyper((-p, 3/4), (7/4,), -b*x**4/a)/3 + e**2*x**5*(1 + b*x**4/a)*
*(-p)*(a + b*x**4)**p*hyper((-p, 5/4), (9/4,), -b*x**4/a)/5
```

Mathematica [A] time = 0.0547916, size = 106, normalized size = 0.71

$$\frac{1}{15}x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(15c^2\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+ex^2\left(10c\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)+3ex^2\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + e*x^2)^2*(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*(15*c^2*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4
)/a)] + e*x^2*(10*c*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)]
 + 3*e*x^2*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)])))/(15*(
1 + (b*x^4)/a)^p)
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)^2\left(bx^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+c)^2*(b*x^4+a)^p,x)
```

```
[Out] int((e*x^2+c)^2*(b*x^4+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)^2\left(bx^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + c)^2*(b*x^4 + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + c)^2*(b*x^4 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e^2x^4+2\,cex^2+c^2\right)\left(bx^4+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + c)^2*(b*x^4 + a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*c*e*x^2 + c^2)*(b*x^4 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+c)**2*(b*x**4+a)**p,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + c)^2 (bx^4 + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + c)^2*(b*x^4 + a)^p,x, algorithm="giac")

[Out] integrate((e*x^2 + c)^2*(b*x^4 + a)^p, x)
```

3.182 $\int (c+ex^2)\left(a+bx^4\right)^p\,dx$

Optimal. Leaf size=96

$$cx\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)$$

```
[Out] (c*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])
/(1 + (b*x^4)/a)^p + (e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4,
-p, 7/4, -((b*x^4)/a)])/(3*(1 + (b*x^4)/a)^p)
```

Rubi [A] time = 0.112348, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294

$$cx\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(c + e*x^2)*(a + b*x^4)^p,x]
```

```
[Out] (c*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])
/(1 + (b*x^4)/a)^p + (e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4,
-p, 7/4, -((b*x^4)/a)])/(3*(1 + (b*x^4)/a)^p)
```

Rubi in Sympy [A] time = 14.8423, size = 76, normalized size = 0.79

$$cx\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p{}_2F_1\left(\begin{matrix}-p,\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|-\frac{bx^4}{a}\right)+\frac{ex^3\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p{}_2F_1\left(\begin{matrix}-p,\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|-\frac{bx^4}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+c)*(b*x**4+a)**p,x)
```

```
[Out] c*x*(1 + b*x**4/a)**(-p)*(a + b*x**4)**p*hyper((-p, 1/4), (5/4,),
 -b*x**4/a) + e*x**3*(1 + b*x**4/a)**(-p)*(a + b*x**4)**p*hyper((
-p, 3/4), (7/4,), -b*x**4/a)/3
```

Mathematica [A] time = 0.0298074, size = 75, normalized size = 0.78

$$\frac{1}{3}x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(3c\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+ex^2\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + e*x^2)*(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*(3*c*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a
)] + e*x^2*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)]))/(3*(1
+ (b*x^4)/a)^p)
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \left(e x^2+c\right) \left(b x^4+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+c)*(b*x^4+a)^p,x)

[Out] int((e*x^2+c)*(b*x^4+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e x^2+c\right) \left(b x^4+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + c)*(b*x^4 + a)^p,x, algorithm="maxima")

[Out] integrate((e*x^2 + c)*(b*x^4 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e x^2+c\right) \left(b x^4+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + c)*(b*x^4 + a)^p,x, algorithm="fricas")

[Out] integral((e*x^2 + c)*(b*x^4 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+c)*(b*x**4+a)**p,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e x^2+c\right) \left(b x^4+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + c)*(b*x^4 + a)^p,x, algorithm="giac")

[Out] integrate((e*x^2 + c)*(b*x^4 + a)^p, x)
```

3.183 $\int \left(a+bx^4\right)^p \, dx$

Optimal. Leaf size=44

$$x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)$$

```
[Out] (x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(
1 + (b*x^4)/a)^p
```

Rubi [A] time = 0.0227041, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(
1 + (b*x^4)/a)^p
```

Rubi in Sympy [A] time = 3.98662, size = 34, normalized size = 0.77

$$x\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p\,{}_2F_1\left(\begin{matrix}-p,\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|-\frac{bx^4}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**p,x)
```

```
[Out] x*(1 + b*x**4/a)**(-p)*(a + b*x**4)**p*hyper((-p, 1/4), (5/4,), -
b*x**4/a)
```

Mathematica [A] time = 0.0100062, size = 44, normalized size = 1.

$$x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(
1 + (b*x^4)/a)^p
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^p,x)
```

```
[Out] int((b*x^4+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^4 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^p, x)
```

Sympy [A] time = 38.5955, size = 34, normalized size = 0.77

$$\frac{a^p x \left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4 \left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**p,x)
```

```
[Out] a**p*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)
/a)/(4*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p, x)
```

3.184 $\int \frac{(a+bx^4)^p}{c+ex^2} \, dx$

Optimal. Leaf size=123

$$\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{1}{4};-p,1;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c}-\frac{ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{3}{4};-p,1;\frac{7}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^2}$$

```
[Out] (x*(a + b*x^4)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^4)/a), (e^2*x^4
)/c^2])/(c*(1 + (b*x^4)/a)^p) - (e*x^3*(a + b*x^4)^p*AppellF1[3/4
, -p, 1, 7/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(3*c^2*(1 + (b*x^4)/a
)^p)
```

Rubi [A] time = 0.280909, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{1}{4};-p,1;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c}-\frac{ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{3}{4};-p,1;\frac{7}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^p/(c + e*x^2),x]
```

```
[Out] (x*(a + b*x^4)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^4)/a), (e^2*x^4
)/c^2])/(c*(1 + (b*x^4)/a)^p) - (e*x^3*(a + b*x^4)^p*AppellF1[3/4
, -p, 1, 7/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(3*c^2*(1 + (b*x^4)/a
)^p)
```

Rubi in Sympy [A] time = 57.1198, size = 97, normalized size = 0.79

$$\frac{x\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p\operatorname{appellf}_1\left(\frac{1}{4},1,-p,\frac{5}{4},\frac{e^2x^4}{c^2},-\frac{bx^4}{a}\right)}{c}-\frac{ex^3\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p\operatorname{appellf}_1\left(\frac{3}{4},1,-p,\frac{7}{4},\frac{e^2x^4}{c^2},-\frac{bx^4}{a}\right)}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**p/(e*x**2+c),x)
```

```
[Out] x*(1 + b*x**4/a)**(-p)*(a + b*x**4)**p*appellf1(1/4, 1, -p, 5/4,
e**2*x**4/c**2, -b*x**4/a)/c - e*x**3*(1 + b*x**4/a)**(-p)*(a + b
*x**4)**p*appellf1(3/4, 1, -p, 7/4, e**2*x**4/c**2, -b*x**4/a)/(3
*c**2)
```

Mathematica [A] time = 0.0454001, size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^4\right)^p}{c+ex^2} \, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a + b*x^4)^p/(c + e*x^2),x]
```

```
[Out] Integrate[(a + b*x^4)^p/(c + e*x^2), x]
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^p}{ex^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^p/(e*x^2+c),x)
```

```
[Out] int((b*x^4+a)^p/(e*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^p}{ex^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^p/(e*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^p/(e*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^4 + a)^p}{ex^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^p/(e*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^p/(e*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**p/(e*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^p}{ex^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^p/(e*x^2 + c),x, algorithm="giac")

[Out] integrate((b*x^4 + a)^p/(e*x^2 + c), x)
```

3.185 $\int \frac{(a+bx^4)^p}{(c+ex^2)^2} dx$

Optimal. Leaf size=189

$$\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{1}{4};-p,2;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c^2}+\frac{e^2x^5\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{5}{4};-p,2;\frac{9}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{5c^4}-\frac{2ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{3}{4};-p,2;\frac{7}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^3}$$

```
[Out] (x*(a + b*x^4)^p*AppellF1[1/4, -p, 2, 5/4, -((b*x^4)/a), (e^2*x^4
)/c^2])/(c^2*(1 + (b*x^4)/a)^p) - (2*e*x^3*(a + b*x^4)^p*AppellF1
[3/4, -p, 2, 7/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(3*c^3*(1 + (b*x^
4)/a)^p) + (e^2*x^5*(a + b*x^4)^p*AppellF1[5/4, -p, 2, 9/4, -((b*
x^4)/a), (e^2*x^4)/c^2])/(5*c^4*(1 + (b*x^4)/a)^p)
```

Rubi [A] time = 0.462122, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{1}{4};-p,2;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c^2}+\frac{e^2x^5\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{5}{4};-p,2;\frac{9}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{5c^4}-\frac{2ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{3}{4};-p,2;\frac{7}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^p/(c + e*x^2)^2,x]
```

```
[Out] (x*(a + b*x^4)^p*AppellF1[1/4, -p, 2, 5/4, -((b*x^4)/a), (e^2*x^4
)/c^2])/(c^2*(1 + (b*x^4)/a)^p) - (2*e*x^3*(a + b*x^4)^p*AppellF1
[3/4, -p, 2, 7/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(3*c^3*(1 + (b*x^
4)/a)^p) + (e^2*x^5*(a + b*x^4)^p*AppellF1[5/4, -p, 2, 9/4, -((b*
x^4)/a), (e^2*x^4)/c^2])/(5*c^4*(1 + (b*x^4)/a)^p)
```

Rubi in Sympy [A] time = 110.37, size = 155, normalized size = 0.82

$$\frac{x\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p\operatorname{appellf}_1\left(\frac{1}{4},2,-p,\frac{5}{4},\frac{e^2x^4}{c^2},-\frac{bx^4}{a}\right)}{c^2}-\frac{2ex^3\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p\operatorname{appellf}_1\left(\frac{3}{4},2,-p,\frac{7}{4},\frac{e^2x^4}{c^2},-\frac{bx^4}{a}\right)}{3c^3}+\frac{e^2x^5\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p\operatorname{appellf}_1\left(\frac{5}{4},2,-p,\frac{9}{4},\frac{e^2x^4}{c^2},-\frac{bx^4}{a}\right)}{5c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**p/(e*x**2+c)**2,x)
```

```
[Out] x*(1 + b*x**4/a)**(-p)*(a + b*x**4)**p*appellf1(1/4, 2, -p, 5/4,
e**2*x**4/c**2, -b*x**4/a)/c**2 - 2*e*x**3*(1 + b*x**4/a)**(-p)*(
```

```
a + b*x**4)**p*appellf1(3/4, 2, -p, 7/4, e**2*x**4/c**2, -b*x**4/
a)/(3*c**3) + e**2*x**5*(1 + b*x**4/a)**(-p)*(a + b*x**4)**p*appe
llf1(5/4, 2, -p, 9/4, e**2*x**4/c**2, -b*x**4/a)/(5*c**4)
```

Mathematica [A] time = 0.0634808, size = 0, normalized size = 0.

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a + b*x^4)^p/(c + e*x^2)^2,x]

[Out] Integrate[(a + b*x^4)^p/(c + e*x^2)^2, x]
```

Maple [F] time = 0.081, size = 0, normalized size = 0.

$$\int \frac{(bx^4+a)^p}{(ex^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^p/(e*x^2+c)^2,x)

[Out] int((b*x^4+a)^p/(e*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+a)^p}{(ex^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^p/(e*x^2 + c)^2,x, algorithm="maxima")

[Out] integrate((b*x^4 + a)^p/(e*x^2 + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^4+a)^p}{e^2x^4+2\,cex^2+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^p/(e*x^2 + c)^2,x, algorithm="fricas")

[Out] integral((b*x^4 + a)^p/(e^2*x^4 + 2*c*e*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**p/(e*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^p}{(ex^2 + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^p/(e*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p/(e*x^2 + c)^2, x)
```

3.186 $\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p \, dx$

Optimal. Leaf size=108

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)+\frac{3}{5}x^5\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-bx^4\right)+\frac{x^3(1-b(4p+7))\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)}{b(4p+7)}-\frac{x^3\left(bx^4+1\right)^{p+1}}{b(4p+7)}$$

```
[Out] -((x^3*(1 + b*x^4)^(1 + p))/(b*(7 + 4*p))) + x*Hypergeometric2F1[
1/4, -p, 5/4, -(b*x^4)] + ((1 - b*(7 + 4*p))*x^3*Hypergeometric2F
1[3/4, -p, 7/4, -(b*x^4)])/(b*(7 + 4*p)) + (3*x^5*Hypergeometric2
F1[5/4, -p, 9/4, -(b*x^4)])/5
```

Rubi [A] time = 0.204596, antiderivative size = 103, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)+\frac{3}{5}x^5\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-bx^4\right)-x^3\left(1-\frac{1}{4bp+7b}\right)\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)-\frac{x^3\left(bx^4+1\right)^{p+1}}{b(4p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)^3*(1 + b*x^4)^p,x]
```

```
[Out] -((x^3*(1 + b*x^4)^(1 + p))/(b*(7 + 4*p))) + x*Hypergeometric2F1[
1/4, -p, 5/4, -(b*x^4)] - (1 - (7*b + 4*b*p)^(-1))*x^3*Hypergeome
tric2F1[3/4, -p, 7/4, -(b*x^4)] + (3*x^5*Hypergeometric2F1[5/4, -
p, 9/4, -(b*x^4)])/5
```

Rubi in Sympy [A] time = 14.2858, size = 70, normalized size = 0.65

$$-\frac{x^7\,{}_2F_1\left(\begin{matrix}-p,\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|-bx^4\right)}{7}+\frac{3x^5\,{}_2F_1\left(\begin{matrix}-p,\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|-bx^4\right)}{5}-x^3\,{}_2F_1\left(\begin{matrix}-p,\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|-bx^4\right)+x\,{}_2F_1\left(\begin{matrix}-p,\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|-bx^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-x**2+1)**3*(b*x**4+1)**p,x)
```

```
[Out] -x**7*hyper((-p, 7/4), (11/4,), -b*x**4)/7 + 3*x**5*hyper((-p, 5/
4), (9/4,), -b*x**4)/5 - x**3*hyper((-p, 3/4), (7/4,), -b*x**4) +
 x*hyper((-p, 1/4), (5/4,), -b*x**4)
```

Mathematica [A] time = 0.0333192, size = 86, normalized size = 0.8

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)-\frac{1}{7}x^7\,{}_2F_1\left(\frac{7}{4},-p;\frac{11}{4};-bx^4\right)+\frac{3}{5}x^5\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-bx^4\right)-x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)^3*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - x^3*Hypergeometric2
F1[3/4, -p, 7/4, -(b*x^4)] + (3*x^5*Hypergeometric2F1[5/4, -p, 9/
```

```
4, -(b*x^4)])/5 - (x^7*Hypergeometric2F1[7/4, -p, 11/4, -(b*x^4)]
)/7
```

Maple [A] time = 0.424, size = 75, normalized size = 0.7

$$-\frac{x^7}{7}{}_2F_1(\frac{7}{4},-p;\ \frac{11}{4};\ -bx^4)+\frac{3x^5}{5}{}_2F_1(\frac{5}{4},-p;\ \frac{9}{4};\ -bx^4)-x^3{}_2F_1(\frac{3}{4},-p;\ \frac{7}{4};\ -bx^4)+x{}_2F_1(\frac{1}{4},-p;\ \frac{5}{4};\ -bx^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^3*(b*x^4+1)^p,x)
```

```
[Out] -1/7*x^7*hypergeom([7/4,-p],[11/4],-b*x^4)+3/5*x^5*hypergeom([5/4
,-p],[9/4],-b*x^4)-x^3*hypergeom([3/4,-p],[7/4],-b*x^4)+x*hyperge
om([1/4,-p],[5/4],-b*x^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int (x^2-1)^3(bx^4+1)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)^3*(b*x^4 + 1)^p,x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-(x^6-3x^4+3x^2-1)(bx^4+1)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)^3*(b*x^4 + 1)^p,x, algorithm="fricas")
```

```
[Out] integral(-(x^6 - 3*x^4 + 3*x^2 - 1)*(b*x^4 + 1)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**3*(b*x**4+1)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -(x^2-1)^3(bx^4+1)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)^3*(b*x^4 + 1)^p,x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

3.187 $\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p \, dx$

Optimal. Leaf size=86

$$-\frac{x(1-b(4p+5))\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)}{b(4p+5)}-\frac{2}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)+\frac{x\left(bx^4+1\right)^{p+1}}{b(4p+5)}$$

```
[Out] (x*(1 + b*x^4)^(1 + p))/(b*(5 + 4*p)) - ((1 - b*(5 + 4*p))*x*Hype
rgeometric2F1[1/4, -p, 5/4, -(b*x^4)])/(b*(5 + 4*p)) - (2*x^3*Hyp
ergeometric2F1[3/4, -p, 7/4, -(b*x^4)])/3
```

Rubi [A] time = 0.148459, antiderivative size = 79, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$x\left(1-\frac{1}{4bp+5b}\right)\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)-\frac{2}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)+\frac{x\left(bx^4+1\right)^{p+1}}{b(4p+5)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^2)^2*(1 + b*x^4)^p,x]
```

```
[Out] (x*(1 + b*x^4)^(1 + p))/(b*(5 + 4*p)) + (1 - (5*b + 4*b*p)^(-1))*
x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (2*x^3*Hypergeometr
ic2F1[3/4, -p, 7/4, -(b*x^4)])/3
```

Rubi in Sympy [A] time = 12.3513, size = 53, normalized size = 0.62

$$\frac{x^5{}_2F_1\left(\begin{matrix}-p,\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|-bx^4\right)}{5}-\frac{2x^3{}_2F_1\left(\begin{matrix}-p,\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|-bx^4\right)}{3}+x{}_2F_1\left(\begin{matrix}-p,\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|-bx^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)**2*(b*x**4+1)**p,x)
```

```
[Out] x**5*hyper((-p, 5/4), (9/4,), -b*x**4)/5 - 2*x**3*hyper((-p, 3/4)
, (7/4,), -b*x**4)/3 + x*hyper((-p, 1/4), (5/4,), -b*x**4)
```

Mathematica [A] time = 0.0214872, size = 65, normalized size = 0.76

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)+\frac{1}{5}x^5\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-bx^4\right)-\frac{2}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^2)^2*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (2*x^3*Hypergeometr
ic2F1[3/4, -p, 7/4, -(b*x^4)])/3 + (x^5*Hypergeometric2F1[5/4, -p
, 9/4, -(b*x^4)])/5
```

Maple [A] time = 0.069, size = 56, normalized size = 0.7

$$\frac{x^5}{5}{}_2F_1\left(\frac{5}{4},-p;\ \frac{9}{4};\ -bx^4\right)-\frac{2\,x^3}{3}{}_2F_1\left(\frac{3}{4},-p;\ \frac{7}{4};\ -bx^4\right)+x\,{}_2F_1\left(\frac{1}{4},-p;\ \frac{5}{4};\ -bx^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^2*(b*x^4+1)^p,x)
```

```
[Out] 1/5*x^5*hypergeom([5/4,-p],[9/4],-b*x^4)-2/3*x^3*hypergeom([3/4,-
p],[7/4],-b*x^4)+x*hypergeom([1/4,-p],[5/4],-b*x^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^2-1\right)^2\left(bx^4+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^2*(b*x^4 + 1)^p,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(x^4-2\,x^2+1\right)\left(bx^4+1\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^2*(b*x^4 + 1)^p,x, algorithm="fricas")
```

```
[Out] integral((x^4 - 2*x^2 + 1)*(b*x^4 + 1)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**2*(b*x**4+1)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^2-1\right)^2\left(bx^4+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^2*(b*x^4 + 1)^p,x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

3.188 $\int (1-x^2)\left(1+bx^4\right)^p\,dx$

Optimal. Leaf size=42

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)-\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)$$

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric
2F1[3/4, -p, 7/4, -(b*x^4)])/3
```

Rubi [A] time = 0.047833, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)-\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^2)*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric
2F1[3/4, -p, 7/4, -(b*x^4)])/3
```

Rubi in Sympy [A] time = 6.66806, size = 32, normalized size = 0.76

$$-\frac{x^3{}_2F_1\left(\begin{matrix}-p,\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|-bx^4\right)}{3}+x{}_2F_1\left(\begin{matrix}-p,\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|-bx^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)*(b*x**4+1)**p,x)
```

```
[Out] -x**3*hyper((-p, 3/4), (7/4,), -b*x**4)/3 + x*hyper((-p, 1/4), (5
/4,), -b*x**4)
```

Mathematica [A] time = 0.0163358, size = 42, normalized size = 1.

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)-\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^2)*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric
2F1[3/4, -p, 7/4, -(b*x^4)])/3
```

Maple [A] time = 0.042, size = 37, normalized size = 0.9

$$x{}_2\mathrm{F}_1(\frac{1}{4},-p;\ \frac{5}{4};\ -bx^4)-\frac{x^3}{3}{}_2\mathrm{F}_1(\frac{3}{4},-p;\ \frac{7}{4};\ -bx^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)*(b*x^4+1)^p,x)
```

```
[Out] x*hypergeom([1/4,-p],[5/4],-b*x^4)-1/3*x^3*hypergeom([3/4,-p],[7/
4],-b*x^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int (x^2-1)\left(bx^4+1\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)*(b*x^4 + 1)^p,x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)*(b*x^4 + 1)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-(x^2-1)\left(bx^4+1\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)*(b*x^4 + 1)^p,x, algorithm="fricas")
```

```
[Out] integral(-(x^2 - 1)*(b*x^4 + 1)^p, x)
```

Sympy [A] time = 155.555, size = 61, normalized size = 1.45

$$-\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)*(b*x**4+1)**p,x)
```

```
[Out] -x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi))
/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*ex
p_polar(I*pi))/(4*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -(x^2-1)\left(bx^4+1\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)*(b*x^4 + 1)^p,x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)*(b*x^4 + 1)^p, x)
```

3.189 $\int (1 + bx^4)^p \, dx$

Optimal. Leaf size=18

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)$$

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]
```

Rubi [A] time = 0.010481, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]
```

Rubi in Sympy [A] time = 1.2766, size = 14, normalized size = 0.78

$$x{}_2F_1\left(\begin{matrix} -p, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| -bx^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+1)**p,x)
```

```
[Out] x*hyper((-p, 1/4), (5/4,), -b*x**4)
```

Mathematica [A] time = 0.00727353, size = 18, normalized size = 1.

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]
```

Maple [A] time = 0.03, size = 17, normalized size = 0.9

$$x{}_2\mathrm{F}_1(\frac{1}{4},-p;\ \frac{5}{4};\ -bx^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+1)^p,x)
```

```
[Out] x*hypergeom([1/4,-p],[5/4],-b*x^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4 + 1\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + 1)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + 1)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^4 + 1\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + 1)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + 1)^p, x)
```

Sympy [A] time = 32.4571, size = 29, normalized size = 1.61

$$\frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| bx^4 e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+1)**p,x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))/(4*
gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4 + 1\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + 1)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + 1)^p, x)
```

3.190 $\int \frac{(1+bx^4)^p}{1-x^2} dx$

Optimal. Leaf size=50

$$xF_1\left(\frac{1}{4};1,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{1}{3}x^3F_1\left(\frac{3}{4};1,-p;\frac{7}{4};x^4,-bx^4\right)$$

```
[Out] x*AppellF1[1/4, 1, -p, 5/4, x^4, -(b*x^4)] + (x^3*AppellF1[3/4, 1
, -p, 7/4, x^4, -(b*x^4)])/3
```

Rubi [A] time = 0.121223, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$xF_1\left(\frac{1}{4};1,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{1}{3}x^3F_1\left(\frac{3}{4};1,-p;\frac{7}{4};x^4,-bx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + b*x^4)^p/(1 - x^2),x]
```

```
[Out] x*AppellF1[1/4, 1, -p, 5/4, x^4, -(b*x^4)] + (x^3*AppellF1[3/4, 1
, -p, 7/4, x^4, -(b*x^4)])/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(bx^4+1)^p}{x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+1)**p/(-x**2+1),x)
```

```
[Out] -Integral((b*x**4 + 1)**p/(x**2 - 1), x)
```

Mathematica [A] time = 0.043094, size = 0, normalized size = 0.

$$\int \frac{(1+bx^4)^p}{1-x^2} dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(1 + b*x^4)^p/(1 - x^2),x]
```

```
[Out] Integrate[(1 + b*x^4)^p/(1 - x^2), x]
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{(bx^4+1)^p}{-x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+1)^p/(-x^2+1),x)

[Out] int((b*x^4+1)^p/(-x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(bx^4+1)^p}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^4 + 1)^p/(x^2 - 1),x, algorithm="maxima")

[Out] -integrate((b*x^4 + 1)^p/(x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(bx^4+1)^p}{x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^4 + 1)^p/(x^2 - 1),x, algorithm="fricas")

[Out] integral(-(b*x^4 + 1)^p/(x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+1)**p/(-x**2+1),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(bx^4+1)^p}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^4 + 1)^p/(x^2 - 1),x, algorithm="giac")

[Out] integrate(-(b*x^4 + 1)^p/(x^2 - 1), x)
```

3.191 $\int \frac{(1+bx^4)^p}{(1-x^2)^2} dx$

Optimal. Leaf size=77

$$xF_1\left(\frac{1}{4};2,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{1}{5}x^5F_1\left(\frac{5}{4};2,-p;\frac{9}{4};x^4,-bx^4\right)+\frac{2}{3}x^3F_1\left(\frac{3}{4};2,-p;\frac{7}{4};x^4,-bx^4\right)$$

```
[Out] x*AppellF1[1/4, 2, -p, 5/4, x^4, -(b*x^4)] + (2*x^3*AppellF1[3/4,
 2, -p, 7/4, x^4, -(b*x^4)])/3 + (x^5*AppellF1[5/4, 2, -p, 9/4, x
^4, -(b*x^4)])/5
```

Rubi [A] time = 0.198221, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$xF_1\left(\frac{1}{4};2,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{1}{5}x^5F_1\left(\frac{5}{4};2,-p;\frac{9}{4};x^4,-bx^4\right)+\frac{2}{3}x^3F_1\left(\frac{3}{4};2,-p;\frac{7}{4};x^4,-bx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + b*x^4)^p/(1 - x^2)^2,x]
```

```
[Out] x*AppellF1[1/4, 2, -p, 5/4, x^4, -(b*x^4)] + (2*x^3*AppellF1[3/4,
 2, -p, 7/4, x^4, -(b*x^4)])/3 + (x^5*AppellF1[5/4, 2, -p, 9/4, x
^4, -(b*x^4)])/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(bx^4+1)^p}{(x-1)^2}\,dx}{4}+\frac{\int \frac{(bx^4+1)^p}{(x+1)^2}\,dx}{4}-\frac{\int \frac{(bx^4+1)^p}{x^2-1}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+1)**p/(-x**2+1)**2,x)
```

```
[Out] Integral((b*x**4 + 1)**p/(x - 1)**2, x)/4 + Integral((b*x**4 + 1)
**p/(x + 1)**2, x)/4 - Integral((b*x**4 + 1)**p/(x**2 - 1), x)/2
```

Mathematica [A] time = 0.0484403, size = 0, normalized size = 0.

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(1 + b*x^4)^p/(1 - x^2)^2,x]
```

```
[Out] Integrate[(1 + b*x^4)^p/(1 - x^2)^2, x]
```

Maple [F] time = 0.081, size = 0, normalized size = 0.

$$\int \frac{(bx^4+1)^p}{(-x^2+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+1)^p/(-x^2+1)^2,x)
```

```
[Out] int((b*x^4+1)^p/(-x^2+1)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+1)^p}{(x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + 1)^p/(x^2 - 1)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^4+1)^p}{x^4-2\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + 1)^p/(x^2 - 1)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + 1)^p/(x^4 - 2*x^2 + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+1)**p/(-x**2+1)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+1)^p}{(x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + 1)^p/(x^2 - 1)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

3.192 $\int \frac{(1+bx^4)^p}{(1-x^2)^3} dx$

Optimal. Leaf size=101

$$xF_1\left(\frac{1}{4};3,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{1}{7}x^7F_1\left(\frac{7}{4};3,-p;\frac{11}{4};x^4,-bx^4\right)$$
$$+\frac{3}{5}x^5F_1\left(\frac{5}{4};3,-p;\frac{9}{4};x^4,-bx^4\right)+x^3F_1\left(\frac{3}{4};3,-p;\frac{7}{4};x^4,-bx^4\right)$$

```
[Out] x*AppellF1[1/4, 3, -p, 5/4, x^4, -(b*x^4)] + x^3*AppellF1[3/4, 3,
 -p, 7/4, x^4, -(b*x^4)] + (3*x^5*AppellF1[5/4, 3, -p, 9/4, x^4,
-(b*x^4)])/5 + (x^7*AppellF1[7/4, 3, -p, 11/4, x^4, -(b*x^4)])/7
```

Rubi [A] time = 0.269839, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$xF_1\left(\frac{1}{4};3,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{1}{7}x^7F_1\left(\frac{7}{4};3,-p;\frac{11}{4};x^4,-bx^4\right)$$
$$+\frac{3}{5}x^5F_1\left(\frac{5}{4};3,-p;\frac{9}{4};x^4,-bx^4\right)+x^3F_1\left(\frac{3}{4};3,-p;\frac{7}{4};x^4,-bx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + b*x^4)^p/(1 - x^2)^3,x]
```

```
[Out] x*AppellF1[1/4, 3, -p, 5/4, x^4, -(b*x^4)] + x^3*AppellF1[3/4, 3,
 -p, 7/4, x^4, -(b*x^4)] + (3*x^5*AppellF1[5/4, 3, -p, 9/4, x^4,
-(b*x^4)])/5 + (x^7*AppellF1[7/4, 3, -p, 11/4, x^4, -(b*x^4)])/7
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+1)**p/(-x**2+1)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0852825, size = 0, normalized size = 0.

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3} dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(1 + b*x^4)^p/(1 - x^2)^3,x]
```

```
[Out] Integrate[(1 + b*x^4)^p/(1 - x^2)^3, x]
```

Maple [F] time = 0.107, size = 0, normalized size = 0.

$$\int \frac{(bx^4+1)^p}{(-x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+1)^p/(-x^2+1)^3,x)

[Out] int((b*x^4+1)^p/(-x^2+1)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(bx^4+1)^p}{(x^2-1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^4 + 1)^p/(x^2 - 1)^3,x, algorithm="maxima")

[Out] -integrate((b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(bx^4+1)^p}{x^6-3\,x^4+3\,x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^4 + 1)^p/(x^2 - 1)^3,x, algorithm="fricas")

[Out] integral(-(b*x^4 + 1)^p/(x^6 - 3*x^4 + 3*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+1)**p/(-x**2+1)**3,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(bx^4+1)^p}{(x^2-1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^4 + 1)^p/(x^2 - 1)^3,x, algorithm="giac")

[Out] integrate(-(b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

3.193 $\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=51

$$\frac{8d^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-7d^2x-\frac{4}{3}dex^3-\frac{1}{5}e^2x^5$$

```
[Out] -7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^(5/2)*ArcTanh[(Sqrt
[e]*x)/Sqrt[d]])/Sqrt[e]
```

Rubi [A] time = 0.0724048, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{8d^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-7d^2x-\frac{4}{3}dex^3-\frac{1}{5}e^2x^5$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^4/(d^2 - e^2*x^4),x]
```

```
[Out] -7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^(5/2)*ArcTanh[(Sqrt
[e]*x)/Sqrt[d]])/Sqrt[e]
```

Rubi in Sympy [A] time = 16.6056, size = 49, normalized size = 0.96

$$\frac{8d^{\frac{5}{2}}\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-7d^2x-\frac{4dex^3}{3}-\frac{e^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**4/(-e**2*x**4+d**2),x)
```

```
[Out] 8*d**(5/2)*atanh(sqrt(e)*x/sqrt(d))/sqrt(e) - 7*d**2*x - 4*d*e*x*
*3/3 - e**2*x**5/5
```

Mathematica [A] time = 0.037631, size = 51, normalized size = 1.

$$\frac{8d^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-7d^2x-\frac{4}{3}dex^3-\frac{1}{5}e^2x^5$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^4/(d^2 - e^2*x^4),x]
```

```
[Out] -7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^(5/2)*ArcTanh[(Sqrt
[e]*x)/Sqrt[d]])/Sqrt[e]
```

Maple [A] time = 0.005, size = 42, normalized size = 0.8

$$-\frac{e^2x^5}{5}-\frac{4\,dex^3}{3}-7\,d^2x+8\,\frac{d^3}{\sqrt{de}}Artanh\left(\frac{ex}{\sqrt{de}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^4/(-e^2*x^4+d^2),x)
```

```
[Out] -1/5*e^2*x^5-4/3*d*e*x^3-7*d^2*x+8*d^3/(d*e)^(1/2)*arctanh(x*e/(d
*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d)^4/(e^2*x^4 - d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.297931, size = 1, normalized size = 0.02

$$\left[-\frac{1}{5}\,e^2x^5-\frac{4}{3}\,dex^3+4\,d^2\sqrt{\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{\frac{d}{e}}+d}{ex^2-d}\right)-7\,d^2x,\ -\frac{1}{5}\,e^2x^5-\frac{4}{3}\,dex^3+8\,d^2\sqrt{-\frac{d}{e}}\arctan\left(\frac{x}{\sqrt{-\frac{d}{e}}}\right)-7\,d^2x\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d)^4/(e^2*x^4 - d^2),x, algorithm="fricas")
```

```
[Out] [-1/5*e^2*x^5 - 4/3*d*e*x^3 + 4*d^2*sqrt(d/e)*log((e*x^2 + 2*e*x*
sqrt(d/e) + d)/(e*x^2 - d)) - 7*d^2*x, -1/5*e^2*x^5 - 4/3*d*e*x^3
 + 8*d^2*sqrt(-d/e)*arctan(x/sqrt(-d/e)) - 7*d^2*x]
```

Sympy [A] time = 1.73923, size = 75, normalized size = 1.47

$$-7d^2x-\frac{4dex^3}{3}-\frac{e^2x^5}{5}-4\sqrt{\frac{d^5}{e}}\log\left(x-\frac{\sqrt{\frac{d^5}{e}}}{d^2}\right)+4\sqrt{\frac{d^5}{e}}\log\left(x+\frac{\sqrt{\frac{d^5}{e}}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**4/(-e**2*x**4+d**2),x)
```

```
[Out] -7*d**2*x - 4*d*e*x**3/3 - e**2*x**5/5 - 4*sqrt(d**5/e)*log(x - s
qrt(d**5/e)/d**2) + 4*sqrt(d**5/e)*log(x + sqrt(d**5/e)/d**2)
```

GIAC/XCAS [A] time = 0.281494, size = 194, normalized size = 3.8

$$4\left(\left(d^{2}\right)^{\frac{1}{4}}d^{2}e^{\frac{11}{2}} - \left(d^{2}\right)^{\frac{1}{4}}d|d|e^{\frac{11}{2}}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\left(d^{2}\right)^{\frac{1}{4}}}\right)e^{(-6)} + 2\left(\left(d^{2}\right)^{\frac{1}{4}}d^{2}e^{\frac{15}{2}} + \left(d^{2}\right)^{\frac{3}{4}}de^{\frac{15}{2}}\right)e^{(-8)}\ln\left(\left|\left(d^{2}\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)} + x\right|\right) - 2\left(\left(d^{2}\right)^{\frac{1}{4}}d^{2}e^{\frac{11}{2}} + \left(d^{2}\right)^{\frac{1}{4}}d|d|e^{\frac{11}{2}}\right)e^{(-6)}\ln\left(\left|-\left(d^{2}\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)} + x\right|\right) - \frac{1}{15}\left(3x^{5}e^{12} + 20dx^{3}e^{11} + 105d^{2}xe^{10}\right)e^{(-10)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d)^4/(e^2*x^4 - d^2),x, algorithm="giac")
```

```
[Out] 4*((d^2)^(1/4)*d^2*e^(11/2) - (d^2)^(1/4)*d*abs(d)*e^(11/2))*arct
an(x*e^(1/2)/(d^2)^(1/4))*e^(-6) + 2*((d^2)^(1/4)*d^2*e^(15/2) +
(d^2)^(3/4)*d*e^(15/2))*e^(-8)*ln(abs((d^2)^(1/4)*e^(-1/2) + x))
- 2*((d^2)^(1/4)*d^2*e^(11/2) + (d^2)^(1/4)*d*abs(d)*e^(11/2))*e^
(-6)*ln(abs(-(d^2)^(1/4)*e^(-1/2) + x)) - 1/15*(3*x^5*e^12 + 20*d
*x^3*e^11 + 105*d^2*x*e^10)*e^(-10)
```

3.194 $\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=38

$$\frac{4d^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-3dx-\frac{ex^3}{3}$$

```
[Out] -3*d*x - (e*x^3)/3 + (4*d^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqr
t[e]
```

Rubi [A] time = 0.0614831, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{4d^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-3dx-\frac{ex^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^3/(d^2 - e^2*x^4),x]
```

```
[Out] -3*d*x - (e*x^3)/3 + (4*d^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqr
t[e]
```

Rubi in Sympy [A] time = 15.4055, size = 36, normalized size = 0.95

$$\frac{4d^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-3dx-\frac{ex^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**3/(-e**2*x**4+d**2),x)
```

```
[Out] 4*d**(3/2)*atanh(sqrt(e)*x/sqrt(d))/sqrt(e) - 3*d*x - e*x**3/3
```

Mathematica [A] time = 0.0273825, size = 38, normalized size = 1.

$$\frac{4d^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-3dx-\frac{ex^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^3/(d^2 - e^2*x^4),x]
```

```
[Out] -3*d*x - (e*x^3)/3 + (4*d^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqr
t[e]
```

Maple [A] time = 0.003, size = 31, normalized size = 0.8

$$-\frac{ex^3}{3}-3\,dx+4\,\frac{d^2}{\sqrt{de}}Artanh\left(\frac{ex}{\sqrt{de}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^3/(-e^2*x^4+d^2),x)
```

```
[Out] -1/3*e*x^3-3*d*x+4*d^2/(d*e)^(1/2)*arctanh(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d)^3/(e^2*x^4 - d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272389, size = 1, normalized size = 0.03

$$\left[-\frac{1}{3}\,ex^{3}+2\,d\sqrt{\frac{d}{e}}\log\left(\frac{ex^{2}+2\,ex\sqrt{\frac{d}{e}}+d}{ex^{2}-d}\right)-3\,dx,-\frac{1}{3}\,ex^{3}+4\,d\sqrt{-\frac{d}{e}}\arctan\left(\frac{x}{\sqrt{-\frac{d}{e}}}\right)-3\,dx\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d)^3/(e^2*x^4 - d^2),x, algorithm="fricas")
```

```
[Out] [-1/3*e*x^3 + 2*d*sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e) + d)/(e*
x^2 - d)) - 3*d*x, -1/3*e*x^3 + 4*d*sqrt(-d/e)*arctan(x/sqrt(-d/e
)) - 3*d*x]
```

Sympy [A] time = 1.517, size = 58, normalized size = 1.53

$$-3dx-\frac{ex^{3}}{3}-2\sqrt{\frac{d^{3}}{e}}\log\left(x-\frac{\sqrt{\frac{d^{3}}{e}}}{d}\right)+2\sqrt{\frac{d^{3}}{e}}\log\left(x+\frac{\sqrt{\frac{d^{3}}{e}}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**3/(-e**2*x**4+d**2),x)
```

```
[Out] -3*d*x - e*x**3/3 - 2*sqrt(d**3/e)*log(x - sqrt(d**3/e)/d) + 2*sq
rt(d**3/e)*log(x + sqrt(d**3/e)/d)
```

GIAC/XCAS [A] time = 0.279749, size = 166, normalized size = 4.37

$$2\left(\left(d^{2}\right)^{\frac{1}{4}}de^{\frac{11}{2}}-\left(d^{2}\right)^{\frac{1}{4}}|d|e^{\frac{11}{2}}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\left(d^{2}\right)^{\frac{1}{4}}}\right)e^{(-6)}$$
$$+\left(\left(d^{2}\right)^{\frac{1}{4}}de^{\frac{15}{2}}+\left(d^{2}\right)^{\frac{3}{4}}e^{\frac{15}{2}}\right)e^{(-8)}\ln\left(\left|\left(d^{2}\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}+x\right|\right)$$
$$-\left(\left(d^{2}\right)^{\frac{1}{4}}de^{\frac{11}{2}}+\left(d^{2}\right)^{\frac{1}{4}}|d|e^{\frac{11}{2}}\right)e^{(-6)}\ln\left(\left|-\left(d^{2}\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}+x\right|\right)-\frac{1}{3}\left(x^{3}e^{7}+9\,dxe^{6}\right)e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d)^3/(e^2*x^4 - d^2),x, algorithm="giac")
```

```
[Out] 2*((d^2)^(1/4)*d*e^(11/2) - (d^2)^(1/4)*abs(d)*e^(11/2))*arctan(x
*e^(1/2)/(d^2)^(1/4))*e^(-6) + ((d^2)^(1/4)*d*e^(15/2) + (d^2)^(3
/4)*e^(15/2))*e^(-8)*ln(abs((d^2)^(1/4)*e^(-1/2) + x)) - ((d^2)^(
1/4)*d*e^(11/2) + (d^2)^(1/4)*abs(d)*e^(11/2))*e^(-6)*ln(abs(-(d^
2)^(1/4)*e^(-1/2) + x)) - 1/3*(x^3*e^7 + 9*d*x*e^6)*e^(-6)
```

3.195 $\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=29

$$\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-x$$

```
[Out] -x + (2*Sqrt[d]*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Rubi [A] time = 0.0407428, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-x$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^2/(d^2 - e^2*x^4),x]

[Out] -x + (2*Sqrt[d]*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Rubi in Sympy [A] time = 11.7222, size = 26, normalized size = 0.9

$$\frac{2\sqrt{d}\,\text{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2/(-e**2*x**4+d**2),x)

[Out] 2*sqrt(d)*atanh(sqrt(e)*x/sqrt(d))/sqrt(e) - x
```

Mathematica [A] time = 0.0138527, size = 29, normalized size = 1.

$$\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^2/(d^2 - e^2*x^4),x]

[Out] -x + (2*Sqrt[d]*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Maple [A] time = 0.003, size = 22, normalized size = 0.8

$$-x+2\,\frac{d}{\sqrt{de}}\mathit{Artanh}\left(\frac{ex}{\sqrt{de}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^2/(-e^2*x^4+d^2),x)

[Out] -x+2*d/(d*e)^(1/2)*arctanh(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d)^2/(e^2*x^4 - d^2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277895, size = 1, normalized size = 0.03

$$\left[\sqrt{\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{\frac{d}{e}}+d}{ex^2-d}\right)-x,\,2\,\sqrt{-\frac{d}{e}}\arctan\left(\frac{x}{\sqrt{-\frac{d}{e}}}\right)-x\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d)^2/(e^2*x^4 - d^2),x, algorithm="fricas")

[Out] [sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e) + d)/(e*x^2 - d)) - x, 2*
sqrt(-d/e)*arctan(x/sqrt(-d/e)) - x]
```

Sympy [A] time = 1.31547, size = 34, normalized size = 1.17

$$-x-\sqrt{\frac{d}{e}}\log\left(x-\sqrt{\frac{d}{e}}\right)+\sqrt{\frac{d}{e}}\log\left(x+\sqrt{\frac{d}{e}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**2/(-e**2*x**4+d**2),x)

[Out] -x - sqrt(d/e)*log(x - sqrt(d/e)) + sqrt(d/e)*log(x + sqrt(d/e))
```

GIAC/XCAS [A] time = 0.280529, size = 159, normalized size = 5.48

$$\frac{\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{7}{2}}-\left(d^2\right)^{\frac{1}{4}}|d|e^{\frac{7}{2}}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\left(d^2\right)^{\frac{1}{4}}}\right)e^{(-4)}}{d}+\frac{\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}+\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)e^{(-6)}\ln\left(\left|\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}+x\right|\right)}{2\,d}-\frac{\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{7}{2}}+\left(d^2\right)^{\frac{1}{4}}|d|e^{\frac{7}{2}}\right)e^{(-4)}\ln\left(\left|-\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}+x\right|\right)}{2\,d}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d)^2/(e^2*x^4 - d^2),x, algorithm="giac")
```

```
[Out] ((d^2)^(1/4)*d*e^(7/2) - (d^2)^(1/4)*abs(d)*e^(7/2))*arctan(x*e^(
1/2)/(d^2)^(1/4))*e^(-4)/d + 1/2*((d^2)^(1/4)*d*e^(11/2) + (d^2)^
(3/4)*e^(11/2))*e^(-6)*ln(abs((d^2)^(1/4)*e^(-1/2) + x))/d - 1/2*
((d^2)^(1/4)*d*e^(7/2) + (d^2)^(1/4)*abs(d)*e^(7/2))*e^(-4)*ln(ab
s(-(d^2)^(1/4)*e^(-1/2) + x))/d - x
```

3.196 $\int \frac{d+ex^2}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=24

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

```
[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])
```

Rubi [A] time = 0.0227719, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(d^2 - e^2*x^4),x]

[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])
```

Rubi in Sympy [A] time = 5.60839, size = 22, normalized size = 0.92

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(-e**2*x**4+d**2),x)

[Out] atanh(sqrt(e)*x/sqrt(d))/(sqrt(d)*sqrt(e))
```

Mathematica [A] time = 0.00653757, size = 24, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/(d^2 - e^2*x^4),x]

[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])
```

Maple [A] time = 0.002, size = 16, normalized size = 0.7

$$1 Artanh\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(-e^2*x^4+d^2),x)
```

```
[Out] 1/(d*e)^(1/2)*arctanh(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d)/(e^2*x^4 - d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271492, size = 1, normalized size = 0.04

$$\left[\frac{\log\left(\frac{2\,dex+(ex^2+d)\sqrt{de}}{ex^2-d}\right)}{2\sqrt{de}}, \frac{\arctan\left(\frac{\sqrt{-de}x}{d}\right)}{\sqrt{-de}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d)/(e^2*x^4 - d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*d*e*x + (e*x^2 + d)*sqrt(d*e))/(e*x^2 - d))/sqrt(d*e)
, arctan(sqrt(-d*e)*x/d)/sqrt(-d*e)]
```

Sympy [A] time = 0.33632, size = 46, normalized size = 1.92

$$-\frac{\sqrt{\frac{1}{de}}\log\left(-d\sqrt{\frac{1}{de}}+x\right)}{2}+\frac{\sqrt{\frac{1}{de}}\log\left(d\sqrt{\frac{1}{de}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(-e**2*x**4+d**2),x)
```

```
[Out] -sqrt(1/(d*e))*log(-d*sqrt(1/(d*e)) + x)/2 + sqrt(1/(d*e))*log(d*
sqrt(1/(d*e)) + x)/2
```

GIAC/XCAS [A] time = 0.27893, size = 157, normalized size = 6.54

$$\frac{\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{7}{2}}-\left(d^2\right)^{\frac{1}{4}}|d|e^{\frac{7}{2}}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\left(d^2\right)^{\frac{1}{4}}}\right)e^{(-4)}}{2\,d^2}+\frac{\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}+\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)e^{(-6)}\ln\left(\left|\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}+x\right|\right)}{4\,d^2}-\frac{\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{7}{2}}+\left(d^2\right)^{\frac{1}{4}}|d|e^{\frac{7}{2}}\right)e^{(-4)}\ln\left(\left|-\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}+x\right|\right)}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d)/(e^2*x^4 - d^2),x, algorithm="giac")
```

```
[Out] 1/2*((d^2)^(1/4)*d*e^(7/2) - (d^2)^(1/4)*abs(d)*e^(7/2))*arctan(x
*e^(1/2)/(d^2)^(1/4))*e^(-4)/d^2 + 1/4*((d^2)^(1/4)*d*e^(11/2) +
(d^2)^(3/4)*e^(11/2))*e^(-6)*ln(abs((d^2)^(1/4)*e^(-1/2) + x))/d^
2 - 1/4*((d^2)^(1/4)*d*e^(7/2) + (d^2)^(1/4)*abs(d)*e^(7/2))*e^(-
4)*ln(abs(-(d^2)^(1/4)*e^(-1/2) + x))/d^2
```

3.197 $\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx$

Optimal. Leaf size=72

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}+\frac{x}{4d^2\left(d+ex^2\right)}$$

```
[Out] x/(4*d^2*(d + e*x^2)) + ArcTan[(Sqrt[e]*x)/Sqrt[d]]/(2*d^(5/2)*Sq
rt[e]) + ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(4*d^(5/2)*Sqrt[e])
```

Rubi [A] time = 0.141373, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}+\frac{x}{4d^2\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(4*d^2*(d + e*x^2)) + ArcTan[(Sqrt[e]*x)/Sqrt[d]]/(2*d^(5/2)*Sq
rt[e]) + ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(4*d^(5/2)*Sqrt[e])
```

Rubi in Sympy [A] time = 35.0936, size = 63, normalized size = 0.88

$$\frac{x}{4d^2\left(d+ex^2\right)}+\frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{\frac{5}{2}}\sqrt{e}}+\frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4d^{\frac{5}{2}}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)/(-e**2*x**4+d**2),x)
```

```
[Out] x/(4*d**2*(d + e*x**2)) + atan(sqrt(e)*x/sqrt(d))/(2*d**(5/2)*sqr
t(e)) + atanh(sqrt(e)*x/sqrt(d))/(4*d**(5/2)*sqrt(e))
```

Mathematica [A] time = 0.0590362, size = 65, normalized size = 0.9

$$\frac{\frac{\sqrt{d}x}{d+ex^2}+\frac{2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}}{4d^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)*(d^2 - e^2*x^4)),x]
```

```
[Out] ((Sqrt[d]*x)/(d + e*x^2) + (2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e
] + ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/Sqrt[e])/(4*d^(5/2))
```

Maple [A] time = 0.014, size = 55, normalized size = 0.8

$$\frac{1}{4\,d^2}Artanh\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{x}{4\,d^2\left(ex^2+d\right)}+\frac{1}{2\,d^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)/(-e^2*x^4+d^2),x)
```

```
[Out] 1/4/d^2/(d*e)^(1/2)*arctanh(x*e/(d*e)^(1/2))+1/4*x/d^2/(e*x^2+d)+
1/2/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((e^2*x^4 - d^2)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.290197, size = 1, normalized size = 0.01

$$\left[\frac{4\left(ex^2+d\right)\arctan\left(\frac{\sqrt{de}\,x}{d}\right)+\left(ex^2+d\right)\log\left(\frac{2\,dex+\left(ex^2+d\right)\sqrt{de}}{ex^2-d}\right)+2\sqrt{de}\,x}{8\left(d^2ex^2+d^3\right)\sqrt{de}}, \frac{\left(ex^2+d\right)\arctan\left(\frac{\sqrt{-de}\,x}{d}\right)+\left(ex^2+d\right)\log\left(\frac{2\,dex+\right.}{}\right.}{4\left(d^2ex^2+d^3\right)\sqrt{-de}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((e^2*x^4 - d^2)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(e*x^2 + d)*arctan(sqrt(d*e)*x/d) + (e*x^2 + d)*log((2*d*
e*x + (e*x^2 + d)*sqrt(d*e))/(e*x^2 - d)) + 2*sqrt(d*e)*x)/((d^2*
e*x^2 + d^3)*sqrt(d*e)), 1/4*((e*x^2 + d)*arctan(sqrt(-d*e)*x/d)
+ (e*x^2 + d)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d))
 + sqrt(-d*e)*x)/((d^2*e*x^2 + d^3)*sqrt(-d*e))]
```

Sympy [A] time = 2.37143, size = 226, normalized size = 3.14

$$\frac{x}{4d^3+4d^2ex^2}-\frac{\sqrt{\frac{1}{d^5e}}\log\left(-\frac{d^8e\left(\frac{1}{d^5e}\right)^{\frac{3}{2}}}{10}-\frac{9d^3\sqrt{\frac{1}{d^5e}}}{10}+x\right)}{8}+\frac{\sqrt{\frac{1}{d^5e}}\log\left(\frac{d^8e\left(\frac{1}{d^5e}\right)^{\frac{3}{2}}}{10}+\frac{9d^3\sqrt{\frac{1}{d^5e}}}{10}+x\right)}{8}$$
$$-\frac{\sqrt{-\frac{1}{d^5e}}\log\left(-\frac{4d^8e\left(-\frac{1}{d^5e}\right)^{\frac{3}{2}}}{5}-\frac{9d^3\sqrt{-\frac{1}{d^5e}}}{5}+x\right)}{4}+\frac{\sqrt{-\frac{1}{d^5e}}\log\left(\frac{4d^8e\left(-\frac{1}{d^5e}\right)^{\frac{3}{2}}}{5}+\frac{9d^3\sqrt{-\frac{1}{d^5e}}}{5}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)/(-e**2*x**4+d**2),x)
```

```
[Out] x/(4*d**3 + 4*d**2*e*x**2) - sqrt(1/(d**5*e))*log(-d**8*e*(1/(d**
5*e))**(3/2)/10 - 9*d**3*sqrt(1/(d**5*e))/10 + x)/8 + sqrt(1/(d**
5*e))*log(d**8*e*(1/(d**5*e))**(3/2)/10 + 9*d**3*sqrt(1/(d**5*e))
/10 + x)/8 - sqrt(-1/(d**5*e))*log(-4*d**8*e*(-1/(d**5*e))**(3/2)
/5 - 9*d**3*sqrt(-1/(d**5*e))/5 + x)/4 + sqrt(-1/(d**5*e))*log(4*
d**8*e*(-1/(d**5*e))**(3/2)/5 + 9*d**3*sqrt(-1/(d**5*e))/5 + x)/4
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((e^2*x^4 - d^2)*(e^x^2 + d)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.198 $\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx$

Optimal. Leaf size=89

$$\frac{7\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}+\frac{5x}{16d^3(d+ex^2)}+\frac{x}{8d^2(d+ex^2)^2}$$

```
[Out] x/(8*d^2*(d + e*x^2)^2) + (5*x)/(16*d^3*(d + e*x^2)) + (7*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*Sqrt[e]) + ArcTanh[(Sqrt[e]*x)/
Sqrt[d]]/(8*d^(7/2)*Sqrt[e])
```

Rubi [A] time = 0.211968, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{7\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}+\frac{5x}{16d^3(d+ex^2)}+\frac{x}{8d^2(d+ex^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)^2*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(8*d^2*(d + e*x^2)^2) + (5*x)/(16*d^3*(d + e*x^2)) + (7*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*Sqrt[e]) + ArcTanh[(Sqrt[e]*x)/
Sqrt[d]]/(8*d^(7/2)*Sqrt[e])
```

Rubi in Sympy [A] time = 58.1471, size = 82, normalized size = 0.92

$$\frac{x}{8d^2(d+ex^2)^2}+\frac{5x}{16d^3(d+ex^2)}+\frac{7\,\mathrm{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{\frac{7}{2}}\sqrt{e}}+\frac{\mathrm{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{\frac{7}{2}}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)**2/(-e**2*x**4+d**2),x)
```

```
[Out] x/(8*d**2*(d + e*x**2)**2) + 5*x/(16*d**3*(d + e*x**2)) + 7*atan(
sqrt(e)*x/sqrt(d))/(16*d**(7/2)*sqrt(e)) + atanh(sqrt(e)*x/sqrt(d
))/(8*d**(7/2)*sqrt(e))
```

Mathematica [A] time = 0.100333, size = 76, normalized size = 0.85

$$\frac{\frac{\sqrt{d}x(7d+5ex^2)}{(d+ex^2)^2}+\frac{7\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}}{16d^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)^2*(d^2 - e^2*x^4)),x]
```

```
[Out] ((Sqrt[d]*x*(7*d + 5*e*x^2))/(d + e*x^2)^2 + (7*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/Sqrt[e] + (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e])/(
16*d^(7/2))
```

Maple [A] time = 0.015, size = 73, normalized size = 0.8

$$\frac{1}{8\,d^3}\,Artanh\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{5\,ex^3}{16\,d^3\left(ex^2+d\right)^2}+\frac{7\,x}{16\,d^2\left(ex^2+d\right)^2}+\frac{7}{16\,d^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x)
```

```
[Out] 1/8/d^3/(d*e)^(1/2)*arctanh(x*e/(d*e)^(1/2))+5/16/d^3/(e*x^2+d)^2
*x^3*e+7/16*x/d^2/(e*x^2+d)^2+7/16/d^3/(d*e)^(1/2)*arctan(x*e/(d*
e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((e^2*x^4 - d^2)*(e*x^2 + d)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.295003, size = 1, normalized size = 0.01

$$\left[\frac{7\left(e^2x^4+2\,dex^2+d^2\right)\arctan\left(\frac{\sqrt{de}x}{d}\right)+\left(e^2x^4+2\,dex^2+d^2\right)\log\left(\frac{2\,dex+\left(ex^2+d\right)\sqrt{de}}{ex^2-d}\right)+\left(5\,ex^3+7\,dx\right)\sqrt{de}}{16\left(d^3e^2x^4+2\,d^4ex^2+d^5\right)\sqrt{de}},\ \frac{4\left(e^2x^4+2\,dex^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((e^2*x^4 - d^2)*(e*x^2 + d)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(7*(e^2*x^4 + 2*d*e*x^2 + d^2)*arctan(sqrt(d*e)*x/d) + (e^2
*x^4 + 2*d*e*x^2 + d^2)*log((2*d*e*x + (e*x^2 + d)*sqrt(d*e))/(e*
x^2 - d)) + (5*e*x^3 + 7*d*x)*sqrt(d*e))/((d^3*e^2*x^4 + 2*d^4*e*
x^2 + d^5)*sqrt(d*e)), 1/32*(4*(e^2*x^4 + 2*d*e*x^2 + d^2)*arctan
(sqrt(-d*e)*x/d) + 7*(e^2*x^4 + 2*d*e*x^2 + d^2)*log((2*d*e*x + (
e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d)) + 2*(5*e*x^3 + 7*d*x)*sqrt(-d
*e))/((d^3*e^2*x^4 + 2*d^4*e*x^2 + d^5)*sqrt(-d*e))]
```

Sympy [A] time = 3.23939, size = 255, normalized size = 2.87

$$-\frac{\sqrt{\frac{1}{d^7e}}\log\left(-\frac{20d^{11}e\left(\frac{1}{d^7e}\right)^{\frac{3}{2}}}{371}-\frac{351d^4\sqrt{\frac{1}{d^7e}}}{371}+x\right)}{16}+\frac{\sqrt{\frac{1}{d^7e}}\log\left(\frac{20d^{11}e\left(\frac{1}{d^7e}\right)^{\frac{3}{2}}}{371}+\frac{351d^4\sqrt{\frac{1}{d^7e}}}{371}+x\right)}{16}$$
$$-\frac{7\sqrt{-\frac{1}{d^7e}}\log\left(-\frac{245d^{11}e\left(-\frac{1}{d^7e}\right)^{\frac{3}{2}}}{106}-\frac{351d^4\sqrt{-\frac{1}{d^7e}}}{106}+x\right)}{32}$$
$$+\frac{7\sqrt{-\frac{1}{d^7e}}\log\left(\frac{245d^{11}e\left(-\frac{1}{d^7e}\right)^{\frac{3}{2}}}{106}+\frac{351d^4\sqrt{-\frac{1}{d^7e}}}{106}+x\right)}{32}+\frac{7dx+5ex^3}{16d^5+32d^4ex^2+16d^3e^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)**2/(-e**2*x**4+d**2),x)
```

```
[Out] -sqrt(1/(d**7*e))*log(-20*d**11*e*(1/(d**7*e))**(3/2)/371 - 351*d
**4*sqrt(1/(d**7*e))/371 + x)/16 + sqrt(1/(d**7*e))*log(20*d**11*
e*(1/(d**7*e))**(3/2)/371 + 351*d**4*sqrt(1/(d**7*e))/371 + x)/16
 - 7*sqrt(-1/(d**7*e))*log(-245*d**11*e*(-1/(d**7*e))**(3/2)/106
- 351*d**4*sqrt(-1/(d**7*e))/106 + x)/32 + 7*sqrt(-1/(d**7*e))*lo
g(245*d**11*e*(-1/(d**7*e))**(3/2)/106 + 351*d**4*sqrt(-1/(d**7*e
))/106 + x)/32 + (7*d*x + 5*e*x**3)/(16*d**5 + 32*d**4*e*x**2 + 1
6*d**3*e**2*x**4)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((e^2*x^4 - d^2)*(e*x^2 + d)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.199 $\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

```
[Out] -(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[e]) + (Sqrt[2]*ArcTan
h[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e]
```

Rubi [A] time = 0.120745, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[e]) + (Sqrt[2]*ArcTan
h[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e]
```

Rubi in Sympy [A] time = 21.5816, size = 56, normalized size = 0.9

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}+\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**(3/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -atanh(sqrt(e)*x/sqrt(d + e*x**2))/sqrt(e) + sqrt(2)*atanh(sqrt(2
)*sqrt(e)*x/sqrt(d + e*x**2))/sqrt(e)
```

Mathematica [A] time = 0.0421149, size = 61, normalized size = 0.98

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\log\left(\sqrt{e}\sqrt{d+ex^2}+ex\right)}{\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]] - Log[e*x +
 Sqrt[e]*Sqrt[d + e*x^2]])/Sqrt[e]
```

Maple [B] time = 0.066, size = 1442, normalized size = 23.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x)
```

```
[Out] 1/6*e/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)^(1/2))/(d*e
)^(1/2)*((x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*
d)^(3/2)+1/4*e/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)^(1
/2))*((x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^
(1/2)*x+5/4*e^(1/2)/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*
e)^(1/2))*d*ln(((x-(d*e)^(1/2)/e)*e+(d*e)^(1/2))/e^(1/2)+((x-(d*e
)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^(1/2))+e/((d*
e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)^(1/2))/(d*e)^(1/2)*d*
((x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^(1/2)
-e/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)^(1/2))/(d*e)^(
1/2)*d^(3/2)*2^(1/2)*ln((4*d+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*2^
(1/2)*d^(1/2)*((x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)
/e)+2*d)^(1/2))/(x-(d*e)^(1/2)/e))-1/6*e/((d*e)^(1/2)+(-d*e)^(1/2
))/(-(d*e)^(1/2)+(-d*e)^(1/2))/(d*e)^(1/2)*((x+(d*e)^(1/2)/e)^2*e
-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(3/2)+1/4*e/((d*e)^(1/2)+(-
d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)^(1/2))*((x+(d*e)^(1/2)/e)^2*e-2*
(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2)*x+5/4*e^(1/2)/((d*e)^(1/
2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)^(1/2))*d*ln(((x+(d*e)^(1/2)
/e)*e-(d*e)^(1/2))/e^(1/2)+((x+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(
x+(d*e)^(1/2)/e)+2*d)^(1/2))-e/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)
^(1/2)+(-d*e)^(1/2))/(d*e)^(1/2)*d*((x+(d*e)^(1/2)/e)^2*e-2*(d*e)
^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2)+e/((d*e)^(1/2)+(-d*e)^(1/2))/
(-(d*e)^(1/2)+(-d*e)^(1/2))/(d*e)^(1/2)*d^(3/2)*2^(1/2)*ln((4*d-2
*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*2^(1/2)*d^(1/2)*((x+(d*e)^(1/2)/
e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2))/(x+(d*e)^(1/2)
/e))-1/6*e/(-d*e)^(1/2)/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+
(-d*e)^(1/2))*((x-1/e*(-d*e)^(1/2))^2*e+2*(-d*e)^(1/2)*(x-1/e*(-d
*e)^(1/2)))^(3/2)-1/4*e/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+
(-d*e)^(1/2))*((x-1/e*(-d*e)^(1/2))^2*e+2*(-d*e)^(1/2)*(x-1/e*(-d
*e)^(1/2)))^(1/2)*x-1/4*e^(1/2)/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e
)^(1/2)+(-d*e)^(1/2))*d*ln(((x-1/e*(-d*e)^(1/2))*e+(-d*e)^(1/2))/
e^(1/2)+((x-1/e*(-d*e)^(1/2))^2*e+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1
/2)))^(1/2))+1/6*e/(-d*e)^(1/2)/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e
)^(1/2)+(-d*e)^(1/2))*((x+1/e*(-d*e)^(1/2))^2*e-2*(-d*e)^(1/2)*(x
+1/e*(-d*e)^(1/2)))^(3/2)-1/4*e/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e
)^(1/2)+(-d*e)^(1/2))*((x+1/e*(-d*e)^(1/2))^2*e-2*(-d*e)^(1/2)*(x
+1/e*(-d*e)^(1/2)))^(1/2)*x-1/4*e^(1/2)/((d*e)^(1/2)+(-d*e)^(1/2)
)/(-(d*e)^(1/2)+(-d*e)^(1/2))*d*ln(((x+1/e*(-d*e)^(1/2))*e-(-d*e)
^(1/2))/e^(1/2)+((x+1/e*(-d*e)^(1/2))^2*e-2*(-d*e)^(1/2)*(x+1/e*(
-d*e)^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d)^(3/2)/(e^2*x^4 - d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.307087, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{2}\log\left(\frac{17\,e^2x^4+14\,dex^2+d^2+\frac{4\sqrt{2}\left(3\,e^2x^3+dex\right)\sqrt{ex^2+d}}{\sqrt{e}}}{e^2x^4-2\,dex^2+d^2}\right)+2\,\log\left(2\,\sqrt{ex^2+d}ex-\left(2\,ex^2+d\right)\sqrt{e}\right)}{4\,\sqrt{e}}, \frac{\sqrt{2}\sqrt{-e}\sqrt{-\frac{1}{e}}\arctan\left(\frac{\sqrt{2}\left(3\,ex^2+d\right)}{4\,\sqrt{ex^2+d}ex\sqrt{-\right.}}\right.}{2\,\sqrt{-e}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d)^(3/2)/(e^2*x^4 - d^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*log((17*e^2*x^4 + 14*d*e*x^2 + d^2 + 4*sqrt(2)*(3*e
^2*x^3 + d*e*x)*sqrt(e*x^2 + d)/sqrt(e))/(e^2*x^4 - 2*d*e*x^2 + d
^2)) + 2*log(2*sqrt(e*x^2 + d)*e*x - (2*e*x^2 + d)*sqrt(e)))/sqrt
(e), 1/2*(sqrt(2)*sqrt(-e)*sqrt(-1/e)*arctan(1/4*sqrt(2)*(3*e*x^2
 + d)/(sqrt(e*x^2 + d)*e*x*sqrt(-1/e))) - 2*arctan(sqrt(-e)*x/sqr
t(e*x^2 + d)))/sqrt(-e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{d + ex^2}}{-d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**(3/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(sqrt(d + e*x**2)/(-d + e*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(ex^2 + d\right)^{\frac{3}{2}}}{e^2x^4 - d^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d)^(3/2)/(e^2*x^4 - d^2),x, algorithm="giac")
```

```
[Out] integrate(-(e*x^2 + d)^(3/2)/(e^2*x^4 - d^2), x)
```

3.200 $\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}d\sqrt{e}}$$

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(Sqrt[2]*d*Sqrt[e])
```

Rubi [A] time = 0.0762615, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d + e*x^2]/(d^2 - e^2*x^4),x]

[Out] ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(Sqrt[2]*d*Sqrt[e])
```

Rubi in Sympy [A] time = 14.4736, size = 36, normalized size = 0.95

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2d\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**(1/2)/(-e**2*x**4+d**2),x)

[Out] sqrt(2)*atanh(sqrt(2)*sqrt(e)*x/sqrt(d + e*x**2))/(2*d*sqrt(e))
```

Mathematica [A] time = 0.0248995, size = 38, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d + e*x^2]/(d^2 - e^2*x^4),x]

[Out] ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(Sqrt[2]*d*Sqrt[e])
```

Maple [B] time = 0.034, size = 986, normalized size = 26.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x)
```

```
[Out] 1/2*e/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)^(1/2))/(d*e
)^(1/2)*((x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*
d)^(1/2)+1/2*e^(1/2)/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d
*e)^(1/2))*ln(((x-(d*e)^(1/2)/e)*e+(d*e)^(1/2))/e^(1/2)+((x-(d*e)
^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^(1/2))-1/2*e/(
(d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)^(1/2))/(d*e)^(1/2)
*d^(1/2)*2^(1/2)*ln((4*d+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*2^(1/2
)*d^(1/2)*((x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+
2*d)^(1/2))/(x-(d*e)^(1/2)/e))-1/2*e/((d*e)^(1/2)+(-d*e)^(1/2))/(
-(d*e)^(1/2)+(-d*e)^(1/2))/(d*e)^(1/2)*((x+(d*e)^(1/2)/e)^2*e-2*(
d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2)+1/2*e^(1/2)/((d*e)^(1/2)+
(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)^(1/2))*ln(((x+(d*e)^(1/2)/e)*e
-(d*e)^(1/2))/e^(1/2)+((x+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*
e)^(1/2)/e)+2*d)^(1/2))+1/2*e/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^
(1/2)+(-d*e)^(1/2))/(d*e)^(1/2)*d^(1/2)*2^(1/2)*ln((4*d-2*(d*e)^(
1/2)*(x+(d*e)^(1/2)/e)+2*2^(1/2)*d^(1/2)*((x+(d*e)^(1/2)/e)^2*e-2
*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2))/(x+(d*e)^(1/2)/e))-1/2
*e/(-d*e)^(1/2)/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)^(
1/2))*((x-1/e*(-d*e)^(1/2))^2*e+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1/2
)))^(1/2)-1/2*e^(1/2)/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-
d*e)^(1/2))*ln(((x-1/e*(-d*e)^(1/2))*e+(-d*e)^(1/2))/e^(1/2)+((x-
1/e*(-d*e)^(1/2))^2*e+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(1/2))
+1/2*e/(-d*e)^(1/2)/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*
e)^(1/2))*((x+1/e*(-d*e)^(1/2))^2*e-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^
(1/2)))^(1/2)-1/2*e^(1/2)/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2
)+(-d*e)^(1/2))*ln(((x+1/e*(-d*e)^(1/2))*e-(-d*e)^(1/2))/e^(1/2)+
((x+1/e*(-d*e)^(1/2))^2*e-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{ex^2 + d}}{e^2x^4 - d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(e*x^2 + d)/(e^2*x^4 - d^2),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(e*x^2 + d)/(e^2*x^4 - d^2), x)
```

Fricas [A] time = 0.29583, size = 1, normalized size = 0.03

$$\left[\frac{\sqrt{2}\log\left(\frac{\sqrt{2}\left(17\,e^2x^4+14\,dex^2+d^2\right)\sqrt{e}+8\left(3\,e^2x^3+dex\right)\sqrt{ex^2+d}}{e^2x^4-2\,dex^2+d^2}\right)}{8\,d\sqrt{e}}, \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(3\,ex^2+d\right)\sqrt{-e}}{4\sqrt{ex^2+d}ex}\right)}{4\,d\sqrt{-e}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(e*x^2 + d)/(e^2*x^4 - d^2),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(2)*log((sqrt(2)*(17*e^2*x^4 + 14*d*e*x^2 + d^2)*sqrt(e)
 + 8*(3*e^2*x^3 + d*e*x)*sqrt(e*x^2 + d))/(e^2*x^4 - 2*d*e*x^2 +
d^2))/(d*sqrt(e)), 1/4*sqrt(2)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*s
qrt(-e)/(sqrt(e*x^2 + d)*e*x))/(d*sqrt(-e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-d\sqrt{d+ex^2}+ex^2\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**(1/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(1/(-d*sqrt(d + e*x**2) + e*x**2*sqrt(d + e*x**2)), x)
```

GIAC/XCAS [A] time = 0.326614, size = 177, normalized size = 4.66

$$-\frac{\left(\sqrt{2}i\arctan\left(\frac{e^{\frac{1}{2}}}{\sqrt{-\frac{de+\sqrt{d^2}e}{d}}}\right)e^{\frac{1}{2}}-\sqrt{2}i\arctan\left(\frac{e^{\frac{1}{2}}}{\sqrt{-\frac{de-\sqrt{d^2}e}{d}}}\right)e^{\frac{1}{2}}\right)e^{(-1)}\mathrm{sign}\left(x\right)}{4\left|d\right|}+\frac{\sqrt{2}i\arctan\left(\frac{\sqrt{\frac{d}{x^2}+e}}{\sqrt{-\frac{de\,\mathrm{sign}(x)+\sqrt{d^2}e}{d\,\mathrm{sign}(x)}}}\right)e^{\left(-\frac{1}{2}\right)}}{2\left|d\right|\left|\mathrm{sign}\left(x\right)\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(e*x^2 + d)/(e^2*x^4 - d^2),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(2)*i*arctan(e^(1/2)/sqrt(-(d*e + sqrt(d^2)*e)/d))*e^(1
/2) - sqrt(2)*i*arctan(e^(1/2)/sqrt(-(d*e - sqrt(d^2)*e)/d))*e^(1
/2))*e^(-1)*sign(x)/abs(d) + 1/2*sqrt(2)*i*arctan(sqrt(d/x^2 + e)
/sqrt(-(d*e*sign(x) + sqrt(d^2)*e)/(d*sign(x))))*e^(-1/2)/(abs(d)
*abs(sign(x)))
```

3.201 $\int \frac{1}{\sqrt{d+ex^2}(d^2-e^2x^4)}\,dx$

Optimal. Leaf size=61

$$\frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}$$

```
[Out] x/(2*d^2*Sqrt[d + e*x^2]) + ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d +
e*x^2]]/(2*Sqrt[2]*d^2*Sqrt[e])
```

Rubi [A] time = 0.114444, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[d + e*x^2]*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(2*d^2*Sqrt[d + e*x^2]) + ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d +
e*x^2]]/(2*Sqrt[2]*d^2*Sqrt[e])
```

Rubi in Sympy [A] time = 21.3912, size = 54, normalized size = 0.89

$$\frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\sqrt{2}\,\text{atanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4d^2\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)**(1/2)/(-e**2*x**4+d**2),x)
```

```
[Out] x/(2*d**2*sqrt(d + e*x**2)) + sqrt(2)*atanh(sqrt(2)*sqrt(e)*x/sqr
t(d + e*x**2))/(4*d**2*sqrt(e))
```

Mathematica [A] time = 0.0516846, size = 57, normalized size = 0.93

$$\frac{\frac{2x}{\sqrt{d+ex^2}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}}{4d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[d + e*x^2]*(d^2 - e^2*x^4)),x]
```

```
[Out] ((2*x)/Sqrt[d + e*x^2] + (Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqr
t[d + e*x^2]])/Sqrt[e])/(4*d^2)
```

Maple [B] time = 0.03, size = 441, normalized size = 7.2

$$-\frac{e\sqrt{2}}{4}\ln\left(1\left(4d+2\sqrt{de}\left(x-\frac{\sqrt{de}}{e}\right)+2\sqrt{2}\sqrt{d}\sqrt{\left(x-\frac{\sqrt{de}}{e}\right)^{2}e+2\sqrt{de}\left(x-\frac{\sqrt{de}}{e}\right)+2d}\right)\left(x-\frac{1}{e}\sqrt{de}\right)^{-1}\right)\left(\sqrt{de}+\sqrt{-de}\right)^{-1}$$
$$+\frac{e\sqrt{2}}{4}\ln\left(1\left(4d-2\sqrt{de}\left(x+\frac{\sqrt{de}}{e}\right)+2\sqrt{2}\sqrt{d}\sqrt{\left(x+\frac{\sqrt{de}}{e}\right)^{2}e-2\sqrt{de}\left(x+\frac{\sqrt{de}}{e}\right)+2d}\right)\left(x+\frac{1}{e}\sqrt{de}\right)^{-1}\right)\left(\sqrt{de}+\sqrt{-de}\right)^{-1}$$
$$-\frac{1}{2d}\sqrt{\left(x-\frac{1}{e}\sqrt{-de}\right)^{2}e+2\sqrt{-de}\left(x-\frac{\sqrt{-de}}{e}\right)}\left(\sqrt{de}+\sqrt{-de}\right)^{-1}\left(-\sqrt{de}+\sqrt{-de}\right)^{-1}\left(x-\frac{1}{e}\sqrt{-de}\right)^{-1}$$
$$-\frac{1}{2d}\sqrt{\left(x+\frac{1}{e}\sqrt{-de}\right)^{2}e-2\sqrt{-de}\left(x+\frac{\sqrt{-de}}{e}\right)}\left(\sqrt{de}+\sqrt{-de}\right)^{-1}\left(-\sqrt{de}+\sqrt{-de}\right)^{-1}\left(x+\frac{1}{e}\sqrt{-de}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x)
```

```
[Out] -1/4*e/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)^(1/2))/(d*
e)^(1/2)*2^(1/2)/d^(1/2)*ln((4*d+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+
2*2^(1/2)*d^(1/2)*((x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(
1/2)/e)+2*d)^(1/2))/(x-(d*e)^(1/2)/e))+1/4*e/((d*e)^(1/2)+(-d*e)^
(1/2))/(-(d*e)^(1/2)+(-d*e)^(1/2))/(d*e)^(1/2)*2^(1/2)/d^(1/2)*ln
((4*d-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*2^(1/2)*d^(1/2)*((x+(d*e)
^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2))/(x+(d*e
)^(1/2)/e))-1/2/d/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)
^(1/2))/(x-1/e*(-d*e)^(1/2))*((x-1/e*(-d*e)^(1/2))^2*e+2*(-d*e)^(
1/2)*(x-1/e*(-d*e)^(1/2)))^(1/2)-1/2/d/((d*e)^(1/2)+(-d*e)^(1/2))
/(-(d*e)^(1/2)+(-d*e)^(1/2))/(x+1/e*(-d*e)^(1/2))*((x+1/e*(-d*e)^
(1/2))^2*e-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(e^2x^4-d^2)\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((e^2*x^4 - d^2)*sqrt(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((e^2*x^4 - d^2)*sqrt(e*x^2 + d)), x)
```

Fricas [A] time = 0.292522, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{2}\left(4\sqrt{2}\sqrt{ex^2+d}\sqrt{e}x+\left(ex^2+d\right)\log\left(\frac{\sqrt{2}\left(17e^2x^4+14dex^2+d^2\right)\sqrt{e}+8\left(3e^2x^3+dex\right)\sqrt{ex^2+d}}{e^2x^4-2dex^2+d^2}\right)\right)}{16\left(d^2ex^2+d^3\right)\sqrt{e}},\ \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ex^2+d}\sqrt{-e}x+\left(ex^2+d\right)\right.}{8\left(d^2ex^2+d^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((e^2*x^4 - d^2)*sqrt(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/16*sqrt(2)*(4*sqrt(2)*sqrt(e*x^2 + d)*sqrt(e)*x + (e*x^2 + d)*
log((sqrt(2)*(17*e^2*x^4 + 14*d*e*x^2 + d^2)*sqrt(e) + 8*(3*e^2*x
^3 + d*e*x)*sqrt(e*x^2 + d))/(e^2*x^4 - 2*d*e*x^2 + d^2)))/((d^2*
```

```
e*x^2 + d^3)*sqrt(e)), 1/8*sqrt(2)*(2*sqrt(2)*sqrt(e*x^2 + d)*sqr
t(-e)*x + (e*x^2 + d)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(-e)/(
sqrt(e*x^2 + d)*e*x)))/((d^2*e*x^2 + d^3)*sqrt(-e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-d^2\sqrt{d+ex^2}+e^2x^4\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)**(1/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(1/(-d**2*sqrt(d + e*x**2) + e**2*x**4*sqrt(d + e*x**2))
, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((e^2*x^4 - d^2)*sqrt(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.202 $\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx$

Optimal. Leaf size=80

$$\frac{7x}{12d^3\sqrt{d+ex^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}d^3\sqrt{e}}+\frac{x}{6d^2\left(d+ex^2\right)^{3/2}}$$

```
[Out] x/(6*d^2*(d + e*x^2)^(3/2)) + (7*x)/(12*d^3*Sqrt[d + e*x^2]) + Ar
cTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(4*Sqrt[2]*d^3*Sqrt[e]
)
```

Rubi [A] time = 0.207191, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{7x}{12d^3\sqrt{d+ex^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}d^3\sqrt{e}}+\frac{x}{6d^2\left(d+ex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)^(3/2)*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(6*d^2*(d + e*x^2)^(3/2)) + (7*x)/(12*d^3*Sqrt[d + e*x^2]) + Ar
cTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(4*Sqrt[2]*d^3*Sqrt[e]
)
```

Rubi in Sympy [A] time = 42.7073, size = 73, normalized size = 0.91

$$\frac{x}{6d^2\left(d+ex^2\right)^{\frac{3}{2}}}+\frac{7x}{12d^3\sqrt{d+ex^2}}+\frac{\sqrt{2}\,\text{atanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8d^3\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)**(3/2)/(-e**2*x**4+d**2),x)
```

```
[Out] x/(6*d**2*(d + e*x**2)**(3/2)) + 7*x/(12*d**3*sqrt(d + e*x**2)) +
 sqrt(2)*atanh(sqrt(2)*sqrt(e)*x/sqrt(d + e*x**2))/(8*d**3*sqrt(e
))
```

Mathematica [A] time = 0.154576, size = 68, normalized size = 0.85

$$\frac{\frac{3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}+\frac{2\left(9dx+7ex^3\right)}{\left(d+ex^2\right)^{3/2}}}{24d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)^(3/2)*(d^2 - e^2*x^4)),x]
```

```
[Out] ((2*(9*d*x + 7*e*x^3))/(d + e*x^2)^(3/2) + (3*Sqrt[2]*ArcTanh[(Sq
rt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e])/(24*d^3)
```

Maple [B] time = 0.033, size = 911, normalized size = 11.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x)
```

```
[Out] 1/4*e/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)^(1/2))/(d*e
)^(1/2)/d/((x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+
2*d)^(1/2)-1/4*e/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)^
(1/2))/d^2/((x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)
+2*d)^(1/2)*x-1/8*e/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*
e)^(1/2))/(d*e)^(1/2)/d^(3/2)*2^(1/2)*ln((4*d+2*(d*e)^(1/2)*(x-(d
*e)^(1/2)/e)+2*2^(1/2)*d^(1/2)*((x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/
2)*(x-(d*e)^(1/2)/e)+2*d)^(1/2))/(x-(d*e)^(1/2)/e))-1/4*e/((d*e)^
(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)^(1/2))/(d*e)^(1/2)/d/((x
+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2)-1/
4*e/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)^(1/2))/d^2/((
x+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2)*x
+1/8*e/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)^(1/2))/(d*
e)^(1/2)/d^(3/2)*2^(1/2)*ln((4*d-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+
2*2^(1/2)*d^(1/2)*((x+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(
1/2)/e)+2*d)^(1/2))/(x+(d*e)^(1/2)/e))-1/6/d/((d*e)^(1/2)+(-d*e)^
(1/2))/(-(d*e)^(1/2)+(-d*e)^(1/2))/(x-1/e*(-d*e)^(1/2))/((x-1/e*(
-d*e)^(1/2))^2*e+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(1/2)-1/3*e
/d^2/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)^(1/2))/((x-1
/e*(-d*e)^(1/2))^2*e+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(1/2)*x
-1/6/d/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e)^(1/2)+(-d*e)^(1/2))/(x+
1/e*(-d*e)^(1/2))/((x+1/e*(-d*e)^(1/2))^2*e-2*(-d*e)^(1/2)*(x+1/e
*(-d*e)^(1/2)))^(1/2)-1/3*e/d^2/((d*e)^(1/2)+(-d*e)^(1/2))/(-(d*e
)^(1/2)+(-d*e)^(1/2))/((x+1/e*(-d*e)^(1/2))^2*e-2*(-d*e)^(1/2)*(x
+1/e*(-d*e)^(1/2)))^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(e^2x^4 - d^2)(ex^2 + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((e^2*x^4 - d^2)*(e*x^2 + d)^(3/2)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((e^2*x^4 - d^2)*(e*x^2 + d)^(3/2)), x)
```

Fricas [A] time = 0.327713, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{2}\left(4\sqrt{2}\left(7\,ex^3 + 9\,dx\right)\sqrt{ex^2 + d}\sqrt{e} + 3\left(e^2x^4 + 2\,dex^2 + d^2\right)\log\left(\frac{\sqrt{2}\left(17\,e^2x^4 + 14\,dex^2 + d^2\right)\sqrt{e} + 8\left(3\,e^2x^3 + dex\right)\sqrt{ex^2 + d}}{e^2x^4 - 2\,dex^2 + d^2}\right)\right)}{96\left(d^3e^2x^4 + 2\,d^4ex^2 + d^5\right)\sqrt{e}}, \frac{\sqrt{2}\left(2\sqrt{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((e^2*x^4 - d^2)*(e*x^2 + d)^(3/2)),x, algorithm="fricas")
```

```
[Out] [1/96*sqrt(2)*(4*sqrt(2)*(7*e*x^3 + 9*d*x)*sqrt(e*x^2 + d)*sqrt(e
) + 3*(e^2*x^4 + 2*d*e*x^2 + d^2)*log((sqrt(2)*(17*e^2*x^4 + 14*d
```

```
*e*x^2 + d^2)*sqrt(e) + 8*(3*e^2*x^3 + d*e*x)*sqrt(e*x^2 + d))/(e
^2*x^4 - 2*d*e*x^2 + d^2)))/((d^3*e^2*x^4 + 2*d^4*e*x^2 + d^5)*sq
rt(e)), 1/48*sqrt(2)*(2*sqrt(2)*(7*e*x^3 + 9*d*x)*sqrt(e*x^2 + d)
*sqrt(-e) + 3*(e^2*x^4 + 2*d*e*x^2 + d^2)*arctan(1/4*sqrt(2)*(3*e
*x^2 + d)*sqrt(-e)/(sqrt(e*x^2 + d)*e*x)))/((d^3*e^2*x^4 + 2*d^4*
e*x^2 + d^5)*sqrt(-e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-d^3\sqrt{d+ex^2} - d^2ex^2\sqrt{d+ex^2} + de^2x^4\sqrt{d+ex^2} + e^3x^6\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)**(3/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(1/(-d**3*sqrt(d + e*x**2) - d**2*e*x**2*sqrt(d + e*x**2
) + d*e**2*x**4*sqrt(d + e*x**2) + e**3*x**6*sqrt(d + e*x**2)), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$[undef, undef, undef, -1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((e^2*x^4 - d^2)*(e*x^2 + d)^(3/2)),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, -1]
```

3.203 $\int \frac{\left(a+bx^2\right)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=153

$$-\frac{9ax\left(a-bx^2\right)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}}-\frac{x\left(a-bx^2\right)\left(a+bx^2\right)^{3/2}}{4\sqrt{a^2-b^2x^4}}+\frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (-9*a*x*(a - b*x^2)*Sqrt[a + b*x^2])/(8*Sqrt[a^2 - b^2*x^4]) - (x
*(a - b*x^2)*(a + b*x^2)^(3/2))/(4*Sqrt[a^2 - b^2*x^4]) + (19*a^2
*Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^
2]])/(8*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.157747, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$

$$-\frac{9ax\left(a-bx^2\right)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}}-\frac{x\left(a-bx^2\right)\left(a+bx^2\right)^{3/2}}{4\sqrt{a^2-b^2x^4}}+\frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (-9*a*x*(a - b*x^2)*Sqrt[a + b*x^2])/(8*Sqrt[a^2 - b^2*x^4]) - (x
*(a - b*x^2)*(a + b*x^2)^(3/2))/(4*Sqrt[a^2 - b^2*x^4]) + (19*a^2
*Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^
2]])/(8*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi in Sympy [A] time = 28.0154, size = 121, normalized size = 0.79

$$\frac{19a^2\sqrt{a^2-b^2x^4}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}-\frac{9ax\sqrt{a^2-b^2x^4}}{8\sqrt{a+bx^2}}-\frac{x\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] 19*a**2*sqrt(a**2 - b**2*x**4)*atan(sqrt(b)*x/sqrt(a - b*x**2))/(
8*sqrt(b)*sqrt(a - b*x**2)*sqrt(a + b*x**2)) - 9*a*x*sqrt(a**2 -
b**2*x**4)/(8*sqrt(a + b*x**2)) - x*sqrt(a + b*x**2)*sqrt(a**2 -
b**2*x**4)/4
```

Mathematica [C] time = 0.228307, size = 98, normalized size = 0.64

$$-\frac{\left(11ax+2bx^3\right)\sqrt{a^2-b^2x^4}}{8\sqrt{a+bx^2}}+\frac{19ia^2\log\left(\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}}-2i\sqrt{b}x\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -((11*a*x + 2*b*x^3)*Sqrt[a^2 - b^2*x^4])/(8*Sqrt[a + b*x^2]) + (
((19*I)/8)*a^2*Log[(-2*I)*Sqrt[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqr
```

```
t[a + b*x^2]])/Sqrt[b]
```

Maple [A] time = 0.079, size = 132, normalized size = 0.9

$$-\frac{1}{8}\sqrt{-b^2x^4+a^2}\left(2\,x^3b^{3/2}\sqrt{-bx^2+a}+11\,ax\sqrt{-bx^2+a}\sqrt{b}+13\,a^2\arctan\left(\frac{x\sqrt{b}}{\sqrt{-bx^2+a}}\right)-32\,a^2\arctan\left(x\sqrt{b}\frac{1}{\sqrt{\frac{\left(-bx+\sqrt{ab}\right)\left(b\right.}{b}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/8*(-b^2*x^4+a^2)^(1/2)*(2*x^3*b^(3/2)*(-b*x^2+a)^(1/2)+11*a*x*
(-b*x^2+a)^(1/2)*b^(1/2)+13*a^2*arctan(b^(1/2)*x/(-b*x^2+a)^(1/2)
)-32*a^2*arctan(b^(1/2)*x/(1/b*(-b*x+(a*b)^(1/2))*(b*x+(a*b)^(1/2
)))^(1/2)))/(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.305802, size = 1, normalized size = 0.01

$$\left[-\frac{2\sqrt{-b^2x^4+a^2}\left(2\,bx^3+11\,ax\right)\sqrt{bx^2+a}\sqrt{-b}-19\left(a^2bx^2+a^3\right)\log\left(-\frac{2\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}bx+\left(2\,b^2x^4+abx^2-a^2\right)\sqrt{-b}}{bx^2+a}\right)}{16\left(bx^2+a\right)\sqrt{-b}},\right.$$
$$\left.-\frac{\sqrt{-b^2x^4+a^2}\left(2\,bx^3+11\,ax\right)\sqrt{bx^2+a}\sqrt{b}+19\left(a^2bx^2+a^3\right)\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{b^2x^3+abx}\right)}{8\left(bx^2+a\right)\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2),x, algorithm="fricas")
```

```
[Out] [-1/16*(2*sqrt(-b^2*x^4 + a^2)*(2*b*x^3 + 11*a*x)*sqrt(b*x^2 + a)
*sqrt(-b) - 19*(a^2*b*x^2 + a^3)*log(-(2*sqrt(-b^2*x^4 + a^2)*sqr
t(b*x^2 + a)*b*x + (2*b^2*x^4 + a*b*x^2 - a^2)*sqrt(-b))/(b*x^2 +
 a)))/((b*x^2 + a)*sqrt(-b)), -1/8*(sqrt(-b^2*x^4 + a^2)*(2*b*x^3
 + 11*a*x)*sqrt(b*x^2 + a)*sqrt(b) + 19*(a^2*b*x^2 + a^3)*arctan(
sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x)))/
((b*x^2 + a)*sqrt(b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2\right)^{\frac{5}{2}}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

3.204 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=110

$$\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}-\frac{x\left(a-bx^2\right)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}$$

```
[Out] -(x*(a - b*x^2)*Sqrt[a + b*x^2])/(2*Sqrt[a^2 - b^2*x^4]) + (3*a*S
qrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]
])/(2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.102016, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}-\frac{x\left(a-bx^2\right)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -(x*(a - b*x^2)*Sqrt[a + b*x^2])/(2*Sqrt[a^2 - b^2*x^4]) + (3*a*S
qrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]
])/(2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi in Sympy [A] time = 15.963, size = 88, normalized size = 0.8

$$\frac{3a\sqrt{a^2-b^2x^4}\,\text{atan}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}-\frac{x\sqrt{a^2-b^2x^4}}{2\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] 3*a*sqrt(a**2 - b**2*x**4)*atan(sqrt(b)*x/sqrt(a - b*x**2))/(2*sq
rt(b)*sqrt(a - b*x**2)*sqrt(a + b*x**2)) - x*sqrt(a**2 - b**2*x**
4)/(2*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.0742761, size = 86, normalized size = 0.78

$$-\frac{x\sqrt{a^2-b^2x^4}}{2\sqrt{a+bx^2}}+\frac{3ia\log\left(\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}}-2i\sqrt{b}x\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -(x*Sqrt[a^2 - b^2*x^4])/(2*Sqrt[a + b*x^2]) + (((3*I)/2)*a*Log[(
-2*I)*Sqrt[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqrt[
b]
```

Maple [A] time = 0.026, size = 109, normalized size = 1.

$$\frac{1}{2}\sqrt{-b^2x^4+a^2}\left(-x\sqrt{-bx^2+a}\sqrt{b}+4\,a\arctan\left(x\sqrt{b}\frac{1}{\sqrt{\frac{\left(-bx+\sqrt{ab}\right)\left(bx+\sqrt{ab}\right)}{b}}}\right)-a\arctan\left(x\sqrt{b}\frac{1}{\sqrt{-bx^2+a}}\right)\right)\frac{1}{\sqrt{bx^2+a}}\frac{1}{\sqrt{-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] 1/2/(b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*(-x*(-b*x^2+a)^(1/2)*b^(
1/2)+4*a*arctan(b^(1/2)*x/(1/b*(-b*x+(a*b)^(1/2))*(b*x+(a*b)^(1/2
)))^(1/2))-a*arctan(b^(1/2)*x/(-b*x^2+a)^(1/2)))/(-b*x^2+a)^(1/2)
/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.302229, size = 1, normalized size = 0.01

$$\left[-\frac{2\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-b}x-3\left(abx^2+a^2\right)\log\left(-\frac{2\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}bx+\left(2\,b^2x^4+abx^2-a^2\right)\sqrt{-b}}{bx^2+a}\right)}{4\,(bx^2+a)\sqrt{-b}},\right.$$
$$\left.-\frac{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}x+3\left(abx^2+a^2\right)\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{b^2x^3+abx}\right)}{2\,(bx^2+a)\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - 3*(a*b
*x^2 + a^2)*log(-(2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*b*x + (2
*b^2*x^4 + a*b*x^2 - a^2)*sqrt(-b))/(b*x^2 + a)))/((b*x^2 + a)*sq
rt(-b)), -1/2*(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)*x + 3
*(a*b*x^2 + a^2)*arctan(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt
(b)/(b^2*x^3 + a*b*x)))/((b*x^2 + a)*sqrt(b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2\right)^{\frac{3}{2}}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

3.205 $\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^
2]])/(Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.0654669, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^
2]])/(Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi in Sympy [A] time = 9.67731, size = 56, normalized size = 0.86

$$\frac{\sqrt{a^2-b^2x^4}\,\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] sqrt(a**2 - b**2*x**4)*atan(sqrt(b)*x/sqrt(a - b*x**2))/(sqrt(b)*
sqrt(a - b*x**2)*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.0367116, size = 50, normalized size = 0.77

$$\frac{i\log\left(\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}}-2i\sqrt{b}x\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (I*Log[(-2*I)*Sqrt[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]
])/Sqrt[b]
```

Maple [A] time = 0.025, size = 69, normalized size = 1.1

$$1\sqrt{-b^2x^4+a^2}\arctan\left(x\sqrt{b}\frac{1}{\sqrt{\frac{1}{b}\left(-bx+\sqrt{ab}\right)\left(bx+\sqrt{ab}\right)}}\right)\frac{1}{\sqrt{bx^2+a}}\frac{1}{\sqrt{-bx^2+a}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] 1/(b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)/(-b*x^2+a)^(1/2)/b^(1/2)*a
rctan(b^(1/2)*x/(1/b*(-b*x+(a*b)^(1/2))*(b*x+(a*b)^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)/sqrt(-b^2*x^4 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.279092, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{2\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}bx+\left(2b^2x^4+abx^2-a^2\right)\sqrt{-b}}{bx^2+a}\right)}{2\sqrt{-b}}, -\frac{\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{b^2x^3+abx}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)/sqrt(-b^2*x^4 + a^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*b*x + (2*b^2*x^
4 + a*b*x^2 - a^2)*sqrt(-b))/(b*x^2 + a))/sqrt(-b), -arctan(sqrt(
-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x))/sqrt(b
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a+bx^2}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

<u>**GIAC/XCAS**</u> **[F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{-b^2x^4 + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/sqrt(-b^2*x^4 + a^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

3.206 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[
a - b*x^2]])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.102868, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[
a - b*x^2]])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi in Sympy [A] time = 15.4851, size = 70, normalized size = 0.9

$$\frac{\sqrt{2}\sqrt{a^2-b^2x^4}\,\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{2a\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(a**2 - b**2*x**4)*atan(sqrt(2)*sqrt(b)*x/sqrt(a - b*
x**2))/(2*a*sqrt(b)*sqrt(a - b*x**2)*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.0500079, size = 78, normalized size = 1.

$$\frac{\sqrt{a^2-b^2x^4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])
/(Sqrt[2]*a*Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a + b*x^2])
```

Maple [B] time = 0.058, size = 248, normalized size = 3.2

$$-\frac{1}{2}\sqrt{-b^2x^4+a^2}\left(b\sqrt{a}\sqrt{2}\ln\left(2\,\frac{b\left(\sqrt{2}\sqrt{a}\sqrt{-bx^2+a}-\sqrt{-ab}x+a\right)}{bx-\sqrt{-ab}}\right)-b\sqrt{a}\sqrt{2}\ln\left(2\,\frac{b\left(\sqrt{2}\sqrt{a}\sqrt{-bx^2+a}+\sqrt{-ab}x+a\right)}{bx+\sqrt{-ab}}\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/2/(b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*(b*a^(1/2)*2^(1/2)*ln(2
*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b
)^(1/2)))-b*a^(1/2)*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1
/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))-2*b^(1/2)*arctan(b^(1/2
)*x/(1/b*(-b*x+(a*b)^(1/2))*(b*x+(a*b)^(1/2)))^(1/2))*(-a*b)^(1/2
)+2*b^(1/2)*arctan(b^(1/2)*x/(-b*x^2+a)^(1/2))*(-a*b)^(1/2))/(-b*
x^2+a)^(1/2)/(-a*b)^(1/2)/((a*b)^(1/2)+(-a*b)^(1/2))/((a*b)^(1/2)
-(-a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281267, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{2}\log\left(-\frac{4\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}bx+\sqrt{2}\left(3\,b^2x^4+2\,abx^2-a^2\right)\sqrt{-b}}{b^2x^4+2\,abx^2+a^2}\right)}{4\,a\sqrt{-b}},\,-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{2\,(b^2x^3+abx)}\right)}{2\,a\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*log(-(4*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*b*x + s
qrt(2)*(3*b^2*x^4 + 2*a*b*x^2 - a^2)*sqrt(-b))/(b^2*x^4 + 2*a*b*x
^2 + a^2))/(a*sqrt(-b)), -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-b^
2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x))/(a*sqrt(b
))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(-a+bx^2)\,(a+bx^2)}\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*sqrt(a + b*x**2)),
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}\sqrt{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)), x)
```

3.207 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=125

$$\frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a+bx^2}\sqrt{a-bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (x*(a - b*x^2))/(4*a^2*Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]) + (3*
Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a
 - b*x^2]])/(4*Sqrt[2]*a^2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.147113, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a+bx^2}\sqrt{a-bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a - b*x^2))/(4*a^2*Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]) + (3*
Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a
 - b*x^2]])/(4*Sqrt[2]*a^2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi in Sympy [A] time = 22.4534, size = 104, normalized size = 0.83

$$\frac{x\sqrt{a^2-b^2x^4}}{4a^2(a+bx^2)^{\frac{3}{2}}}+\frac{3\sqrt{2}\sqrt{a^2-b^2x^4}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{8a^2\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] x*sqrt(a**2 - b**2*x**4)/(4*a**2*(a + b*x**2)**(3/2)) + 3*sqrt(2)
*sqrt(a**2 - b**2*x**4)*atan(sqrt(2)*sqrt(b)*x/sqrt(a - b*x**2))/
(8*a**2*sqrt(b)*sqrt(a - b*x**2)*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.0949873, size = 111, normalized size = 0.89

$$\frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}x\sqrt{a-bx^2}+3\sqrt{2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)\right)}{8a^2\sqrt{b}\sqrt{a-bx^2}(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a - b*x^2] + 3*Sqrt[2]*(a
+ b*x^2)*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]]))/(8*a^2*Sqr
t[b]*Sqrt[a - b*x^2]*(a + b*x^2)^(3/2))
```

Maple [B] time = 0.089, size = 504, normalized size = 4.

$$-\frac{b^2}{4}\sqrt{-b^2x^4+a^2}\left(4\arctan\left(\frac{x\sqrt{b}}{\sqrt{-bx^2+a}}\right)x^2b^{3/2}\sqrt{a}\sqrt{-ab}-4\arctan\left(x\sqrt{b}\frac{1}{\sqrt{\frac{\left(-bx+\sqrt{ab}\right)\left(bx+\sqrt{ab}\right)}{b}}}\right)x^2b^{3/2}\sqrt{a}\sqrt{-ab}-3\ln\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/4*(-b^2*x^4+a^2)^(1/2)*b^2*(4*arctan(b^(1/2)*x/(-b*x^2+a)^(1/2
))*x^2*b^(3/2)*a^(1/2)*(-a*b)^(1/2)-4*arctan(b^(1/2)*x/(1/b*(-b*x
+(a*b)^(1/2))*(b*x+(a*b)^(1/2)))^(1/2))*x^2*b^(3/2)*a^(1/2)*(-a*b
)^(1/2)-3*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x
+a)/(b*x+(-a*b)^(1/2)))*2^(1/2)*x^2*a*b^2+3*ln(2*b*(2^(1/2)*a^(1/
2)*(-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*2^(1/2)
*x^2*a*b^2+4*arctan(b^(1/2)*x/(-b*x^2+a)^(1/2))*a^(3/2)*b^(1/2)*(
-a*b)^(1/2)-4*arctan(b^(1/2)*x/(1/b*(-b*x+(a*b)^(1/2))*(b*x+(a*b)
^(1/2)))^(1/2))*a^(3/2)*b^(1/2)*(-a*b)^(1/2)-3*ln(2*b*(2^(1/2)*a^
(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*2^(1
/2)*a^2*b+3*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)-(-a*b)^(1/2)
*x+a)/(b*x-(-a*b)^(1/2)))*2^(1/2)*a^2*b-4*a^(1/2)*b*(-a*b)^(1/2)*
(-b*x^2+a)^(1/2)*x)/(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/((a*b)^(1/2)
+(-a*b)^(1/2))^2/((a*b)^(1/2)-(-a*b)^(1/2))^2/(-a*b)^(1/2)/a^(1/2
)/(b*x-(-a*b)^(1/2))/(b*x+(-a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-b^2x^4+a^2}(bx^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(3/2)), x)
```

Fricas [A] time = 0.289883, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-b}x+3\left(b^2x^4+2abx^2+a^2\right)\log\left(-\frac{4\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}bx+\sqrt{2}\left(3b^2x^4+2abx^2-a^2\right)\sqrt{-b}}{b^2x^4+2abx^2+a^2}\right)\right)}{16\left(a^2b^2x^4+2a^3bx^2+a^4\right)\sqrt{-b}},\frac{\sqrt{2}\left(\sqrt{\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(3/2)),x, algorithm="fricas")
```

```
[Out] [1/16*sqrt(2)*(2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqr
t(-b)*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(-(4*sqrt(-b^2*x^4 + a
^2)*sqrt(b*x^2 + a)*b*x + sqrt(2)*(3*b^2*x^4 + 2*a*b*x^2 - a^2)*s
qrt(-b))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((a^2*b^2*x^4 + 2*a^3*b*x^
2 + a^4)*sqrt(-b)), 1/8*sqrt(2)*(sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqr
t(b*x^2 + a)*sqrt(b)*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*arctan(1/2
*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)/(b^2*x^3 +
a*b*x)))/((a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*sqrt(b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\left(a+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a + b*x**2)**(3/2)
), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4+a^2}(bx^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(3/2)), x)
```

3.208 $\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=168

$$\frac{x\left(a-bx^2\right)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{9x\left(a-bx^2\right)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a+bx^2}\sqrt{a-bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (x*(a - b*x^2))/(8*a^2*(a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]) + (
9*x*(a - b*x^2))/(32*a^3*Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]) + (
19*Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqr
t[a - b*x^2]])/(32*Sqrt[2]*a^3*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.247323, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{x\left(a-bx^2\right)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{9x\left(a-bx^2\right)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a+bx^2}\sqrt{a-bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a - b*x^2))/(8*a^2*(a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]) + (
9*x*(a - b*x^2))/(32*a^3*Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]) + (
19*Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqr
t[a - b*x^2]])/(32*Sqrt[2]*a^3*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi in Sympy [A] time = 43.8884, size = 136, normalized size = 0.81

$$\frac{x\sqrt{a^2-b^2x^4}}{8a^2\left(a+bx^2\right)^{\frac{5}{2}}}+\frac{9x\sqrt{a^2-b^2x^4}}{32a^3\left(a+bx^2\right)^{\frac{3}{2}}}+\frac{19\sqrt{2}\sqrt{a^2-b^2x^4}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{64a^3\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] x*sqrt(a**2 - b**2*x**4)/(8*a**2*(a + b*x**2)**(5/2)) + 9*x*sqrt(
a**2 - b**2*x**4)/(32*a**3*(a + b*x**2)**(3/2)) + 19*sqrt(2)*sqrt
(a**2 - b**2*x**4)*atan(sqrt(2)*sqrt(b)*x/sqrt(a - b*x**2))/(64*a
**3*sqrt(b)*sqrt(a - b*x**2)*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.123945, size = 123, normalized size = 0.73

$$\frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}x\sqrt{a-bx^2}\left(13a+9bx^2\right)+19\sqrt{2}\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)\right)}{64a^3\sqrt{b}\sqrt{a-bx^2}\left(a+bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a - b*x^2]*(13*a + 9*b*x^2
) + 19*Sqrt[2]*(a + b*x^2)^2*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a -
```

```
b*x^2]]))/(64*a^3*Sqrt[b]*Sqrt[a - b*x^2]*(a + b*x^2)^(5/2))
```

Maple [B] time = 0.072, size = 729, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/16*(-b^2*x^4+a^2)^(1/2)*b^4*(19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2
)*(-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*x^4*a*b^
3-19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2
)*x+a)/(b*x+(-a*b)^(1/2)))*x^4*a*b^3-16*arctan(b^(1/2)*x/(1/b*(-b
*x+(a*b)^(1/2))*(b*x+(a*b)^(1/2)))^(1/2))*x^4*b^(5/2)*a^(1/2)*(-a
*b)^(1/2)+16*arctan(b^(1/2)*x/(-b*x^2+a)^(1/2))*x^4*b^(5/2)*a^(1/
2)*(-a*b)^(1/2)+38*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/
2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*x^2*a^2*b^2-38*2^(1/2)*l
n(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-
a*b)^(1/2)))*x^2*a^2*b^2-32*arctan(b^(1/2)*x/(1/b*(-b*x+(a*b)^(1/
2))*(b*x+(a*b)^(1/2)))^(1/2))*x^2*a^(3/2)*b^(3/2)*(-a*b)^(1/2)+32
*arctan(b^(1/2)*x/(-b*x^2+a)^(1/2))*x^2*a^(3/2)*b^(3/2)*(-a*b)^(1
/2)-36*a^(1/2)*b^2*(-a*b)^(1/2)*(-b*x^2+a)^(1/2)*x^3+19*2^(1/2)*l
n(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-
a*b)^(1/2)))*a^3*b-19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^
(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*a^3*b-16*arctan(b^(1/
2)*x/(1/b*(-b*x+(a*b)^(1/2))*(b*x+(a*b)^(1/2)))^(1/2))*a^(5/2)*b^
(1/2)*(-a*b)^(1/2)+16*arctan(b^(1/2)*x/(-b*x^2+a)^(1/2))*a^(5/2)*
b^(1/2)*(-a*b)^(1/2)-52*a^(3/2)*b*(-a*b)^(1/2)*(-b*x^2+a)^(1/2)*x
)/(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/((a*b)^(1/2)+(-a*b)^(1/2))^3/(
(a*b)^(1/2)-(-a*b)^(1/2))^3/a^(1/2)/(b*x-(-a*b)^(1/2))^2/(b*x+(-a
*b)^(1/2))^2/(-a*b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}(bx^2 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(5/2)), x)
```

Fricas [A] time = 0.279335, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{-b^2x^4 + a^2}\left(9\,bx^3 + 13\,ax\right)\sqrt{bx^2 + a}\sqrt{-b} + 19\,\left(b^3x^6 + 3\,ab^2x^4 + 3\,a^2bx^2 + a^3\right)\log\left(-\frac{4\,\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}bx+\sqrt{2}(3\,b^2 \cdots}{b^2x^4+2\,abx^2+a \cdots}\right.\right.}{128\,(a^3b^3x^6 + 3\,a^4b^2x^4 + 3\,a^5bx^2 + a^6)\sqrt{-b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(5/2)),x, algorithm="fricas")
```

```
[Out] [1/128*sqrt(2)*(2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*(9*b*x^3 + 13*a*x)
*sqrt(b*x^2 + a)*sqrt(-b) + 19*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x
```

```
^2 + a^3)*log(-(4*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*b*x + sqrt
(2)*(3*b^2*x^4 + 2*a*b*x^2 - a^2)*sqrt(-b))/(b^2*x^4 + 2*a*b*x^2
+ a^2)))/((a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)*sqrt(
-b)), 1/64*sqrt(2)*(sqrt(2)*sqrt(-b^2*x^4 + a^2)*(9*b*x^3 + 13*a*
x)*sqrt(b*x^2 + a)*sqrt(b) - 19*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*
x^2 + a^3)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a
)*sqrt(b)/(b^2*x^3 + a*b*x)))/((a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a
^5*b*x^2 + a^6)*sqrt(b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\left(a+bx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a + b*x**2)**(5/2)
), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4+a^2}\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(5/2)), x)
```

3.209 $\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=152

$$-\frac{9ax\sqrt{a-bx^2}\left(a+bx^2\right)}{8\sqrt{a^2-b^2x^4}}-\frac{x\left(a-bx^2\right)^{3/2}\left(a+bx^2\right)}{4\sqrt{a^2-b^2x^4}}+\frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (-9*a*x*Sqrt[a - b*x^2]*(a + b*x^2))/(8*Sqrt[a^2 - b^2*x^4]) - (x
*(a - b*x^2)^(3/2)*(a + b*x^2))/(4*Sqrt[a^2 - b^2*x^4]) + (19*a^2
*Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(8*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.1507, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172

$$-\frac{9ax\sqrt{a-bx^2}\left(a+bx^2\right)}{8\sqrt{a^2-b^2x^4}}-\frac{x\left(a-bx^2\right)^{3/2}\left(a+bx^2\right)}{4\sqrt{a^2-b^2x^4}}+\frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (-9*a*x*Sqrt[a - b*x^2]*(a + b*x^2))/(8*Sqrt[a^2 - b^2*x^4]) - (x
*(a - b*x^2)^(3/2)*(a + b*x^2))/(4*Sqrt[a^2 - b^2*x^4]) + (19*a^2
*Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(8*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi in Sympy [A] time = 28.7303, size = 121, normalized size = 0.8

$$\frac{19a^2\sqrt{a^2-b^2x^4}\,\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}-\frac{9ax\sqrt{a^2-b^2x^4}}{8\sqrt{a-bx^2}}-\frac{x\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] 19*a**2*sqrt(a**2 - b**2*x**4)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/
(8*sqrt(b)*sqrt(a - b*x**2)*sqrt(a + b*x**2)) - 9*a*x*sqrt(a**2 -
 b**2*x**4)/(8*sqrt(a - b*x**2)) - x*sqrt(a - b*x**2)*sqrt(a**2 -
 b**2*x**4)/4
```

Mathematica [A] time = 0.297825, size = 123, normalized size = 0.81

$$\frac{1}{8}\left(\frac{x\left(2bx^2-11a\right)\sqrt{a^2-b^2x^4}}{\sqrt{a-bx^2}}+\frac{19a^2\log\left(\sqrt{b}\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}+abx-b^2x^3\right)}{\sqrt{b}}-\frac{19a^2\log\left(bx^2-a\right)}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a - b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] ((x*(-11*a + 2*b*x^2)*Sqrt[a^2 - b^2*x^4])/Sqrt[a - b*x^2] - (19*
a^2*Log[-a + b*x^2])/Sqrt[b] + (19*a^2*Log[a*b*x - b^2*x^3 + Sqrt
[b]*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]])/Sqrt[b])/8
```

Maple [A] time = 0.022, size = 105, normalized size = 0.7

$$-\frac{1}{8\,b{x}^{2}-8\,a}\sqrt{-b{x}^{2}+a}\sqrt{-{b}^{2}{x}^{4}+{a}^{2}}\left(2\,{b}^{3/2}{x}^{3}\sqrt{b{x}^{2}+a}-11\,ax\sqrt{b{x}^{2}+a}\sqrt{b}+19\,{a}^{2}\ln\left(x\sqrt{b}+\sqrt{b{x}^{2}+a}\right)\right)\frac{1}{\sqrt{b{x}^{2}+a}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/8*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*(2*b^(3/2)*x^3*(b*x^2+
a)^(1/2)-11*a*x*(b*x^2+a)^(1/2)*b^(1/2)+19*a^2*ln(x*b^(1/2)+(b*x^
2+a)^(1/2)))/(b*x^2-a)/(b*x^2+a)^(1/2)/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282586, size = 1, normalized size = 0.01

$$\left[-\frac{2\,\sqrt{-b^{2}x^{4}+a^{2}}\left(2\,bx^{3}-11\,ax\right)\sqrt{-bx^{2}+a}\sqrt{b}-19\,\left(a^{2}bx^{2}-a^{3}\right)\log\left(-\frac{2\,\sqrt{-b^{2}x^{4}+a^{2}}\sqrt{-bx^{2}+a}bx-\left(2\,b^{2}x^{4}-abx^{2}-a^{2}\right)\sqrt{b}}{bx^{2}-a}\right)}{16\,\left(bx^{2}-a\right)\sqrt{b}},\right.$$
$$\left.-\frac{\sqrt{-b^{2}x^{4}+a^{2}}\left(2\,bx^{3}-11\,ax\right)\sqrt{-bx^{2}+a}\sqrt{-b}+19\,\left(a^{2}bx^{2}-a^{3}\right)\arctan\left(\frac{\sqrt{-b^{2}x^{4}+a^{2}}\sqrt{-bx^{2}+a}\sqrt{-b}}{b^{2}x^{3}-abx}\right)}{8\,\left(bx^{2}-a\right)\sqrt{-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2),x, algorithm="fricas")
```

```
[Out] [-1/16*(2*sqrt(-b^2*x^4 + a^2)*(2*b*x^3 - 11*a*x)*sqrt(-b*x^2 + a
)*sqrt(b) - 19*(a^2*b*x^2 - a^3)*log(-(2*sqrt(-b^2*x^4 + a^2)*sqr
t(-b*x^2 + a)*b*x - (2*b^2*x^4 - a*b*x^2 - a^2)*sqrt(b))/(b*x^2 -
 a)))/((b*x^2 - a)*sqrt(b)), -1/8*(sqrt(-b^2*x^4 + a^2)*(2*b*x^3
- 11*a*x)*sqrt(-b*x^2 + a)*sqrt(-b) + 19*(a^2*b*x^2 - a^3)*arctan
(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x)
))/((b*x^2 - a)*sqrt(-b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a-bx^{2}\right)^{\frac{5}{2}}}{\sqrt{-\left(-a+bx^{2}\right)\left(a+bx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral((a - b*x**2)**(5/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{5}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

3.210 $\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=109

$$\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}-\frac{x\sqrt{a-bx^2}\,(a+bx^2)}{2\sqrt{a^2-b^2x^4}}$$

```
[Out] -(x*Sqrt[a - b*x^2]*(a + b*x^2))/(2*Sqrt[a^2 - b^2*x^4]) + (3*a*S
qrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2
]])/(2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.101543, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}-\frac{x\sqrt{a-bx^2}\,(a+bx^2)}{2\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -(x*Sqrt[a - b*x^2]*(a + b*x^2))/(2*Sqrt[a^2 - b^2*x^4]) + (3*a*S
qrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2
]])/(2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi in Sympy [A] time = 17.3618, size = 88, normalized size = 0.81

$$\frac{3a\sqrt{a^2-b^2x^4}\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}-\frac{x\sqrt{a^2-b^2x^4}}{2\sqrt{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] 3*a*sqrt(a**2 - b**2*x**4)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(2*s
qrt(b)*sqrt(a - b*x**2)*sqrt(a + b*x**2)) - x*sqrt(a**2 - b**2*x*
*4)/(2*sqrt(a - b*x**2))
```

Mathematica [A] time = 0.152985, size = 110, normalized size = 1.01

$$\frac{1}{2}\left(-\frac{x\sqrt{a^2-b^2x^4}}{\sqrt{a-bx^2}}+\frac{3a\log\left(\sqrt{b}\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}+abx-b^2x^3\right)}{\sqrt{b}}-\frac{3a\log\left(bx^2-a\right)}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a - b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (-((x*Sqrt[a^2 - b^2*x^4])/Sqrt[a - b*x^2]) - (3*a*Log[-a + b*x^2
])/Sqrt[b] + (3*a*Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]*S
qrt[a^2 - b^2*x^4]])/Sqrt[b])/2
```

Maple [A] time = 0.017, size = 85, normalized size = 0.8

$$-\frac{1}{2\,b{x}^{2}-2\,a}\sqrt{-b{x}^{2}+a}\sqrt{-{b}^{2}{x}^{4}+{a}^{2}}\left(-x\sqrt{b{x}^{2}+a}\sqrt{b}+3\,a\ln\left(x\sqrt{b}+\sqrt{b{x}^{2}+a}\right)\right)\frac{1}{\sqrt{b{x}^{2}+a}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/2*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*(-x*(b*x^2+a)^(1/2)*b^
(1/2)+3*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))/(b*x^2-a)/(b*x^2+a)^(1/2
)/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.286728, size = 1, normalized size = 0.01

$$\left[\frac{2\,\sqrt{-b^{2}x^{4}+a^{2}}\sqrt{-bx^{2}+a}\sqrt{b}x+3\left(abx^{2}-a^{2}\right)\log\left(-\frac{2\,\sqrt{-b^{2}x^{4}+a^{2}}\sqrt{-bx^{2}+a}bx-\left(2\,b^{2}x^{4}-abx^{2}-a^{2}\right)\sqrt{b}}{bx^{2}-a}\right)}{4\left(bx^{2}-a\right)\sqrt{b}},\ \frac{\sqrt{-b^{2}x^{4}+a^{2}}\sqrt{-bx^{2}+a}\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(b)*x + 3*(a*b*
x^2 - a^2)*log(-(2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*b*x - (2
*b^2*x^4 - a*b*x^2 - a^2)*sqrt(b))/(b*x^2 - a)))/((b*x^2 - a)*sqr
t(b)), 1/2*(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b)*x - 3*
(a*b*x^2 - a^2)*arctan(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt
(-b)/(b^2*x^3 - a*b*x)))/((b*x^2 - a)*sqrt(-b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a-bx^{2}\right)^{\frac{3}{2}}}{\sqrt{-\left(-a+bx^{2}\right)\left(a+bx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral((a - b*x**2)**(3/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{3}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

3.211 $\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.0691381, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a - b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi in Sympy [A] time = 12.7305, size = 56, normalized size = 0.88

$$\frac{\sqrt{a^2-b^2x^4}\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] sqrt(a**2 - b**2*x**4)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(sqrt(b)
*sqrt(a - b*x**2)*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.0280174, size = 67, normalized size = 1.05

$$\frac{\log\left(\sqrt{b}\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}+abx-b^2x^3\right)-\log\left(bx^2-a\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a - b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (-Log[-a + b*x^2] + Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]
*Sqrt[a^2 - b^2*x^4]])/Sqrt[b]
```

Maple [A] time = 0.015, size = 54, normalized size = 0.8

$$1\sqrt{-b^2x^4+a^2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{-bx^2+a}}\frac{1}{\sqrt{bx^2+a}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] 1/(-b*x^2+a)^(1/2)/(b*x^2+a)^(1/2)/b^(1/2)*(-b^2*x^4+a^2)^(1/2)*l
n(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-b*x^2 + a)/sqrt(-b^2*x^4 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276606, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{2\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}bx-(2b^2x^4-abx^2-a^2)\sqrt{b}}{bx^2-a}\right)}{2\sqrt{b}}, -\frac{\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{-b}}{b^2x^3-abx}\right)}{\sqrt{-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-b*x^2 + a)/sqrt(-b^2*x^4 + a^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*b*x - (2*b^2*x
^4 - a*b*x^2 - a^2)*sqrt(b))/(b*x^2 - a))/sqrt(b), -arctan(sqrt(-
b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x))/sqrt(
-b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-(-a+bx^2)(a+bx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2+a}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-b*x^2 + a)/sqrt(-b^2*x^4 + a^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

3.212 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt
[a + b*x^2]])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.100935, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt
[a + b*x^2]])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi in Sympy [A] time = 21.5253, size = 70, normalized size = 0.91

$$\frac{\sqrt{2}\sqrt{a^2-b^2x^4}\,\text{atanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2a\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(a**2 - b**2*x**4)*atanh(sqrt(2)*sqrt(b)*x/sqrt(a + b
*x**2))/(2*a*sqrt(b)*sqrt(a - b*x**2)*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.0472205, size = 77, normalized size = 1.

$$\frac{\sqrt{a^2-b^2x^4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]]
)/(Sqrt[2]*a*Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a + b*x^2])
```

Maple [B] time = 0.074, size = 266, normalized size = 3.5

$$\frac{1}{2\,b x^2 - 2\,a}\sqrt{-b x^2 + a}\sqrt{-b^2 x^4 + a^2}\left(b\sqrt{a}\sqrt{2}\ln\left(2\,\frac{b\left(\sqrt{2}\sqrt{a}\sqrt{b x^2 + a} + x\sqrt{ab} + a\right)}{bx - \sqrt{ab}}\right) - b\sqrt{a}\sqrt{2}\ln\left(2\,\frac{b\left(\sqrt{2}\sqrt{a}\sqrt{b x^2 + a} - x\sqrt{ab} + a\right)}{bx + \sqrt{ab}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] 1/2*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*(b*a^(1/2)*2^(1/2)*ln(2
*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+x*(a*b)^(1/2)+a)/(b*x-(a*b)^(
1/2)))-b*a^(1/2)*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-
x*(a*b)^(1/2)+a)/(b*x+(a*b)^(1/2)))-2*b^(1/2)*ln((b^(1/2)*(b*x^2+
a)^(1/2)+b*x)/b^(1/2))*(a*b)^(1/2)+2*b^(1/2)*ln((b^(1/2)*(-(b*x+(
-a*b)^(1/2))/b*(-b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*(a*b)^(1/
2))/(b*x^2-a)/(b*x^2+a)^(1/2)/((a*b)^(1/2)+(-a*b)^(1/2))/((-a*b)^
(1/2)-(a*b)^(1/2))/(a*b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.279983, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{2}\log\left(\frac{4\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}bx-\sqrt{2}\left(3\,b^2x^4-2\,abx^2-a^2\right)\sqrt{b}}{b^2x^4-2\,abx^2+a^2}\right)}{4\,a\sqrt{b}},-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{-b}}{2\left(b^2x^3-abx\right)}\right)}{2\,a\sqrt{-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*log((4*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*b*x - s
qrt(2)*(3*b^2*x^4 - 2*a*b*x^2 - a^2)*sqrt(b))/(b^2*x^4 - 2*a*b*x^
2 + a^2))/(a*sqrt(b)), -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-b^2*
x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x))/(a*sqrt(-
b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(-a + bx^2\right)\left(a + bx^2\right)}\sqrt{a - bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*sqrt(a - b*x**2)),
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)), x)
```

3.213 $\int \frac{1}{(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=124

$$\frac{x\left(a+bx^2\right)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (x*(a + b*x^2))/(4*a^2*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]) + (3*
Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[
a + b*x^2]])/(4*Sqrt[2]*a^2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.148741, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\frac{x\left(a+bx^2\right)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a + b*x^2))/(4*a^2*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]) + (3*
Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[
a + b*x^2]])/(4*Sqrt[2]*a^2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi in Sympy [A] time = 29.1302, size = 104, normalized size = 0.84

$$\frac{x\sqrt{a^2-b^2x^4}}{4a^2\left(a-bx^2\right)^{\frac{3}{2}}}+\frac{3\sqrt{2}\sqrt{a^2-b^2x^4}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8a^2\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] x*sqrt(a**2 - b**2*x**4)/(4*a**2*(a - b*x**2)**(3/2)) + 3*sqrt(2)
*sqrt(a**2 - b**2*x**4)*atanh(sqrt(2)*sqrt(b)*x/sqrt(a + b*x**2))
/(8*a**2*sqrt(b)*sqrt(a - b*x**2)*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.0932392, size = 110, normalized size = 0.89

$$\frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}x\sqrt{a+bx^2}+3\sqrt{2}\left(a-bx^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)\right)}{8a^2\sqrt{b}\left(a-bx^2\right)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a + b*x^2] + 3*Sqrt[2]*(a
- b*x^2)*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]]))/(8*a^2*Sq
rt[b]*(a - b*x^2)^(3/2)*Sqrt[a + b*x^2])
```

Maple [B] time = 0.062, size = 526, normalized size = 4.2

$$-\frac{b^{2}}{4\,b x^{2}-4\,a}\sqrt{-b x^{2}+a}\sqrt{-b^{2} x^{4}+a^{2}}\left(3\,\sqrt{2}\ln\left(2\,\frac{b\left(\sqrt{2}\sqrt{a}\sqrt{b x^{2}+a}+x\sqrt{ab}+a\right)}{bx-\sqrt{ab}}\right)x^{2}ab^{2}-3\,\sqrt{2}\ln\left(2\,\frac{b\left(\sqrt{2}\sqrt{a}\sqrt{b x^{2}+a}-x\right.}{bx+\sqrt{ab}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/4*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*b^2*(3*2^(1/2)*ln(2*b*
(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+x*(a*b)^(1/2)+a)/(b*x-(a*b)^(1/2
)))*x^2*a*b^2-3*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-x
*(a*b)^(1/2)+a)/(b*x+(a*b)^(1/2)))*x^2*a*b^2-4*ln((b^(1/2)*(b*x^2
+a)^(1/2)+b*x)/b^(1/2))*x^2*b^(3/2)*(a*b)^(1/2)*a^(1/2)+4*ln((b^(
1/2)*(-(b*x+(-a*b)^(1/2))/b*(-b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/
2))*x^2*b^(3/2)*(a*b)^(1/2)*a^(1/2)-3*2^(1/2)*ln(2*b*(2^(1/2)*a^(
1/2)*(b*x^2+a)^(1/2)+x*(a*b)^(1/2)+a)/(b*x-(a*b)^(1/2)))*a^2*b+3*
2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-x*(a*b)^(1/2)+a)/
(b*x+(a*b)^(1/2)))*a^2*b+4*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/
2))*a^(3/2)*b^(1/2)*(a*b)^(1/2)-4*ln((b^(1/2)*(-(b*x+(-a*b)^(1/2)
)/b*(-b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*a^(3/2)*b^(1/2)*(a*b
)^(1/2)-4*(a*b)^(1/2)*a^(1/2)*b*(b*x^2+a)^(1/2)*x)/(b*x^2-a)/(b*x
^2+a)^(1/2)/((a*b)^(1/2)+(-a*b)^(1/2))^2/((-a*b)^(1/2)-(a*b)^(1/2
))^2/(a*b)^(1/2)/a^(1/2)/(b*x-(a*b)^(1/2))/(b*x+(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-b^{2} x^{4}+a^{2}}(-b x^{2}+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(3/2)), x)
```

Fricas [A] time = 0.282146, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{-b^{2} x^{4}+a^{2}}\sqrt{-b x^{2}+a}\sqrt{b}x+3\,\left(b^{2} x^{4}-2\,ab x^{2}+a^{2}\right)\log\left(\frac{4\,\sqrt{-b^{2} x^{4}+a^{2}}\sqrt{-b x^{2}+a}bx-\sqrt{2}\left(3\,b^{2} x^{4}-2\,ab x^{2}-a^{2}\right)\sqrt{b}}{b^{2} x^{4}-2\,ab x^{2}+a^{2}}\right)\right)}{16\left(a^{2} b^{2} x^{4}-2\,a^{3} b x^{2}+a^{4}\right)\sqrt{b}},\ \frac{\sqrt{2}\left(\sqrt{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(3/2)),x, algorithm="fricas")
```

```
[Out] [1/16*sqrt(2)*(2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sq
rt(b)*x + 3*(b^2*x^4 - 2*a*b*x^2 + a^2)*log((4*sqrt(-b^2*x^4 + a^
2)*sqrt(-b*x^2 + a)*b*x - sqrt(2)*(3*b^2*x^4 - 2*a*b*x^2 - a^2)*s
qrt(b))/(b^2*x^4 - 2*a*b*x^2 + a^2)))/((a^2*b^2*x^4 - 2*a^3*b*x^2
 + a^4)*sqrt(b)), 1/8*sqrt(2)*(sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(
-b*x^2 + a)*sqrt(-b)*x - 3*(b^2*x^4 - 2*a*b*x^2 + a^2)*arctan(1/2
*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3
- a*b*x)))/((a^2*b^2*x^4 - 2*a^3*b*x^2 + a^4)*sqrt(-b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\left(a-bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a - b*x**2)**(3/2)
), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4+a^2}(-bx^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(3/2)), x)
```

3.214 $\int \frac{1}{(a-bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=167

$$\frac{x\left(a+bx^2\right)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{9x\left(a+bx^2\right)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (x*(a + b*x^2))/(8*a^2*(a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]) + (
9*x*(a + b*x^2))/(32*a^3*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]) + (
19*Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(32*Sqrt[2]*a^3*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.247705, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$\frac{x\left(a+bx^2\right)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{9x\left(a+bx^2\right)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a - b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a + b*x^2))/(8*a^2*(a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]) + (
9*x*(a + b*x^2))/(32*a^3*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]) + (
19*Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(32*Sqrt[2]*a^3*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi in Sympy [A] time = 52.7301, size = 136, normalized size = 0.81

$$\frac{x\sqrt{a^2-b^2x^4}}{8a^2\left(a-bx^2\right)^{\frac{5}{2}}}+\frac{9x\sqrt{a^2-b^2x^4}}{32a^3\left(a-bx^2\right)^{\frac{3}{2}}}+\frac{19\sqrt{2}\sqrt{a^2-b^2x^4}\,\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{64a^3\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] x*sqrt(a**2 - b**2*x**4)/(8*a**2*(a - b*x**2)**(5/2)) + 9*x*sqrt(
a**2 - b**2*x**4)/(32*a**3*(a - b*x**2)**(3/2)) + 19*sqrt(2)*sqrt
(a**2 - b**2*x**4)*atanh(sqrt(2)*sqrt(b)*x/sqrt(a + b*x**2))/(64*
a**3*sqrt(b)*sqrt(a - b*x**2)*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.124062, size = 122, normalized size = 0.73

$$\frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}x\left(13a-9bx^2\right)\sqrt{a+bx^2}+19\sqrt{2}\left(a-bx^2\right)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)\right)}{64a^3\sqrt{b}\left(a-bx^2\right)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a - b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*(13*a - 9*b*x^2)*Sqrt[a + b*x^2
] + 19*Sqrt[2]*(a - b*x^2)^2*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a +
```

```
b*x^2]]))/(64*a^3*Sqrt[b]*(a - b*x^2)^(5/2)*Sqrt[a + b*x^2])
```

Maple [B] time = 0.067, size = 757, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] 1/16*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*b^4*(19*2^(1/2)*ln(2*b
*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+x*(a*b)^(1/2)+a)/(b*x-(a*b)^(1/
2)))*x^4*a*b^3-19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)
-x*(a*b)^(1/2)+a)/(b*x+(a*b)^(1/2)))*x^4*a*b^3+16*ln((b^(1/2)*(-(
b*x+(-a*b)^(1/2))/b*(-b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*x^4*
b^(5/2)*a^(1/2)*(a*b)^(1/2)-16*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b
^(1/2))*x^4*b^(5/2)*a^(1/2)*(a*b)^(1/2)-38*2^(1/2)*ln(2*b*(2^(1/2
)*a^(1/2)*(b*x^2+a)^(1/2)+x*(a*b)^(1/2)+a)/(b*x-(a*b)^(1/2)))*x^2
*a^2*b^2+38*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-x*(a*
b)^(1/2)+a)/(b*x+(a*b)^(1/2)))*x^2*a^2*b^2-32*ln((b^(1/2)*(-(b*x+
(-a*b)^(1/2))/b*(-b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*x^2*a^(3
/2)*b^(3/2)*(a*b)^(1/2)+32*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/
2))*x^2*a^(3/2)*b^(3/2)*(a*b)^(1/2)-36*b^2*(a*b)^(1/2)*(b*x^2+a)^
(1/2)*a^(1/2)*x^3+19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1
/2)+x*(a*b)^(1/2)+a)/(b*x-(a*b)^(1/2)))*a^3*b-19*2^(1/2)*ln(2*b*(
2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-x*(a*b)^(1/2)+a)/(b*x+(a*b)^(1/2)
))*a^3*b+16*ln((b^(1/2)*(-(b*x+(-a*b)^(1/2))/b*(-b*x+(-a*b)^(1/2)
))^(1/2)+b*x)/b^(1/2))*a^(5/2)*b^(1/2)*(a*b)^(1/2)-16*ln((b^(1/2)
*(b*x^2+a)^(1/2)+b*x)/b^(1/2))*a^(5/2)*b^(1/2)*(a*b)^(1/2)+52*(a*
b)^(1/2)*a^(3/2)*b*(b*x^2+a)^(1/2)*x)/(b*x^2-a)/(b*x^2+a)^(1/2)/(
(a*b)^(1/2)+(-a*b)^(1/2))^3/((-a*b)^(1/2)-(a*b)^(1/2))^3/(a*b)^(1
/2)/a^(1/2)/(b*x-(a*b)^(1/2))^2/(b*x+(a*b)^(1/2))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}(-bx^2 + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(5/2)), x)
```

Fricas [A] time = 0.285128, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-b^2x^4 + a^2}\left(9\,bx^3 - 13\,ax\right)\sqrt{-bx^2 + a}\sqrt{b} + 19\left(b^3x^6 - 3\,ab^2x^4 + 3\,a^2bx^2 - a^3\right)\log\left(\frac{4\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}bx - \sqrt{2}(3\,b^2}{b^2x^4 - 2\,abx^2 + a^2}\right.\right.}{128\left(a^3b^3x^6 - 3\,a^4b^2x^4 + 3\,a^5bx^2 - a^6\right)\sqrt{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(5/2)),x, algorithm="fricas")
```

```
[Out] [1/128*sqrt(2)*(2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*(9*b*x^3 - 13*a*x)
*sqrt(-b*x^2 + a)*sqrt(b) + 19*(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x
```

```
^2 - a^3)*log((4*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*b*x - sqrt
(2)*(3*b^2*x^4 - 2*a*b*x^2 - a^2)*sqrt(b))/(b^2*x^4 - 2*a*b*x^2 +
 a^2)))/((a^3*b^3*x^6 - 3*a^4*b^2*x^4 + 3*a^5*b*x^2 - a^6)*sqrt(b
)), 1/64*sqrt(2)*(sqrt(2)*sqrt(-b^2*x^4 + a^2)*(9*b*x^3 - 13*a*x)
*sqrt(-b*x^2 + a)*sqrt(-b) - 19*(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*
x^2 - a^3)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 +
a)*sqrt(-b)/(b^2*x^3 - a*b*x)))/((a^3*b^3*x^6 - 3*a^4*b^2*x^4 + 3
*a^5*b*x^2 - a^6)*sqrt(-b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(-a+bx^2)(a+bx^2)}(a-bx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a - b*x**2)**(5/2)
), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4+a^2}(-bx^2+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(5/2)), x)
```

3.215 $\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}} \, dx$

Optimal. Leaf size=30

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\sinh^{-1}(x)}{\sqrt{x^4-1}}$$

```
[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]
```

Rubi [A] time = 0.0290058, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\sinh^{-1}(x)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^2]/Sqrt[-1 + x^4],x]

[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]
```

Rubi in Sympy [A] time = 5.6062, size = 27, normalized size = 0.9

$$\frac{\sqrt{x^4-1}\,\operatorname{asinh}(x)}{\sqrt{x^2-1}\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2-1)**(1/2)/(x**4-1)**(1/2),x)

[Out] sqrt(x**4 - 1)*asinh(x)/(sqrt(x**2 - 1)*sqrt(x**2 + 1))
```

Mathematica [A] time = 0.014237, size = 38, normalized size = 1.27

$$\log\left(x^3+\sqrt{x^2-1}\sqrt{x^4-1}-x\right)-\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^2]/Sqrt[-1 + x^4],x]

[Out] -Log[1 - x^2] + Log[-x + x^3 + Sqrt[-1 + x^2]*Sqrt[-1 + x^4]]
```

Maple [A] time = 0.014, size = 25, normalized size = 0.8

$$Arcsinh(x)\sqrt{x^4-1}\frac{1}{\sqrt{x^2-1}}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(1/2)/(x^4-1)^(1/2),x)
```

```
[Out] 1/(x^2-1)^(1/2)*(x^4-1)^(1/2)/(x^2+1)^(1/2)*arcsinh(x)
```

Maxima [A] time = 0.849519, size = 3, normalized size = 0.1

$$\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1),x, algorithm="maxima")
```

```
[Out] arcsinh(x)
```

Fricas [A] time = 0.288848, size = 99, normalized size = 3.3

$$\frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right)-\frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1),x, algorithm="fricas")
```

```
[Out] 1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 - 1) - x)/(x^3 - x)) - 1/2*
log(-(x^3 - sqrt(x^4 - 1)*sqrt(x^2 - 1) - x)/(x^3 - x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{(x-1)(x+1)}}{\sqrt{(x-1)(x+1)(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-1)**(1/2)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1))/sqrt((x - 1)*(x + 1)*(x**2 + 1)),
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1), x)
```

3.216 $\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=24

$$-\frac{\sqrt{x^4-1}\sin^{-1}(x)}{\sqrt{1-x^4}}$$

```
[Out] -((Sqrt[-1 + x^4]*ArcSin[x])/Sqrt[1 - x^4])
```

Rubi [A] time = 0.0332101, antiderivative size = 40, normalized size of antiderivative = 1.67, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + x^2]/Sqrt[-1 + x^4],x]

[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1
+ x^4]
```

Rubi in Sympy [A] time = 4.92557, size = 36, normalized size = 1.5

$$\frac{\sqrt{x^4-1}\,\operatorname{atanh}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^2-1}\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)**(1/2)/(x**4-1)**(1/2),x)

[Out] sqrt(x**4 - 1)*atanh(x/sqrt(x**2 - 1))/(sqrt(x**2 - 1)*sqrt(x**2
+ 1))
```

Mathematica [A] time = 0.0133049, size = 34, normalized size = 1.42

$$\log\left(x^3+\sqrt{x^2+1}\sqrt{x^4-1}+x\right)-\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + x^2]/Sqrt[-1 + x^4],x]

[Out] -Log[1 + x^2] + Log[x + x^3 + Sqrt[1 + x^2]*Sqrt[-1 + x^4]]
```

Maple [A] time = 0.011, size = 33, normalized size = 1.4

$$1\sqrt{x^4-1}\ln\left(x+\sqrt{x^2-1}\right)\frac{1}{\sqrt{x^2+1}}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^(1/2)/(x^4-1)^(1/2),x)
```

```
[Out] 1/(x^2+1)^(1/2)*(x^4-1)^(1/2)/(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [A] time = 0.875546, size = 19, normalized size = 0.79

$$\log\left(2\,x + 2\,\sqrt{x^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 - 1))
```

Fricas [A] time = 0.270665, size = 88, normalized size = 3.67

$$\frac{1}{2}\,\log\left(\frac{x^3 + \sqrt{x^4 - 1}\sqrt{x^2 + 1} + x}{x^3 + x}\right) - \frac{1}{2}\,\log\left(-\frac{x^3 - \sqrt{x^4 - 1}\sqrt{x^2 + 1} + x}{x^3 + x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1),x, algorithm="fricas")
```

```
[Out] 1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*
log(-(x^3 - sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + 1}}{\sqrt{(x - 1)\,(x + 1)\,(x^2 + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**(1/2)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + 1}}{\sqrt{x^4 - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1), x)
```

3.217 $\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=73

$$\frac{\sqrt{x^2-1}\sqrt{x^4-1}\sinh^{-1}(x)}{(1-x^2)\sqrt{x^2+1}}-\frac{\sqrt{x^4-1}\sin^{-1}(x)}{\sqrt{1-x^2}\sqrt{x^2+1}}$$

```
[Out] -((Sqrt[-1 + x^4]*ArcSin[x])/(Sqrt[1 - x^2]*Sqrt[1 + x^2])) + (Sq
rt[-1 + x^2]*Sqrt[-1 + x^4]*ArcSinh[x])/((1 - x^2)*Sqrt[1 + x^2])
```

Rubi [A] time = 0.204226, antiderivative size = 72, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^4-1}}-\frac{\sqrt{x^2-1}\sqrt{x^2+1}\sinh^{-1}(x)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4],x]
```

```
[Out] -((Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]) + (Sq
rt[-1 + x^2]*Sqrt[1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x
^4]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-\sqrt{x^2-1}+\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-(x**2-1)**(1/2)+(x**2+1)**(1/2))/(x**4-1)**(1/2),x)
```

```
[Out] Integral((-sqrt(x**2 - 1) + sqrt(x**2 + 1))/sqrt(x**4 - 1), x)
```

Mathematica [A] time = 0.0212421, size = 71, normalized size = 0.97

$$\log\left(1-x^2\right)-\log\left(x^2+1\right)-\log\left(x^3+\sqrt{x^2-1}\sqrt{x^4-1}-x\right)+\log\left(x^3+\sqrt{x^2+1}\sqrt{x^4-1}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4],x]
```

```
[Out] Log[1 - x^2] - Log[1 + x^2] - Log[-x + x^3 + Sqrt[-1 + x^2]*Sqrt[
-1 + x^4]] + Log[x + x^3 + Sqrt[1 + x^2]*Sqrt[-1 + x^4]]
```

Maple [A] time = 0.003, size = 59, normalized size = 0.8

$$-Arcsinh\,(x)\sqrt{x^4-1}\frac{1}{\sqrt{x^2-1}}\frac{1}{\sqrt{x^2+1}}+1\sqrt{x^4-1}\ln\left(x+\sqrt{x^2-1}\right)\frac{1}{\sqrt{x^2-1}}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x)
```

```
[Out] -1/(x^2-1)^(1/2)*(x^4-1)^(1/2)/(x^2+1)^(1/2)*arcsinh(x)+1/(x^2+1)
^(1/2)*(x^4-1)^(1/2)/(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [A] time = 0.881393, size = 26, normalized size = 0.36

$$-\operatorname{arsinh}(x)+\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1),x, algorithm="maxima")
```

```
[Out] -arcsinh(x) + log(2*x + 2*sqrt(x^2 - 1))
```

Fricas [A] time = 0.286901, size = 185, normalized size = 2.53

$$\frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right)-\frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right)-\frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right)+\frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1),x, algorithm="fricas")
```

```
[Out] 1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*
log(-(x^3 - sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log
((x^3 + sqrt(x^4 - 1)*sqrt(x^2 - 1) - x)/(x^3 - x)) + 1/2*log(-(x
^3 - sqrt(x^4 - 1)*sqrt(x^2 - 1) - x)/(x^3 - x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{-\sqrt{x^2-1}+\sqrt{x^2+1}}{\sqrt{(x-1)\,(x+1)\,(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-(x**2-1)**(1/2)+(x**2+1)**(1/2))/(x**4-1)**(1/2),x)
```

```
[Out] Integral((-sqrt(x**2 - 1) + sqrt(x**2 + 1))/sqrt((x - 1)*(x + 1)*
(x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1),x, algorithm="giac")

[Out] integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1), x)
```

3.218 $\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=121

$$\frac{x\left(b^2e^2-5bcde+7c^2d^2\right)}{c^3}-\frac{(2cd-be)^3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\sqrt{cd-be}}+\frac{ex^3(4cd-be)}{3c^2}+\frac{e^2x^5}{5c}$$

```
[Out] ((7*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*x)/c^3 + (e*(4*c*d - b*e)*x^3)
/(3*c^2) + (e^2*x^5)/(5*c) - ((2*c*d - b*e)^3*ArcTanh[(Sqrt[c]*Sq
rt[e]*x)/Sqrt[c*d - b*e]])/(c^(7/2)*Sqrt[e]*Sqrt[c*d - b*e])
```

Rubi [A] time = 0.275817, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{x\left(b^2e^2-5bcde+7c^2d^2\right)}{c^3}-\frac{(2cd-be)^3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\sqrt{cd-be}}+\frac{ex^3(4cd-be)}{3c^2}+\frac{e^2x^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```


```
[Out] ((7*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*x)/c^3 + (e*(4*c*d - b*e)*x^3)
/(3*c^2) + (e^2*x^5)/(5*c) - ((2*c*d - b*e)^3*ArcTanh[(Sqrt[c]*Sq
rt[e]*x)/Sqrt[c*d - b*e]])/(c^(7/2)*Sqrt[e]*Sqrt[c*d - b*e])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\left(b^2e^2-5bcde+7c^2d^2\right)\int \frac{1}{c^3}\,dx+\frac{e^2x^5}{5c}-\frac{ex^3\,(be-4cd)}{3c^2}-\frac{(be-2cd)^3\,\mathrm{atan}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{c^{\frac{7}{2}}\sqrt{e}\sqrt{be-cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((e*x**2+d)**4/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] (b**2*e**2 - 5*b*c*d*e + 7*c**2*d**2)*Integral(c**(-3), x) + e**2
*x**5/(5*c) - e*x**3*(b*e - 4*c*d)/(3*c**2) - (b*e - 2*c*d)**3*at
an(sqrt(c)*sqrt(e)*x/sqrt(b*e - c*d))/(c**(7/2)*sqrt(e)*sqrt(b*e
- c*d))
```

Mathematica [A] time = 0.123384, size = 121, normalized size = 1.

$$-\frac{x\left(-b^2e^2+5bcde-7c^2d^2\right)}{c^3}-\frac{(be-2cd)^3\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{c^{7/2}\sqrt{e}\sqrt{be-cd}}-\frac{ex^3(be-4cd)}{3c^2}+\frac{e^2x^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(((-7*c^2*d^2 + 5*b*c*d*e - b^2*e^2)*x)/c^3) - (e*(-4*c*d + b*e)
*x^3)/(3*c^2) + (e^2*x^5)/(5*c) - ((-2*c*d + b*e)^3*ArcTan[(Sqrt[
c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(c^(7/2)*Sqrt[e]*Sqrt[-(c*d) +
```

```
b*e])
```

Maple [B] time = 0.014, size = 226, normalized size = 1.9

$$\frac{e^2x^5}{5c}-\frac{bx^3e^2}{3c^2}+\frac{4\,dex^3}{3c}+\frac{b^2e^2x}{c^3}-5\,\frac{bdex}{c^2}+7\,\frac{d^2x}{c}-\frac{b^3e^3}{c^3}\arctan\left(cex\frac{1}{\sqrt{(be-cd)\,ce}}\right)\frac{1}{\sqrt{(be-cd)\,ce}}+6\,\frac{b^2de^2}{c^2\sqrt{(be-cd)\,ce}}\arctan\left(\frac{cex}{\sqrt{(be-cd)\,ce}}\right)-12\,\frac{bd^2e}{c\sqrt{(be-cd)\,ce}}\arctan\left(\frac{cex}{\sqrt{(be-cd)\,ce}}\right)+8\,\frac{d^3}{\sqrt{(be-cd)\,ce}}\arctan\left(\frac{cex}{\sqrt{(be-cd)\,ce}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/5*e^2*x^5/c-1/3/c^2*x^3*b*e^2+4/3/c*x^3*d*e+1/c^3*b^2*e^2*x-5/c
^2*b*d*e*x+7/c*d^2*x-1/c^3/((b*e-c*d)*c*e)^(1/2)*arctan(x*c*e/((b
*e-c*d)*c*e)^(1/2))*b^3*e^3+6/c^2/((b*e-c*d)*c*e)^(1/2)*arctan(x*
c*e/((b*e-c*d)*c*e)^(1/2))*b^2*d*e^2-12/c/((b*e-c*d)*c*e)^(1/2)*a
rctan(x*c*e/((b*e-c*d)*c*e)^(1/2))*b*d^2*e+8/((b*e-c*d)*c*e)^(1/2
)*arctan(x*c*e/((b*e-c*d)*c*e)^(1/2))*d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^4/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm="max
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281362, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(8\,c^3d^3-12\,bc^2d^2e+6\,b^2cde^2-b^3e^3\right)\log\left(\frac{2\left(c^2de-bce^2\right)x+\sqrt{c^2de-bce^2}\left(cex^2+cd-be\right)}{cex^2-cd+be}\right)-2\left(3\,c^2e^2x^5+5\left(4\,c^2de-bce^2\right)x^3\right.}{30\sqrt{c^2de-bce^2}c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^4/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm="fri
```

```
[Out] [-1/30*(15*(8*c^3*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*e^3)
*log((2*(c^2*d*e - b*c*e^2)*x + sqrt(c^2*d*e - b*c*e^2)*(c*e*x^2
+ c*d - b*e))/(c*e*x^2 - c*d + b*e)) - 2*(3*c^2*e^2*x^5 + 5*(4*c^
2*d*e - b*c*e^2)*x^3 + 15*(7*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*x)*sq
rt(c^2*d*e - b*c*e^2))/(sqrt(c^2*d*e - b*c*e^2)*c^3), 1/15*(15*(8
*c^3*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*e^3)*arctan(-sqrt
(-c^2*d*e + b*c*e^2)*x/(c*d - b*e)) + (3*c^2*e^2*x^5 + 5*(4*c^2*d
*e - b*c*e^2)*x^3 + 15*(7*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*x)*sqrt(
-c^2*d*e + b*c*e^2))/(sqrt(-c^2*d*e + b*c*e^2)*c^3)]
```

Sympy [A] time = 3.15263, size = 343, normalized size = 2.83

$$\frac{\sqrt{-\frac{1}{c^7e(be-cd)}}\,(be-2cd)^3\log\left(x+\frac{-bc^3e\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3+c^4d\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3}{b^3e^3-6b^2cde^2+12bc^2d^2e-8c^3d^3}\right)}{2} - \frac{\sqrt{-\frac{1}{c^7e(be-cd)}}\,(be-2cd)^3\log\left(x+\frac{bc^3e\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3-c^4d\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3}{b^3e^3-6b^2cde^2+12bc^2d^2e-8c^3d^3}\right)}{2} + \frac{e^2x^5}{5c} - \frac{x^3\left(be^2-4cde\right)}{3c^2} + \frac{x\left(b^2e^2-5bcde+7c^2d^2\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**4/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3*log(x + (-b*c**3*e
*sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3 + c**4*d*sqrt(-1/
(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3)/(b**3*e**3 - 6*b**2*c*d*e
**2 + 12*b*c**2*d**2*e - 8*c**3*d**3))/2 - sqrt(-1/(c**7*e*(b*e -
 c*d)))*(b*e - 2*c*d)**3*log(x + (b*c**3*e*sqrt(-1/(c**7*e*(b*e -
 c*d)))*(b*e - 2*c*d)**3 - c**4*d*sqrt(-1/(c**7*e*(b*e - c*d)))*(
b*e - 2*c*d)**3)/(b**3*e**3 - 6*b**2*c*d*e**2 + 12*b*c**2*d**2*e
- 8*c**3*d**3))/2 + e**2*x**5/(5*c) - x**3*(b*e**2 - 4*c*d*e)/(3*
c**2) + x*(b**2*e**2 - 5*b*c*d*e + 7*c**2*d**2)/c**3
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^4/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm="gia
```

```
[Out] Timed out
```

3.219 $\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=86

$$-\frac{(2cd-be)^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\sqrt{cd-be}}+\frac{x(3cd-be)}{c^2}+\frac{ex^3}{3c}$$

```
[Out] ((3*c*d - b*e)*x)/c^2 + (e*x^3)/(3*c) - ((2*c*d - b*e)^2*ArcTanh[
(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(c^(5/2)*Sqrt[e]*Sqrt[c*d -
 b*e])
```

Rubi [A] time = 0.183219, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{(2cd-be)^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\sqrt{cd-be}}+\frac{x(3cd-be)}{c^2}+\frac{ex^3}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^3/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] ((3*c*d - b*e)*x)/c^2 + (e*x^3)/(3*c) - ((2*c*d - b*e)^2*ArcTanh[
(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(c^(5/2)*Sqrt[e]*Sqrt[c*d -
 b*e])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-(be-3cd)\int \frac{1}{c^2}\,dx+\frac{ex^3}{3c}+\frac{(be-2cd)^2\,\mathrm{atan}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{c^{\frac{5}{2}}\sqrt{e}\sqrt{be-cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**3/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] -(b*e - 3*c*d)*Integral(c**(-2), x) + e*x**3/(3*c) + (b*e - 2*c*d
)**2*atan(sqrt(c)*sqrt(e)*x/sqrt(b*e - c*d))/(c**(5/2)*sqrt(e)*sq
rt(b*e - c*d))
```

Mathematica [A] time = 0.073463, size = 84, normalized size = 0.98

$$\frac{(be-2cd)^2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{c^{5/2}\sqrt{e}\sqrt{be-cd}}-\frac{x(be-3cd)}{c^2}+\frac{ex^3}{3c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^3/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(((-3*c*d + b*e)*x)/c^2) + (e*x^3)/(3*c) + ((-2*c*d + b*e)^2*Arc
Tan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(c^(5/2)*Sqrt[e]*Sqr
t[-(c*d) + b*e])
```

Maple [A] time = 0.004, size = 142, normalized size = 1.7

$$\frac{ex^3}{3c} - \frac{bex}{c^2} + 3\frac{dx}{c} + \frac{b^2e^2}{c^2}\arctan\left(cex\frac{1}{\sqrt{(be-cd)\,ce}}\right)\frac{1}{\sqrt{(be-cd)\,ce}} - 4\frac{bde}{c\sqrt{(be-cd)\,ce}}\arctan\left(\frac{cex}{\sqrt{(be-cd)\,ce}}\right) + 4\frac{d^2}{\sqrt{(be-cd)\,ce}}\arctan\left(\frac{cex}{\sqrt{(be-cd)\,ce}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/3*e*x^3/c-1/c^2*b*e*x+3/c*d*x+1/c^2/((b*e-c*d)*c*e)^(1/2)*arcta
n(x*c*e/((b*e-c*d)*c*e)^(1/2))*b^2*e^2-4/c/((b*e-c*d)*c*e)^(1/2)*
arctan(x*c*e/((b*e-c*d)*c*e)^(1/2))*b*d*e+4/((b*e-c*d)*c*e)^(1/2)
*arctan(x*c*e/((b*e-c*d)*c*e)^(1/2))*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm="max
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.287878, size = 1, normalized size = 0.01

$$\left[\frac{3\left(4\,c^2d^2 - 4\,bcde + b^2e^2\right)\log\left(-\frac{2\left(c^2de-bce^2\right)x-\sqrt{c^2de-bce^2}\left(cex^2+cd-be\right)}{cex^2-cd+be}\right) + 2\left(cex^3 + 3\left(3\,cd - be\right)x\right)\sqrt{c^2de - bce^2}}{6\sqrt{c^2de - bce^2}c^2}, \frac{3\left(4\,c^2d^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm="fri
```

```
[Out] [1/6*(3*(4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*log(-(2*(c^2*d*e - b*c*
e^2)*x - sqrt(c^2*d*e - b*c*e^2)*(c*e*x^2 + c*d - b*e))/(c*e*x^2
- c*d + b*e)) + 2*(c*e*x^3 + 3*(3*c*d - b*e)*x)*sqrt(c^2*d*e - b*
c*e^2))/(sqrt(c^2*d*e - b*c*e^2)*c^2), 1/3*(3*(4*c^2*d^2 - 4*b*c*
d*e + b^2*e^2)*arctan(-sqrt(-c^2*d*e + b*c*e^2)*x/(c*d - b*e)) +
(c*e*x^3 + 3*(3*c*d - b*e)*x)*sqrt(-c^2*d*e + b*c*e^2))/(sqrt(-c^
2*d*e + b*c*e^2)*c^2)]
```

Sympy [A] time = 2.57207, size = 275, normalized size = 3.2

$$-\frac{\sqrt{-\frac{1}{c^5e(be-cd)}}\,(be-2cd)^2\log\left(x+\frac{-bc^2e\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2+c^3d\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2}{b^2e^2-4bcde+4c^2d^2}\right)}{2}$$
$$+\frac{\sqrt{-\frac{1}{c^5e(be-cd)}}\,(be-2cd)^2\log\left(x+\frac{bc^2e\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2-c^3d\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2}{b^2e^2-4bcde+4c^2d^2}\right)}{2}$$
$$+\frac{ex^3}{3c}-\frac{x\,(be-3cd)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**3/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] -sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2*log(x + (-b*c**2*
e*sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2 + c**3*d*sqrt(-1
/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2)/(b**2*e**2 - 4*b*c*d*e +
 4*c**2*d**2))/2 + sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2
*log(x + (b*c**2*e*sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2
 - c**3*d*sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2)/(b**2*e
**2 - 4*b*c*d*e + 4*c**2*d**2))/2 + e*x**3/(3*c) - x*(b*e - 3*c*d
)/c**2
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm="gia
```

```
[Out] Timed out
```

3.220 $\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=64

$$\frac{x}{c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\sqrt{cd-be}}$$

```
[Out] x/c - ((2*c*d - b*e)*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]]
)/(c^(3/2)*Sqrt[e]*Sqrt[c*d - b*e])
```

Rubi [A] time = 0.121866, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{x}{c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\sqrt{cd-be}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^2/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] x/c - ((2*c*d - b*e)*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]]
)/(c^(3/2)*Sqrt[e]*Sqrt[c*d - b*e])
```

Rubi in Sympy [A] time = 27.18, size = 54, normalized size = 0.84

$$\frac{x}{c}-\frac{(be-2cd)\,\mathrm{atan}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{c^{\frac{3}{2}}\sqrt{e}\sqrt{be-cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] x/c - (b*e - 2*c*d)*atan(sqrt(c)*sqrt(e)*x/sqrt(b*e - c*d))/(c**(
3/2)*sqrt(e)*sqrt(b*e - c*d))
```

Mathematica [A] time = 0.0925596, size = 63, normalized size = 0.98

$$\frac{x}{c}-\frac{(be-2cd)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{c^{3/2}\sqrt{e}\sqrt{be-cd}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^2/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] x/c - ((-2*c*d + b*e)*ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*
e]])/(c^(3/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])
```

Maple [A] time = 0.003, size = 79, normalized size = 1.2

$$\frac{x}{c}-\frac{be}{c}\arctan\left(cex\frac{1}{\sqrt{(be-cd)\,ce}}\right)\frac{1}{\sqrt{(be-cd)\,ce}}+2\,\frac{d}{\sqrt{(be-cd)\,ce}}\arctan\left(\frac{cex}{\sqrt{(be-cd)\,ce}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] x/c-1/c/((b*e-c*d)*c*e)^(1/2)*arctan(x*c*e/((b*e-c*d)*c*e)^(1/2))
*b*e+2/((b*e-c*d)*c*e)^(1/2)*arctan(x*c*e/((b*e-c*d)*c*e)^(1/2))*
d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^2/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm="max
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276516, size = 1, normalized size = 0.02

$$\left[-\frac{(2\,cd-be)\log\left(\frac{2\,(c^2de-bce^2)x+\sqrt{c^2de-bce^2}(cex^2+cd-be)}{cex^2-cd+be}\right)-2\,\sqrt{c^2de-bce^2}x}{2\,\sqrt{c^2de-bce^2}c},\ \frac{(2\,cd-be)\arctan\left(-\frac{\sqrt{-c^2de+bce^2}x}{cd-be}\right)+\sqrt{-c^2de}}{\sqrt{-c^2de+bce^2}c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^2/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm="fri
```

```
[Out] [-1/2*((2*c*d - b*e)*log((2*(c^2*d*e - b*c*e^2)*x + sqrt(c^2*d*e
- b*c*e^2)*(c*e*x^2 + c*d - b*e))/(c*e*x^2 - c*d + b*e)) - 2*sqrt
(c^2*d*e - b*c*e^2)*x)/(sqrt(c^2*d*e - b*c*e^2)*c), ((2*c*d - b*e
)*arctan(-sqrt(-c^2*d*e + b*c*e^2)*x/(c*d - b*e)) + sqrt(-c^2*d*e
 + b*c*e^2)*x)/(sqrt(-c^2*d*e + b*c*e^2)*c)]
```

Sympy [A] time = 1.95221, size = 212, normalized size = 3.31

$$\frac{\sqrt{-\frac{1}{c^3e(be-cd)}}\,(be-2cd)\log\left(x+\frac{-bce\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)+c^2d\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)}{be-2cd}\right)}{2} - \frac{\sqrt{-\frac{1}{c^3e(be-cd)}}\,(be-2cd)\log\left(x+\frac{bce\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)-c^2d\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)}{be-2cd}\right)}{2}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**2/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)*log(x + (-b*c*e*sqrt(
-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d) + c**2*d*sqrt(-1/(c**3*e*(
b*e - c*d)))*(b*e - 2*c*d))/(b*e - 2*c*d))/2 - sqrt(-1/(c**3*e*(b
*e - c*d)))*(b*e - 2*c*d)*log(x + (b*c*e*sqrt(-1/(c**3*e*(b*e - c
*d)))*(b*e - 2*c*d) - c**2*d*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e -
 2*c*d))/(b*e - 2*c*d))/2 + x/c
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^2/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm="gia
```

```
[Out] Timed out
```

3.221 $\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}}$$

```
[Out] -(ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]]/(Sqrt[c]*Sqrt[e]*S
qrt[c*d - b*e]))
```

Rubi [A] time = 0.0524903, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]]/(Sqrt[c]*Sqrt[e]*S
qrt[c*d - b*e]))
```

Rubi in Sympy [A] time = 15.3808, size = 42, normalized size = 0.86

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{\sqrt{c}\sqrt{e}\sqrt{be-cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] atan(sqrt(c)*sqrt(e)*x/sqrt(b*e - c*d))/(sqrt(c)*sqrt(e)*sqrt(b*e
 - c*d))
```

Mathematica [A] time = 0.020428, size = 48, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{\sqrt{c}\sqrt{e}\sqrt{be-cd}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]]/(Sqrt[c]*Sqrt[e]*S
qrt[-(c*d) + b*e])
```

Maple [A] time = 0.001, size = 33, normalized size = 0.7

$$1\arctan\left(cex\frac{1}{\sqrt{(be-cd)\,ce}}\right)\frac{1}{\sqrt{(be-cd)\,ce}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/((b*e-c*d)*c*e)^(1/2)*arctan(x*c*e/((b*e-c*d)*c*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm="maxim
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268387, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{2\left(c^2de-bce^2\right)x-\sqrt{c^2de-bce^2}\left(cex^2+cd-be\right)}{cex^2-cd+be}\right)}{2\sqrt{c^2de-bce^2}}, \frac{\arctan\left(-\frac{\sqrt{-c^2de+bce^2}x}{cd-be}\right)}{\sqrt{-c^2de+bce^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm="frica
```

```
[Out] [1/2*log(-(2*(c^2*d*e - b*c*e^2)*x - sqrt(c^2*d*e - b*c*e^2)*(c*e
*x^2 + c*d - b*e))/(c*e*x^2 - c*d + b*e))/sqrt(c^2*d*e - b*c*e^2)
, arctan(-sqrt(-c^2*d*e + b*c*e^2)*x/(c*d - b*e))/sqrt(-c^2*d*e +
 b*c*e^2)]
```

Sympy [A] time = 0.564659, size = 124, normalized size = 2.53

$$-\frac{\sqrt{-\frac{1}{ce(be-cd)}}\log\left(-be\sqrt{-\frac{1}{ce(be-cd)}}+cd\sqrt{-\frac{1}{ce(be-cd)}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ce(be-cd)}}\log\left(be\sqrt{-\frac{1}{ce(be-cd)}}-cd\sqrt{-\frac{1}{ce(be-cd)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] -sqrt(-1/(c*e*(b*e - c*d)))*log(-b*e*sqrt(-1/(c*e*(b*e - c*d))) +
 c*d*sqrt(-1/(c*e*(b*e - c*d))) + x)/2 + sqrt(-1/(c*e*(b*e - c*d)
))*log(b*e*sqrt(-1/(c*e*(b*e - c*d))) - c*d*sqrt(-1/(c*e*(b*e - c
*d))) + x)/2
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm="giac"
```

```
[Out] Timed out
```

3.222 $\int \frac{1}{(d+ex^2)(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx$

Optimal. Leaf size=136

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^2}-\frac{(4cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2}-\frac{x}{2d\,(d+ex^2)\,(2cd-be)}$$

```
[Out] -x/(2*d*(2*c*d - b*e)*(d + e*x^2)) - ((4*c*d - b*e)*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(2*d^(3/2)*Sqrt[e]*(2*c*d - b*e)^2) - (c^(3/2)*Ar
cTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(Sqrt[e]*Sqrt[c*d - b
*e]*(2*c*d - b*e)^2)
```

Rubi [A] time = 0.39953, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^2}-\frac{(4cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2}-\frac{x}{2d\,(d+ex^2)\,(2cd-be)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -x/(2*d*(2*c*d - b*e)*(d + e*x^2)) - ((4*c*d - b*e)*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(2*d^(3/2)*Sqrt[e]*(2*c*d - b*e)^2) - (c^(3/2)*Ar
cTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(Sqrt[e]*Sqrt[c*d - b
*e]*(2*c*d - b*e)^2)
```

Rubi in Sympy [A] time = 87.4403, size = 117, normalized size = 0.86

$$\frac{c^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{\sqrt{e}\,(be-2cd)^2\sqrt{be-cd}}+\frac{x}{2d\,(d+ex^2)\,(be-2cd)}+\frac{(be-4cd)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{\frac{3}{2}}\sqrt{e}\,(be-2cd)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(e*x**2+d)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] c**(3/2)*atan(sqrt(c)*sqrt(e)*x/sqrt(b*e - c*d))/(sqrt(e)*(b*e -
2*c*d)**2*sqrt(b*e - c*d)) + x/(2*d*(d + e*x**2)*(b*e - 2*c*d)) +
 (b*e - 4*c*d)*atan(sqrt(e)*x/sqrt(d))/(2*d**(3/2)*sqrt(e)*(b*e -
 2*c*d)**2)
```

Mathematica [A] time = 0.388379, size = 133, normalized size = 0.98

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{\sqrt{e}(be-2cd)^2\sqrt{be-cd}}+\frac{(be-4cd)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2}-\frac{x}{2d\,(d+ex^2)\,(2cd-be)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -x/(2*d*(2*c*d - b*e)*(d + e*x^2)) + ((-4*c*d + b*e)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(2*d^(3/2)*Sqrt[e]*(2*c*d - b*e)^2) + (c^(3/2)*A
```

```
rcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(Sqrt[e]*(-2*c*d +
 b*e)^2*Sqrt[-(c*d) + b*e])
```

Maple [A] time = 0.024, size = 155, normalized size = 1.1

$$\frac{c^2}{(be-2\,cd)^2}\arctan\left(cex\frac{1}{\sqrt{(be-cd)\,ce}}\right)\frac{1}{\sqrt{(be-cd)\,ce}}+\frac{bxe}{2\,(be-2\,cd)^2\,d\,(ex^2+d)}-\frac{cx}{(be-2\,cd)^2\,(ex^2+d)}+\frac{be}{2\,(be-2\,cd)^2\,d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-2\,\frac{c}{(be-2\,cd)^2\,\sqrt{de}}\arctan\left(\frac{ex}{\sqrt{de}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] c^2/(b*e-2*c*d)^2/((b*e-c*d)*c*e)^(1/2)*arctan(x*c*e/((b*e-c*d)*c
*e)^(1/2))+1/2/(b*e-2*c*d)^2/d*x/(e*x^2+d)*b*e-1/(b*e-2*c*d)^2*x/
(e*x^2+d)*c+1/2/(b*e-2*c*d)^2/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2
))*b*e-2/(b*e-2*c*d)^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*(e*x^2 + d)),x, algorithm="m
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.419647, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*(e*x^2 + d)),x, algorithm="f
```

```
[Out] [1/4*(2*(c*d*e*x^2 + c*d^2)*sqrt(-d*e)*sqrt(c/(c*d*e - b*e^2))*lo
g((c*e*x^2 - 2*(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d -
b*e)/(c*e*x^2 - c*d + b*e)) - 2*(2*c*d - b*e)*sqrt(-d*e)*x - (4*c
*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*log((2*d*e*x + (e*x^2 - d)*
sqrt(-d*e))/(e*x^2 + d)))/((4*c^2*d^4 - 4*b*c*d^3*e + b^2*d^2*e^2
 + (4*c^2*d^3*e - 4*b*c*d^2*e^2 + b^2*d*e^3)*x^2)*sqrt(-d*e)), 1/
2*((c*d*e*x^2 + c*d^2)*sqrt(d*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e
*x^2 - 2*(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(
c*e*x^2 - c*d + b*e)) - (2*c*d - b*e)*sqrt(d*e)*x - (4*c*d^2 - b*
d*e + (4*c*d*e - b*e^2)*x^2)*arctan(sqrt(d*e)*x/d))/((4*c^2*d^4 -
 4*b*c*d^3*e + b^2*d^2*e^2 + (4*c^2*d^3*e - 4*b*c*d^2*e^2 + b^2*d
*e^3)*x^2)*sqrt(d*e)), 1/4*(4*(c*d*e*x^2 + c*d^2)*sqrt(-d*e)*sqrt
(-c/(c*d*e - b*e^2))*arctan(-c*x/((c*d - b*e)*sqrt(-c/(c*d*e - b*
e^2)))) - 2*(2*c*d - b*e)*sqrt(-d*e)*x - (4*c*d^2 - b*d*e + (4*c*
d*e - b*e^2)*x^2)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2 +
 d)))/((4*c^2*d^4 - 4*b*c*d^3*e + b^2*d^2*e^2 + (4*c^2*d^3*e - 4*
b*c*d^2*e^2 + b^2*d*e^3)*x^2)*sqrt(-d*e)), 1/2*(2*(c*d*e*x^2 + c*
d^2)*sqrt(d*e)*sqrt(-c/(c*d*e - b*e^2))*arctan(-c*x/((c*d - b*e)*
sqrt(-c/(c*d*e - b*e^2)))) - (2*c*d - b*e)*sqrt(d*e)*x - (4*c*d^2
 - b*d*e + (4*c*d*e - b*e^2)*x^2)*arctan(sqrt(d*e)*x/d))/((4*c^2*
```

```
d^4 - 4*b*c*d^3*e + b^2*d^2*e^2 + (4*c^2*d^3*e - 4*b*c*d^2*e^2 +
b^2*d*e^3)*x^2)*sqrt(d*e))]
```

Sympy [A] time = 60.3676, size = 2664, normalized size = 19.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] x/(2*b*d**2*e - 4*c*d**3 + x**2*(2*b*d*e**2 - 4*c*d**2*e)) - sqrt
(-1/(d**3*e))*(b*e - 4*c*d)*log(x + (-b**7*d**3*e**8*(-1/(d**3*e)
)**(3/2)*(b*e - 4*c*d)**3/(2*(b*e - 2*c*d)**6) + 7*b**6*c*d**4*e*
*7*(-1/(d**3*e))**(3/2)*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 - 79*b*
*5*c**2*d**5*e**6*(-1/(d**3*e))**(3/2)*(b*e - 4*c*d)**3/(2*(b*e -
 2*c*d)**6) - b**5*e**5*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(2*(b*e -
 2*c*d)**2) + 117*b**4*c**3*d**6*e**5*(-1/(d**3*e))**(3/2)*(b*e -
 4*c*d)**3/(b*e - 2*c*d)**6 + 7*b**4*c*d*e**4*sqrt(-1/(d**3*e))*(
b*e - 4*c*d)/(b*e - 2*c*d)**2 - 196*b**3*c**4*d**7*e**4*(-1/(d**3
*e))**(3/2)*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 - 73*b**3*c**2*d**2
*e**3*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(2*(b*e - 2*c*d)**2) + 184*
b**2*c**5*d**8*e**3*(-1/(d**3*e))**(3/2)*(b*e - 4*c*d)**3/(b*e -
2*c*d)**6 + 86*b**2*c**3*d**3*e**2*sqrt(-1/(d**3*e))*(b*e - 4*c*d
)/(b*e - 2*c*d)**2 - 88*b*c**6*d**9*e**2*(-1/(d**3*e))**(3/2)*(b*
e - 4*c*d)**3/(b*e - 2*c*d)**6 - 88*b*c**4*d**4*e*sqrt(-1/(d**3*e
))*(b*e - 4*c*d)/(b*e - 2*c*d)**2 + 16*c**7*d**10*e*(-1/(d**3*e))
**(3/2)*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 + 36*c**5*d**5*sqrt(-1/
(d**3*e))*(b*e - 4*c*d)/(b*e - 2*c*d)**2)/(b**2*c**2*e**2 - 9*b*c
**3*d*e + 20*c**4*d**2))/(4*(b*e - 2*c*d)**2) + sqrt(-1/(d**3*e))
*(b*e - 4*c*d)*log(x + (b**7*d**3*e**8*(-1/(d**3*e))**(3/2)*(b*e
- 4*c*d)**3/(2*(b*e - 2*c*d)**6) - 7*b**6*c*d**4*e**7*(-1/(d**3*e
))**(3/2)*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 + 79*b**5*c**2*d**5*e
**6*(-1/(d**3*e))**(3/2)*(b*e - 4*c*d)**3/(2*(b*e - 2*c*d)**6) +
b**5*e**5*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(2*(b*e - 2*c*d)**2) -
117*b**4*c**3*d**6*e**5*(-1/(d**3*e))**(3/2)*(b*e - 4*c*d)**3/(b*
e - 2*c*d)**6 - 7*b**4*c*d*e**4*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(
b*e - 2*c*d)**2 + 196*b**3*c**4*d**7*e**4*(-1/(d**3*e))**(3/2)*(b
*e - 4*c*d)**3/(b*e - 2*c*d)**6 + 73*b**3*c**2*d**2*e**3*sqrt(-1/
(d**3*e))*(b*e - 4*c*d)/(2*(b*e - 2*c*d)**2) - 184*b**2*c**5*d**8
*e**3*(-1/(d**3*e))**(3/2)*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 - 86
*b**2*c**3*d**3*e**2*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(b*e - 2*c*d
)**2 + 88*b*c**6*d**9*e**2*(-1/(d**3*e))**(3/2)*(b*e - 4*c*d)**3/
(b*e - 2*c*d)**6 + 88*b*c**4*d**4*e*sqrt(-1/(d**3*e))*(b*e - 4*c*
d)/(b*e - 2*c*d)**2 - 16*c**7*d**10*e*(-1/(d**3*e))**(3/2)*(b*e -
 4*c*d)**3/(b*e - 2*c*d)**6 - 36*c**5*d**5*sqrt(-1/(d**3*e))*(b*e
 - 4*c*d)/(b*e - 2*c*d)**2)/(b**2*c**2*e**2 - 9*b*c**3*d*e + 20*c
**4*d**2))/(4*(b*e - 2*c*d)**2) - sqrt(-c**3/(e*(b*e - c*d)))*log
(x + (-4*b**7*d**3*e**8*(-c**3/(e*(b*e - c*d)))**(3/2)/(b*e - 2*c
*d)**6 + 56*b**6*c*d**4*e**7*(-c**3/(e*(b*e - c*d)))**(3/2)/(b*e
- 2*c*d)**6 - 316*b**5*c**2*d**5*e**6*(-c**3/(e*(b*e - c*d)))**(3
/2)/(b*e - 2*c*d)**6 - b**5*e**5*sqrt(-c**3/(e*(b*e - c*d)))/(b*e
 - 2*c*d)**2 + 936*b**4*c**3*d**6*e**5*(-c**3/(e*(b*e - c*d)))**(
3/2)/(b*e - 2*c*d)**6 + 14*b**4*c*d*e**4*sqrt(-c**3/(e*(b*e - c*d
)))/(b*e - 2*c*d)**2 - 1568*b**3*c**4*d**7*e**4*(-c**3/(e*(b*e -
c*d)))**(3/2)/(b*e - 2*c*d)**6 - 73*b**3*c**2*d**2*e**3*sqrt(-c**
3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 + 1472*b**2*c**5*d**8*e**3*(-
c**3/(e*(b*e - c*d)))**(3/2)/(b*e - 2*c*d)**6 + 172*b**2*c**3*d**
3*e**2*sqrt(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 - 704*b*c**6*
d**9*e**2*(-c**3/(e*(b*e - c*d)))**(3/2)/(b*e - 2*c*d)**6 - 176*b
*c**4*d**4*e*sqrt(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 + 128*c
**7*d**10*e*(-c**3/(e*(b*e - c*d)))**(3/2)/(b*e - 2*c*d)**6 + 72*
c**5*d**5*sqrt(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2)/(b**2*c**
2*e**2 - 9*b*c**3*d*e + 20*c**4*d**2))/(2*(b*e - 2*c*d)**2) + sqr
t(-c**3/(e*(b*e - c*d)))*log(x + (4*b**7*d**3*e**8*(-c**3/(e*(b*e
 - c*d)))**(3/2)/(b*e - 2*c*d)**6 - 56*b**6*c*d**4*e**7*(-c**3/(e
*(b*e - c*d)))**(3/2)/(b*e - 2*c*d)**6 + 316*b**5*c**2*d**5*e**6*
```

```
(-c**3/(e*(b*e - c*d)))**(3/2)/(b*e - 2*c*d)**6 + b**5*e**5*sqrt(
-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 - 936*b**4*c**3*d**6*e**5
*(-c**3/(e*(b*e - c*d)))**(3/2)/(b*e - 2*c*d)**6 - 14*b**4*c*d*e*
*4*sqrt(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 + 1568*b**3*c**4*
d**7*e**4*(-c**3/(e*(b*e - c*d)))**(3/2)/(b*e - 2*c*d)**6 + 73*b*
*3*c**2*d**2*e**3*sqrt(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 -
1472*b**2*c**5*d**8*e**3*(-c**3/(e*(b*e - c*d)))**(3/2)/(b*e - 2*
c*d)**6 - 172*b**2*c**3*d**3*e**2*sqrt(-c**3/(e*(b*e - c*d)))/(b*
e - 2*c*d)**2 + 704*b*c**6*d**9*e**2*(-c**3/(e*(b*e - c*d)))**(3/
2)/(b*e - 2*c*d)**6 + 176*b*c**4*d**4*e*sqrt(-c**3/(e*(b*e - c*d)
))/(b*e - 2*c*d)**2 - 128*c**7*d**10*e*(-c**3/(e*(b*e - c*d)))**(
3/2)/(b*e - 2*c*d)**6 - 72*c**5*d**5*sqrt(-c**3/(e*(b*e - c*d)))/
(b*e - 2*c*d)**2)/(b**2*c**2*e**2 - 9*b*c**3*d*e + 20*c**4*d**2))
/(2*(b*e - 2*c*d)**2)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*(e*x^2 + d)),x, algorithm="g
```

```
[Out] Timed out
```

3.223 $\int \frac{1}{(d+ex^2)^2(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx$

Optimal. Leaf size=187

$$-\frac{(3b^2e^2-16bcde+28c^2d^2)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}(2cd-be)^3}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^3}-\frac{x(10cd-3be)}{8d^2(d+ex^2)(2cd-be)^2}-\frac{x}{4d(d+ex^2)^2(2cd-be)}$$

```
[Out] -x/(4*d*(2*c*d - b*e)*(d + e*x^2)^2) - ((10*c*d - 3*b*e)*x)/(8*d^
2*(2*c*d - b*e)^2*(d + e*x^2)) - ((28*c^2*d^2 - 16*b*c*d*e + 3*b^
2*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]*(2*c*d - b
*e)^3) - (c^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(
Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^3)
```

Rubi [A] time = 0.609655, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{(3b^2e^2-16bcde+28c^2d^2)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}(2cd-be)^3}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^3}-\frac{x(10cd-3be)}{8d^2(d+ex^2)(2cd-be)^2}-\frac{x}{4d(d+ex^2)^2(2cd-be)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)^2*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -x/(4*d*(2*c*d - b*e)*(d + e*x^2)^2) - ((10*c*d - 3*b*e)*x)/(8*d^
2*(2*c*d - b*e)^2*(d + e*x^2)) - ((28*c^2*d^2 - 16*b*c*d*e + 3*b^
2*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]*(2*c*d - b
*e)^3) - (c^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(
Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^3)
```

Rubi in Sympy [A] time = 135.329, size = 170, normalized size = 0.91

$$-\frac{c^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{\sqrt{e}\,(be-2cd)^3\sqrt{be-cd}}+\frac{x}{4d\,(d+ex^2)^2\,(be-2cd)}+\frac{x\,(3be-10cd)}{8d^2\,(d+ex^2)\,(be-2cd)^2}+\frac{(3b^2e^2-16bcde+28c^2d^2)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{\frac{5}{2}}\sqrt{e}\,(be-2cd)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)**2/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] -c**(5/2)*atan(sqrt(c)*sqrt(e)*x/sqrt(b*e - c*d))/(sqrt(e)*(b*e -
 2*c*d)**3*sqrt(b*e - c*d)) + x/(4*d*(d + e*x**2)**2*(b*e - 2*c*d
)) + x*(3*b*e - 10*c*d)/(8*d**2*(d + e*x**2)*(b*e - 2*c*d)**2) +
(3*b**2*e**2 - 16*b*c*d*e + 28*c**2*d**2)*atan(sqrt(e)*x/sqrt(d))
/(8*d**(5/2)*sqrt(e)*(b*e - 2*c*d)**3)
```

Mathematica [A] time = 0.777968, size = 177, normalized size = 0.95

$$\frac{1}{8}\left(-\frac{\left(3b^2e^2-16bcde+28c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\sqrt{e}(2cd-be)^3}-\frac{\frac{8c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{\sqrt{e}\sqrt{be-cd}}+\frac{x(be-2cd)\left(2cd\left(7d+5ex^2\right)-be\left(5d+3ex^2\right)\right)}{d^2\left(d+ex^2\right)^2}}{(be-2cd)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] (-(((28*c^2*d^2 - 16*b*c*d*e + 3*b^2*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(d^(5/2)*Sqrt[e]*(2*c*d - b*e)^3)) - (((-2*c*d + b*e)*x*(-(
b*e*(5*d + 3*e*x^2)) + 2*c*d*(7*d + 5*e*x^2)))/(d^2*(d + e*x^2)^2
) + (8*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(S
qrt[e]*Sqrt[-(c*d) + b*e]))/(-2*c*d + b*e)^3)/8
```

Maple [A] time = 0.017, size = 319, normalized size = 1.7

$$-\frac{c^3}{(be-2\,cd)^3}\arctan\left(cex\frac{1}{\sqrt{(be-cd)\,ce}}\right)\frac{1}{\sqrt{(be-cd)\,ce}}+\frac{3\,e^3x^3b^2}{8\,(be-2\,cd)^3\,(ex^2+d)^2\,d^2}$$
$$-2\,\frac{e^2x^3bc}{(be-2\,cd)^3\,(ex^2+d)^2\,d}+\frac{5\,ex^3c^2}{2\,(be-2\,cd)^3\,(ex^2+d)^2}+\frac{5\,xb^2e^2}{8\,(be-2\,cd)^3\,(ex^2+d)^2\,d}$$
$$-3\,\frac{bxce}{(be-2\,cd)^3\,(ex^2+d)^2}+\frac{7\,dxc^2}{2\,(be-2\,cd)^3\,(ex^2+d)^2}+\frac{3\,b^2e^2}{8\,(be-2\,cd)^3\,d^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$
$$-2\,\frac{bce}{(be-2\,cd)^3\,d\sqrt{de}}\arctan\left(\frac{ex}{\sqrt{de}}\right)+\frac{7\,c^2}{2\,(be-2\,cd)^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] -c^3/(b*e-2*c*d)^3/((b*e-c*d)*c*e)^(1/2)*arctan(x*c*e/((b*e-c*d)*
c*e)^(1/2))+3/8/(b*e-2*c*d)^3/(e*x^2+d)^2*e^3/d^2*x^3*b^2-2/(b*e-
2*c*d)^3/(e*x^2+d)^2*e^2/d*x^3*b*c+5/2/(b*e-2*c*d)^3/(e*x^2+d)^2*
e*x^3*c^2+5/8/(b*e-2*c*d)^3/(e*x^2+d)^2/d*x*b^2*e^2-3/(b*e-2*c*d)
^3/(e*x^2+d)^2*x*b*c*e+7/2/(b*e-2*c*d)^3/(e*x^2+d)^2*d*x*c^2+3/8/
(b*e-2*c*d)^3/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b^2*e^2-2/(
b*e-2*c*d)^3/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b*c*e+7/2/(b*e
-2*c*d)^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*(e*x^2 + d)^2),x, algorithm=
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32221, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*(e*x^2 + d)^2),x, algorithm=
```

```
[Out] [-1/16*(8*(c^2*d^2*e^2*x^4 + 2*c^2*d^3*e*x^2 + c^2*d^4)*sqrt(-d*e
)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x^2 + 2*(c*d*e - b*e^2)*x*sqrt
(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*d + b*e)) + (28*c^2
*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 - 16*b*c*d*
e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d
*e^3)*x^2)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d)) +
2*((20*c^2*d^2*e - 16*b*c*d*e^2 + 3*b^2*e^3)*x^3 + (28*c^2*d^3 -
24*b*c*d^2*e + 5*b^2*d*e^2)*x)*sqrt(-d*e))/((8*c^3*d^7 - 12*b*c^2
*d^6*e + 6*b^2*c*d^5*e^2 - b^3*d^4*e^3 + (8*c^3*d^5*e^2 - 12*b*c^
2*d^4*e^3 + 6*b^2*c*d^3*e^4 - b^3*d^2*e^5)*x^4 + 2*(8*c^3*d^6*e -
 12*b*c^2*d^5*e^2 + 6*b^2*c*d^4*e^3 - b^3*d^3*e^4)*x^2)*sqrt(-d*e
)), -1/8*(4*(c^2*d^2*e^2*x^4 + 2*c^2*d^3*e*x^2 + c^2*d^4)*sqrt(d*
e)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x^2 + 2*(c*d*e - b*e^2)*x*sqr
t(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*d + b*e)) + (28*c^
2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 - 16*b*c*d
*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*
d*e^3)*x^2)*arctan(sqrt(d*e)*x/d) + ((20*c^2*d^2*e - 16*b*c*d*e^2
 + 3*b^2*e^3)*x^3 + (28*c^2*d^3 - 24*b*c*d^2*e + 5*b^2*d*e^2)*x)*
sqrt(d*e))/((8*c^3*d^7 - 12*b*c^2*d^6*e + 6*b^2*c*d^5*e^2 - b^3*d
^4*e^3 + (8*c^3*d^5*e^2 - 12*b*c^2*d^4*e^3 + 6*b^2*c*d^3*e^4 - b^
3*d^2*e^5)*x^4 + 2*(8*c^3*d^6*e - 12*b*c^2*d^5*e^2 + 6*b^2*c*d^4*
e^3 - b^3*d^3*e^4)*x^2)*sqrt(d*e)), 1/16*(16*(c^2*d^2*e^2*x^4 + 2
*c^2*d^3*e*x^2 + c^2*d^4)*sqrt(-d*e)*sqrt(-c/(c*d*e - b*e^2))*arc
tan(-c*x/((c*d - b*e)*sqrt(-c/(c*d*e - b*e^2)))) - (28*c^2*d^4 -
16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 - 16*b*c*d*e^3 + 3
*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*e^3)*x
^2)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d)) - 2*((20*
c^2*d^2*e - 16*b*c*d*e^2 + 3*b^2*e^3)*x^3 + (28*c^2*d^3 - 24*b*c*
d^2*e + 5*b^2*d*e^2)*x)*sqrt(-d*e))/((8*c^3*d^7 - 12*b*c^2*d^6*e
+ 6*b^2*c*d^5*e^2 - b^3*d^4*e^3 + (8*c^3*d^5*e^2 - 12*b*c^2*d^4*e
^3 + 6*b^2*c*d^3*e^4 - b^3*d^2*e^5)*x^4 + 2*(8*c^3*d^6*e - 12*b*c
^2*d^5*e^2 + 6*b^2*c*d^4*e^3 - b^3*d^3*e^4)*x^2)*sqrt(-d*e)), 1/8
*(8*(c^2*d^2*e^2*x^4 + 2*c^2*d^3*e*x^2 + c^2*d^4)*sqrt(d*e)*sqrt(
-c/(c*d*e - b*e^2))*arctan(-c*x/((c*d - b*e)*sqrt(-c/(c*d*e - b*e
^2)))) - (28*c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2
*e^2 - 16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d
^2*e^2 + 3*b^2*d*e^3)*x^2)*arctan(sqrt(d*e)*x/d) - ((20*c^2*d^2*e
 - 16*b*c*d*e^2 + 3*b^2*e^3)*x^3 + (28*c^2*d^3 - 24*b*c*d^2*e + 5
*b^2*d*e^2)*x)*sqrt(d*e))/((8*c^3*d^7 - 12*b*c^2*d^6*e + 6*b^2*c*
d^5*e^2 - b^3*d^4*e^3 + (8*c^3*d^5*e^2 - 12*b*c^2*d^4*e^3 + 6*b^2
*c*d^3*e^4 - b^3*d^2*e^5)*x^4 + 2*(8*c^3*d^6*e - 12*b*c^2*d^5*e^2
 + 6*b^2*c*d^4*e^3 - b^3*d^3*e^4)*x^2)*sqrt(d*e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*(e*x^2 + d)^2),x, algorithm=
```

```
[Out] Timed out
```

3.224 $\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=139

$$-\frac{(2cd-be)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c^2\sqrt{e}\sqrt{cd-be}}+\frac{(5cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}}+\frac{x\sqrt{d+ex^2}}{2c}$$

```
[Out] (x*Sqrt[d + e*x^2])/(2*c) + ((5*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/
Sqrt[d + e*x^2]])/(2*c^2*Sqrt[e]) - ((2*c*d - b*e)^(3/2)*ArcTanh[
(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])
/(c^2*Sqrt[e]*Sqrt[c*d - b*e])
```

Rubi [A] time = 0.551695, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171

$$-\frac{(2cd-be)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c^2\sqrt{e}\sqrt{cd-be}}+\frac{(5cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}}+\frac{x\sqrt{d+ex^2}}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^(5/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c) + ((5*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/
Sqrt[d + e*x^2]])/(2*c^2*Sqrt[e]) - ((2*c*d - b*e)^(3/2)*ArcTanh[
(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])
/(c^2*Sqrt[e]*Sqrt[c*d - b*e])
```

Rubi in Sympy [A] time = 102.221, size = 124, normalized size = 0.89

$$\frac{x\sqrt{d+ex^2}}{2c}+\frac{(be-2cd)^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{e}x\sqrt{be-2cd}}{\sqrt{d+ex^2}\sqrt{be-cd}}\right)}{c^2\sqrt{e}\sqrt{be-cd}}-\frac{(2be-5cd)\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((e*x**2+d)**(5/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-
c*d**2),x)
```

```
[Out] x*sqrt(d + e*x**2)/(2*c) + (b*e - 2*c*d)**(3/2)*atanh(sqrt(e)*x*s
qrt(b*e - 2*c*d)/(sqrt(d + e*x**2)*sqrt(b*e - c*d)))/(c**2*sqrt(e
)*sqrt(b*e - c*d)) - (2*b*e - 5*c*d)*atanh(sqrt(e)*x/sqrt(d + e*x
**2))/(2*c**2*sqrt(e))
```

Mathematica [A] time = 0.392921, size = 134, normalized size = 0.96

$$-\frac{\frac{(2be-5cd)\log\left(\sqrt{e}\sqrt{d+ex^2}+ex\right)}{\sqrt{e}}-\frac{2(be-2cd)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{be-2cd}}{\sqrt{d+ex^2}\sqrt{be-cd}}\right)}{\sqrt{e}\sqrt{be-cd}}-cx\sqrt{d+ex^2}}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^(5/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(-(c*x*Sqrt[d + e*x^2]) - (2*(-2*c*d + b*e)^(3/2)*ArcTanh[(Sqrt[
e]*Sqrt[-2*c*d + b*e]*x)/(Sqrt[-(c*d) + b*e]*Sqrt[d + e*x^2])])/(
Sqrt[e]*Sqrt[-(c*d) + b*e]) + ((-5*c*d + 2*b*e)*Log[e*x + Sqrt[e]
*Sqrt[d + e*x^2]])/Sqrt[e])/(2*c^2)
```

Maple [B] time = 0.069, size = 7043, normalized size = 50.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(5/2)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm=
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.783323, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(5/2)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm=
```

```
[Out] [1/4*(2*sqrt(e*x^2 + d)*c*sqrt(e)*x - (2*c*d - b*e)*sqrt(e)*sqrt(
(2*c*d - b*e)/(c*d*e - b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d
^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c
^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 + 4*((3*c^2*d^2*e^2
- 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c*d^2*e^2 + b^2
*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2)))/(
c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^
2)*x^2)) - (5*c*d - 2*b*e)*log(2*sqrt(e*x^2 + d)*e*x - (2*e*x^2 +
 d)*sqrt(e)))/(c^2*sqrt(e)), 1/4*(2*sqrt(e*x^2 + d)*c*sqrt(-e)*x
- (2*c*d - b*e)*sqrt(-e)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2))*log(
(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d
*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d
*e^3)*x^2 + 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c
^2*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*
c*d - b*e)/(c*d*e - b*e^2)))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e +
 b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)) + 2*(5*c*d - 2*b*e)*arctan
(sqrt(-e)*x/sqrt(e*x^2 + d)))/(c^2*sqrt(-e)), 1/4*(2*sqrt(e*x^2 +
 d)*c*sqrt(e)*x - 2*(2*c*d - b*e)*sqrt(e)*sqrt(-(2*c*d - b*e)/(c*
d*e - b*e^2))*arctan(1/2*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2
)/((c*d*e - b*e^2)*sqrt(e*x^2 + d)*x*sqrt(-(2*c*d - b*e)/(c*d*e -
 b*e^2)))) - (5*c*d - 2*b*e)*log(2*sqrt(e*x^2 + d)*e*x - (2*e*x^2
 + d)*sqrt(e)))/(c^2*sqrt(e)), 1/2*(sqrt(e*x^2 + d)*c*sqrt(-e)*x
- (2*c*d - b*e)*sqrt(-e)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))*arc
tan(1/2*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)/((c*d*e - b*e^2
)*sqrt(e*x^2 + d)*x*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2)))) + (5*c
```

```
*d - 2*b*e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(c^2*sqrt(-e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex^{2}\right)^{\frac{3}{2}}}{be - cd + cex^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(5/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)

[Out] Integral((d + e*x**2)**(3/2)/(b*e - c*d + c*e*x**2), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(5/2)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm=

[Out] Timed out
```

3.225 $\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=108

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{\sqrt{2cd-be}\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c\sqrt{e}\sqrt{cd-be}}$$

```
[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(c*Sqrt[e]) - (Sqrt[2*c*d -
b*e]*ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[
d + e*x^2])])/(c*Sqrt[e]*Sqrt[c*d - b*e])
```

Rubi [A] time = 0.257815, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{\sqrt{2cd-be}\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c\sqrt{e}\sqrt{cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(c*Sqrt[e]) - (Sqrt[2*c*d -
b*e]*ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[
d + e*x^2])])/(c*Sqrt[e]*Sqrt[c*d - b*e])
```

Rubi in Sympy [A] time = 66.5288, size = 94, normalized size = 0.87

$$-\frac{\sqrt{be-2cd}\,\operatorname{atanh}\left(\frac{\sqrt{e}x\sqrt{be-2cd}}{\sqrt{d+ex^2}\sqrt{be-cd}}\right)}{c\sqrt{e}\sqrt{be-cd}}+\frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((e*x**2+d)**(3/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-
c*d**2),x)
```

```
[Out] -sqrt(b*e - 2*c*d)*atanh(sqrt(e)*x*sqrt(b*e - 2*c*d)/(sqrt(d + e*
x**2)*sqrt(b*e - c*d)))/(c*sqrt(e)*sqrt(b*e - c*d)) + atanh(sqrt(
e)*x/sqrt(d + e*x**2))/(c*sqrt(e))
```

Mathematica [A] time = 0.0925788, size = 103, normalized size = 0.95

$$-\frac{\frac{\sqrt{be-2cd}\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{be-2cd}}{\sqrt{d+ex^2}\sqrt{be-cd}}\right)}{\sqrt{be-cd}}-\log\left(\sqrt{e}\sqrt{d+ex^2}+ex\right)}{c\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(((Sqrt[-2*c*d + b*e]*ArcTanh[(Sqrt[e]*Sqrt[-2*c*d + b*e]*x)/(Sq
rt[-(c*d) + b*e]*Sqrt[d + e*x^2])])/Sqrt[-(c*d) + b*e] - Log[e*x
```

```
+ Sqrt[e]*Sqrt[d + e*x^2]])/(c*Sqrt[e]))
```

Maple [B] time = 0.031, size = 4332, normalized size = 40.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/6*e*c/(-d*e)^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d
*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))*((x-1/e*(-d*e)^(1/2))^2*e+2*(
-d*e)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(3/2)+1/4*e*c/((-d*e)^(1/2)*c+(
-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))*((
x-1/e*(-d*e)^(1/2))^2*e+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(1/2
)*x+1/4*e^(1/2)*c/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)
^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))*d*ln(((x-1/e*(-d*e)^(1/2))*e+(-d
*e)^(1/2))/e^(1/2)+((x-1/e*(-d*e)^(1/2))^2*e+2*(-d*e)^(1/2)*(x-1/
e*(-d*e)^(1/2)))^(1/2))-1/6*e*c/(-d*e)^(1/2)/((-d*e)^(1/2)*c+(-(b
*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))*((x+1
/e*(-d*e)^(1/2))^2*e-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(3/2)+1
/4*e*c/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-
(b*e-c*d)*c*e)^(1/2))*((x+1/e*(-d*e)^(1/2))^2*e-2*(-d*e)^(1/2)*(x
+1/e*(-d*e)^(1/2)))^(1/2)*x+1/4*e^(1/2)*c/((-d*e)^(1/2)*c+(-(b*e-
c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))*d*ln(((x
+1/e*(-d*e)^(1/2))*e-(-d*e)^(1/2))/e^(1/2)+((x+1/e*(-d*e)^(1/2))^
2*e-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(1/2))-1/6*e*c^2/((-d*e)
^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)
^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*
e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-
2*c*d)/c)^(3/2)-1/4*e*c/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(
(-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))*((x-(-(b*e-c*d)*c*e)^(1/2)
/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/
e)-(b*e-2*c*d)/c)^(1/2)*x-5/4*e^(1/2)*c/((-d*e)^(1/2)*c+(-(b*e-c*
d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))*ln(((-(b*e
-c*d)*c*e)^(1/2)/c+(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)*e)/e^(1/2)+((x-
(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(
b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))*d+1/2*e^2*c/((-d*e
)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e
)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2
*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e
-2*c*d)/c)^(1/2)*b-e*c^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/
((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((
x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(
-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)*d+1/2*e^(3/2)/((-
d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*
c*e)^(1/2))*ln(((-(b*e-c*d)*c*e)^(1/2)/c+(x-(-(b*e-c*d)*c*e)^(1/2
)/c/e)*e)/e^(1/2)+((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*
d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/
2))*b+1/2*e^3/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/
2)*c-(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)
/c)^(1/2)*ln((-2*(b*e-2*c*d)/c+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b
*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*((x-(-(b*e-c*d)*
c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e
)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)
)*b^2-2*e^2*c/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/
2)*c-(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)
/c)^(1/2)*ln((-2*(b*e-2*c*d)/c+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b
*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*((x-(-(b*e-c*d)*
c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e
)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)
)*b*d+2*e*c^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/
2)*c-(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)
/c)^(1/2)*ln((-2*(b*e-2*c*d)/c+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b
*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*((x-(-(b*e-c*d)*
c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e
)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)
```

```
)*d^2+1/6*e*c^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(
1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((x+(-(b*e-
c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d
)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(3/2)-1/4*e*c/((-d*e)^(1/2)*c+(-
(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))*((x
+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-
(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)*x-5/4*e^(1/2)*c/((
-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)
*c*e)^(1/2))*ln((-(-(b*e-c*d)*c*e)^(1/2)/c+(x+(-(b*e-c*d)*c*e)^(1
/2)/c/e)*e)/e^(1/2)+((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-
c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(
1/2))*d-1/2*e^2*c/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)
^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((x+(-(b*
e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c
*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)*b+e*c^2/((-d*e)^(1/2)*c+
(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/(
-(b*e-c*d)*c*e)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*
e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)
^(1/2)*d+1/2*e^(3/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d
*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))*ln((-(-(b*e-c*d)*c*e)^(1/2)/c
+(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)*e)/e^(1/2)+((x+(-(b*e-c*d)*c*e)^(
1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2
)/c/e)-(b*e-2*c*d)/c)^(1/2))*b-1/2*e^3/((-d*e)^(1/2)*c+(-(b*e-c*d
)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)
*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-(b*e-
c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/
c)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(
1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-
(b*e-c*d)*c*e)^(1/2)/c/e))*b^2+2*e^2*c/((-d*e)^(1/2)*c+(-(b*e-c*d
)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)
*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-(b*e-
c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/
c)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(
1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-
(b*e-c*d)*c*e)^(1/2)/c/e))*b*d-2*e*c^2/((-d*e)^(1/2)*c+(-(b*e-c*d
)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)
*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-(b*e-
c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/
c)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(
1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-
(b*e-c*d)*c*e)^(1/2)/c/e))*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm=
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.385129, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm=
```

```
[Out] [1/4*(sqrt(e)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2))*log((c^2*d^4 -
2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^
2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 -
```

```
 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e -
2*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/
(c*d*e - b*e^2)))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 -
2*(c^2*d*e - b*c*e^2)*x^2)) + 2*log(-2*sqrt(e*x^2 + d)*e*x - (2*e
*x^2 + d)*sqrt(e)))/(c*sqrt(e)), 1/4*(sqrt(-e)*sqrt((2*c*d - b*e)
/(c*d*e - b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*
c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11
*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 - 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3
 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x)*sq
rt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2)))/(c^2*e^2*x^4 +
 c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)) + 4*
arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(c*sqrt(-e)), -1/2*(sqrt(e)*s
qrt(-(2*c*d - b*e)/(c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b*d*e + (
3*c*d*e - 2*b*e^2)*x^2)/((c*d*e - b*e^2)*sqrt(e*x^2 + d)*x*sqrt(-
(2*c*d - b*e)/(c*d*e - b*e^2)))) - log(-2*sqrt(e*x^2 + d)*e*x - (
2*e*x^2 + d)*sqrt(e)))/(c*sqrt(e)), -1/2*(sqrt(-e)*sqrt(-(2*c*d -
 b*e)/(c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b*d*e + (3*c*d*e - 2*b
*e^2)*x^2)/((c*d*e - b*e^2)*sqrt(e*x^2 + d)*x*sqrt(-(2*c*d - b*e)
/(c*d*e - b*e^2)))) - 2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(c*sq
rt(-e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d + ex^2}}{be - cd + cex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(b*e - c*d + c*e*x**2), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm=
```

```
[Out] Timed out
```

3.226 $\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=76

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

```
[Out] -(ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d +
 e*x^2])]/(Sqrt[e]*Sqrt[c*d - b*e]*Sqrt[2*c*d - b*e]))
```

Rubi [A] time = 0.159295, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.073

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d +
 e*x^2])]/(Sqrt[e]*Sqrt[c*d - b*e]*Sqrt[2*c*d - b*e]))
```

Rubi in Sympy [A] time = 50.8589, size = 66, normalized size = 0.87

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{e}x\sqrt{be-2cd}}{\sqrt{d+ex^2}\sqrt{be-cd}}\right)}{\sqrt{e}\sqrt{be-2cd}\sqrt{be-cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((e*x**2+d)**(1/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-
c*d**2),x)
```

```
[Out] atanh(sqrt(e)*x*sqrt(b*e - 2*c*d)/(sqrt(d + e*x**2)*sqrt(b*e - c*
d)))/(sqrt(e)*sqrt(b*e - 2*c*d)*sqrt(b*e - c*d))
```

Mathematica [A] time = 0.061687, size = 73, normalized size = 0.96

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{be-2cd}}{\sqrt{d+ex^2}\sqrt{be-cd}}\right)}{\sqrt{e}\sqrt{be-2cd}\sqrt{be-cd}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[e]*Sqrt[-2*c*d + b*e]*x)/(Sqrt[-(c*d) + b*e]*Sqrt[d
 + e*x^2])]/(Sqrt[e]*Sqrt[-2*c*d + b*e]*Sqrt[-(c*d) + b*e])
```

Maple [B] time = 0.031, size = 2264, normalized size = 29.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/2*e*c/(-d*e)^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d
*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))*((x-1/e*(-d*e)^(1/2))^2*e+2*(
-d*e)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(1/2)+1/2*e^(1/2)*c/((-d*e)^(1/
2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/
2))*ln(((x-1/e*(-d*e)^(1/2))*e+(-d*e)^(1/2))/e^(1/2)+((x-1/e*(-d*
e)^(1/2))^2*e+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(1/2))-1/2*e*c
/(-d*e)^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/
2)*c-(-(b*e-c*d)*c*e)^(1/2))*((x+1/e*(-d*e)^(1/2))^2*e-2*(-d*e)^(
1/2)*(x+1/e*(-d*e)^(1/2)))^(1/2)+1/2*e^(1/2)*c/((-d*e)^(1/2)*c+(-
(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))*ln(
((x+1/e*(-d*e)^(1/2))*e-(-d*e)^(1/2))/e^(1/2)+((x+1/e*(-d*e)^(1/2
))^2*e-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(1/2))-1/2*e*c^2/((-d
*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c
*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)
^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b
*e-2*c*d)/c)^(1/2)-1/2*e^(1/2)*c/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)
^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))*ln(((-(b*e-c*d)*c
*e)^(1/2)/c+(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)*e)/e^(1/2)+((x-(-(b*e-
c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d
)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))-1/2*e^2*c/((-d*e)^(1/2)*c
+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/
(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/
c+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(
b*e-2*c*d)/c)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-
c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(
1/2))/(x-(-(b*e-c*d)*c*e)^(1/2)/c/e))*b+e*c^2/((-d*e)^(1/2)*c+(-(
b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/(-(b
*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c+2*
(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-
2*c*d)/c)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)
*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)
)/(x-(-(b*e-c*d)*c*e)^(1/2)/c/e))*d+1/2*e*c^2/((-d*e)^(1/2)*c+(-(
b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/(-(b
*e-c*d)*c*e)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c
*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1
/2)-1/2*e^(1/2)*c/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)
^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))*ln((-(-(b*e-c*d)*c*e)^(1/2)/c+(x
+(-(b*e-c*d)*c*e)^(1/2)/c/e)*e)/e^(1/2)+((x+(-(b*e-c*d)*c*e)^(1/2
)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c
/e)-(b*e-2*c*d)/c)^(1/2))+1/2*e^2*c/((-d*e)^(1/2)*c+(-(b*e-c*d)*c
*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*
e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-(b*e-c*d
)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^
(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2
)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-(b*
e-c*d)*c*e)^(1/2)/c/e))*b-e*c^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^
(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(
1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-(b*e-c*d)*c*
e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2
)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*
(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-(b*e-c*
d)*c*e)^(1/2)/c/e))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}}{ce^2x^4+be^2x^2-cd^2+bde}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x^2 + d)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm="m
```

```
[Out] integrate(sqrt(e*x^2 + d)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)
```

```
, x)
```

Fricas [A] time = 0.367445, size = 1, normalized size = 0.01

$$\left[\frac{\log\left(\frac{\left(c^2d^4-2\,bcd^3e+b^2d^2e^2+\left(17\,c^2d^2e^2-24\,bcde^3+8\,b^2e^4\right)x^4+2\,\left(7\,c^2d^3e-11\,bcd^2e^2+4\,b^2de^3\right)x^2\right)\sqrt{2\,c^2d^2e-3\,bcde^2+b^2e^3}-4\,\left(\left(6\,c^3d^3e^2-13\,bc^2d^2e^3+9\,b^2\right.\right.}{c^2e^2x^4+c^2d^2-2\,bcde+b^2e^2-2\,(c^2de-bce^2)x^2}\right.}{4\,\sqrt{2\,c^2d^2e-3\,bcde^2+b^2e^3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm="f
```

```
[Out] [1/4*log(((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2
- 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2
 + 4*b^2*d*e^3)*x^2)*sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3) -
4*((6*c^3*d^3*e^2 - 13*b*c^2*d^2*e^3 + 9*b^2*c*d*e^4 - 2*b^3*e^5)
*x^3 + (2*c^3*d^4*e - 5*b*c^2*d^3*e^2 + 4*b^2*c*d^2*e^3 - b^3*d*e
^4)*x)*sqrt(e*x^2 + d))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*
e^2 - 2*(c^2*d*e - b*c*e^2)*x^2))/sqrt(2*c^2*d^2*e - 3*b*c*d*e^2
+ b^2*e^3), 1/2*arctan(-1/2*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2
*e^3)*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)/((2*c^2*d^2*e - 3
*b*c*d*e^2 + b^2*e^3)*sqrt(e*x^2 + d)*x))/sqrt(-2*c^2*d^2*e + 3*b
*c*d*e^2 - b^2*e^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex^2}\,(be-cd+cex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x**2)*(b*e - c*d + c*e*x**2)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e),x, algorithm="g
```

```
[Out] Timed out
```

3.227 $\int \frac{1}{\sqrt{d+ex^2}(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx$

Optimal. Leaf size=106

$$-\frac{x}{d\sqrt{d+ex^2}(2cd-be)}-\frac{c\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{3/2}}$$

```
[Out] -(x/(d*(2*c*d - b*e)*Sqrt[d + e*x^2])) - (c*ArcTanh[(Sqrt[e]*Sqrt
[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqr
t[c*d - b*e]*(2*c*d - b*e)^(3/2))
```

Rubi [A] time = 0.273206, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098

$$-\frac{x}{d\sqrt{d+ex^2}(2cd-be)}-\frac{c\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x^2]*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -(x/(d*(2*c*d - b*e)*Sqrt[d + e*x^2])) - (c*ArcTanh[(Sqrt[e]*Sqrt
[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqr
t[c*d - b*e]*(2*c*d - b*e)^(3/2))
```

Rubi in Sympy [A] time = 63.2106, size = 90, normalized size = 0.85

$$-\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{e}x\sqrt{be-2cd}}{\sqrt{d+ex^2}\sqrt{be-cd}}\right)}{\sqrt{e}\,(be-2cd)^{\frac{3}{2}}\,\sqrt{be-cd}}+\frac{x}{d\sqrt{d+ex^2}\,(be-2cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(e*x**2+d)**(1/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-
c*d**2),x)
```

```
[Out] -c*atanh(sqrt(e)*x*sqrt(b*e - 2*c*d)/(sqrt(d + e*x**2)*sqrt(b*e -
 c*d)))/(sqrt(e)*(b*e - 2*c*d)**(3/2)*sqrt(b*e - c*d)) + x/(d*sqr
t(d + e*x**2)*(b*e - 2*c*d))
```

Mathematica [A] time = 0.254416, size = 106, normalized size = 1.

$$-\frac{\frac{x}{\sqrt{d+ex^2}}-\frac{cd\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{be-2cd}}{\sqrt{d+ex^2}\sqrt{be-cd}}\right)}{\sqrt{e}\sqrt{be-2cd}\sqrt{be-cd}}}{2cd^2-bde}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -((x/Sqrt[d + e*x^2] - (c*d*ArcTanh[(Sqrt[e]*Sqrt[-2*c*d + b*e]*x
)/(Sqrt[-(c*d) + b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[-2*c*d + b
```

```
*e]*Sqrt[-(c*d) + b*e]))/(2*c*d^2 - b*d*e))
```

Maple [B] time = 0.029, size = 775, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/2*c/d/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(
-(b*e-c*d)*c*e)^(1/2))/(x-1/e*(-d*e)^(1/2))*((x-1/e*(-d*e)^(1/2))
^2*e+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(1/2)+1/2*c/d/((-d*e)^(
1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(
1/2))/(x+1/e*(-d*e)^(1/2))*((x+1/e*(-d*e)^(1/2))^2*e-2*(-d*e)^(1/
2)*(x+1/e*(-d*e)^(1/2)))^(1/2)+1/2*e*c^2/((-d*e)^(1/2)*c+(-(b*e-c
*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*
d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c+2*(-(b*
e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d
)/c)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)
^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x-
(-(b*e-c*d)*c*e)^(1/2)/c/e))-1/2*e*c^2/((-d*e)^(1/2)*c+(-(b*e-c*d
)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)
*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-(b*e-
c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/
c)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(
1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-
(b*e-c*d)*c*e)^(1/2)/c/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ce^2x^4 + be^2x^2 - cd^2 + bde)\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*sqrt(e*x^2 + d)),x, algorith
```

```
[Out] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*sqrt(e*x^2 +
 d)), x)
```

Fricas [A] time = 0.430583, size = 1, normalized size = 0.01

$$\left[-\frac{4\sqrt{2\,c^2d^2e - 3\,bcde^2 + b^2e^3}\sqrt{ex^2 + d}x + \left(cdex^2 + cd^2\right)\log\left(\frac{\left(c^2d^4 - 2\,bcd^3e + b^2d^2e^2 + \left(17\,c^2d^2e^2 - 24\,bcde^3 + 8\,b^2e^4\right)x^4 + 2\left(7\,c^2d^3e - 11\,bcd\right.\right.}{}\right.}{4\sqrt{2\,c^2d^2e - 3\,bcde^2 + b^2e^3}(2\,cd^3 -}\right.$$

$$\left.-\frac{2\sqrt{-2\,c^2d^2e + 3\,bcde^2 - b^2e^3}\sqrt{ex^2 + d}x - \left(cdex^2 + cd^2\right)\arctan\left(-\frac{\sqrt{-2\,c^2d^2e + 3\,bcde^2 - b^2e^3}\left(cd^2 - bde + \left(3\,cde - 2\,be^2\right)x^2\right)}{2\left(2\,c^2d^2e - 3\,bcde^2 + b^2e^3\right)\sqrt{ex^2 + d}x}\right)}{2\sqrt{-2\,c^2d^2e + 3\,bcde^2 - b^2e^3}(2\,cd^3 - bd^2e + (2\,cd^2e - bde^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*sqrt(e*x^2 + d)),x, algorith
```

```
[Out] [-1/4*(4*sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*sqrt(e*x^2 + d
)*x + (c*d*e*x^2 + c*d^2)*log(((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e
^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d
^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2)*sqrt(2*c^2*d^2*e - 3*b*
c*d*e^2 + b^2*e^3) + 4*((6*c^3*d^3*e^2 - 13*b*c^2*d^2*e^3 + 9*b^2
*c*d*e^4 - 2*b^3*e^5)*x^3 + (2*c^3*d^4*e - 5*b*c^2*d^3*e^2 + 4*b^
2*c*d^2*e^3 - b^3*d*e^4)*x)*sqrt(e*x^2 + d))/(c^2*e^2*x^4 + c^2*d
^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)))/(sqrt(2*c
^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*(2*c*d^3 - b*d^2*e + (2*c*d^2*e
 - b*d*e^2)*x^2)), -1/2*(2*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*
e^3)*sqrt(e*x^2 + d)*x - (c*d*e*x^2 + c*d^2)*arctan(-1/2*sqrt(-2*
c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*(c*d^2 - b*d*e + (3*c*d*e - 2*
b*e^2)*x^2)/((2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*sqrt(e*x^2 + d
)*x)))/(sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*(2*c*d^3 - b*d
^2*e + (2*c*d^2*e - b*d*e^2)*x^2))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^{2}\right)^{\frac{3}{2}}\left(be-cd+cex^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(1/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral(1/((d + e*x**2)**(3/2)*(b*e - c*d + c*e*x**2)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*sqrt(e*x^2 + d)),x, algorith
```

```
[Out] Timed out
```

3.228 $\int \frac{1}{(d+ex^2)^{3/2}(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx$

Optimal. Leaf size=149

$$-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{5/2}}-\frac{x(7cd-2be)}{3d^2\sqrt{d+ex^2}(2cd-be)^2}-\frac{x}{3d\left(d+ex^2\right)^{3/2}(2cd-be)}$$

```
[Out] -x/(3*d*(2*c*d - b*e)*(d + e*x^2)^(3/2)) - ((7*c*d - 2*b*e)*x)/(3
*d^2*(2*c*d - b*e)^2*Sqrt[d + e*x^2]) - (c^2*ArcTanh[(Sqrt[e]*Sqr
t[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sq
rt[c*d - b*e]*(2*c*d - b*e)^(5/2))
```

Rubi [A] time = 0.604432, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$

$$-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{5/2}}-\frac{x(7cd-2be)}{3d^2\sqrt{d+ex^2}(2cd-be)^2}-\frac{x}{3d\left(d+ex^2\right)^{3/2}(2cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^(3/2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```



```
[Out] -x/(3*d*(2*c*d - b*e)*(d + e*x^2)^(3/2)) - ((7*c*d - 2*b*e)*x)/(3
*d^2*(2*c*d - b*e)^2*Sqrt[d + e*x^2]) - (c^2*ArcTanh[(Sqrt[e]*Sqr
t[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sq
rt[c*d - b*e]*(2*c*d - b*e)^(5/2))
```

Rubi in Sympy [A] time = 118.803, size = 131, normalized size = 0.88

$$\frac{c^2\,\text{atanh}\left(\frac{\sqrt{e}x\sqrt{be-2cd}}{\sqrt{d+ex^2}\sqrt{be-cd}}\right)}{\sqrt{e}\,(be-2cd)^{\frac{5}{2}}\sqrt{be-cd}}+\frac{x}{3d\left(d+ex^2\right)^{\frac{3}{2}}(be-2cd)}+\frac{x\,(2be-7cd)}{3d^2\sqrt{d+ex^2}\,(be-2cd)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(e*x**2+d)**(3/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-
c*d**2),x)
```

```
[Out] c**2*atanh(sqrt(e)*x*sqrt(b*e - 2*c*d)/(sqrt(d + e*x**2)*sqrt(b*e
 - c*d)))/(sqrt(e)*(b*e - 2*c*d)**(5/2)*sqrt(b*e - c*d)) + x/(3*d
*(d + e*x**2)**(3/2)*(b*e - 2*c*d)) + x*(2*b*e - 7*c*d)/(3*d**2*s
qrt(d + e*x**2)*(b*e - 2*c*d)**2)
```

Mathematica [A] time = 0.552906, size = 134, normalized size = 0.9

$$\frac{c^2\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{be-2cd}}{\sqrt{d+ex^2}\sqrt{be-cd}}\right)}{\sqrt{e}(be-2cd)^{5/2}\sqrt{be-cd}}-\frac{x\left(cd\left(9d+7ex^2\right)-be\left(3d+2ex^2\right)\right)}{3d^2\left(d+ex^2\right)^{3/2}(be-2cd)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -(x*(-(b*e*(3*d + 2*e*x^2)) + c*d*(9*d + 7*e*x^2)))/(3*d^2*(-2*c*
d + b*e)^2*(d + e*x^2)^(3/2)) + (c^2*ArcTanh[(Sqrt[e]*Sqrt[-2*c*d
 + b*e]*x)/(Sqrt[-(c*d) + b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*(-2*c*
d + b*e)^(5/2)*Sqrt[-(c*d) + b*e])
```

Maple [B] time = 0.032, size = 1647, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/6*c/d/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(
-(b*e-c*d)*c*e)^(1/2))/(x-1/e*(-d*e)^(1/2))/((x-1/e*(-d*e)^(1/2))
^2*e+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(1/2)+1/3*e*c/d^2/((-d*
e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*
e)^(1/2))/((x-1/e*(-d*e)^(1/2))^2*e+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^
(1/2)))^(1/2)*x+1/6*c/d/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(
(-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/(x+1/e*(-d*e)^(1/2))/((x+1
/e*(-d*e)^(1/2))^2*e-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(1/2)+1
/3*e*c/d^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*
c-(-(b*e-c*d)*c*e)^(1/2))/((x+1/e*(-d*e)^(1/2))^2*e-2*(-d*e)^(1/2
)*(x+1/e*(-d*e)^(1/2)))^(1/2)*x+1/2*e*c^3/((-d*e)^(1/2)*c+(-(b*e-
c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c
*d)*c*e)^(1/2)/(b*e-2*c*d)/((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*
(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*
d)/c)^(1/2)-1/2*e*c^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-
d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/(b*e-2*c*d)/d/((x-(-(b*e-c*d
)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c
*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)*x-1/2*e*c^3/((-d*e)^(1/2)*c+(
-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/(-
(b*e-c*d)*c*e)^(1/2)/(b*e-2*c*d)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b
*e-2*c*d)/c+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/
c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e
+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2
*c*d)/c)^(1/2))/(x-(-(b*e-c*d)*c*e)^(1/2)/c/e))-1/2*e*c^3/((-d*e)
^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)
^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(b*e-2*c*d)/((x+(-(b*e-c*d)*c*e)^(
1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2
)/c/e)-(b*e-2*c*d)/c)^(1/2)-1/2*e*c^2/((-d*e)^(1/2)*c+(-(b*e-c*d)
*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))/(b*e-2*c*d)/
d/((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*
(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)*x+1/2*e*c^3/(
(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d
)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(b*e-2*c*d)/(-(b*e-2*c*d)/c)
^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-
c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*((x+(-(b*e-c*d)*c*e
)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(
1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-(b*e-c*d)*c*e)^(1/2)/c/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ce^2x^4 + be^2x^2 - cd^2 + bde)(ex^2 + d)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*(e*x^2 + d)^(3/2)),x, algori
```

```
[Out] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*(e*x^2 + d)^
(3/2)), x)
```

Fricas [A] time = 0.704455, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*(e*x^2 + d)^(3/2)),x, algori
```

```
[Out] [-1/12*(4*sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*((7*c*d*e - 2
*b*e^2)*x^3 + 3*(3*c*d^2 - b*d*e)*x)*sqrt(e*x^2 + d) - 3*(c^2*d^2
*e^2*x^4 + 2*c^2*d^3*e*x^2 + c^2*d^4)*log(((c^2*d^4 - 2*b*c*d^3*e
 + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4
+ 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2)*sqrt(2*c^2*
d^2*e - 3*b*c*d*e^2 + b^2*e^3) - 4*((6*c^3*d^3*e^2 - 13*b*c^2*d^2
*e^3 + 9*b^2*c*d*e^4 - 2*b^3*e^5)*x^3 + (2*c^3*d^4*e - 5*b*c^2*d^
3*e^2 + 4*b^2*c*d^2*e^3 - b^3*d*e^4)*x)*sqrt(e*x^2 + d))/(c^2*e^2
*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)
))/((4*c^2*d^6 - 4*b*c*d^5*e + b^2*d^4*e^2 + (4*c^2*d^4*e^2 - 4*b
*c*d^3*e^3 + b^2*d^2*e^4)*x^4 + 2*(4*c^2*d^5*e - 4*b*c*d^4*e^2 +
b^2*d^3*e^3)*x^2)*sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)), -1/
6*(2*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*((7*c*d*e - 2*b*e
^2)*x^3 + 3*(3*c*d^2 - b*d*e)*x)*sqrt(e*x^2 + d) - 3*(c^2*d^2*e^2
*x^4 + 2*c^2*d^3*e*x^2 + c^2*d^4)*arctan(-1/2*sqrt(-2*c^2*d^2*e +
 3*b*c*d*e^2 - b^2*e^3)*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)
/((2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*sqrt(e*x^2 + d)*x)))/((4*
c^2*d^6 - 4*b*c*d^5*e + b^2*d^4*e^2 + (4*c^2*d^4*e^2 - 4*b*c*d^3*
e^3 + b^2*d^2*e^4)*x^4 + 2*(4*c^2*d^5*e - 4*b*c*d^4*e^2 + b^2*d^3
*e^3)*x^2)*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^2)^{\frac{5}{2}}\,(be-cd+cex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral(1/((d + e*x**2)**(5/2)*(b*e - c*d + c*e*x**2)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*(e*x^2 + d)^(3/2)),x, algori
```

```
[Out] Timed out
```

3.229 $\int \left(1+x^2\right)^3 \sqrt{1+x^2+x^4}\,dx$

Optimal. Leaf size=183

$$\frac{1}{3}\left(x^4+x^2+1\right)^{3/2}x+\frac{2}{45}\left(6x^2+7\right)\sqrt{x^4+x^2+1}x+\frac{26\sqrt{x^4+x^2+1}x}{45\left(x^2+1\right)}+\frac{7\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{15\sqrt{x^4+x^2+1}}-\frac{26\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{45\sqrt{x^4+x^2+1}}+\frac{1}{9}\left(x^4+x^2+1\right)^{3/2}x^3$$

```
[Out] (26*x*Sqrt[1 + x^2 + x^4])/(45*(1 + x^2)) + (2*x*(7 + 6*x^2)*Sqrt
[1 + x^2 + x^4])/45 + (x*(1 + x^2 + x^4)^(3/2))/3 + (x^3*(1 + x^2
 + x^4)^(3/2))/9 - (26*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2
]*EllipticE[2*ArcTan[x], 1/4])/(45*Sqrt[1 + x^2 + x^4]) + (7*(1 +
 x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/
4])/(15*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.179909, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{1}{3}\left(x^4+x^2+1\right)^{3/2}x+\frac{2}{45}\left(6x^2+7\right)\sqrt{x^4+x^2+1}x+\frac{26\sqrt{x^4+x^2+1}x}{45\left(x^2+1\right)}+\frac{7\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{15\sqrt{x^4+x^2+1}}-\frac{26\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{45\sqrt{x^4+x^2+1}}+\frac{1}{9}\left(x^4+x^2+1\right)^{3/2}x^3$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)^3*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (26*x*Sqrt[1 + x^2 + x^4])/(45*(1 + x^2)) + (2*x*(7 + 6*x^2)*Sqrt
[1 + x^2 + x^4])/45 + (x*(1 + x^2 + x^4)^(3/2))/3 + (x^3*(1 + x^2
 + x^4)^(3/2))/9 - (26*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2
]*EllipticE[2*ArcTan[x], 1/4])/(45*Sqrt[1 + x^2 + x^4]) + (7*(1 +
 x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/
4])/(15*Sqrt[1 + x^2 + x^4])
```

Rubi in Sympy [A] time = 37.5185, size = 172, normalized size = 0.94

$$\frac{x^3\left(x^4+x^2+1\right)^{\frac{3}{2}}}{9}+\frac{x\left(4x^2+\frac{14}{3}\right)\sqrt{x^4+x^2+1}}{15}+\frac{x\left(x^4+x^2+1\right)^{\frac{3}{2}}}{3}+\frac{26x\sqrt{x^4+x^2+1}}{45\left(x^2+1\right)}-\frac{26\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\left(x^2+1\right)E\left(2\operatorname{atan}(x)\middle|\frac{1}{4}\right)}{45\sqrt{x^4+x^2+1}}+\frac{7\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\left(x^2+1\right)F\left(2\operatorname{atan}(x)\middle|\frac{1}{4}\right)}{15\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)**3*(x**4+x**2+1)**(1/2),x)
```

```
[Out] x**3*(x**4 + x**2 + 1)**(3/2)/9 + x*(4*x**2 + 14/3)*sqrt(x**4 + x
**2 + 1)/15 + x*(x**4 + x**2 + 1)**(3/2)/3 + 26*x*sqrt(x**4 + x**
2 + 1)/(45*(x**2 + 1)) - 26*sqrt((x**4 + x**2 + 1)/(x**2 + 1)**2)
*(x**2 + 1)*elliptic_e(2*atan(x), 1/4)/(45*sqrt(x**4 + x**2 + 1))
 + 7*sqrt((x**4 + x**2 + 1)/(x**2 + 1)**2)*(x**2 + 1)*elliptic_f(
2*atan(x), 1/4)/(15*sqrt(x**4 + x**2 + 1))
```

Mathematica [C] time = 0.372671, size = 169, normalized size = 0.92

$$\frac{2(-1)^{5/6}\left(4\sqrt{3}+9i\right)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+26\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}E\left(i\,\mathrm{si}\right.}{45\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)^3*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*(29 + 61*x^2 + 81*x^4 + 57*x^6 + 25*x^8 + 5*x^10) + 26*(-1)^(1
/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I
*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2*(-1)^(5/6)*(9*I + 4*Sqrt[
3])*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I
*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(45*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.226, size = 263, normalized size = 1.4

$$\frac{29x}{45}\sqrt{x^4+x^2+1}$$
$$+\frac{32}{45\sqrt{-2+2i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}$$
$$-\frac{104}{45\sqrt{-2+2i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)-Ellip\right.$$
$$+\frac{x^7}{9}\sqrt{x^4+x^2+1}+\frac{4x^5}{9}\sqrt{x^4+x^2+1}+\frac{32x^3}{45}\sqrt{x^4+x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^3*(x^4+x^2+1)^(1/2),x)
```

```
[Out] 29/45*x*(x^4+x^2+1)^(1/2)+32/45/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1
/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+
x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3
^(1/2))^(1/2))-104/45/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/
2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/
2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+
2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-
2+2*I*3^(1/2))^(1/2)))+1/9*x^7*(x^4+x^2+1)^(1/2)+4/9*x^5*(x^4+x^2
+1)^(1/2)+32/45*x^3*(x^4+x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+x^2+1}\left(x^2+1\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(x^6+3x^4+3x^2+1\right)\sqrt{x^4+x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3,x, algorithm="fricas")
```

```
[Out] integral((x^6 + 3*x^4 + 3*x^2 + 1)*sqrt(x^4 + x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^2 - x + 1)(x^2 + x + 1)}\left(x^2 + 1\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**3*(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + x^2 + 1}\left(x^2 + 1\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3, x)
```

3.230 $\int \left(1+x^2\right)^2 \sqrt{1+x^2+x^4}\,dx$

Optimal. Leaf size=164

$$\frac{1}{7}x\left(x^4+x^2+1\right)^{3/2}+\frac{2}{21}x\left(3x^2+4\right)\sqrt{x^4+x^2+1}+\frac{2x\sqrt{x^4+x^2+1}}{3\left(x^2+1\right)}+\frac{4\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{7\sqrt{x^4+x^2+1}}-\frac{2\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

```
[Out] (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (2*x*(4 + 3*x^2)*Sqrt[1
 + x^2 + x^4])/21 + (x*(1 + x^2 + x^4)^(3/2))/7 - (2*(1 + x^2)*Sq
rt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*S
qrt[1 + x^2 + x^4]) + (4*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticF[2*ArcTan[x], 1/4])/(7*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.140456, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{1}{7}x\left(x^4+x^2+1\right)^{3/2}+\frac{2}{21}x\left(3x^2+4\right)\sqrt{x^4+x^2+1}+\frac{2x\sqrt{x^4+x^2+1}}{3\left(x^2+1\right)}+\frac{4\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{7\sqrt{x^4+x^2+1}}-\frac{2\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)^2*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (2*x*(4 + 3*x^2)*Sqrt[1
 + x^2 + x^4])/21 + (x*(1 + x^2 + x^4)^(3/2))/7 - (2*(1 + x^2)*Sq
rt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*S
qrt[1 + x^2 + x^4]) + (4*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticF[2*ArcTan[x], 1/4])/(7*Sqrt[1 + x^2 + x^4])
```

Rubi in Sympy [A] time = 28.8372, size = 156, normalized size = 0.95

$$\frac{x\left(\frac{30x^2}{7}+\frac{40}{7}\right)\sqrt{x^4+x^2+1}}{15}+\frac{x\left(x^4+x^2+1\right)^{\frac{3}{2}}}{7}+\frac{2x\sqrt{x^4+x^2+1}}{3\left(x^2+1\right)}-\frac{2\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\left(x^2+1\right)E\left(2\operatorname{atan}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}+\frac{4\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\left(x^2+1\right)F\left(2\operatorname{atan}(x)|\frac{1}{4}\right)}{7\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)**2*(x**4+x**2+1)**(1/2),x)
```

```
[Out] x*(30*x**2/7 + 40/7)*sqrt(x**4 + x**2 + 1)/15 + x*(x**4 + x**2 +
1)**(3/2)/7 + 2*x*sqrt(x**4 + x**2 + 1)/(3*(x**2 + 1)) - 2*sqrt((
x**4 + x**2 + 1)/(x**2 + 1)**2)*(x**2 + 1)*elliptic_e(2*atan(x),
1/4)/(3*sqrt(x**4 + x**2 + 1)) + 4*sqrt((x**4 + x**2 + 1)/(x**2 +
 1)**2)*(x**2 + 1)*elliptic_f(2*atan(x), 1/4)/(7*sqrt(x**4 + x**2
 + 1))
```

Mathematica [C] time = 0.232731, size = 162, normalized size = 0.99

$$\frac{2\sqrt[3]{-1}\left(5\sqrt[3]{-1}-7\right)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+14\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}E\left(i\sin\right.}{21\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)^2*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*(11 + 20*x^2 + 23*x^4 + 12*x^6 + 3*x^8) + 14*(-1)^(1/3)*Sqrt[1
 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(
-1)^(5/6)*x], (-1)^(2/3)] + 2*(-1)^(1/3)*(-7 + 5*(-1)^(1/3))*Sqrt
[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh
[(-1)^(5/6)*x], (-1)^(2/3)])/(21*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.011, size = 248, normalized size = 1.5

$$\frac{11x}{21}\sqrt{x^4+x^2+1}$$
$$+\frac{20}{21\sqrt{-2+2i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}$$
$$-\frac{8}{3\sqrt{-2+2i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)-Ellipti\right.$$
$$+\frac{x^5}{7}\sqrt{x^4+x^2+1}+\frac{3x^3}{7}\sqrt{x^4+x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^2*(x^4+x^2+1)^(1/2),x)
```

```
[Out] 11/21*x*(x^4+x^2+1)^(1/2)+20/21/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1
/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+
x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3
^(1/2))^(1/2))-8/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))
*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/
(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I
*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2
*I*3^(1/2))^(1/2)))+1/7*x^5*(x^4+x^2+1)^(1/2)+3/7*x^3*(x^4+x^2+1)
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^4+x^2+1}\left(x^2+1\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(x^4+2x^2+1\right)\sqrt{x^4+x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2,x, algorithm="fricas")
```

```
[Out] integral((x^4 + 2*x^2 + 1)*sqrt(x^4 + x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^2 - x + 1)(x^2 + x + 1)}\left(x^2 + 1\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**2*(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + x^2 + 1}\left(x^2 + 1\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2, x)
```

3.231 $\int (1+x^2)\sqrt{1+x^2+x^4}\,dx$

Optimal. Leaf size=145

$$\frac{1}{5}\left(x^2+2\right)\sqrt{x^4+x^2+1}x+\frac{3\sqrt{x^4+x^2+1}x}{5\left(x^2+1\right)}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}-\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}$$

```
[Out] (3*x*Sqrt[1 + x^2 + x^4])/(5*(1 + x^2)) + (x*(2 + x^2)*Sqrt[1 + x
^2 + x^4])/5 - (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Ell
ipticE[2*ArcTan[x], 1/4])/(5*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*
Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(5
*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.103504, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{1}{5}\left(x^2+2\right)\sqrt{x^4+x^2+1}x+\frac{3\sqrt{x^4+x^2+1}x}{5\left(x^2+1\right)}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}-\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (3*x*Sqrt[1 + x^2 + x^4])/(5*(1 + x^2)) + (x*(2 + x^2)*Sqrt[1 + x
^2 + x^4])/5 - (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Ell
ipticE[2*ArcTan[x], 1/4])/(5*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*
Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(5
*Sqrt[1 + x^2 + x^4])
```

Rubi in Sympy [A] time = 18.4321, size = 138, normalized size = 0.95

$$\frac{x\left(3x^2+6\right)\sqrt{x^4+x^2+1}}{15}+\frac{3x\sqrt{x^4+x^2+1}}{5\left(x^2+1\right)}-\frac{3\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\left(x^2+1\right)E\left(2\operatorname{atan}(x)\middle|\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}+\frac{3\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\left(x^2+1\right)F\left(2\operatorname{atan}(x)\middle|\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)*(x**4+x**2+1)**(1/2),x)
```

```
[Out] x*(3*x**2 + 6)*sqrt(x**4 + x**2 + 1)/15 + 3*x*sqrt(x**4 + x**2 +
1)/(5*(x**2 + 1)) - 3*sqrt((x**4 + x**2 + 1)/(x**2 + 1)**2)*(x**2
 + 1)*elliptic_e(2*atan(x), 1/4)/(5*sqrt(x**4 + x**2 + 1)) + 3*sq
rt((x**4 + x**2 + 1)/(x**2 + 1)**2)*(x**2 + 1)*elliptic_f(2*atan(
x), 1/4)/(5*sqrt(x**4 + x**2 + 1))
```

Mathematica [C] time = 0.327481, size = 168, normalized size = 1.16

$$\frac{x^7+3x^5+3x^3+\frac{3}{2}\sqrt{2+\left(1-i\sqrt{3}\right)x^2}\sqrt{2+\left(1+i\sqrt{3}\right)x^2}F\left(\sin^{-1}\left(\frac{1}{2}\left(i\sqrt{3}x+x\right)\right)|\frac{1}{2}i\left(i+\sqrt{3}\right)\right)+3\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(}}{5\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (2*x + 3*x^3 + 3*x^5 + x^7 + 3*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2
]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1
)^(2/3)] + (3*Sqrt[2 + (1 - I*Sqrt[3])*x^2]*Sqrt[2 + (1 + I*Sqrt[
3])*x^2]*EllipticF[ArcSin[(x + I*Sqrt[3]*x)/2], (I/2)*(I + Sqrt[3
])])/2)/(5*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.008, size = 233, normalized size = 1.6

$$\frac{2x}{5}\sqrt{x^4+x^2+1} + \frac{6}{5\sqrt{-2+2i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}} - \frac{12}{5\sqrt{-2+2i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right) - Ellipti\right. + \frac{x^3}{5}\sqrt{x^4+x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)*(x^4+x^2+1)^(1/2),x)
```

```
[Out] 2/5*x*(x^4+x^2+1)^(1/2)+6/5/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I
*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+
1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/
2))^(1/2))-12/5/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^
2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(I*
3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^
(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*
3^(1/2))^(1/2)))+1/5*x^3*(x^4+x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+x^2+1}\left(x^2+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x^4+x^2+1}\left(x^2+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)*(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^2 - x + 1)(x^2 + x + 1)}\,(x^2 + 1)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)*(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + x^2 + 1}(x^2 + 1)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1), x)
```

3.232 $\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx$

Optimal. Leaf size=137

$$\frac{\sqrt{x^4+x^2+1}\,x}{x^2+1}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}$$

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(1 + x^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]
/2 - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*Arc
Tan[x], 1/4])/Sqrt[1 + x^2 + x^4] + (3*(1 + x^2)*Sqrt[(1 + x^2 +
x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 +
x^4])
```

Rubi [A] time = 0.180849, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$\frac{\sqrt{x^4+x^2+1}\,x}{x^2+1}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + x^2 + x^4]/(1 + x^2),x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(1 + x^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]
/2 - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*Arc
Tan[x], 1/4])/Sqrt[1 + x^2 + x^4] + (3*(1 + x^2)*Sqrt[(1 + x^2 +
x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 +
x^4])
```

Rubi in Sympy [A] time = 91.864, size = 320, normalized size = 2.34

$$\frac{x\sqrt{x^4+x^2+1}}{x^2+1}-\frac{\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\left(x^2+1\right)E\left(2\operatorname{atan}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}+\frac{\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\left(x^2+1\right)F\left(2\operatorname{atan}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}+\frac{2\sqrt{x^4+x^2+1}F\left(\operatorname{atan}\left(x\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)\right)\middle|\frac{3}{2}+\frac{\sqrt{3}i}{2}\right)}{\sqrt{\frac{x^2\left(\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)+1}{x^2\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)+1}}\left(\sqrt{3}-i\right)\left(x^2\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)+1\right)}+\frac{\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)\sqrt{x^4+x^2+1}\,\Pi\left(\frac{1}{2}-\frac{\sqrt{3}i}{2};\operatorname{atan}\left(x\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)\right)\middle|\frac{3}{2}-\frac{\sqrt{3}i}{2}\right)}{\sqrt{\frac{x^2\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)+1}{x^2\left(\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)+1}}\left(x^2\left(\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+x**2+1)**(1/2)/(x**2+1),x)
```

```
[Out] x*sqrt(x**4 + x**2 + 1)/(x**2 + 1) - sqrt((x**4 + x**2 + 1)/(x**2
 + 1)**2)*(x**2 + 1)*elliptic_e(2*atan(x), 1/4)/sqrt(x**4 + x**2
```

```
+ 1) + sqrt((x**4 + x**2 + 1)/(x**2 + 1)**2)*(x**2 + 1)*elliptic_
f(2*atan(x), 1/4)/(2*sqrt(x**4 + x**2 + 1)) + 2*sqrt(x**4 + x**2
+ 1)*elliptic_f(atan(x*(sqrt(3)/2 + I/2)), 3/2 + sqrt(3)*I/2)/(sq
rt((x**2*(1/2 - sqrt(3)*I/2) + 1)/(x**2*(1/2 + sqrt(3)*I/2) + 1))
*(sqrt(3) - I)*(x**2*(1/2 + sqrt(3)*I/2) + 1)) + (sqrt(3)/2 - I/2
)*sqrt(x**4 + x**2 + 1)*elliptic_pi(1/2 - sqrt(3)*I/2, atan(x*(sq
rt(3)/2 - I/2)), 3/2 - sqrt(3)*I/2)/(sqrt((x**2*(1/2 + sqrt(3)*I/
2) + 1)/(x**2*(1/2 - sqrt(3)*I/2) + 1))*(x**2*(1/2 - sqrt(3)*I/2)
 + 1))
```

Mathematica [C] time = 0.100507, size = 118, normalized size = 0.86

$$-\frac{\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+\sqrt[3]{-1}\left(\sqrt[3]{-1};-i\sinh\right.\right.}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2),x]
```

```
[Out] -(((-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(
-EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*Arc
Sinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3)*EllipticPi[(-1)^(1/3
), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]))/Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.091, size = 293, normalized size = 2.1

$$-4\,\frac{\sqrt{1+1/2\,x^2-i/2x^2\sqrt{3}}\sqrt{1+1/2\,x^2+i/2x^2\sqrt{3}}EllipticF\left(1/2\,x\sqrt{-2+2\,i\sqrt{3}},1/2\,\sqrt{-2+2\,i\sqrt{3}}\right)}{\sqrt{-2+2\,i\sqrt{3}}\sqrt{x^4+x^2+1}\left(i\sqrt{3}+1\right)}$$
$$+4\,\frac{\sqrt{1+1/2\,x^2-i/2x^2\sqrt{3}}\sqrt{1+1/2\,x^2+i/2x^2\sqrt{3}}EllipticE\left(1/2\,x\sqrt{-2+2\,i\sqrt{3}},1/2\,\sqrt{-2+2\,i\sqrt{3}}\right)}{\sqrt{-2+2\,i\sqrt{3}}\sqrt{x^4+x^2+1}\left(i\sqrt{3}+1\right)}$$
$$+\frac{1}{\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}EllipticPi\left(\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}x,-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)^{-1},\frac{\sqrt{-\frac{1}{2}-\frac{i}{2}\sqrt{3}}}{\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{x^4+x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+x^2+1)^(1/2)/(x^2+1),x)
```

```
[Out] -4/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+
1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*
EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)
)+4/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1
+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)
*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2
))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/
2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*Elliptic
Pi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2
*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^4+x^2+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + x^2 + 1}}{x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2 - x + 1)(x^2 + x + 1)}}{x^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+x**2+1)**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + x^2 + 1}}{x^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1), x)
```

3.233 $\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2} \, dx$

Optimal. Leaf size=49

$$\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}$$

```
[Out] ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x
], 1/4])/(2*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.0345742, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^2,x]
```

```
[Out] ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x
], 1/4])/(2*Sqrt[1 + x^2 + x^4])
```

Rubi in Sympy [A] time = 140.011, size = 541, normalized size = 11.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+x**2+1)**(1/2)/(x**2+1)**2,x)
```

```
[Out] -x*(1/4 + sqrt(3)*I/4)*sqrt(x**4 + x**2 + 1)/(x**2*(1/2 + sqrt(3)
*I/2) + 1) + x*sqrt(x**4 + x**2 + 1)/(2*(x**2 + 1)) + sqrt(x**4 +
 x**2 + 1)*elliptic_e(atan(x*(sqrt(3)/2 + I/2)), 3/2 + sqrt(3)*I/
2)/(sqrt((x**2*(1/2 - sqrt(3)*I/2) + 1)/(x**2*(1/2 + sqrt(3)*I/2)
 + 1))*(sqrt(3) - I)*(x**2*(1/2 + sqrt(3)*I/2) + 1)) - 4*I*(-2/((
1 - sqrt(3)*I)*(1 + sqrt(3)*I)) + 1/2)*sqrt(x**4 + x**2 + 1)*im(1
/((1 + sqrt(3)*I)*(1 - 2/(1 + sqrt(3)*I))*(sqrt(3) - I)))*ellipti
c_f(atan(x*(sqrt(3)/2 - I/2)), 3/2 - sqrt(3)*I/2)/(sqrt((x**2*(1/
2 + sqrt(3)*I/2) + 1)/(x**2*(1/2 - sqrt(3)*I/2) + 1))*(x**2*(1/2
- sqrt(3)*I/2) + 1)) + sqrt(x**4 + x**2 + 1)*elliptic_f(atan(x*(s
qrt(3)/2 - I/2)), 3/2 - sqrt(3)*I/2)/(sqrt((x**2*(1/2 + sqrt(3)*I
/2) + 1)/(x**2*(1/2 - sqrt(3)*I/2) + 1))*(sqrt(3) - I)*(x**2*(1/2
 - sqrt(3)*I/2) + 1)) + (sqrt(3)/2 - I/2)*(-2/((1 - sqrt(3)*I)*(1
 + sqrt(3)*I)) + 1/2)*sqrt(x**4 + x**2 + 1)*elliptic_pi(1/2 - sqr
t(3)*I/2, atan(x*(sqrt(3)/2 - I/2)), 3/2 - sqrt(3)*I/2)/(sqrt((x*
*2*(1/2 + sqrt(3)*I/2) + 1)/(x**2*(1/2 - sqrt(3)*I/2) + 1))*(x**2
*(1/2 - sqrt(3)*I/2) + 1))
```

Mathematica [C] time = 0.535733, size = 164, normalized size = 3.35

$$\frac{(-1)^{2/3}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)+\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(F\left(i\sinh^{-1}\left((-1)^{5/6}x\right.\right.\right.}{2\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^2,x]
```

```
[Out] ((x + x^3 + x^5)/(1 + x^2) + (-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*
Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^
(2/3)] + (-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*
x^2]*(-EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + EllipticF
[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]))/(2*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.025, size = 224, normalized size = 4.6

$$\frac{x}{2x^2+2}\sqrt{x^4+x^2+1}+\frac{1}{\sqrt{-2+2i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}EllipticF\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}+2\frac{\sqrt{1-\left(-1/2+i/2\sqrt{3}\right)x^2}\sqrt{1-\left(-1/2-i/2\sqrt{3}\right)x^2}\left(EllipticF\left(1/2\,x\sqrt{-2+2i\sqrt{3}},1/2\sqrt{-2+2i\sqrt{3}}\right)-EllipticE\left(1/2\,x\sqrt{\cdot\right.\right.}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}\left(i\sqrt{3}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+x^2+1)^(1/2)/(x^2+1)^2,x)
```

```
[Out] 1/2*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2
+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^
4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I
*3^(1/2))^(1/2))+2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))
*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/
(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I
*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2
*I*3^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+x^2+1}}{\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^4+2x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^4 + 2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2 - x + 1)(x^2 + x + 1)}}{(x^2 + 1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+x**2+1)**(1/2)/(x**2+1)**2,x)

[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + x^2 + 1}}{(x^2 + 1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^2,x, algorithm="giac")

[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^2, x)
```

3.234 $\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3}\,dx$

Optimal. Leaf size=93

$$\frac{\sqrt{x^4+x^2+1}x}{4\left(x^2+1\right)^2}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 +
 x^4]]/4 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE
[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.748212, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$

$$\frac{\sqrt{x^4+x^2+1}x}{4\left(x^2+1\right)^2}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^3,x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 +
 x^4]]/4 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE
[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+x**2+1)**(1/2)/(x**2+1)**3,x)
```

```
[Out] Exception raised: TypeError
```

Mathematica [C] time = 0.349285, size = 176, normalized size = 1.89

$$\frac{-2(-1)^{2/3}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(\sqrt[3]{-1};-i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(F\left(i\sinh^{-1}\right.\right.}{4\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^3,x]
```

```
[Out] ((x*(2 + x^2)*(1 + x^2 + x^4))/(1 + x^2)^2 + (-1)^(1/3)*Sqrt[1 +
(-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[(-
1)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1
)^(2/3)]) - 2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(
2/3)*x^2]*EllipticPi[(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)
^(2/3)])/(4*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.033, size = 333, normalized size = 3.6

$$\frac{x}{4\,(x^2+1)^2}\sqrt{x^4+x^2+1}+\frac{x}{4\,x^2+4}\sqrt{x^4+x^2+1}$$
$$+\frac{1}{\sqrt{-2+2\,i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}\mathit{EllipticF}\left(\frac{x\sqrt{-2+2\,i\sqrt{3}}}{2},\frac{\sqrt{-2+2\,i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}$$
$$-\frac{1}{\sqrt{-2+2\,i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}\mathit{EllipticE}\left(\frac{x\sqrt{-2+2\,i\sqrt{3}}}{2},\frac{\sqrt{-2+2\,i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}$$
$$+\frac{1}{2\sqrt{-1/2+i/2\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}\mathit{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}x,-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)^{-1},\frac{\sqrt{-\frac{1}{2}-\frac{i}{2}\sqrt{3}}}{\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{x^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+x^2+1)^(1/2)/(x^2+1)^3,x)
```

```
[Out] 1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)
+1/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+
1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*
EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)
)-1/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1
+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)
*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2
))+1/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(
1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*Ellipt
icPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1
/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^4+x^2+1}}{(x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^6+3\,x^4+3\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^6 + 3*x^4 + 3*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{(x^2-x+1)(x^2+x+1)}}{(x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+x**2+1)**(1/2)/(x**2+1)**3,x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + x^2 + 1}}{(x^2 + 1)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^3, x)
```

3.235 $\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^4}\,dx$

Optimal. Leaf size=166

$$\frac{\sqrt{x^4+x^2+1}\,x}{6\left(x^2+1\right)^2}+\frac{\sqrt{x^4+x^2+1}\,x}{6\left(x^2+1\right)^3}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{8\sqrt{x^4+x^2+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(6*(1 + x^2)^3) + (x*Sqrt[1 + x^2 + x^4])
/(6*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 + ((1 + x^2)*S
qrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*
Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^
2]*EllipticF[2*ArcTan[x], 1/4])/(8*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.93407, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.65$

$$\frac{\sqrt{x^4+x^2+1}\,x}{6\left(x^2+1\right)^2}+\frac{\sqrt{x^4+x^2+1}\,x}{6\left(x^2+1\right)^3}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{8\sqrt{x^4+x^2+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^4,x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(6*(1 + x^2)^3) + (x*Sqrt[1 + x^2 + x^4])
/(6*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 + ((1 + x^2)*S
qrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*
Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^
2]*EllipticF[2*ArcTan[x], 1/4])/(8*Sqrt[1 + x^2 + x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+x**2+1)**(1/2)/(x**2+1)**4,x)
```

```
[Out] Exception raised: TypeError
```

Mathematica [C] time = 0.471568, size = 240, normalized size = 1.45

$$\frac{-(-1)^{2/3}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-3(-1)^{2/3}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(\sqrt[3]{-1};-i\sinh^{-}\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^4,x]
```

```
[Out] ((x*(1 + x^2 + x^4)*(4 + 5*x^2 + 2*x^4))/(1 + x^2)^3 - 2*(-1)^(1/
3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I
*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - EllipticF[I*ArcSinh[(-1)^(5
/6)*x], (-1)^(2/3)]) - (-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1
 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]
 - 3*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]
*EllipticPi[(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/
(6*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.034, size = 438, normalized size = 2.6

$$\frac{x}{6\,(x^2+1)^3}\sqrt{x^4+x^2+1}+\frac{x}{6\,(x^2+1)^2}\sqrt{x^4+x^2+1}+\frac{x}{3\,x^2+3}\sqrt{x^4+x^2+1}$$
$$-\frac{1}{3\sqrt{-2+2\,i\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}EllipticF\left(\frac{x\sqrt{-2+2\,i\sqrt{3}}}{2},\frac{\sqrt{-2+2\,i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}$$
$$+\frac{4}{3\sqrt{-2+2\,i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}EllipticF\left(\frac{x\sqrt{-2+2\,i\sqrt{3}}}{2},\frac{\sqrt{-2+2\,i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}$$
$$-\frac{4}{3\sqrt{-2+2\,i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}EllipticE\left(\frac{x\sqrt{-2+2\,i\sqrt{3}}}{2},\frac{\sqrt{-2+2\,i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}$$
$$+\frac{1}{2\sqrt{-1/2+i/2\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}EllipticPi\left(\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}x,-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)^{-1},\frac{\sqrt{-\frac{1}{2}-\frac{i}{2}\sqrt{3}}}{\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{x^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+x^2+1)^(1/2)/(x^2+1)^4,x)
```

```
[Out] 1/6*x*(x^4+x^2+1)^(1/2)/(x^2+1)^3+1/6*x*(x^4+x^2+1)^(1/2)/(x^2+1)
^2+1/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)-1/3/(-2+2*I*3^(1/2))^(1/2)*(1+
1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1
/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*
(-2+2*I*3^(1/2))^(1/2))+4/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2
*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^
2+1)^(1/2)/(I*3^(1/2)+1)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1
/2*(-2+2*I*3^(1/2))^(1/2))-4/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-
1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4
+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2
),1/2*(-2+2*I*3^(1/2))^(1/2))+1/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1
/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/
2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(
-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2
))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^8+4\,x^6+6\,x^4+4\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^4,x, algorithm="fricas")

[Out] integral(sqrt(x^4 + x^2 + 1)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{(x^2-x+1)(x^2+x+1)}}{(x^2+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+x**2+1)**(1/2)/(x**2+1)**4,x)

[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**4, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^4+x^2+1}}{(x^2+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^4,x, algorithm="giac")

[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^4, x)
```

3.236 $\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=159

$$\frac{14\sqrt{x^4+x^2+1}x}{15(x^2+1)}+\frac{11}{15}\sqrt{x^4+x^2+1}x+\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}-\frac{14(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{15\sqrt{x^4+x^2+1}}+\frac{1}{5}\sqrt{x^4+x^2+1}x^3$$

```
[Out] (11*x*Sqrt[1 + x^2 + x^4])/15 + (x^3*Sqrt[1 + x^2 + x^4])/5 + (14
*x*Sqrt[1 + x^2 + x^4])/(15*(1 + x^2)) - (14*(1 + x^2)*Sqrt[(1 +
x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(15*Sqrt[1 +
 x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Ell
ipticF[2*ArcTan[x], 1/4])/(5*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.14811, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{14\sqrt{x^4+x^2+1}x}{15(x^2+1)}+\frac{11}{15}\sqrt{x^4+x^2+1}x+\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}-\frac{14(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{15\sqrt{x^4+x^2+1}}+\frac{1}{5}\sqrt{x^4+x^2+1}x^3$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)^3/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (11*x*Sqrt[1 + x^2 + x^4])/15 + (x^3*Sqrt[1 + x^2 + x^4])/5 + (14
*x*Sqrt[1 + x^2 + x^4])/(15*(1 + x^2)) - (14*(1 + x^2)*Sqrt[(1 +
x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(15*Sqrt[1 +
 x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Ell
ipticF[2*ArcTan[x], 1/4])/(5*Sqrt[1 + x^2 + x^4])
```

Rubi in Sympy [A] time = 30.5789, size = 150, normalized size = 0.94

$$\frac{x^3\sqrt{x^4+x^2+1}}{5}+\frac{11x\sqrt{x^4+x^2+1}}{15}+\frac{14x\sqrt{x^4+x^2+1}}{15(x^2+1)}-\frac{14\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}(x^2+1)E\left(2\operatorname{atan}(x)\middle|\frac{1}{4}\right)}{15\sqrt{x^4+x^2+1}}+\frac{3\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}(x^2+1)F\left(2\operatorname{atan}(x)\middle|\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)**3/(x**4+x**2+1)**(1/2),x)
```

```
[Out] x**3*sqrt(x**4 + x**2 + 1)/5 + 11*x*sqrt(x**4 + x**2 + 1)/15 + 14
*x*sqrt(x**4 + x**2 + 1)/(15*(x**2 + 1)) - 14*sqrt((x**4 + x**2 +
 1)/(x**2 + 1)**2)*(x**2 + 1)*elliptic_e(2*atan(x), 1/4)/(15*sqrt
(x**4 + x**2 + 1)) + 3*sqrt((x**4 + x**2 + 1)/(x**2 + 1)**2)*(x**
2 + 1)*elliptic_f(2*atan(x), 1/4)/(5*sqrt(x**4 + x**2 + 1))
```

Mathematica [C] time = 0.219053, size = 157, normalized size = 0.99

$$\frac{2\sqrt[3]{-1}\left(2\sqrt[3]{-1}-7\right)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+14\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}E\left(i\sin\right.}{15\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)^3/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*(11 + 14*x^2 + 14*x^4 + 3*x^6) + 14*(-1)^(1/3)*Sqrt[1 + (-1)^(
1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)
*x], (-1)^(2/3)] + 2*(-1)^(1/3)*(-7 + 2*(-1)^(1/3))*Sqrt[1 + (-1)
^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/
6)*x], (-1)^(2/3)])/(15*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.037, size = 233, normalized size = 1.5

$$\frac{8}{15\sqrt{-2+2i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}EllipticF\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}$$
$$+\frac{x^3}{5}\sqrt{x^4+x^2+1}+\frac{11x}{15}\sqrt{x^4+x^2+1}$$
$$-\frac{56}{15\sqrt{-2+2i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)-Ellip\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^3/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 8/15/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1
-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*
x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+1/5*x^3*(x^4
+x^2+1)^(1/2)+11/15*x*(x^4+x^2+1)^(1/2)-56/15/(-2+2*I*3^(1/2))^(1
/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^
2)^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I
*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2
*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^2+1\right)^3}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^3/sqrt(x^4 + x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^3/sqrt(x^4 + x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6+3x^4+3x^2+1}{\sqrt{x^4+x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^3/sqrt(x^4 + x^2 + 1),x, algorithm="fricas")

[Out] integral((x^6 + 3*x^4 + 3*x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+1\right)^3}{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**3/(x**4+x**2+1)**(1/2),x)

[Out] Integral((x**2 + 1)**3/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+1\right)^3}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^3/sqrt(x^4 + x^2 + 1),x, algorithm="giac")

[Out] integrate((x^2 + 1)^3/sqrt(x^4 + x^2 + 1), x)
```

3.237 $\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=137

$$\frac{4\sqrt{x^4+x^2+1}x}{3(x^2+1)}+\frac{1}{3}\sqrt{x^4+x^2+1}x+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{4(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

```
[Out] (x*Sqrt[1 + x^2 + x^4])/3 + (4*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2
)) - (4*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*A
rcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^
2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 +
 x^4]
```

Rubi [A] time = 0.110453, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{4\sqrt{x^4+x^2+1}x}{3(x^2+1)}+\frac{1}{3}\sqrt{x^4+x^2+1}x+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{4(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)^2/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/3 + (4*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2
)) - (4*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*A
rcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^
2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 +
 x^4]
```

Rubi in Sympy [A] time = 20.9895, size = 128, normalized size = 0.93

$$\frac{x\sqrt{x^4+x^2+1}}{3}+\frac{4x\sqrt{x^4+x^2+1}}{3(x^2+1)}-\frac{4\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}(x^2+1)E\left(2\,\text{atan}(x)\middle|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}+\frac{\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}(x^2+1)F\left(2\,\text{atan}(x)\middle|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)**2/(x**4+x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(x**4 + x**2 + 1)/3 + 4*x*sqrt(x**4 + x**2 + 1)/(3*(x**2 +
1)) - 4*sqrt((x**4 + x**2 + 1)/(x**2 + 1)**2)*(x**2 + 1)*elliptic
_e(2*atan(x), 1/4)/(3*sqrt(x**4 + x**2 + 1)) + sqrt((x**4 + x**2
+ 1)/(x**2 + 1)**2)*(x**2 + 1)*elliptic_f(2*atan(x), 1/4)/sqrt(x*
*4 + x**2 + 1)
```

Mathematica [C] time = 0.193832, size = 143, normalized size = 1.04

$$\frac{x^5+x^3+2\sqrt[3]{-1}\left(\sqrt[3]{-1}-2\right)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+4\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)^2/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x + x^3 + x^5 + 4*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (
-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2
*(-1)^(1/3)*(-2 + (-1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (
-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(3
*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.01, size = 218, normalized size = 1.6

$$\frac{4}{3\sqrt{-2+2i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}$$
$$+\frac{x}{3}\sqrt{x^4+x^2+1}$$
$$-\frac{16}{3\sqrt{-2+2i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)-Ellipti\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^2/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 4/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-
(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x
*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+1/3*x*(x^4+x^
2+1)^(1/2)-16/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^
2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(I*
3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^
(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*
3^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+1\right)^2}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^2/sqrt(x^4 + x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^2/sqrt(x^4 + x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4+2x^2+1}{\sqrt{x^4+x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^2/sqrt(x^4 + x^2 + 1),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 2*x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+1\right)^2}{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**2/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral((x**2 + 1)**2/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+1\right)^2}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^2/sqrt(x^4 + x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^2/sqrt(x^4 + x^2 + 1), x)
```

3.238 $\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=115

$$\frac{\sqrt{x^4+x^2+1}x}{x^2+1}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}$$

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^2 + x^
4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]
+ ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan
[x], 1/4])/Sqrt[1 + x^2 + x^4]
```

Rubi [A] time = 0.0648186, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\sqrt{x^4+x^2+1}x}{x^2+1}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^2 + x^
4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]
+ ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan
[x], 1/4])/Sqrt[1 + x^2 + x^4]
```

Rubi in Sympy [A] time = 11.7741, size = 105, normalized size = 0.91

$$\frac{x\sqrt{x^4+x^2+1}}{x^2+1}-\frac{\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\left(x^2+1\right)E\left(2\operatorname{atan}(x)\middle|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}+\frac{\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\left(x^2+1\right)F\left(2\operatorname{atan}(x)\middle|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(x**4 + x**2 + 1)/(x**2 + 1) - sqrt((x**4 + x**2 + 1)/(x**2
 + 1)**2)*(x**2 + 1)*elliptic_e(2*atan(x), 1/4)/sqrt(x**4 + x**2
+ 1) + sqrt((x**4 + x**2 + 1)/(x**2 + 1)**2)*(x**2 + 1)*elliptic_
f(2*atan(x), 1/4)/sqrt(x**4 + x**2 + 1)
```

Mathematica [C] time = 0.0901072, size = 94, normalized size = 0.82

$$\frac{\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(\left(\sqrt[3]{-1}-1\right)F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)\right)}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] ((-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(El
lipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1 + (-1)^(1/3))*
```

```
EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]))/Sqrt[1 + x^2 + x
^4]
```

Maple [C] time = 0.007, size = 205, normalized size = 1.8

$$2\frac{\sqrt{1-\left(-1/2+i/2\sqrt{3}\right)x^2}\sqrt{1-\left(-1/2-i/2\sqrt{3}\right)x^2}EllipticF\left(1/2\,x\sqrt{-2+2\,i\sqrt{3}},1/2\,\sqrt{-2+2\,i\sqrt{3}}\right)}{\sqrt{-2+2\,i\sqrt{3}}\sqrt{x^4+x^2+1}} -4\frac{\sqrt{1-\left(-1/2+i/2\sqrt{3}\right)x^2}\sqrt{1-\left(-1/2-i/2\sqrt{3}\right)x^2}\left(EllipticF\left(1/2\,x\sqrt{-2+2\,i\sqrt{3}},1/2\,\sqrt{-2+2\,i\sqrt{3}}\right)-EllipticE\left(1/2\,x\sqrt{\right.\right.}{\sqrt{-2+2\,i\sqrt{3}}\sqrt{x^4+x^2+1}\left(i\sqrt{3}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-
1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(
-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-4/(-2+2*I*3^(1/
2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/
2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(
-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x
*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/sqrt(x^4 + x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2+1}{\sqrt{x^4+x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/sqrt(x^4 + x^2 + 1),x, algorithm="fricas")
```

```
[Out] integral((x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{\sqrt{(x^2-x+1)(x^2+x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**4+x**2+1)**(1/2),x)

[Out] Integral((x**2 + 1)/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/sqrt(x^4 + x^2 + 1),x, algorithm="giac")

[Out] integrate((x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

3.239 $\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=69

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

```
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)
/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4]
)
```

Rubi [A] time = 0.109755, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)
/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4]
)
```

Rubi in Sympy [A] time = 66.1126, size = 236, normalized size = 3.42

$$-\frac{4i\sqrt{x^4+x^2+1}\,\Im\left(\frac{1}{\left(1+\sqrt{3}i\right)\left(1-\frac{2}{1+\sqrt{3}i}\right)\left(\sqrt{3}-i\right)}\right)F\left(\operatorname{atan}\left(x\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)\right)\middle|\frac{3}{2}-\frac{\sqrt{3}i}{2}\right)}{\sqrt{\frac{x^2\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)+1}{x^2\left(\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)+1}}\left(x^2\left(\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)+1\right)}+\frac{\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)\sqrt{x^4+x^2+1}\,\Pi\left(\frac{1}{2}-\frac{\sqrt{3}i}{2};\operatorname{atan}\left(x\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)\right)\middle|\frac{3}{2}-\frac{\sqrt{3}i}{2}\right)}{\sqrt{\frac{x^2\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)+1}{x^2\left(\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)+1}}\left(x^2\left(\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] -4*I*sqrt(x**4 + x**2 + 1)*im(1/((1 + sqrt(3)*I)*(1 - 2/(1 + sqrt
(3)*I))*(sqrt(3) - I)))*elliptic_f(atan(x*(sqrt(3)/2 - I/2)), 3/2
 - sqrt(3)*I/2)/(sqrt((x**2*(1/2 + sqrt(3)*I/2) + 1)/(x**2*(1/2 -
 sqrt(3)*I/2) + 1))*(x**2*(1/2 - sqrt(3)*I/2) + 1)) + (sqrt(3)/2
- I/2)*sqrt(x**4 + x**2 + 1)*elliptic_pi(1/2 - sqrt(3)*I/2, atan(
x*(sqrt(3)/2 - I/2)), 3/2 - sqrt(3)*I/2)/(sqrt((x**2*(1/2 + sqrt(
3)*I/2) + 1)/(x**2*(1/2 - sqrt(3)*I/2) + 1))*(x**2*(1/2 - sqrt(3)
*I/2) + 1))
```

Mathematica [C] time = 0.0381989, size = 73, normalized size = 1.06

$$-\frac{(-1)^{2/3}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\,\Pi\left(\sqrt[3]{-1};-i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] -(((-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*E
llipticPi[(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/Sq
rt[1 + x^2 + x^4])
```

Maple [C] time = 0.022, size = 104, normalized size = 1.5

$$\frac{1}{\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}EllipticPi\left(\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}x,-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)^{-1},\frac{\sqrt{-\frac{1}{2}-\frac{i}{2}\sqrt{3}}}{\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*
(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi(
(-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*
3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{x^4+x^2+1}(x^2+1)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2-x+1)(x^2+x+1)}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)/(x**4+x**2+1)**(1/2),x)

[Out] Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + x^2 + 1}(x^2 + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)),x, algorithm="giac")

[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

3.240 $\int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=118

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}$$

```
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)
/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4]
) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcT
an[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.197711, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)
/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4]
) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcT
an[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rubi in Sympy [A] time = 80.0594, size = 328, normalized size = 2.78

$$\frac{\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\left(x^2+1\right)E\left(2\operatorname{atan}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}-\frac{\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\left(x^2+1\right)F\left(2\operatorname{atan}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}$$
$$-\frac{4i\sqrt{x^4+x^2+1}\,\Im\left(\frac{1}{\left(1+\sqrt{3}i\right)\left(1-\frac{2}{1+\sqrt{3}i}\right)\left(\sqrt{3}-i\right)}\right)F\left(\operatorname{atan}\left(x\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)\right)\middle|\frac{3}{2}-\frac{\sqrt{3}i}{2}\right)}{\sqrt{\frac{x^2\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)+1}{x^2\left(\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)+1}}\left(x^2\left(\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)+1\right)}$$
$$+\frac{\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)\sqrt{x^4+x^2+1}\,\Pi\left(\frac{1}{2}-\frac{\sqrt{3}i}{2};\operatorname{atan}\left(x\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)\right)\middle|\frac{3}{2}-\frac{\sqrt{3}i}{2}\right)}{\sqrt{\frac{x^2\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)+1}{x^2\left(\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)+1}}\left(x^2\left(\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)**2/(x**4+x**2+1)**(1/2),x)
```

```
[Out] sqrt((x**4 + x**2 + 1)/(x**2 + 1)**2)*(x**2 + 1)*elliptic_e(2*ata
n(x), 1/4)/(2*sqrt(x**4 + x**2 + 1)) - sqrt((x**4 + x**2 + 1)/(x*
*2 + 1)**2)*(x**2 + 1)*elliptic_f(2*atan(x), 1/4)/(2*sqrt(x**4 +
x**2 + 1)) - 4*I*sqrt(x**4 + x**2 + 1)*im(1/((1 + sqrt(3)*I)*(1 -
 2/(1 + sqrt(3)*I))*(sqrt(3) - I)))*elliptic_f(atan(x*(sqrt(3)/2
- I/2)), 3/2 - sqrt(3)*I/2)/(sqrt((x**2*(1/2 + sqrt(3)*I/2) + 1)/
(x**2*(1/2 - sqrt(3)*I/2) + 1))*(x**2*(1/2 - sqrt(3)*I/2) + 1)) +
 (sqrt(3)/2 - I/2)*sqrt(x**4 + x**2 + 1)*elliptic_pi(1/2 - sqrt(3
)*I/2, atan(x*(sqrt(3)/2 - I/2)), 3/2 - sqrt(3)*I/2)/(sqrt((x**2*
(1/2 + sqrt(3)*I/2) + 1)/(x**2*(1/2 - sqrt(3)*I/2) + 1))*(x**2*(1
```

```
/2 - sqrt(3)*I/2) + 1))
```

Mathematica [C] time = 0.742727, size = 226, normalized size = 1.92

$$\frac{-(-1)^{2/3}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-2(-1)^{2/3}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(\sqrt[3]{-1};-i\sinh^{-}\right.}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ((x + x^3 + x^5)/(1 + x^2) - (-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*
Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^
(2/3)] + (-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*
x^2]*(-EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + EllipticF
[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) - 2*(-1)^(2/3)*Sqrt[1 + (-
1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), (-I
)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(2*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.03, size = 397, normalized size = 3.4

$$\frac{x}{2x^2+2}\sqrt{x^4+x^2+1}$$
$$-\frac{1}{\sqrt{-2+2i\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}\mathit{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}$$
$$+2\frac{\sqrt{1+1/2x^2-i/2x^2\sqrt{3}}\sqrt{1+1/2x^2+i/2x^2\sqrt{3}}\mathit{EllipticF}\left(1/2x\sqrt{-2+2i\sqrt{3}},1/2\sqrt{-2+2i\sqrt{3}}\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}\left(i\sqrt{3}+1\right)}$$
$$-2\frac{\sqrt{1+1/2x^2-i/2x^2\sqrt{3}}\sqrt{1+1/2x^2+i/2x^2\sqrt{3}}\mathit{EllipticE}\left(1/2x\sqrt{-2+2i\sqrt{3}},1/2\sqrt{-2+2i\sqrt{3}}\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}\left(i\sqrt{3}+1\right)}$$
$$+\frac{1}{\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}\mathit{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}x,-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)^{-1},\frac{\sqrt{-\frac{1}{2}-\frac{i}{2}\sqrt{3}}}{\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{x^4+x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/2*x*(x^4+x^2+1)^(1/2)/(x^2+1)-1/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x
^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(
x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2
*I*3^(1/2))^(1/2))+2/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*
3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1
/2)/(I*3^(1/2)+1)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+
2*I*3^(1/2))^(1/2))-2/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2
*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(
1/2)/(I*3^(1/2)+1)*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2
+2*I*3^(1/2))^(1/2))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*
I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2
+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*
3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+x^2+1}(x^2+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(x^4 + 2\,x^2 + 1)\sqrt{x^4 + x^2 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2),x, algorithm="fricas")
```

```
[Out] integral(1/((x^4 + 2*x^2 + 1)*sqrt(x^4 + x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 - x + 1)\,(x^2 + x + 1)}\,(x^2 + 1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)**2/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**2), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + x^2 + 1}(x^2 + 1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2), x)
```

3.241 $\int \frac{1}{(1+x^2)^3 \sqrt{1+x^2+x^4}} \, dx$

Optimal. Leaf size=142

$$\frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}+\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 +
 x^4]]/4 + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Ellipti
cE[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[(
1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(2*Sqrt[
1 + x^2 + x^4])
```

Rubi [A] time = 0.432877, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}+\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x^2)^3*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 +
 x^4]]/4 + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Ellipti
cE[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[(
1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(2*Sqrt[
1 + x^2 + x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)**3/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.413462, size = 235, normalized size = 1.65

$$\frac{-2(-1)^{2/3}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-2(-1)^{2/3}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(\sqrt[3]{-1};-i\sinh\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x^2)^3*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ((x*(4 + 3*x^2)*(1 + x^2 + x^4))/(1 + x^2)^2 - 3*(-1)^(1/3)*Sqrt[
1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh
[(-1)^(5/6)*x], (-1)^(2/3)] - EllipticF[I*ArcSinh[(-1)^(5/6)*x],
(-1)^(2/3)]) - 2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1
)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - 2*(
-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*Ellip
ticPi[(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(4*Sqr
t[1 + x^2 + x^4])
```

Maple [C] time = 0.031, size = 418, normalized size = 2.9

$$\frac{x}{4\left(x^2+1\right)^2}\sqrt{x^4+x^2+1}+\frac{3x}{4x^2+4}\sqrt{x^4+x^2+1}-\frac{1}{\sqrt{-2+2i\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}EllipticF\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}+3\frac{\sqrt{1+1/2x^2-i/2x^2\sqrt{3}}\sqrt{1+1/2x^2+i/2x^2\sqrt{3}}EllipticF\left(1/2x\sqrt{-2+2i\sqrt{3}},1/2\sqrt{-2+2i\sqrt{3}}\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}\left(i\sqrt{3}+1\right)}-3\frac{\sqrt{1+1/2x^2-i/2x^2\sqrt{3}}\sqrt{1+1/2x^2+i/2x^2\sqrt{3}}EllipticE\left(1/2x\sqrt{-2+2i\sqrt{3}},1/2\sqrt{-2+2i\sqrt{3}}\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}\left(i\sqrt{3}+1\right)}+\frac{1}{2\sqrt{-1/2+i/2\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}EllipticPi\left(\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}x,-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)^{-1},\frac{\sqrt{-\frac{1}{2}-\frac{i}{2}\sqrt{3}}}{\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{x^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+3/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)
-1/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+
1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*
x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+3/(-2+2*I*3^
(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I
*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticF(1/2
*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-3/(-2+2*I*3
^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*
I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticE(1/
2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+1/2/(-1/2+
1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x
^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/
2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))
^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+x^2+1}\left(x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(x^{6}+3\,x^{4}+3\,x^{2}+1\right)\sqrt{x^{4}+x^{2}+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3),x, algorithm="fricas")
```

```
[Out] integral(1/((x^6 + 3*x^4 + 3*x^2 + 1)*sqrt(x^4 + x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(x^{2}-x+1\right)\left(x^{2}+x+1\right)}\left(x^{2}+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)**3/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**3), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^{4}+x^{2}+1}\left(x^{2}+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3), x)
```

3.242 $\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=144

$$\frac{2\sqrt{x^4+x^2+1}x}{3(x^2+1)}-\frac{(1-x^2)x}{3\sqrt{x^4+x^2+1}}+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

```
[Out] -(x*(1 - x^2))/(3*Sqrt[1 + x^2 + x^4]) + (2*x*Sqrt[1 + x^2 + x^4]
)/(3*(1 + x^2)) - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*
EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + ((1 + x^2)
*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/S
qrt[1 + x^2 + x^4]
```

Rubi [A] time = 0.114214, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2\sqrt{x^4+x^2+1}x}{3(x^2+1)}-\frac{(1-x^2)x}{3\sqrt{x^4+x^2+1}}+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)^3/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] -(x*(1 - x^2))/(3*Sqrt[1 + x^2 + x^4]) + (2*x*Sqrt[1 + x^2 + x^4]
)/(3*(1 + x^2)) - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*
EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + ((1 + x^2)
*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/S
qrt[1 + x^2 + x^4]
```

Rubi in Sympy [A] time = 22.2267, size = 133, normalized size = 0.92

$$-\frac{x(-x^2+1)}{3\sqrt{x^4+x^2+1}}+\frac{2x\sqrt{x^4+x^2+1}}{3(x^2+1)}-\frac{2\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}(x^2+1)E\left(2\operatorname{atan}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}+\frac{\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}(x^2+1)F\left(2\operatorname{atan}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)**3/(x**4+x**2+1)**(3/2),x)
```

```
[Out] -x*(-x**2 + 1)/(3*sqrt(x**4 + x**2 + 1)) + 2*x*sqrt(x**4 + x**2 +
 1)/(3*(x**2 + 1)) - 2*sqrt((x**4 + x**2 + 1)/(x**2 + 1)**2)*(x**
2 + 1)*elliptic_e(2*atan(x), 1/4)/(3*sqrt(x**4 + x**2 + 1)) + sqr
t((x**4 + x**2 + 1)/(x**2 + 1)**2)*(x**2 + 1)*elliptic_f(2*atan(x
), 1/4)/sqrt(x**4 + x**2 + 1)
```

Mathematica [C] time = 0.272696, size = 136, normalized size = 0.94

$$\frac{x^3+2(-1)^{5/6}\sqrt{3\sqrt[3]{-1}x^2+3}\sqrt{1-(-1)^{2/3}x^2}F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+2\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}E\left(i\sinh^{-1}\left((\right.\right.}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)^3/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] (-x + x^3 + 2*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(
2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2*(-1)
^(5/6)*Sqrt[3 + 3*(-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*Ellipt
icF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(3*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.044, size = 268, normalized size = 1.9

$$-4\frac{-x/6+1/6\,x^3}{\sqrt{x^4+x^2+1}}+\frac{8}{3\sqrt{-2+2\,i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-2+2\,i\sqrt{3}}}{2},\frac{\sqrt{-2+2\,i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}-\frac{8}{3\sqrt{-2+2\,i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-2+2\,i\sqrt{3}}}{2},\frac{\sqrt{-2+2\,i\sqrt{3}}}{2}\right)-Ellipti\right.-6\frac{-1/3\,x^3-x/6}{\sqrt{x^4+x^2+1}}-6\frac{1/6\,x^3+x/3}{\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^3/(x^4+x^2+1)^(3/2),x)
```

```
[Out] -4*(-1/6*x+1/6*x^3)/(x^4+x^2+1)^(1/2)+8/3/(-2+2*I*3^(1/2))^(1/2)*
(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(
1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2
*(-2+2*I*3^(1/2))^(1/2))-8/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*
I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2
+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1
/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2)
,1/2*(-2+2*I*3^(1/2))^(1/2)))-6*(-1/3*x^3-1/6*x)/(x^4+x^2+1)^(1/2
)-6*(1/6*x^3+1/3*x)/(x^4+x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+1\right)^3}{\left(x^4+x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^3/(x^4 + x^2 + 1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^3/(x^4 + x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6+3\,x^4+3\,x^2+1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^3/(x^4 + x^2 + 1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x^6 + 3*x^4 + 3*x^2 + 1)/(x^4 + x^2 + 1)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2+1)^3}{((x^2-x+1)(x^2+x+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**3/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral((x**2 + 1)**3/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2+1)^3}{(x^4+x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^3/(x^4 + x^2 + 1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^3/(x^4 + x^2 + 1)^(3/2), x)
```

3.243 $\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2\sqrt{x^4+x^2+1}x}{3(x^2+1)}+\frac{(2x^2+1)x}{3\sqrt{x^4+x^2+1}}+\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

```
[Out] (x*(1 + 2*x^2))/(3*Sqrt[1 + x^2 + x^4]) - (2*x*Sqrt[1 + x^2 + x^4
])/(3*(1 + x^2)) + (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]
*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.072919, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{2\sqrt{x^4+x^2+1}x}{3(x^2+1)}+\frac{(2x^2+1)x}{3\sqrt{x^4+x^2+1}}+\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)^2/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(1 + 2*x^2))/(3*Sqrt[1 + x^2 + x^4]) - (2*x*Sqrt[1 + x^2 + x^4
])/(3*(1 + x^2)) + (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]
*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4])
```

Rubi in Sympy [A] time = 16.2451, size = 90, normalized size = 0.92

$$\frac{x(2x^2+1)}{3\sqrt{x^4+x^2+1}}-\frac{2x\sqrt{x^4+x^2+1}}{3(x^2+1)}+\frac{2\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}(x^2+1)E\left(2\,\mathrm{atan}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)**2/(x**4+x**2+1)**(3/2),x)
```

```
[Out] x*(2*x**2 + 1)/(3*sqrt(x**4 + x**2 + 1)) - 2*x*sqrt(x**4 + x**2 +
 1)/(3*(x**2 + 1)) + 2*sqrt((x**4 + x**2 + 1)/(x**2 + 1)**2)*(x**
2 + 1)*elliptic_e(2*atan(x), 1/4)/(3*sqrt(x**4 + x**2 + 1))
```

Mathematica [C] time = 0.294037, size = 158, normalized size = 1.61

$$\frac{2x^3-i\sqrt{2+\left(1+i\sqrt{3}\right)x^2}\sqrt{6+\left(3-3i\sqrt{3}\right)x^2}F\left(\sin^{-1}\left(\frac{1}{2}\left(i\sqrt{3}x+x\right)\right)|\frac{1}{2}i\left(i+\sqrt{3}\right)\right)-2\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}E}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)^2/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] (x + 2*x^3 - 2*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^
(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - I*Sqr
t[2 + (1 + I*Sqrt[3])*x^2]*Sqrt[6 + (3 - (3*I)*Sqrt[3])*x^2]*Elli
```

```
pticF[ArcSin[(x + I*Sqrt[3]*x)/2], (I/2)*(I + Sqrt[3])])/(3*Sqrt[
1 + x^2 + x^4])
```

Maple [C] time = 0.011, size = 268, normalized size = 2.7

$$-2\frac{-x/6+1/6x^3}{\sqrt{x^4+x^2+1}}+\frac{4}{3\sqrt{-2+2i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}+\frac{8}{3\sqrt{-2+2i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)-Elliptic\right.-2\frac{1/6x^3+x/3}{\sqrt{x^4+x^2+1}}-4\frac{-1/3x^3-x/6}{\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^2/(x^4+x^2+1)^(3/2),x)
```

```
[Out] -2*(-1/6*x+1/6*x^3)/(x^4+x^2+1)^(1/2)+4/3/(-2+2*I*3^(1/2))^(1/2)*
(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(
1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2
*(-2+2*I*3^(1/2))^(1/2))+8/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*
I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2
+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1
/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2)
,1/2*(-2+2*I*3^(1/2))^(1/2)))-2*(1/6*x^3+1/3*x)/(x^4+x^2+1)^(1/2)
-4*(-1/3*x^3-1/6*x)/(x^4+x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^2+1\right)^2}{\left(x^4+x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^2/(x^4 + x^2 + 1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^2/(x^4 + x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4+2x^2+1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^2/(x^4 + x^2 + 1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 2*x^2 + 1)/(x^4 + x^2 + 1)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+1\right)^2}{\left(\left(x^2-x+1\right)\left(x^2+x+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**2/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral((x**2 + 1)**2/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+1\right)^2}{\left(x^4+x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^2/(x^4 + x^2 + 1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^2/(x^4 + x^2 + 1)^(3/2), x)
```

3.244 $\int \frac{1+x^2}{(1+x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{\sqrt{x^4+x^2+1}x}{3(x^2+1)}+\frac{(x^2+2)\,x}{3\sqrt{x^4+x^2+1}}+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

```
[Out] (x*(2 + x^2))/(3*Sqrt[1 + x^2 + x^4]) - (x*Sqrt[1 + x^2 + x^4])/(
3*(1 + x^2)) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Ellip
ticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.0585351, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{\sqrt{x^4+x^2+1}x}{3(x^2+1)}+\frac{(x^2+2)\,x}{3\sqrt{x^4+x^2+1}}+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(2 + x^2))/(3*Sqrt[1 + x^2 + x^4]) - (x*Sqrt[1 + x^2 + x^4])/(
3*(1 + x^2)) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Ellip
ticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4])
```

Rubi in Sympy [A] time = 11.466, size = 85, normalized size = 0.89

$$\frac{x\left(x^2+2\right)}{3\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{3(x^2+1)}+\frac{\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\left(x^2+1\right)E\left(2\,\mathrm{atan}\,(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/(x**4+x**2+1)**(3/2),x)
```

```
[Out] x*(x**2 + 2)/(3*sqrt(x**4 + x**2 + 1)) - x*sqrt(x**4 + x**2 + 1)/
(3*(x**2 + 1)) + sqrt((x**4 + x**2 + 1)/(x**2 + 1)**2)*(x**2 + 1)
*elliptic_e(2*atan(x), 1/4)/(3*sqrt(x**4 + x**2 + 1))
```

Mathematica [C] time = 0.300801, size = 160, normalized size = 1.67

$$\frac{x^3-\frac{1}{2}i\sqrt{2+\left(1+i\sqrt{3}\right)x^2}\sqrt{6+\left(3-3i\sqrt{3}\right)x^2}F\left(\sin^{-1}\left(\frac{1}{2}\left(i\sqrt{3}x+x\right)\right)|\frac{1}{2}i\left(i+\sqrt{3}\right)\right)-\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}E}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] (2*x + x^3 - (-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2
/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - (I/2)*S
qrt[2 + (1 + I*Sqrt[3])*x^2]*Sqrt[6 + (3 - (3*I)*Sqrt[3])*x^2]*El
```

```
lipticF[ArcSin[(x + I*Sqrt[3]*x)/2], (I/2)*(I + Sqrt[3])])/(3*Sqr
t[1 + x^2 + x^4])
```

Maple [C] time = 0.01, size = 247, normalized size = 2.6

$$-2\frac{-x/6+1/6\,x^3}{\sqrt{x^4+x^2+1}} + \frac{2}{3\sqrt{-2+2\,i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-2+2\,i\sqrt{3}}}{2},\frac{\sqrt{-2+2\,i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}} + \frac{4}{3\sqrt{-2+2\,i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-2+2\,i\sqrt{3}}}{2},\frac{\sqrt{-2+2\,i\sqrt{3}}}{2}\right) - Elliptic\right. - 2\frac{-1/3\,x^3-x/6}{\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^4+x^2+1)^(3/2),x)
```

```
[Out] -2*(-1/6*x+1/6*x^3)/(x^4+x^2+1)^(1/2)+2/3/(-2+2*I*3^(1/2))^(1/2)*
(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(
1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2
*(-2+2*I*3^(1/2))^(1/2))+4/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*
I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2
+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1
/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2)
,1/2*(-2+2*I*3^(1/2))^(1/2)))-2*(-1/3*x^3-1/6*x)/(x^4+x^2+1)^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{(x^4+x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + x^2 + 1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2+1}{(x^4+x^2+1)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + x^2 + 1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x^2 + 1)/(x^4 + x^2 + 1)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + 1}{\left((x^2 - x + 1)(x^2 + x + 1)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral((x**2 + 1)/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + 1}{(x^4 + x^2 + 1)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^4 + x^2 + 1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)/(x^4 + x^2 + 1)^(3/2), x)
```

3.245 $\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=166

$$\frac{2\sqrt{x^4+x^2+1}x}{3(x^2+1)}-\frac{(2x^2+1)\,x}{3\sqrt{x^4+x^2+1}}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{2\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

```
[Out] -(x*(1 + 2*x^2))/(3*Sqrt[1 + x^2 + x^4]) + (2*x*Sqrt[1 + x^2 + x^
4])/(3*(1 + x^2)) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 - (2*(1 + x^2
)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/
(3*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 +
x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.223068, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$\frac{2\sqrt{x^4+x^2+1}x}{3(x^2+1)}-\frac{(2x^2+1)\,x}{3\sqrt{x^4+x^2+1}}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{2\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x^2)*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] -(x*(1 + 2*x^2))/(3*Sqrt[1 + x^2 + x^4]) + (2*x*Sqrt[1 + x^2 + x^
4])/(3*(1 + x^2)) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 - (2*(1 + x^2
)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/
(3*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 +
x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rubi in Sympy [A] time = 130.828, size = 348, normalized size = 2.1

$$-\frac{x\left(2x^2+1\right)}{3\sqrt{x^4+x^2+1}}+\frac{2x\sqrt{x^4+x^2+1}}{3\left(x^2+1\right)}-\frac{2\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\left(x^2+1\right)E\left(2\operatorname{atan}(x)\middle|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}+\frac{\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\left(x^2+1\right)F\left(2\operatorname{atan}(x)\middle|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}+\frac{2\sqrt{x^4+x^2+1}F\left(\operatorname{atan}\left(x\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)\right)\middle|\frac{3}{2}+\frac{\sqrt{3}i}{2}\right)}{\sqrt{\frac{x^2\left(\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)+1}{x^2\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)+1}}\left(\sqrt{3}-i\right)\left(x^2\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)+1\right)}+\frac{\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)\sqrt{x^4+x^2+1}\left(\frac{1}{2}-\frac{\sqrt{3}i}{2};\operatorname{atan}\left(x\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)\right)\middle|\frac{3}{2}-\frac{\sqrt{3}i}{2}\right)}{\sqrt{\frac{x^2\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)+1}{x^2\left(\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)+1}}\left(x^2\left(\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)/(x**4+x**2+1)**(3/2),x)
```

```
[Out] -x*(2*x**2 + 1)/(3*sqrt(x**4 + x**2 + 1)) + 2*x*sqrt(x**4 + x**2
+ 1)/(3*(x**2 + 1)) - 2*sqrt((x**4 + x**2 + 1)/(x**2 + 1)**2)*(x*
*2 + 1)*elliptic_e(2*atan(x), 1/4)/(3*sqrt(x**4 + x**2 + 1)) + sq
```

```
rt((x**4 + x**2 + 1)/(x**2 + 1)**2)*(x**2 + 1)*elliptic_f(2*atan(
x), 1/4)/(2*sqrt(x**4 + x**2 + 1)) + 2*sqrt(x**4 + x**2 + 1)*elli
ptic_f(atan(x*(sqrt(3)/2 + I/2)), 3/2 + sqrt(3)*I/2)/(sqrt((x**2*
(1/2 - sqrt(3)*I/2) + 1)/(x**2*(1/2 + sqrt(3)*I/2) + 1))*(sqrt(3)
 - I)*(x**2*(1/2 + sqrt(3)*I/2) + 1)) + (sqrt(3)/2 - I/2)*sqrt(x*
*4 + x**2 + 1)*elliptic_pi(1/2 - sqrt(3)*I/2, atan(x*(sqrt(3)/2 -
 I/2)), 3/2 - sqrt(3)*I/2)/(sqrt((x**2*(1/2 + sqrt(3)*I/2) + 1)/(
x**2*(1/2 - sqrt(3)*I/2) + 1))*(x**2*(1/2 - sqrt(3)*I/2) + 1))
```

Mathematica [C] time = 0.291583, size = 204, normalized size = 1.23

$$\frac{-2x^3 + \sqrt[3]{-1}\left(\sqrt[3]{-1}-2\right)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right) + 2\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}E\left(i\right.}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x^2)*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] (-x - 2*x^3 + 2*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)
^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)
^(1/3)*(-2 + (-1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(
2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - 3*(-1)
^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*Elliptic
Pi[(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(3*Sqrt[1
 + x^2 + x^4])
```

Maple [C] time = 0.026, size = 398, normalized size = 2.4

$$-2\,\frac{1/3\,x^3+x/6}{\sqrt{x^4+x^2+1}}$$
$$+\frac{2}{3\sqrt{-2+2i\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}EllipticF\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}$$
$$-\frac{8}{3\sqrt{-2+2i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}EllipticF\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}$$
$$+\frac{8}{3\sqrt{-2+2i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}EllipticE\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}$$
$$+\frac{1}{\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}EllipticPi\left(\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}x,-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)^{-1},\frac{\sqrt{-\frac{1}{2}-\frac{i}{2}\sqrt{3}}}{\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{x^4+x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)/(x^4+x^2+1)^(3/2),x)
```

```
[Out] -2*(1/3*x^3+1/6*x)/(x^4+x^2+1)^(1/2)+2/3/(-2+2*I*3^(1/2))^(1/2)*(
1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^
(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/
2*(-2+2*I*3^(1/2))^(1/2))-8/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1
/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+
x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2)
,1/2*(-2+2*I*3^(1/2))^(1/2))+8/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^
2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x
^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1
/2),1/2*(-2+2*I*3^(1/2))^(1/2))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1
/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/
2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(
```

```
-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2
))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + x^2 + 1)^{\frac{3}{2}}(x^2 + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(x^6 + 2\,x^4 + 2\,x^2 + 1)\sqrt{x^4 + x^2 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] integral(1/((x^6 + 2*x^4 + 2*x^2 + 1)*sqrt(x^4 + x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2 - x + 1)\,(x^2 + x + 1))^{\frac{3}{2}}\,(x^2 + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + x^2 + 1)^{\frac{3}{2}}(x^2 + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)), x)
```

3.246 $\int \frac{1}{(1+x^2)^2(1+x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=111

$$\frac{\sqrt{x^4+x^2+1}x}{3(x^2+1)}-\frac{(x^2+2)\,x}{3\sqrt{x^4+x^2+1}}+\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{6\sqrt{x^4+x^2+1}}$$

```
[Out] -(x*(2 + x^2))/(3*Sqrt[1 + x^2 + x^4]) + (x*Sqrt[1 + x^2 + x^4])/
(3*(1 + x^2)) + ArcTan[x/Sqrt[1 + x^2 + x^4]] + ((1 + x^2)*Sqrt[(
1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(6*Sqrt[
1 + x^2 + x^4])
```

Rubi [A] time = 0.435122, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45

$$\frac{\sqrt{x^4+x^2+1}x}{3(x^2+1)}-\frac{(x^2+2)\,x}{3\sqrt{x^4+x^2+1}}+\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{6\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x^2)^2*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] -(x*(2 + x^2))/(3*Sqrt[1 + x^2 + x^4]) + (x*Sqrt[1 + x^2 + x^4])/
(3*(1 + x^2)) + ArcTan[x/Sqrt[1 + x^2 + x^4]] + ((1 + x^2)*Sqrt[(
1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(6*Sqrt[
1 + x^2 + x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)**2/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

Mathematica [C] time = 0.574707, size = 168, normalized size = 1.51

$$\frac{-2x(x^2+1)(x^2+2)-\sqrt[3]{-1}(x^2+1)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(\left(5\sqrt[3]{-1}-1\right)F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+E\left(i\sinh\right.\right.}{6(x^2+1)\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x^2)^2*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] (-2*x*(1 + x^2)*(2 + x^2) + 3*x*(1 + x^2 + x^4) - (-1)^(1/3)*(1 +
 x^2)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(Elliptic
E[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1 + 5*(-1)^(1/3))*Elli
pticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 12*(-1)^(1/3)*Ellipt
icPi[(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]))/(6*(1
+ x^2)*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.034, size = 419, normalized size = 3.8

$$\frac{x}{2x^{2}+2}\sqrt{x^{4}+x^{2}+1}-2\,\frac{1/6\,x^{3}+x/3}{\sqrt{x^{4}+x^{2}+1}}$$
$$-\frac{5}{3\sqrt{-2+2\,i\sqrt{3}}}\sqrt{1+\frac{x^{2}}{2}-\frac{i}{2}x^{2}\sqrt{3}}\sqrt{1+\frac{x^{2}}{2}+\frac{i}{2}x^{2}\sqrt{3}}\mathit{EllipticF}\left(\frac{x\sqrt{-2+2\,i\sqrt{3}}}{2},\frac{\sqrt{-2+2\,i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^{4}+x^{2}+1}}$$
$$+\frac{2}{3\sqrt{-2+2\,i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1+\frac{x^{2}}{2}-\frac{i}{2}x^{2}\sqrt{3}}\sqrt{1+\frac{x^{2}}{2}+\frac{i}{2}x^{2}\sqrt{3}}\mathit{EllipticF}\left(\frac{x\sqrt{-2+2\,i\sqrt{3}}}{2},\frac{\sqrt{-2+2\,i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^{4}+x^{2}+1}}$$
$$-\frac{2}{3\sqrt{-2+2\,i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1+\frac{x^{2}}{2}-\frac{i}{2}x^{2}\sqrt{3}}\sqrt{1+\frac{x^{2}}{2}+\frac{i}{2}x^{2}\sqrt{3}}\mathit{EllipticE}\left(\frac{x\sqrt{-2+2\,i\sqrt{3}}}{2},\frac{\sqrt{-2+2\,i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^{4}+x^{2}+1}}$$
$$+2\,\frac{\sqrt{1+1/2\,x^{2}-i/2x^{2}\sqrt{3}}\sqrt{1+1/2\,x^{2}+i/2x^{2}\sqrt{3}}}{\sqrt{-1/2+i/2\sqrt{3}}\sqrt{x^{4}+x^{2}+1}}\mathit{EllipticPi}\left(\sqrt{-1/2+i/2\sqrt{3}}x,-\left(-1/2+i/2\sqrt{3}\right)^{-1},\frac{\sqrt{-1/2-i/2\sqrt{3}}}{\sqrt{-1/2+i/2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x)
```

```
[Out] 1/2*x*(x^4+x^2+1)^(1/2)/(x^2+1)-2*(1/6*x^3+1/3*x)/(x^4+x^2+1)^(1/
2)-5/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)
*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(
1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/3/(-2+
2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2
+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*Ellipti
cF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-2/3/(
-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*
x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*Elli
pticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/
(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1
+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-
1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^
(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^{4}+x^{2}+1)^{\frac{3}{2}}(x^{2}+1)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{(x^{8}+3x^{6}+4x^{4}+3x^{2}+1)\sqrt{x^{4}+x^{2}+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^2),x, algorithm="fricas")
```

```
[Out] integral(1/((x^8 + 3*x^6 + 4*x^4 + 3*x^2 + 1)*sqrt(x^4 + x^2 + 1)
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x^{2}-x+1\right)\left(x^{2}+x+1\right)\right)^{\frac{3}{2}}\left(x^{2}+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)**2/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)**2)
, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^{4}+x^{2}+1\right)^{\frac{3}{2}}\left(x^{2}+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^2), x)
```

3.247 $\int \frac{1}{(1+x^2)^3(1+x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=190

$$-\frac{\sqrt{x^4+x^2+1}\,x}{3\left(x^2+1\right)}+\frac{\sqrt{x^4+x^2+1}\,x}{4\left(x^2+1\right)^2}-\frac{\left(1-x^2\right)x}{3\sqrt{x^4+x^2+1}}+\frac{3}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{5\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+\frac{19\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{12\sqrt{x^4+x^2+1}}$$

```
[Out] -(x*(1 - x^2))/(3*Sqrt[1 + x^2 + x^4]) + (x*Sqrt[1 + x^2 + x^4])/
(4*(1 + x^2)^2) - (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (3*ArcT
an[x/Sqrt[1 + x^2 + x^4]])/4 + (19*(1 + x^2)*Sqrt[(1 + x^2 + x^4)
/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(12*Sqrt[1 + x^2 + x^4
]) - (5*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*A
rcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.912358, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.65

$$-\frac{\sqrt{x^4+x^2+1}\,x}{3\left(x^2+1\right)}+\frac{\sqrt{x^4+x^2+1}\,x}{4\left(x^2+1\right)^2}-\frac{\left(1-x^2\right)x}{3\sqrt{x^4+x^2+1}}+\frac{3}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{5\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+\frac{19\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{12\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x^2)^3*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] -(x*(1 - x^2))/(3*Sqrt[1 + x^2 + x^4]) + (x*Sqrt[1 + x^2 + x^4])/
(4*(1 + x^2)^2) - (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (3*ArcT
an[x/Sqrt[1 + x^2 + x^4]])/4 + (19*(1 + x^2)*Sqrt[(1 + x^2 + x^4)
/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(12*Sqrt[1 + x^2 + x^4
]) - (5*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*A
rcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)**3/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.379815, size = 192, normalized size = 1.01

$$\frac{4x\left(x^2-1\right)\left(x^2+1\right)^2-\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(x^2+1\right)^2\left(\left(-9+10i\sqrt{3}\right)F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)+19E\left(i\right.\right.}{12\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x^2)^3*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] (4*x*(-1 + x^2)*(1 + x^2)^2 + 3*x*(1 + x^2 + x^4) + 15*x*(1 + x^2
)*(1 + x^2 + x^4) - (-1)^(1/3)*(1 + x^2)^2*Sqrt[1 + (-1)^(1/3)*x^
2]*Sqrt[1 - (-1)^(2/3)*x^2]*(19*EllipticE[I*ArcSinh[(-1)^(5/6)*x]
, (-1)^(2/3)] + (-9 + (10*I)*Sqrt[3])*EllipticF[I*ArcSinh[(-1)^(5
/6)*x], (-1)^(2/3)] + 18*(-1)^(1/3)*EllipticPi[(-1)^(1/3), (-I)*A
rcSinh[(-1)^(5/6)*x], (-1)^(2/3)]))/(12*(1 + x^2)^2*Sqrt[1 + x^2
+ x^4])
```

Maple [C] time = 0.035, size = 439, normalized size = 2.3

$$\frac{x}{4\left(x^2+1\right)^2}\sqrt{x^4+x^2+1}+\frac{5x}{4x^2+4}\sqrt{x^4+x^2+1}-2\frac{x/6-1/6x^3}{\sqrt{x^4+x^2+1}}$$
$$-\frac{10}{3\sqrt{-2+2i\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}\mathit{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}$$
$$+\frac{19}{3\sqrt{-2+2i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}\mathit{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}$$
$$-\frac{19}{3\sqrt{-2+2i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}\mathit{EllipticE}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\frac{1}{\sqrt{x^4+x^2+1}}$$
$$+\frac{3}{2\sqrt{-1/2+i/2\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}\mathit{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}x,-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)^{-1},\frac{\sqrt{-\frac{1}{2}-\frac{i}{2}\sqrt{3}}}{\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{x^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x)
```

```
[Out] 1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+5/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)
-2*(1/6*x-1/6*x^3)/(x^4+x^2+1)^(1/2)-10/3/(-2+2*I*3^(1/2))^(1/2)*
(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))
^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1
/2*(-2+2*I*3^(1/2))^(1/2))+19/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2
-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^
4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/
2),1/2*(-2+2*I*3^(1/2))^(1/2))-19/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2
*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)
/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticE(1/2*x*(-2+2*I*3^(1/2))
^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+3/2/(-1/2+1/2*I*3^(1/2))^(1/2)
*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2)
)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x
,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3
^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}\left(x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(x^{10} + 4\,x^8 + 7\,x^6 + 7\,x^4 + 4\,x^2 + 1)\sqrt{x^4 + x^2 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^3),x, algorithm="fricas")
```

```
[Out] integral(1/((x^10 + 4*x^8 + 7*x^6 + 7*x^4 + 4*x^2 + 1)*sqrt(x^4 +
 x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x^2 - x + 1)\,(x^2 + x + 1)\right)^{\frac{3}{2}}\,(x^2 + 1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)**3/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)**3)
, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + x^2 + 1)^{\frac{3}{2}}(x^2 + 1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^3), x)
```

3.248 $\int \left(d+ex^2\right)^4 \left(a+bx^2+cx^4\right)\, dx$

Optimal. Leaf size=135

$$\frac{1}{9}e^2x^9\left(e(ae+4bd)+6cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+4bde+cd^2\right)+\frac{2}{7}dex^7\left(e(2ae+3bd)+2cd^2\right)+\frac{1}{3}d^3x^3(4ae+bd)+ad^4x+\frac{1}{11}e^3x^{11}(be+4cd)+\frac{1}{13}ce^4x^{13}$$

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^3)/3 + (d^2*(c*d^2 + 4*b*d*e + 6*a
*e^2)*x^5)/5 + (2*d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^7)/7 + (e^2
*(6*c*d^2 + e*(4*b*d + a*e))*x^9)/9 + (e^3*(4*c*d + b*e)*x^11)/11
 + (c*e^4*x^13)/13
```

Rubi [A] time = 0.25501, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{1}{9}e^2x^9\left(e(ae+4bd)+6cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+4bde+cd^2\right)+\frac{2}{7}dex^7\left(e(2ae+3bd)+2cd^2\right)+\frac{1}{3}d^3x^3(4ae+bd)+ad^4x+\frac{1}{11}e^3x^{11}(be+4cd)+\frac{1}{13}ce^4x^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^4*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^3)/3 + (d^2*(c*d^2 + 4*b*d*e + 6*a
*e^2)*x^5)/5 + (2*d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^7)/7 + (e^2
*(6*c*d^2 + e*(4*b*d + a*e))*x^9)/9 + (e^3*(4*c*d + b*e)*x^11)/11
 + (c*e^4*x^13)/13
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ce^4x^{13}}{13}+d^4\int a\,dx+\frac{d^3x^3\left(4ae+bd\right)}{3}+\frac{d^2x^5\left(6ae^2+4bde+cd^2\right)}{5}+\frac{2dex^7\left(2ae^2+3bde+2cd^2\right)}{7}+\frac{e^3x^{11}\left(be+4cd\right)}{11}+\frac{e^2x^9\left(ae^2+4bde+6cd^2\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**4*(c*x**4+b*x**2+a),x)
```

```
[Out] c*e**4*x**13/13 + d**4*Integral(a, x) + d**3*x**3*(4*a*e + b*d)/3
 + d**2*x**5*(6*a*e**2 + 4*b*d*e + c*d**2)/5 + 2*d*e*x**7*(2*a*e*
*2 + 3*b*d*e + 2*c*d**2)/7 + e**3*x**11*(b*e + 4*c*d)/11 + e**2*x
**9*(a*e**2 + 4*b*d*e + 6*c*d**2)/9
```

Mathematica [A] time = 0.0705287, size = 135, normalized size = 1.

$$\frac{1}{9}e^2x^9\left(ae^2+4bde+6cd^2\right)+\frac{2}{7}dex^7\left(2ae^2+3bde+2cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+4bde+cd^2\right)+\frac{1}{3}d^3x^3(4ae+bd)+ad^4x+\frac{1}{11}e^3x^{11}(be+4cd)+\frac{1}{13}ce^4x^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^4*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^3)/3 + (d^2*(c*d^2 + 4*b*d*e + 6*a
*e^2)*x^5)/5 + (2*d*e*(2*c*d^2 + 3*b*d*e + 2*a*e^2)*x^7)/7 + (e^2
*(6*c*d^2 + 4*b*d*e + a*e^2)*x^9)/9 + (e^3*(4*c*d + b*e)*x^11)/11
 + (c*e^4*x^13)/13
```

Maple [A] time = 0.002, size = 136, normalized size = 1.

$$\frac{ce^4x^{13}}{13}+\frac{\left(e^4b+4\,de^3c\right)x^{11}}{11}+\frac{\left(ae^4+4\,de^3b+6\,d^2e^2c\right)x^9}{9}+\frac{\left(4\,de^3a+6\,d^2e^2b+4\,d^3ec\right)x^7}{7}+\frac{\left(6\,d^2e^2a+4\,d^3eb+d^4c\right)x^5}{5}+\frac{\left(4\,d^3ea+d^4b\right)x^3}{3}+ad^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^4*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/13*c*e^4*x^13+1/11*(b*e^4+4*c*d*e^3)*x^11+1/9*(a*e^4+4*b*d*e^3+
6*c*d^2*e^2)*x^9+1/7*(4*a*d*e^3+6*b*d^2*e^2+4*c*d^3*e)*x^7+1/5*(6
*a*d^2*e^2+4*b*d^3*e+c*d^4)*x^5+1/3*(4*a*d^3*e+b*d^4)*x^3+a*d^4*x
```

Maxima [A] time = 0.739495, size = 182, normalized size = 1.35

$$\frac{1}{13}\,ce^4x^{13}+\frac{1}{11}\left(4\,cde^3+be^4\right)x^{11}+\frac{1}{9}\left(6\,cd^2e^2+4\,bde^3+ae^4\right)x^9+\frac{2}{7}\left(2\,cd^3e+3\,bd^2e^2+2\,ade^3\right)x^7+ad^4x+\frac{1}{5}\left(cd^4+4\,bd^3e+6\,ad^2e^2\right)x^5+\frac{1}{3}\left(bd^4+4\,ad^3e\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^4,x, algorithm="maxima")
```

```
[Out] 1/13*c*e^4*x^13 + 1/11*(4*c*d*e^3 + b*e^4)*x^11 + 1/9*(6*c*d^2*e^
2 + 4*b*d*e^3 + a*e^4)*x^9 + 2/7*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d
*e^3)*x^7 + a*d^4*x + 1/5*(c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^5 +
 1/3*(b*d^4 + 4*a*d^3*e)*x^3
```

Fricas [A] time = 0.248502, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}e^4c+\frac{4}{11}x^{11}e^3dc+\frac{1}{11}x^{11}e^4b+\frac{2}{3}x^9e^2d^2c+\frac{4}{9}x^9e^3db+\frac{1}{9}x^9e^4a+\frac{4}{7}x^7ed^3c+\frac{6}{7}x^7e^2d^2b+\frac{4}{7}x^7e^3da+\frac{1}{5}x^5d^4c+\frac{4}{5}x^5ed^3b+\frac{6}{5}x^5e^2d^2a+\frac{1}{3}x^3d^4b+\frac{4}{3}x^3ed^3a+xd^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^4,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^4*c + 4/11*x^11*e^3*d*c + 1/11*x^11*e^4*b + 2/3*x^9*e
^2*d^2*c + 4/9*x^9*e^3*d*b + 1/9*x^9*e^4*a + 4/7*x^7*e*d^3*c + 6/
7*x^7*e^2*d^2*b + 4/7*x^7*e^3*d*a + 1/5*x^5*d^4*c + 4/5*x^5*e*d^3
*b + 6/5*x^5*e^2*d^2*a + 1/3*x^3*d^4*b + 4/3*x^3*e*d^3*a + x*d^4*
a
```

Sympy [A] time = 0.154345, size = 156, normalized size = 1.16

$$ad^4x + \frac{ce^4x^{13}}{13} + x^{11}\left(\frac{be^4}{11} + \frac{4cde^3}{11}\right) + x^9\left(\frac{ae^4}{9} + \frac{4bde^3}{9} + \frac{2cd^2e^2}{3}\right) + x^7\left(\frac{4ade^3}{7} + \frac{6bd^2e^2}{7} + \frac{4cd^3e}{7}\right) + x^5\left(\frac{6ad^2e^2}{5} + \frac{4bd^3e}{5} + \frac{cd^4}{5}\right) + x^3\left(\frac{4ad^3e}{3} + \frac{bd^4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**4*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**4*x + c*e**4*x**13/13 + x**11*(b*e**4/11 + 4*c*d*e**3/11) +
x**9*(a*e**4/9 + 4*b*d*e**3/9 + 2*c*d**2*e**2/3) + x**7*(4*a*d*e*
*3/7 + 6*b*d**2*e**2/7 + 4*c*d**3*e/7) + x**5*(6*a*d**2*e**2/5 +
4*b*d**3*e/5 + c*d**4/5) + x**3*(4*a*d**3*e/3 + b*d**4/3)
```

GIAC/XCAS [A] time = 0.267267, size = 192, normalized size = 1.42

$$\frac{1}{13}cx^{13}e^4 + \frac{4}{11}cdx^{11}e^3 + \frac{1}{11}bx^{11}e^4 + \frac{2}{3}cd^2x^9e^2 + \frac{4}{9}bdx^9e^3 + \frac{4}{7}cd^3x^7e + \frac{1}{9}ax^9e^4 + \frac{6}{7}bd^2x^7e^2 + \frac{1}{5}cd^4x^5 + \frac{4}{7}adx^7e^3 + \frac{4}{5}bd^3x^5e + \frac{6}{5}ad^2x^5e^2 + \frac{1}{3}bd^4x^3 + \frac{4}{3}ad^3x^3e + ad^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^4,x, algorithm="giac")
```

```
[Out] 1/13*c*x^13*e^4 + 4/11*c*d*x^11*e^3 + 1/11*b*x^11*e^4 + 2/3*c*d^2
*x^9*e^2 + 4/9*b*d*x^9*e^3 + 4/7*c*d^3*x^7*e + 1/9*a*x^9*e^4 + 6/
7*b*d^2*x^7*e^2 + 1/5*c*d^4*x^5 + 4/7*a*d*x^7*e^3 + 4/5*b*d^3*x^5
*e + 6/5*a*d^2*x^5*e^2 + 1/3*b*d^4*x^3 + 4/3*a*d^3*x^3*e + a*d^4*
x
```

3.249 $\int (d+ex^2)^3 (a+bx^2+cx^4)\, dx$

Optimal. Leaf size=103

$$\frac{1}{7}ex^7\left(e(ae+3bd)+3cd^2\right)+\frac{1}{5}dx^5\left(3e(ae+bd)+cd^2\right)+\frac{1}{3}d^2x^3(3ae+bd)+ad^3x+\frac{1}{9}e^2x^9(be+3cd)+\frac{1}{11}ce^3x^{11}$$

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^3)/3 + (d*(c*d^2 + 3*e*(b*d + a*e)
)*x^5)/5 + (e*(3*c*d^2 + e*(3*b*d + a*e))*x^7)/7 + (e^2*(3*c*d +
b*e)*x^9)/9 + (c*e^3*x^11)/11
```

Rubi [A] time = 0.197033, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{1}{7}ex^7\left(e(ae+3bd)+3cd^2\right)+\frac{1}{5}dx^5\left(3e(ae+bd)+cd^2\right)+\frac{1}{3}d^2x^3(3ae+bd)+ad^3x+\frac{1}{9}e^2x^9(be+3cd)+\frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^3*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^3)/3 + (d*(c*d^2 + 3*e*(b*d + a*e)
)*x^5)/5 + (e*(3*c*d^2 + e*(3*b*d + a*e))*x^7)/7 + (e^2*(3*c*d +
b*e)*x^9)/9 + (c*e^3*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ce^3x^{11}}{11}+d^3\int a\,dx+\frac{d^2x^3\,(3ae+bd)}{3}+\frac{dx^5\left(3ae^2+3bde+cd^2\right)}{5}+\frac{e^2x^9\,(be+3cd)}{9}+\frac{ex^7\left(ae^2+3bde+3cd^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**3*(c*x**4+b*x**2+a),x)
```

```
[Out] c*e**3*x**11/11 + d**3*Integral(a, x) + d**2*x**3*(3*a*e + b*d)/3
 + d*x**5*(3*a*e**2 + 3*b*d*e + c*d**2)/5 + e**2*x**9*(b*e + 3*c*
d)/9 + e*x**7*(a*e**2 + 3*b*d*e + 3*c*d**2)/7
```

Mathematica [A] time = 0.0490723, size = 104, normalized size = 1.01

$$\frac{1}{7}ex^7\left(ae^2+3bde+3cd^2\right)+\frac{1}{5}dx^5\left(3ae^2+3bde+cd^2\right)+\frac{1}{3}d^2x^3(3ae+bd)+ad^3x+\frac{1}{9}e^2x^9(be+3cd)+\frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^3*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^3)/3 + (d*(c*d^2 + 3*b*d*e + 3*a*e
^2)*x^5)/5 + (e*(3*c*d^2 + 3*b*d*e + a*e^2)*x^7)/7 + (e^2*(3*c*d
+ b*e)*x^9)/9 + (c*e^3*x^11)/11
```

Maple [A] time = 0.001, size = 103, normalized size = 1.

$$\frac{ce^3x^{11}}{11}+\frac{\left(e^3b+3\,e^2dc\right)x^9}{9}+\frac{\left(ae^3+3\,e^2db+3\,cd^2e\right)x^7}{7}+\frac{\left(3\,e^2da+3\,d^2eb+d^3c\right)x^5}{5}+\frac{\left(3\,d^2ea+d^3b\right)x^3}{3}+ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((e*x^2+d)^3*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/11*c*e^3*x^11+1/9*(b*e^3+3*c*d*e^2)*x^9+1/7*(a*e^3+3*b*d*e^2+3*
c*d^2*e)*x^7+1/5*(3*a*d*e^2+3*b*d^2*e+c*d^3)*x^5+1/3*(3*a*d^2*e+b
*d^3)*x^3+a*d^3*x
```

Maxima [A] time = 0.762148, size = 138, normalized size = 1.34

$$\frac{1}{11}\,ce^3x^{11}+\frac{1}{9}\left(3\,cde^2+be^3\right)x^9+\frac{1}{7}\left(3\,cd^2e+3\,bde^2+ae^3\right)x^7+\frac{1}{5}\left(cd^3+3\,bd^2e+3\,ade^2\right)x^5+ad^3x+\frac{1}{3}\left(bd^3+3\,ad^2e\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^3,x, algorithm="maxima")
```

```
[Out] 1/11*c*e^3*x^11 + 1/9*(3*c*d*e^2 + b*e^3)*x^9 + 1/7*(3*c*d^2*e +
3*b*d*e^2 + a*e^3)*x^7 + 1/5*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^5
+ a*d^3*x + 1/3*(b*d^3 + 3*a*d^2*e)*x^3
```

Fricas [A] time = 0.243962, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}e^3c+\frac{1}{3}x^9e^2dc+\frac{1}{9}x^9e^3b+\frac{3}{7}x^7ed^2c+\frac{3}{7}x^7e^2db+\frac{1}{7}x^7e^3a+\frac{1}{5}x^5d^3c+\frac{3}{5}x^5ed^2b+\frac{3}{5}x^5e^2da+\frac{1}{3}x^3d^3b+x^3ed^2a+xd^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^3*c + 1/3*x^9*e^2*d*c + 1/9*x^9*e^3*b + 3/7*x^7*e*d^2
*c + 3/7*x^7*e^2*d*b + 1/7*x^7*e^3*a + 1/5*x^5*d^3*c + 3/5*x^5*e*
d^2*b + 3/5*x^5*e^2*d*a + 1/3*x^3*d^3*b + x^3*e*d^2*a + x*d^3*a
```

Sympy [A] time = 0.141501, size = 112, normalized size = 1.09

$$ad^3x+\frac{ce^3x^{11}}{11}+x^9\left(\frac{be^3}{9}+\frac{cde^2}{3}\right)+x^7\left(\frac{ae^3}{7}+\frac{3bde^2}{7}+\frac{3cd^2e}{7}\right)+x^5\left(\frac{3ade^2}{5}+\frac{3bd^2e}{5}+\frac{cd^3}{5}\right)+x^3\left(ad^2e+\frac{bd^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((e*x**2+d)**3*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**3*x + c*e**3*x**11/11 + x**9*(b*e**3/9 + c*d*e**2/3) + x**7*
(a*e**3/7 + 3*b*d*e**2/7 + 3*c*d**2*e/7) + x**5*(3*a*d*e**2/5 + 3
*b*d**2*e/5 + c*d**3/5) + x**3*(a*d**2*e + b*d**3/3)
```

GIAC/XCAS [A] time = 0.266497, size = 146, normalized size = 1.42

$$\frac{1}{11}\,cx^{11}e^3+\frac{1}{3}\,cdx^9e^2+\frac{1}{9}\,bx^9e^3+\frac{3}{7}\,cd^2x^7e+\frac{3}{7}\,bdx^7e^2+\frac{1}{5}\,cd^3x^5+\frac{1}{7}\,ax^7e^3+\frac{3}{5}\,bd^2x^5e+\frac{3}{5}\,adx^5e^2+\frac{1}{3}\,bd^3x^3+ad^2x^3e+ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^3,x, algorithm="giac")
```

```
[Out] 1/11*c*x^11*e^3 + 1/3*c*d*x^9*e^2 + 1/9*b*x^9*e^3 + 3/7*c*d^2*x^7
*e + 3/7*b*d*x^7*e^2 + 1/5*c*d^3*x^5 + 1/7*a*x^7*e^3 + 3/5*b*d^2*
x^5*e + 3/5*a*d*x^5*e^2 + 1/3*b*d^3*x^3 + a*d^2*x^3*e + a*d^3*x
```

3.250 $\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=73

$$\frac{1}{5}x^5\left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^
5)/5 + (e*(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Rubi [A] time = 0.128959, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{1}{5}x^5\left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^
5)/5 + (e*(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ce^2x^9}{9}+d^2\int a\,dx+\frac{dx^3\,(2ae+bd)}{3}+\frac{ex^7\,(be+2cd)}{7}+x^5\left(\frac{ae^2}{5}+\frac{2bde}{5}+\frac{cd^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] c*e**2*x**9/9 + d**2*Integral(a, x) + d*x**3*(2*a*e + b*d)/3 + e*
x**7*(b*e + 2*c*d)/7 + x**5*(a*e**2/5 + 2*b*d*e/5 + c*d**2/5)
```

Mathematica [A] time = 0.0361437, size = 73, normalized size = 1.

$$\frac{1}{5}x^5\left(ae^2+2bde+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + 2*b*d*e + a*e^2)*x^
5)/5 + (e*(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Maple [A] time = 0.002, size = 70, normalized size = 1.

$$\frac{ce^2x^9}{9}+\frac{\left(be^2+2\,cde\right)x^7}{7}+\frac{\left(ae^2+2\,bde+cd^2\right)x^5}{5}+\frac{\left(2\,ade+bd^2\right)x^3}{3}+ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/9*c*e^2*x^9+1/7*(b*e^2+2*c*d*e)*x^7+1/5*(a*e^2+2*b*d*e+c*d^2)*x
^5+1/3*(2*a*d*e+b*d^2)*x^3+a*d^2*x
```

Maxima [A] time = 0.743599, size = 93, normalized size = 1.27

$$\frac{1}{9}ce^2x^9 + \frac{1}{7}\left(2\,cde + be^2\right)x^7 + \frac{1}{5}\left(cd^2 + 2\,bde + ae^2\right)x^5 + ad^2x + \frac{1}{3}\left(bd^2 + 2\,ade\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c*e^2*x^9 + 1/7*(2*c*d*e + b*e^2)*x^7 + 1/5*(c*d^2 + 2*b*d*e
+ a*e^2)*x^5 + a*d^2*x + 1/3*(b*d^2 + 2*a*d*e)*x^3
```

Fricas [A] time = 0.257313, size = 1, normalized size = 0.01

$$\frac{1}{9}x^9e^2c + \frac{2}{7}x^7edc + \frac{1}{7}x^7e^2b + \frac{1}{5}x^5d^2c + \frac{2}{5}x^5edb + \frac{1}{5}x^5e^2a + \frac{1}{3}x^3d^2b + \frac{2}{3}x^3eda + xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^2*c + 2/7*x^7*e*d*c + 1/7*x^7*e^2*b + 1/5*x^5*d^2*c + 2
/5*x^5*e*d*b + 1/5*x^5*e^2*a + 1/3*x^3*d^2*b + 2/3*x^3*e*d*a + x*
d^2*a
```

Sympy [A] time = 0.153886, size = 78, normalized size = 1.07

$$ad^2x + \frac{ce^2x^9}{9} + x^7\left(\frac{be^2}{7} + \frac{2cde}{7}\right) + x^5\left(\frac{ae^2}{5} + \frac{2bde}{5} + \frac{cd^2}{5}\right) + x^3\left(\frac{2ade}{3} + \frac{bd^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x + c*e**2*x**9/9 + x**7*(b*e**2/7 + 2*c*d*e/7) + x**5*(a*
e**2/5 + 2*b*d*e/5 + c*d**2/5) + x**3*(2*a*d*e/3 + b*d**2/3)
```

GIAC/XCAS [A] time = 0.268844, size = 103, normalized size = 1.41

$$\frac{1}{9}cx^9e^2 + \frac{2}{7}cdx^7e + \frac{1}{7}bx^7e^2 + \frac{1}{5}cd^2x^5 + \frac{2}{5}bdx^5e + \frac{1}{5}ax^5e^2 + \frac{1}{3}bd^2x^3 + \frac{2}{3}adx^3e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2,x, algorithm="giac")
```

```
[Out] 1/9*c*x^9*e^2 + 2/7*c*d*x^7*e + 1/7*b*x^7*e^2 + 1/5*c*d^2*x^5 + 2
/5*b*d*x^5*e + 1/5*a*x^5*e^2 + 1/3*b*d^2*x^3 + 2/3*a*d*x^3*e + a*
d^2*x
```

3.251 $\int \left(d+e x^2\right) \left(a+b x^2+c x^4\right) \, dx$

Optimal. Leaf size=42

$$\frac{1}{3}x^3(ae+bd)+adx+\frac{1}{5}x^5(be+cd)+\frac{1}{7}cex^7$$

```
[Out] a*d*x + ((b*d + a*e)*x^3)/3 + ((c*d + b*e)*x^5)/5 + (c*e*x^7)/7
```

Rubi [A] time = 0.0572616, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{1}{3}x^3(ae+bd)+adx+\frac{1}{5}x^5(be+cd)+\frac{1}{7}cex^7$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^3)/3 + ((c*d + b*e)*x^5)/5 + (c*e*x^7)/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{cex^7}{7}+d\int a\,dx+x^5\left(\frac{be}{5}+\frac{cd}{5}\right)+x^3\left(\frac{ae}{3}+\frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] c*e*x**7/7 + d*Integral(a, x) + x**5*(b*e/5 + c*d/5) + x**3*(a*e/
3 + b*d/3)
```

Mathematica [A] time = 0.0144805, size = 42, normalized size = 1.

$$\frac{1}{3}x^3(ae+bd)+adx+\frac{1}{5}x^5(be+cd)+\frac{1}{7}cex^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^3)/3 + ((c*d + b*e)*x^5)/5 + (c*e*x^7)/7
```

Maple [A] time = 0.001, size = 37, normalized size = 0.9

$$adx+\frac{(ae+bd)\,x^3}{3}+\frac{(be+cd)\,x^5}{5}+\frac{cex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)*(c*x^4+b*x^2+a),x)
```

```
[Out] a*d*x+1/3*(a*e+b*d)*x^3+1/5*(b*e+c*d)*x^5+1/7*c*e*x^7
```

Maxima [A] time = 0.736978, size = 49, normalized size = 1.17

$$\frac{1}{7}\,cex^7 + \frac{1}{5}\,(cd + be)x^5 + \frac{1}{3}\,(bd + ae)x^3 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d),x, algorithm="maxima")
```

```
[Out] 1/7*c*e*x^7 + 1/5*(c*d + b*e)*x^5 + 1/3*(b*d + a*e)*x^3 + a*d*x
```

Fricas [A] time = 0.273002, size = 1, normalized size = 0.02

$$\frac{1}{7}x^7ec + \frac{1}{5}x^5dc + \frac{1}{5}x^5eb + \frac{1}{3}x^3db + \frac{1}{3}x^3ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e*c + 1/5*x^5*d*c + 1/5*x^5*e*b + 1/3*x^3*d*b + 1/3*x^3*e
*a + x*d*a
```

Sympy [A] time = 0.117002, size = 39, normalized size = 0.93

$$adx + \frac{cex^7}{7} + x^5\left(\frac{be}{5} + \frac{cd}{5}\right) + x^3\left(\frac{ae}{3} + \frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d*x + c*e*x**7/7 + x**5*(b*e/5 + c*d/5) + x**3*(a*e/3 + b*d/3)
```

GIAC/XCAS [A] time = 0.265885, size = 58, normalized size = 1.38

$$\frac{1}{7}\,cx^7e + \frac{1}{5}\,cdx^5 + \frac{1}{5}\,bx^5e + \frac{1}{3}\,bdx^3 + \frac{1}{3}\,ax^3e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7*e + 1/5*c*d*x^5 + 1/5*b*x^5*e + 1/3*b*d*x^3 + 1/3*a*x^3
*e + a*d*x
```

3.252 $\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx$

Optimal. Leaf size=66

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)}{\sqrt{d}e^{5/2}}-\frac{x(cd-be)}{e^2}+\frac{cx^3}{3e}$$

```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 - b*d*e + a*e^2)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))
```

Rubi [A] time = 0.0961722, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)}{\sqrt{d}e^{5/2}}-\frac{x(cd-be)}{e^2}+\frac{cx^3}{3e}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(d + e*x^2),x]
```

```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 - b*d*e + a*e^2)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))
```

Rubi in Sympy [A] time = 19.9889, size = 58, normalized size = 0.88

$$\frac{cx^3}{3e}+\frac{x\,(be-cd)}{e^2}+\frac{\left(ae^2-bde+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/(e*x**2+d),x)
```

```
[Out] c*x**3/(3*e) + x*(b*e - c*d)/e**2 + (a*e**2 - b*d*e + c*d**2)*ata
n(sqrt(e)*x/sqrt(d))/(sqrt(d)*e**(5/2))
```

Mathematica [A] time = 0.104986, size = 65, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)}{\sqrt{d}e^{5/2}}+\frac{x(be-cd)}{e^2}+\frac{cx^3}{3e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2),x]
```

```
[Out] ((-(c*d) + b*e)*x)/e^2 + (c*x^3)/(3*e) + ((c*d^2 - b*d*e + a*e^2)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))
```

Maple [A] time = 0.003, size = 84, normalized size = 1.3

$$\frac{cx^3}{3\,e}+\frac{bx}{e}-\frac{cdx}{e^2}+a\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{bd}{e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{cd^2}{e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/(e*x^2+d),x)
```

```
[Out] 1/3*c*x^3/e+1/e*b*x-c*d*x/e^2+1/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2
))*a-1/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b*d+1/e^2/(d*e)^(1/2
)*arctan(x*e/(d*e)^(1/2))*c*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285536, size = 1, normalized size = 0.02

$$\left[\frac{3\left(cd^2 - bde + ae^2\right)\log\left(\frac{2\,dex + \left(ex^2 - d\right)\sqrt{-de}}{ex^2 + d}\right) + 2\left(cex^3 - 3\left(cd - be\right)x\right)\sqrt{-de}}{6\sqrt{-de}e^2}, \frac{3\left(cd^2 - bde + ae^2\right)\arctan\left(\frac{\sqrt{de}x}{d}\right) + \left(cex^3 - \right.}{3\sqrt{de}e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(c*d^2 - b*d*e + a*e^2)*log((2*d*e*x + (e*x^2 - d)*sqrt(-
d*e))/(e*x^2 + d)) + 2*(c*e*x^3 - 3*(c*d - b*e)*x)*sqrt(-d*e))/(s
qrt(-d*e)*e^2), 1/3*(3*(c*d^2 - b*d*e + a*e^2)*arctan(sqrt(d*e)*x
/d) + (c*e*x^3 - 3*(c*d - b*e)*x)*sqrt(d*e))/(sqrt(d*e)*e^2)]
```

Sympy [A] time = 2.16484, size = 117, normalized size = 1.77

$$\frac{cx^3}{3e} - \frac{\sqrt{-\frac{1}{de^5}}\left(ae^2 - bde + cd^2\right)\log\left(-de^2\sqrt{-\frac{1}{de^5}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{de^5}}\left(ae^2 - bde + cd^2\right)\log\left(de^2\sqrt{-\frac{1}{de^5}} + x\right)}{2} + \frac{x\left(be - cd\right)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/(e*x**2+d),x)
```

```
[Out] c*x**3/(3*e) - sqrt(-1/(d*e**5))*(a*e**2 - b*d*e + c*d**2)*log(-d
*e**2*sqrt(-1/(d*e**5)) + x)/2 + sqrt(-1/(d*e**5))*(a*e**2 - b*d*
e + c*d**2)*log(d*e**2*sqrt(-1/(d*e**5)) + x)/2 + x*(b*e - c*d)/e
**2
```

GIAC/XCAS [A] time = 0.267936, size = 76, normalized size = 1.15

$$\frac{\left(cd^2 - bde + ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{\sqrt{d}} + \frac{1}{3}\left(cx^3e^2 - 3\,cdxe + 3\,bxe^2\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d),x, algorithm="giac")
```

```
[Out] (c*d^2 - b*d*e + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d
) + 1/3*(c*x^3*e^2 - 3*c*d*x*e + 3*b*x*e^2)*e^(-3)
```

3.253 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3
*c*d^2 - e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e
^(5/2))
```

Rubi [A] time = 0.158282, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3
*c*d^2 - e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e
^(5/2))
```

Rubi in Sympy [A] time = 33.0675, size = 78, normalized size = 0.94

$$\frac{cx}{e^2}+\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)}+\frac{\left(ae^2+bde-3cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{\frac{3}{2}}e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d*e**2*(d + e*x**2)) +
(a*e**2 + b*d*e - 3*c*d**2)*atan(sqrt(e)*x/sqrt(d))/(2*d**(3/2)*e
**(5/2))
```

Mathematica [A] time = 0.0979119, size = 88, normalized size = 1.06

$$\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-ae^2-bde+3cd^2\right)}{2d^{3/2}e^{5/2}}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - (
(3*c*d^2 - b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)
*e^(5/2))
```

Maple [A] time = 0.011, size = 118, normalized size = 1.4

$$\frac{cx}{e^2}+\frac{ax}{2\,d\,(ex^2+d)}-\frac{bx}{2\,e\,(ex^2+d)}+\frac{cdx}{2\,e^2\,(ex^2+d)}+\frac{a}{2\,d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{b}{2\,e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{3\,cd}{2\,e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] c*x/e^2+1/2/d*x/(e*x^2+d)*a-1/2/e*x/(e*x^2+d)*b+1/2/e^2*d*x/(e*x^
2+d)*c+1/2/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+1/2/e/(d*e)^(1
/2)*arctan(x*e/(d*e)^(1/2))*b-3/2/e^2*d/(d*e)^(1/2)*arctan(x*e/(d
*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274499, size = 1, normalized size = 0.01

$$\left[-\frac{\left(3\,cd^3-bd^2e-ade^2+\left(3\,cd^2e-bde^2-ae^3\right)x^2\right)\log\left(\frac{2\,dex+\left(ex^2-d\right)\sqrt{-de}}{ex^2+d}\right)-2\left(2\,cdex^3+\left(3\,cd^2-bde+ae^2\right)x\right)\sqrt{-de}}{4\left(de^3x^2+d^2e^2\right)\sqrt{-de}},\ -\frac{\left(3\,cd^3-bd^2e-ade^2+\left(3\,cd^2e-bde^2-ae^3\right)x^2\right)\arctan\left(\frac{\sqrt{de}x}{d}\right)-\left(2\,cdex^3+\left(3\,cd^2-bde+ae^2\right)x\right)\sqrt{de}}{2\left(de^3x^2+d^2e^2\right)\sqrt{de}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^
3)*x^2)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d)) - 2*(
2*c*d*e*x^3 + (3*c*d^2 - b*d*e + a*e^2)*x)*sqrt(-d*e))/((d*e^3*x^
2 + d^2*e^2)*sqrt(-d*e)), -1/2*((3*c*d^3 - b*d^2*e - a*d*e^2 + (3
*c*d^2*e - b*d*e^2 - a*e^3)*x^2)*arctan(sqrt(d*e)*x/d) - (2*c*d*e
*x^3 + (3*c*d^2 - b*d*e + a*e^2)*x)*sqrt(d*e))/((d*e^3*x^2 + d^2*
e^2)*sqrt(d*e))]
```

Sympy [A] time = 3.76299, size = 153, normalized size = 1.84

$$\frac{cx}{e^2}+\frac{x\left(ae^2-bde+cd^2\right)}{2d^2e^2+2de^3x^2}-\frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2+bde-3cd^2\right)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2+bde-3cd^2\right)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x*
*2) - sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*
e**2*sqrt(-1/(d**3*e**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 +
 b*d*e - 3*c*d**2)*log(d**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4
```

GIAC/XCAS [A] time = 0.271493, size = 101, normalized size = 1.22

$$cxe^{(-2)} - \frac{\left(3\,cd^2 - bde - ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{\left(cd^2x - bdxe + axe^2\right)e^{(-2)}}{2\,(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(
d))*e^(-5/2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/(
(x^2*e + d)*d)
```

3.254 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^3} dx$

Optimal. Leaf size=115

$$-\frac{x\left(5cd^2-e(3ae+bd)\right)}{8d^2e^2\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}$$

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(4*d*(d + e*x^2)^2) - ((5*c*d^2 - e
*(b*d + 3*a*e))*x)/(8*d^2*e^2*(d + e*x^2)) + ((3*c*d^2 + e*(b*d +
 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Rubi [A] time = 0.204612, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{x\left(5cd^2-e(3ae+bd)\right)}{8d^2e^2\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(4*d*(d + e*x^2)^2) - ((5*c*d^2 - e
*(b*d + 3*a*e))*x)/(8*d^2*e^2*(d + e*x^2)) + ((3*c*d^2 + e*(b*d +
 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Rubi in Sympy [A] time = 34.0143, size = 110, normalized size = 0.96

$$\frac{x\left(ae^2-bde+cd^2\right)}{4de^2\left(d+ex^2\right)^2}+\frac{x\left(3ae^2+bde-5cd^2\right)}{8d^2e^2\left(d+ex^2\right)}+\frac{\left(3ae^2+bde+3cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{\frac{5}{2}}e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] x*(a*e**2 - b*d*e + c*d**2)/(4*d*e**2*(d + e*x**2)**2) + x*(3*a*e
**2 + b*d*e - 5*c*d**2)/(8*d**2*e**2*(d + e*x**2)) + (3*a*e**2 +
b*d*e + 3*c*d**2)*atan(sqrt(e)*x/sqrt(d))/(8*d**(5/2)*e**(5/2))
```

Mathematica [A] time = 0.183018, size = 110, normalized size = 0.96

$$\frac{x\left(e\left(ae\left(5d+3ex^2\right)+bd\left(ex^2-d\right)\right)-cd^2\left(3d+5ex^2\right)\right)}{8d^2e^2\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] (x*(-(c*d^2*(3*d + 5*e*x^2)) + e*(b*d*(-d + e*x^2) + a*e*(5*d + 3
*e*x^2))))/(8*d^2*e^2*(d + e*x^2)^2) + ((3*c*d^2 + e*(b*d + 3*a*e
))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Maple [A] time = 0.012, size = 131, normalized size = 1.1

$$\frac{1}{(ex^2+d)^2}\left(\frac{(3\,ae^2+bde-5\,cd^2)\,x^3}{8\,d^2e}+\frac{(5\,ae^2-bde-3\,cd^2)\,x}{8\,e^2d}\right)+\frac{3\,a}{8\,d^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{b}{8\,de}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{3\,c}{8\,e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] (1/8*(3*a*e^2+b*d*e-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-b*d*e-3*c*d^2
)/e^2/d*x)/(e*x^2+d)^2+3/8/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)
)*a+1/8/d/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b+3/8/e^2/(d*e)^(
1/2)*arctan(x*e/(d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276682, size = 1, normalized size = 0.01

$$\left[\frac{\left(3\,cd^4+bd^3e+3\,ad^2e^2+\left(3\,cd^2e^2+bde^3+3\,ae^4\right)x^4+2\left(3\,cd^3e+bd^2e^2+3\,ade^3\right)x^2\right)\log\left(\frac{2\,dex+(ex^2-d)\sqrt{-de}}{ex^2+d}\right)-2\left(\left(5\,cd^2e\right.\right.}{16\,(d^2e^4x^4+2\,d^3e^3x^2+d^4e^2)\sqrt{-de}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^3,x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*d^4 + b*d^3*e + 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3
+ 3*a*e^4)*x^4 + 2*(3*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*x^2)*log((
2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d)) - 2*((5*c*d^2*e -
b*d*e^2 - 3*a*e^3)*x^3 + (3*c*d^3 + b*d^2*e - 5*a*d*e^2)*x)*sqrt(
-d*e))/((d^2*e^4*x^4 + 2*d^3*e^3*x^2 + d^4*e^2)*sqrt(-d*e)), 1/8*
((3*c*d^4 + b*d^3*e + 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*
e^4)*x^4 + 2*(3*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*x^2)*arctan(sqrt
(d*e)*x/d) - ((5*c*d^2*e - b*d*e^2 - 3*a*e^3)*x^3 + (3*c*d^3 + b*
d^2*e - 5*a*d*e^2)*x)*sqrt(d*e))/((d^2*e^4*x^4 + 2*d^3*e^3*x^2 +
d^4*e^2)*sqrt(d*e))]
```

Sympy [A] time = 6.59205, size = 196, normalized size = 1.7

$$-\frac{\sqrt{-\frac{1}{d^5e^5}}\left(3ae^2+bde+3cd^2\right)\log\left(-d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{d^5e^5}}\left(3ae^2+bde+3cd^2\right)\log\left(d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16}+\frac{x^3\left(3ae^3+bde^2-5cd^2e\right)+x\left(5ade^2-bd^2e-3cd^3\right)}{8d^4e^2+16d^3e^3x^2+8d^2e^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] -sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*log(-d**3*e**
2*sqrt(-1/(d**5*e**5)) + x)/16 + sqrt(-1/(d**5*e**5))*(3*a*e**2 +
 b*d*e + 3*c*d**2)*log(d**3*e**2*sqrt(-1/(d**5*e**5)) + x)/16 + (
x**3*(3*a*e**3 + b*d*e**2 - 5*c*d**2*e) + x*(5*a*d*e**2 - b*d**2*
e - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e**3*x**2 + 8*d**2*e**4*x**
4)
```

GIAC/XCAS [A] time = 0.272916, size = 136, normalized size = 1.18

$$\frac{\left(3\,cd^2+bde+3\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{8\,d^{\frac{5}{2}}}-\frac{\left(5\,cd^2x^3e-bdx^3e^2+3\,cd^3x-3\,ax^3e^3+bd^2xe-5\,adxe^2\right)e^{(-2)}}{8\,\left(x^2e+d\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2
)/d^(5/2) - 1/8*(5*c*d^2*x^3*e - b*d*x^3*e^2 + 3*c*d^3*x - 3*a*x^
3*e^3 + b*d^2*x*e - 5*a*d*x*e^2)*e^(-2)/((x^2*e + d)^2*d^2)
```

3.255 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx$

Optimal. Leaf size=150

$$-\frac{x\left(7cd^2-e(5ae+bd)\right)}{24d^2e^2\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(5ae+bd)+cd^2\right)}{16d^{7/2}e^{5/2}}+\frac{x\left(e(5ae+bd)+cd^2\right)}{16d^3e^2\left(d+ex^2\right)}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{6d\left(d+ex^2\right)^3}$$

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(6*d*(d + e*x^2)^3) - ((7*c*d^2 - e
*(b*d + 5*a*e))*x)/(24*d^2*e^2*(d + e*x^2)^2) + ((c*d^2 + e*(b*d
+ 5*a*e))*x)/(16*d^3*e^2*(d + e*x^2)) + ((c*d^2 + e*(b*d + 5*a*e)
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))
```

Rubi [A] time = 0.308372, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{x\left(7cd^2-e(5ae+bd)\right)}{24d^2e^2\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(5ae+bd)+cd^2\right)}{16d^{7/2}e^{5/2}}+\frac{x\left(e(5ae+bd)+cd^2\right)}{16d^3e^2\left(d+ex^2\right)}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{6d\left(d+ex^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(6*d*(d + e*x^2)^3) - ((7*c*d^2 - e
*(b*d + 5*a*e))*x)/(24*d^2*e^2*(d + e*x^2)^2) + ((c*d^2 + e*(b*d
+ 5*a*e))*x)/(16*d^3*e^2*(d + e*x^2)) + ((c*d^2 + e*(b*d + 5*a*e)
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))
```

Rubi in Sympy [A] time = 38.8595, size = 144, normalized size = 0.96

$$\frac{x\left(ae^2-bde+cd^2\right)}{6de^2\left(d+ex^2\right)^3}+\frac{x\left(5ae^2+bde-7cd^2\right)}{24d^2e^2\left(d+ex^2\right)^2}+\frac{x\left(5ae^2+bde+cd^2\right)}{16d^3e^2\left(d+ex^2\right)}+\frac{\left(5ae^2+bde+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{\frac{7}{2}}e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/(e*x**2+d)**4,x)
```

```
[Out] x*(a*e**2 - b*d*e + c*d**2)/(6*d*e**2*(d + e*x**2)**3) + x*(5*a*e
**2 + b*d*e - 7*c*d**2)/(24*d**2*e**2*(d + e*x**2)**2) + x*(5*a*e
**2 + b*d*e + c*d**2)/(16*d**3*e**2*(d + e*x**2)) + (5*a*e**2 + b
*d*e + c*d**2)*atan(sqrt(e)*x/sqrt(d))/(16*d**(7/2)*e**(5/2))
```

Mathematica [A] time = 0.250498, size = 142, normalized size = 0.95

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(5ae+bd)+cd^2\right)}{16d^{7/2}e^{5/2}}+\frac{x\left(e\left(ae\left(33d^2+40dex^2+15e^2x^4\right)+bd\left(-3d^2+8dex^2+3e^2x^4\right)\right)+cd^2\left(-3d^2-8dex^2+3e^2x^4\right)\right)}{48d^3e^2\left(d+ex^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] (x*(c*d^2*(-3*d^2 - 8*d*e*x^2 + 3*e^2*x^4) + e*(b*d*(-3*d^2 + 8*d
*e*x^2 + 3*e^2*x^4) + a*e*(33*d^2 + 40*d*e*x^2 + 15*e^2*x^4))))/(
48*d^3*e^2*(d + e*x^2)^3) + ((c*d^2 + e*(b*d + 5*a*e))*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))
```

Maple [A] time = 0.013, size = 158, normalized size = 1.1

$$\frac{1}{(ex^2+d)^3}\left(\frac{(5\,ae^2+bde+cd^2)\,x^5}{16\,d^3}+\frac{(5\,ae^2+bde-cd^2)\,x^3}{6\,d^2e}+\frac{(11\,ae^2-bde-cd^2)\,x}{16\,e^2d}\right)+\frac{5\,a}{16\,d^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{b}{16\,d^2e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{c}{16\,e^2d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/(e*x^2+d)^4,x)
```

```
[Out] (1/16*(5*a*e^2+b*d*e+c*d^2)/d^3*x^5+1/6*(5*a*e^2+b*d*e-c*d^2)/d^2
/e*x^3+1/16*(11*a*e^2-b*d*e-c*d^2)/e^2/d*x)/(e*x^2+d)^3+5/16/d^3/
(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+1/16/d^2/e/(d*e)^(1/2)*arct
an(x*e/(d*e)^(1/2))*b+1/16/d/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/
2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281113, size = 1, normalized size = 0.01

$$\left[\frac{3\left(\left(cd^2e^3+bde^4+5\,ae^5\right)x^6+cd^5+bd^4e+5\,ad^3e^2+3\left(cd^3e^2+bd^2e^3+5\,ade^4\right)x^4+3\left(cd^4e+bd^3e^2+5\,ad^2e^3\right)x^2\right)\log\left(\frac{2d}{\cdots}\right.}{96\left(d^3e^5x^6+3\,d^4e^4x^4+3\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(3*((c*d^2*e^3 + b*d*e^4 + 5*a*e^5)*x^6 + c*d^5 + b*d^4*e +
 5*a*d^3*e^2 + 3*(c*d^3*e^2 + b*d^2*e^3 + 5*a*d*e^4)*x^4 + 3*(c*d
^4*e + b*d^3*e^2 + 5*a*d^2*e^3)*x^2)*log((2*d*e*x + (e*x^2 - d)*s
qrt(-d*e))/(e*x^2 + d)) + 2*(3*(c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^
5 - 8*(c*d^3*e - b*d^2*e^2 - 5*a*d*e^3)*x^3 - 3*(c*d^4 + b*d^3*e
- 11*a*d^2*e^2)*x)*sqrt(-d*e))/((d^3*e^5*x^6 + 3*d^4*e^4*x^4 + 3*
d^5*e^3*x^2 + d^6*e^2)*sqrt(-d*e)), 1/48*(3*((c*d^2*e^3 + b*d*e^4
 + 5*a*e^5)*x^6 + c*d^5 + b*d^4*e + 5*a*d^3*e^2 + 3*(c*d^3*e^2 +
b*d^2*e^3 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e + b*d^3*e^2 + 5*a*d^2*e^3
)*x^2)*arctan(sqrt(d*e)*x/d) + (3*(c*d^2*e^2 + b*d*e^3 + 5*a*e^4)
*x^5 - 8*(c*d^3*e - b*d^2*e^2 - 5*a*d*e^3)*x^3 - 3*(c*d^4 + b*d^3
*e - 11*a*d^2*e^2)*x)*sqrt(d*e))/((d^3*e^5*x^6 + 3*d^4*e^4*x^4 +
3*d^5*e^3*x^2 + d^6*e^2)*sqrt(d*e))]
```

Sympy [A] time = 11.4936, size = 241, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{d^7e^5}}\left(5ae^2+bde+cd^2\right)\log\left(-d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{d^7e^5}}\left(5ae^2+bde+cd^2\right)\log\left(d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}+\frac{x^5\left(15ae^4+3bde^3+3cd^2e^2\right)+x^3\left(40ade^3+8bd^2e^2-8cd^3e\right)+x\left(33ad^2e^2-3bd^3e-3cd^4\right)}{48d^6e^2+144d^5e^3x^2+144d^4e^4x^4+48d^3e^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/(e*x**2+d)**4,x)
```

```
[Out] -sqrt(-1/(d**7*e**5))*(5*a*e**2 + b*d*e + c*d**2)*log(-d**4*e**2*
sqrt(-1/(d**7*e**5)) + x)/32 + sqrt(-1/(d**7*e**5))*(5*a*e**2 + b
*d*e + c*d**2)*log(d**4*e**2*sqrt(-1/(d**7*e**5)) + x)/32 + (x**5
*(15*a*e**4 + 3*b*d*e**3 + 3*c*d**2*e**2) + x**3*(40*a*d*e**3 + 8
*b*d**2*e**2 - 8*c*d**3*e) + x*(33*a*d**2*e**2 - 3*b*d**3*e - 3*c
*d**4))/(48*d**6*e**2 + 144*d**5*e**3*x**2 + 144*d**4*e**4*x**4 +
 48*d**3*e**5*x**6)
```

GIAC/XCAS [A] time = 0.269919, size = 181, normalized size = 1.21

$$\frac{\left(cd^2+bde+5\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{16\,d^{\frac{7}{2}}}+\frac{\left(3\,cd^2x^5e^2+3\,bdx^5e^3-8\,cd^3x^3e+15\,ax^5e^4+8\,bd^2x^3e^2-3\,cd^4x+40\,adx^3e^3-3\,bd^3xe+33\,ad^2xe^2\right)e^{(-2)}}{48\,(x^2e+d)^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^4,x, algorithm="giac")
```

```
[Out] 1/16*(c*d^2 + b*d*e + 5*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)
/d^(7/2) + 1/48*(3*c*d^2*x^5*e^2 + 3*b*d*x^5*e^3 - 8*c*d^3*x^3*e
+ 15*a*x^5*e^4 + 8*b*d^2*x^3*e^2 - 3*c*d^4*x + 40*a*d*x^3*e^3 - 3
*b*d^3*x*e + 33*a*d^2*x*e^2)*e^(-2)/((x^2*e + d)^3*d^3)
```

3.256 $\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right)^2 dx$

Optimal. Leaf size=223

$$\frac{1}{7}x^7\left(a^2e^3+6abde^2+6acd^2e+3b^2d^2e+2bcd^3\right)+a^2d^3x+\frac{1}{11}ex^{11}\left(2ce(ae+3bd)+b^2e^2+3c^2d^2\right)+\frac{1}{5}dx^5\left(6abde+a\left(3ae^2+2cd^2\right)+b^2d^2\right)+\frac{1}{9}x^9\left(6cde(ae+bd)+be^2(2ae+3bd)+c^2d^3\right)+\frac{1}{3}ad^2x^3(3ae+2bd)+\frac{1}{13}ce^2x^{13}(2be+3cd)+\frac{1}{15}c^2e^3x^{15}$$

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^3)/3 + (d*(b^2*d^2 + 6*a*b*d
*e + a*(2*c*d^2 + 3*a*e^2))*x^5)/5 + ((2*b*c*d^3 + 3*b^2*d^2*e +
6*a*c*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*x^7)/7 + ((c^2*d^3 + 6*c*d*e
*(b*d + a*e) + b*e^2*(3*b*d + 2*a*e))*x^9)/9 + (e*(3*c^2*d^2 + b^
2*e^2 + 2*c*e*(3*b*d + a*e))*x^11)/11 + (c*e^2*(3*c*d + 2*b*e)*x^
13)/13 + (c^2*e^3*x^15)/15
```

Rubi [A] time = 0.424471, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{1}{7}x^7\left(a^2e^3+6abde^2+6acd^2e+3b^2d^2e+2bcd^3\right)+a^2d^3x+\frac{1}{11}ex^{11}\left(2ce(ae+3bd)+b^2e^2+3c^2d^2\right)+\frac{1}{5}dx^5\left(6abde+a\left(3ae^2+2cd^2\right)+b^2d^2\right)+\frac{1}{9}x^9\left(6cde(ae+bd)+be^2(2ae+3bd)+c^2d^3\right)+\frac{1}{3}ad^2x^3(3ae+2bd)+\frac{1}{13}ce^2x^{13}(2be+3cd)+\frac{1}{15}c^2e^3x^{15}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^3)/3 + (d*(b^2*d^2 + 6*a*b*d
*e + a*(2*c*d^2 + 3*a*e^2))*x^5)/5 + ((2*b*c*d^3 + 3*b^2*d^2*e +
6*a*c*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*x^7)/7 + ((c^2*d^3 + 6*c*d*e
*(b*d + a*e) + b*e^2*(3*b*d + 2*a*e))*x^9)/9 + (e*(3*c^2*d^2 + b^
2*e^2 + 2*c*e*(3*b*d + a*e))*x^11)/11 + (c*e^2*(3*c*d + 2*b*e)*x^
13)/13 + (c^2*e^3*x^15)/15
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ad^2x^3\left(3ae+2bd\right)}{3}+\frac{c^2e^3x^{15}}{15}+\frac{ce^2x^{13}\left(2be+3cd\right)}{13}+d^3\int a^2\,dx+\frac{dx^5\left(3a^2e^2+6abde+2acd^2+b^2d^2\right)}{5}+\frac{ex^{11}\left(2ace^2+b^2e^2+6bcde+3c^2d^2\right)}{11}+x^9\left(\frac{2abe^3}{9}+\frac{2acde^2}{3}+\frac{b^2de^2}{3}+\frac{2bcd^2e}{3}+\frac{c^2d^3}{9}\right)+x^7\left(\frac{a^2e^3}{7}+\frac{6abde^2}{7}+\frac{6acd^2e}{7}+\frac{3b^2d^2e}{7}+\frac{2bcd^3}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**3*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a*d**2*x**3*(3*a*e + 2*b*d)/3 + c**2*e**3*x**15/15 + c*e**2*x**13
*(2*b*e + 3*c*d)/13 + d**3*Integral(a**2, x) + d*x**5*(3*a**2*e**
2 + 6*a*b*d*e + 2*a*c*d**2 + b**2*d**2)/5 + e*x**11*(2*a*c*e**2 +
 b**2*e**2 + 6*b*c*d*e + 3*c**2*d**2)/11 + x**9*(2*a*b*e**3/9 + 2
*a*c*d*e**2/3 + b**2*d*e**2/3 + 2*b*c*d**2*e/3 + c**2*d**3/9) + x
**7*(a**2*e**3/7 + 6*a*b*d*e**2/7 + 6*a*c*d**2*e/7 + 3*b**2*d**2*
e/7 + 2*b*c*d**3/7)
```

Mathematica [A] time = 0.176502, size = 223, normalized size = 1.

$$\frac{1}{7}x^7\left(a^2e^3+6abde^2+6acd^2e+3b^2d^2e+2bcd^3\right)+a^2d^3x+\frac{1}{11}ex^{11}\left(2ce(ae+3bd)+b^2e^2+3c^2d^2\right)+\frac{1}{5}dx^5\left(6abde+a\left(3ae^2+2cd^2\right)+b^2d^2\right)+\frac{1}{9}x^9\left(6cde(ae+bd)+be^2(2ae+3bd)+c^2d^3\right)+\frac{1}{3}ad^2x^3(3ae+2bd)+\frac{1}{13}ce^2x^{13}(2be+3cd)+\frac{1}{15}c^2e^3x^{15}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^3)/3 + (d*(b^2*d^2 + 6*a*b*d
*e + a*(2*c*d^2 + 3*a*e^2))*x^5)/5 + ((2*b*c*d^3 + 3*b^2*d^2*e +
6*a*c*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*x^7)/7 + ((c^2*d^3 + 6*c*d*e
*(b*d + a*e) + b*e^2*(3*b*d + 2*a*e))*x^9)/9 + (e*(3*c^2*d^2 + b^
2*e^2 + 2*c*e*(3*b*d + a*e))*x^11)/11 + (c*e^2*(3*c*d + 2*b*e)*x^
13)/13 + (c^2*e^3*x^15)/15
```

Maple [A] time = 0.002, size = 219, normalized size = 1.

$$\frac{c^2e^3x^{15}}{15}+\frac{\left(2\,e^3bc+3\,e^2dc^2\right)x^{13}}{13}+\frac{\left(3\,d^2ec^2+6\,e^2dbc+e^3\left(2\,ac+b^2\right)\right)x^{11}}{11}+\frac{\left(c^2d^3+6\,bcd^2e+3\,e^2d\left(2\,ac+b^2\right)+2\,abe^3\right)x^9}{9}+\frac{\left(2\,bcd^3+3\,d^2e\left(2\,ac+b^2\right)+6\,abde^2+a^2e^3\right)x^7}{7}+\frac{\left(d^3\left(2\,ac+b^2\right)+6\,abd^2e+3\,da^2e^2\right)x^5}{5}+\frac{\left(3\,d^2ea^2+2\,d^3ab\right)x^3}{3}+a^2d^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/15*c^2*e^3*x^15+1/13*(2*b*c*e^3+3*c^2*d*e^2)*x^13+1/11*(3*d^2*e
*c^2+6*e^2*d*b*c+e^3*(2*a*c+b^2))*x^11+1/9*(c^2*d^3+6*b*c*d^2*e+3
*e^2*d*(2*a*c+b^2)+2*a*b*e^3)*x^9+1/7*(2*b*c*d^3+3*d^2*e*(2*a*c+b
^2)+6*a*b*d*e^2+a^2*e^3)*x^7+1/5*(d^3*(2*a*c+b^2)+6*a*b*d^2*e+3*d
*a^2*e^2)*x^5+1/3*(3*a^2*d^2*e+2*a*b*d^3)*x^3+a^2*d^3*x
```

Maxima [A] time = 0.714763, size = 294, normalized size = 1.32

$$\frac{1}{15}\,c^2e^3x^{15}+\frac{1}{13}\left(3\,c^2de^2+2\,bce^3\right)x^{13}+\frac{1}{11}\left(3\,c^2d^2e+6\,bcde^2+\left(b^2+2\,ac\right)e^3\right)x^{11}+\frac{1}{9}\left(c^2d^3+6\,bcd^2e+2\,abe^3+3\left(b^2+2\,ac\right)de^2\right)x^9+\frac{1}{7}\left(2\,bcd^3+6\,abde^2+a^2e^3+3\left(b^2+2\,ac\right)d^2e\right)x^7+a^2d^3x+\frac{1}{5}\left(6\,abd^2e+3\,a^2de^2+\left(b^2+2\,ac\right)d^3\right)x^5+\frac{1}{3}\left(2\,abd^3+3\,a^2d^2e\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(e*x^2 + d)^3,x, algorithm="maxima")
```

```
[Out] 1/15*c^2*e^3*x^15 + 1/13*(3*c^2*d*e^2 + 2*b*c*e^3)*x^13 + 1/11*(3
*c^2*d^2*e + 6*b*c*d*e^2 + (b^2 + 2*a*c)*e^3)*x^11 + 1/9*(c^2*d^3
 + 6*b*c*d^2*e + 2*a*b*e^3 + 3*(b^2 + 2*a*c)*d*e^2)*x^9 + 1/7*(2*
b*c*d^3 + 6*a*b*d*e^2 + a^2*e^3 + 3*(b^2 + 2*a*c)*d^2*e)*x^7 + a^
```

```
2*d^3*x + 1/5*(6*a*b*d^2*e + 3*a^2*d*e^2 + (b^2 + 2*a*c)*d^3)*x^5
 + 1/3*(2*a*b*d^3 + 3*a^2*d^2*e)*x^3
```

Fricas [A] time = 0.244987, size = 1, normalized size = 0.

$$\frac{1}{15}x^{15}e^3c^2+\frac{3}{13}x^{13}e^2dc^2+\frac{2}{13}x^{13}e^3cb+\frac{3}{11}x^{11}ed^2c^2+\frac{6}{11}x^{11}e^2dcb+\frac{1}{11}x^{11}e^3b^2+\frac{2}{11}x^{11}e^3ca+\frac{1}{9}x^9d^3c^2+\frac{2}{3}x^9ed^2cb+\frac{1}{3}x^9e^2db^2+\frac{2}{3}x^9e^2dca+\frac{2}{9}x^9e^3ba+\frac{2}{7}x^7d^3cb+\frac{3}{7}x^7ed^2b^2+\frac{6}{7}x^7ed^2ca+\frac{6}{7}x^7e^2dba+\frac{1}{7}x^7e^3a^2+\frac{1}{5}x^5d^3b^2+\frac{2}{5}x^5d^3ca+\frac{6}{5}x^5ed^2ba+\frac{3}{5}x^5e^2da^2+\frac{2}{3}x^3d^3ba+x^3ed^2a^2+xd^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(e*x^2 + d)^3,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*e^3*c^2 + 3/13*x^13*e^2*d*c^2 + 2/13*x^13*e^3*c*b + 3/1
1*x^11*e*d^2*c^2 + 6/11*x^11*e^2*d*c*b + 1/11*x^11*e^3*b^2 + 2/11
*x^11*e^3*c*a + 1/9*x^9*d^3*c^2 + 2/3*x^9*e*d^2*c*b + 1/3*x^9*e^2
*d*b^2 + 2/3*x^9*e^2*d*c*a + 2/9*x^9*e^3*b*a + 2/7*x^7*d^3*c*b +
3/7*x^7*e*d^2*b^2 + 6/7*x^7*e*d^2*c*a + 6/7*x^7*e^2*d*b*a + 1/7*x
^7*e^3*a^2 + 1/5*x^5*d^3*b^2 + 2/5*x^5*d^3*c*a + 6/5*x^5*e*d^2*b*
a + 3/5*x^5*e^2*d*a^2 + 2/3*x^3*d^3*b*a + x^3*e*d^2*a^2 + x*d^3*a
^2
```

Sympy [A] time = 0.238388, size = 272, normalized size = 1.22

$$a^2d^3x+\frac{c^2e^3x^{15}}{15}+x^{13}\left(\frac{2bce^3}{13}+\frac{3c^2de^2}{13}\right)+x^{11}\left(\frac{2ace^3}{11}+\frac{b^2e^3}{11}+\frac{6bcde^2}{11}+\frac{3c^2d^2e}{11}\right)+x^9\left(\frac{2abe^3}{9}+\frac{2acde^2}{3}+\frac{b^2de^2}{3}+\frac{2bcd^2e}{3}+\frac{c^2d^3}{9}\right)+x^7\left(\frac{a^2e^3}{7}+\frac{6abde^2}{7}+\frac{6acd^2e}{7}+\frac{3b^2d^2e}{7}+\frac{2bcd^3}{7}\right)+x^5\left(\frac{3a^2de^2}{5}+\frac{6abd^2e}{5}+\frac{2acd^3}{5}+\frac{b^2d^3}{5}\right)+x^3\left(a^2d^2e+\frac{2abd^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**3*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d**3*x + c**2*e**3*x**15/15 + x**13*(2*b*c*e**3/13 + 3*c**2*
d*e**2/13) + x**11*(2*a*c*e**3/11 + b**2*e**3/11 + 6*b*c*d*e**2/1
1 + 3*c**2*d**2*e/11) + x**9*(2*a*b*e**3/9 + 2*a*c*d*e**2/3 + b**
2*d*e**2/3 + 2*b*c*d**2*e/3 + c**2*d**3/9) + x**7*(a**2*e**3/7 +
6*a*b*d*e**2/7 + 6*a*c*d**2*e/7 + 3*b**2*d**2*e/7 + 2*b*c*d**3/7)
 + x**5*(3*a**2*d*e**2/5 + 6*a*b*d**2*e/5 + 2*a*c*d**3/5 + b**2*d
**3/5) + x**3*(a**2*d**2*e + 2*a*b*d**3/3)
```

GIAC/XCAS [A] time = 0.268795, size = 344, normalized size = 1.54

$$\frac{1}{15}c^2x^{15}e^3+\frac{3}{13}c^2dx^{13}e^2+\frac{2}{13}bcx^{13}e^3+\frac{3}{11}c^2d^2x^{11}e+\frac{6}{11}bcdx^{11}e^2+\frac{1}{9}c^2d^3x^9+\frac{1}{11}b^2x^{11}e^3+\frac{2}{11}acx^{11}e^3+\frac{2}{3}bcd^2x^9e+\frac{1}{3}b^2dx^9e^2+\frac{2}{3}acdx^9e^2+\frac{2}{7}bcd^3x^7+\frac{2}{9}abx^9e^3+\frac{3}{7}b^2d^2x^7e+\frac{6}{7}acd^2x^7e+\frac{6}{7}abdx^7e^2+\frac{1}{5}b^2d^3x^5+\frac{2}{5}acd^3x^5+\frac{1}{7}a^2x^7e^3+\frac{6}{5}abd^2x^5e+\frac{3}{5}a^2dx^5e^2+\frac{2}{3}abd^3x^3+a^2d^2x^3e+a^2d^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(e*x^2 + d)^3,x, algorithm="giac")
```

```
[Out] 1/15*c^2*x^15*e^3 + 3/13*c^2*d*x^13*e^2 + 2/13*b*c*x^13*e^3 + 3/1
1*c^2*d^2*x^11*e + 6/11*b*c*d*x^11*e^2 + 1/9*c^2*d^3*x^9 + 1/11*b
^2*x^11*e^3 + 2/11*a*c*x^11*e^3 + 2/3*b*c*d^2*x^9*e + 1/3*b^2*d*x
^9*e^2 + 2/3*a*c*d*x^9*e^2 + 2/7*b*c*d^3*x^7 + 2/9*a*b*x^9*e^3 +
3/7*b^2*d^2*x^7*e + 6/7*a*c*d^2*x^7*e + 6/7*a*b*d*x^7*e^2 + 1/5*b
^2*d^3*x^5 + 2/5*a*c*d^3*x^5 + 1/7*a^2*x^7*e^3 + 6/5*a*b*d^2*x^5*
e + 3/5*a^2*d*x^5*e^2 + 2/3*a*b*d^3*x^3 + a^2*d^2*x^3*e + a^2*d^3
*x
```

3.257 $\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=155

$$a^2d^2x+\frac{1}{9}x^9\left(2ce(ae+2bd)+b^2e^2+c^2d^2\right)+\frac{2}{7}x^7\left(abe^2+2acde+b^2de+bcd^2\right)+\frac{1}{5}x^5\left(4abde+a\left(ae^2+2cd^2\right)+b^2d^2\right)+\frac{2}{3}adx^3(ae+bd)+\frac{2}{11}cex^{11}(be+cd)+\frac{1}{13}c^2e^2x^{13}$$

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^3)/3 + ((b^2*d^2 + 4*a*b*d*e + a
*(2*c*d^2 + a*e^2))*x^5)/5 + (2*(b*c*d^2 + b^2*d*e + 2*a*c*d*e +
a*b*e^2)*x^7)/7 + ((c^2*d^2 + b^2*e^2 + 2*c*e*(2*b*d + a*e))*x^9)
/9 + (2*c*e*(c*d + b*e)*x^11)/11 + (c^2*e^2*x^13)/13
```

Rubi [A] time = 0.281059, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$a^2d^2x+\frac{1}{9}x^9\left(2ce(ae+2bd)+b^2e^2+c^2d^2\right)+\frac{2}{7}x^7\left(abe^2+2acde+b^2de+bcd^2\right)+\frac{1}{5}x^5\left(4abde+a\left(ae^2+2cd^2\right)+b^2d^2\right)+\frac{2}{3}adx^3(ae+bd)+\frac{2}{11}cex^{11}(be+cd)+\frac{1}{13}c^2e^2x^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^3)/3 + ((b^2*d^2 + 4*a*b*d*e + a
*(2*c*d^2 + a*e^2))*x^5)/5 + (2*(b*c*d^2 + b^2*d*e + 2*a*c*d*e +
a*b*e^2)*x^7)/7 + ((c^2*d^2 + b^2*e^2 + 2*c*e*(2*b*d + a*e))*x^9)
/9 + (2*c*e*(c*d + b*e)*x^11)/11 + (c^2*e^2*x^13)/13
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2adx^3(ae+bd)}{3}+\frac{c^2e^2x^{13}}{13}+\frac{2cex^{11}(be+cd)}{11}+d^2\int a^2\,dx+x^9\left(\frac{2ace^2}{9}+\frac{b^2e^2}{9}+\frac{4bcde}{9}+\frac{c^2d^2}{9}\right)+x^7\left(\frac{2abe^2}{7}+\frac{4acde}{7}+\frac{2b^2de}{7}+\frac{2bcd^2}{7}\right)+x^5\left(\frac{a^2e^2}{5}+\frac{4abde}{5}+\frac{2acd^2}{5}+\frac{b^2d^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a*d*x**3*(a*e + b*d)/3 + c**2*e**2*x**13/13 + 2*c*e*x**11*(b*e
+ c*d)/11 + d**2*Integral(a**2, x) + x**9*(2*a*c*e**2/9 + b**2*e*
*2/9 + 4*b*c*d*e/9 + c**2*d**2/9) + x**7*(2*a*b*e**2/7 + 4*a*c*d*
e/7 + 2*b**2*d*e/7 + 2*b*c*d**2/7) + x**5*(a**2*e**2/5 + 4*a*b*d*
e/5 + 2*a*c*d**2/5 + b**2*d**2/5)
```

Mathematica [A] time = 0.0999601, size = 156, normalized size = 1.01

$$\frac{1}{5}x^5\left(a^2e^2+4abde+2acd^2+b^2d^2\right)+a^2d^2x+\frac{1}{9}x^9\left(2ace^2+b^2e^2+4bcde+c^2d^2\right)+\frac{2}{7}x^7\left(abe^2+2acde+b^2de+bcd^2\right)+\frac{2}{3}adx^3(ae+bd)+\frac{2}{11}cex^{11}(be+cd)+\frac{1}{13}c^2e^2x^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^3)/3 + ((b^2*d^2 + 2*a*c*d^2 + 4
*a*b*d*e + a^2*e^2)*x^5)/5 + (2*(b*c*d^2 + b^2*d*e + 2*a*c*d*e +
a*b*e^2)*x^7)/7 + ((c^2*d^2 + 4*b*c*d*e + b^2*e^2 + 2*a*c*e^2)*x^
9)/9 + (2*c*e*(c*d + b*e)*x^11)/11 + (c^2*e^2*x^13)/13
```

Maple [A] time = 0.001, size = 155, normalized size = 1.

$$\frac{c^2e^2x^{13}}{13}+\frac{\left(2\,bce^2+2\,dec^2\right)x^{11}}{11}+\frac{\left(c^2d^2+4\,bcde+e^2\left(2\,ac+b^2\right)\right)x^9}{9}+\frac{\left(2\,bcd^2+2\,de\left(2\,ac+b^2\right)+2\,abe^2\right)x^7}{7}+\frac{\left(d^2\left(2\,ac+b^2\right)+4\,abde+a^2e^2\right)x^5}{5}+\frac{\left(2\,dea^2+2\,ad^2b\right)x^3}{3}+a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/13*c^2*e^2*x^13+1/11*(2*b*c*e^2+2*c^2*d*e)*x^11+1/9*(c^2*d^2+4*
b*c*d*e+e^2*(2*a*c+b^2))*x^9+1/7*(2*b*c*d^2+2*d*e*(2*a*c+b^2)+2*a
*b*e^2)*x^7+1/5*(d^2*(2*a*c+b^2)+4*a*b*d*e+a^2*e^2)*x^5+1/3*(2*a^
2*d*e+2*a*b*d^2)*x^3+a^2*d^2*x
```

Maxima [A] time = 0.737121, size = 198, normalized size = 1.28

$$\frac{1}{13}\,c^2e^2x^{13}+\frac{2}{11}\left(c^2de+bce^2\right)x^{11}+\frac{1}{9}\left(c^2d^2+4\,bcde+\left(b^2+2\,ac\right)e^2\right)x^9+\frac{2}{7}\left(bcd^2+abe^2+\left(b^2+2\,ac\right)de\right)x^7+\frac{1}{5}\left(4\,abde+a^2e^2+\left(b^2+2\,ac\right)d^2\right)x^5+a^2d^2x+\frac{2}{3}\left(abd^2+a^2de\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(e*x^2 + d)^2,x, algorithm="maxima")
```

```
[Out] 1/13*c^2*e^2*x^13 + 2/11*(c^2*d*e + b*c*e^2)*x^11 + 1/9*(c^2*d^2
+ 4*b*c*d*e + (b^2 + 2*a*c)*e^2)*x^9 + 2/7*(b*c*d^2 + a*b*e^2 + (
b^2 + 2*a*c)*d*e)*x^7 + 1/5*(4*a*b*d*e + a^2*e^2 + (b^2 + 2*a*c)*
d^2)*x^5 + a^2*d^2*x + 2/3*(a*b*d^2 + a^2*d*e)*x^3
```

Fricas [A] time = 0.242174, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}e^2c^2+\frac{2}{11}x^{11}edc^2+\frac{2}{11}x^{11}e^2cb+\frac{1}{9}x^9d^2c^2+\frac{4}{9}x^9edcb+\frac{1}{9}x^9e^2b^2+\frac{2}{9}x^9e^2ca+\frac{2}{7}x^7d^2cb+\frac{2}{7}x^7edb^2+\frac{4}{7}x^7edca+\frac{2}{7}x^7e^2ba+\frac{1}{5}x^5d^2b^2+\frac{2}{5}x^5d^2ca+\frac{4}{5}x^5edba+\frac{1}{5}x^5e^2a^2+\frac{2}{3}x^3d^2ba+\frac{2}{3}x^3eda^2+xd^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(e*x^2 + d)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^2*c^2 + 2/11*x^11*e*d*c^2 + 2/11*x^11*e^2*c*b + 1/9*x
^9*d^2*c^2 + 4/9*x^9*e*d*c*b + 1/9*x^9*e^2*b^2 + 2/9*x^9*e^2*c*a
+ 2/7*x^7*d^2*c*b + 2/7*x^7*e*d*b^2 + 4/7*x^7*e*d*c*a + 2/7*x^7*e
```

```
^2*b*a + 1/5*x^5*d^2*b^2 + 2/5*x^5*d^2*c*a + 4/5*x^5*e*d*b*a + 1/
5*x^5*e^2*a^2 + 2/3*x^3*d^2*b*a + 2/3*x^3*e*d*a^2 + x*d^2*a^2
```

Sympy [A] time = 0.190843, size = 192, normalized size = 1.24

$$a^2d^2x + \frac{c^2e^2x^{13}}{13} + x^{11}\left(\frac{2bce^2}{11} + \frac{2c^2de}{11}\right) + x^9\left(\frac{2ace^2}{9} + \frac{b^2e^2}{9} + \frac{4bcde}{9} + \frac{c^2d^2}{9}\right) + x^7\left(\frac{2abe^2}{7} + \frac{4acde}{7} + \frac{2b^2de}{7} + \frac{2bcd^2}{7}\right) + x^5\left(\frac{a^2e^2}{5} + \frac{4abde}{5} + \frac{2acd^2}{5} + \frac{b^2d^2}{5}\right) + x^3\left(\frac{2a^2de}{3} + \frac{2abd^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d**2*x + c**2*e**2*x**13/13 + x**11*(2*b*c*e**2/11 + 2*c**2*
d*e/11) + x**9*(2*a*c*e**2/9 + b**2*e**2/9 + 4*b*c*d*e/9 + c**2*d
**2/9) + x**7*(2*a*b*e**2/7 + 4*a*c*d*e/7 + 2*b**2*d*e/7 + 2*b*c*
d**2/7) + x**5*(a**2*e**2/5 + 4*a*b*d*e/5 + 2*a*c*d**2/5 + b**2*d
**2/5) + x**3*(2*a**2*d*e/3 + 2*a*b*d**2/3)
```

GIAC/XCAS [A] time = 0.267394, size = 244, normalized size = 1.57

$$\frac{1}{13}c^2x^{13}e^2 + \frac{2}{11}c^2dx^{11}e + \frac{2}{11}bcx^{11}e^2 + \frac{1}{9}c^2d^2x^9 + \frac{4}{9}bcdx^9e + \frac{1}{9}b^2x^9e^2 + \frac{2}{9}acx^9e^2 + \frac{2}{7}bcd^2x^7 + \frac{2}{7}b^2dx^7e + \frac{4}{7}acdx^7e + \frac{2}{7}abx^7e^2 + \frac{1}{5}b^2d^2x^5 + \frac{2}{5}acd^2x^5 + \frac{4}{5}abdx^5e + \frac{1}{5}a^2x^5e^2 + \frac{2}{3}abd^2x^3 + \frac{2}{3}a^2dx^3e + a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(e*x^2 + d)^2,x, algorithm="giac")
```

```
[Out] 1/13*c^2*x^13*e^2 + 2/11*c^2*d*x^11*e + 2/11*b*c*x^11*e^2 + 1/9*c
^2*d^2*x^9 + 4/9*b*c*d*x^9*e + 1/9*b^2*x^9*e^2 + 2/9*a*c*x^9*e^2
+ 2/7*b*c*d^2*x^7 + 2/7*b^2*d*x^7*e + 4/7*a*c*d*x^7*e + 2/7*a*b*x
^7*e^2 + 1/5*b^2*d^2*x^5 + 2/5*a*c*d^2*x^5 + 4/5*a*b*d*x^5*e + 1/
5*a^2*x^5*e^2 + 2/3*a*b*d^2*x^3 + 2/3*a^2*d*x^3*e + a^2*d^2*x
```

3.258 $\int \left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=96

$$a^2dx+\frac{1}{7}x^7\left(2ace+b^2e+2bcd\right)+\frac{1}{5}x^5\left(2abe+2acd+b^2d\right)+\frac{1}{3}ax^3(ae+2bd)+\frac{1}{9}cx^9(2be+cd)+\frac{1}{11}c^2ex^{11}$$

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^3)/3 + ((b^2*d + 2*a*c*d + 2*a*b*e)*
x^5)/5 + ((2*b*c*d + b^2*e + 2*a*c*e)*x^7)/7 + (c*(c*d + 2*b*e)*x
^9)/9 + (c^2*e*x^11)/11
```

Rubi [A] time = 0.137177, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$a^2dx+\frac{1}{7}x^7\left(2ace+b^2e+2bcd\right)+\frac{1}{5}x^5\left(2abe+2acd+b^2d\right)+\frac{1}{3}ax^3(ae+2bd)+\frac{1}{9}cx^9(2be+cd)+\frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^3)/3 + ((b^2*d + 2*a*c*d + 2*a*b*e)*
x^5)/5 + ((2*b*c*d + b^2*e + 2*a*c*e)*x^7)/7 + (c*(c*d + 2*b*e)*x
^9)/9 + (c^2*e*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\int d\,dx+\frac{ax^3\,(ae+2bd)}{3}+\frac{c^2ex^{11}}{11}+\frac{cx^9\,(2be+cd)}{9}+x^7\left(\frac{2ace}{7}+\frac{b^2e}{7}+\frac{2bcd}{7}\right)+x^5\left(\frac{2abe}{5}+\frac{2acd}{5}+\frac{b^2d}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*Integral(d, x) + a*x**3*(a*e + 2*b*d)/3 + c**2*e*x**11/11 +
c*x**9*(2*b*e + c*d)/9 + x**7*(2*a*c*e/7 + b**2*e/7 + 2*b*c*d/7)
+ x**5*(2*a*b*e/5 + 2*a*c*d/5 + b**2*d/5)
```

Mathematica [A] time = 0.0428684, size = 96, normalized size = 1.

$$a^2dx+\frac{1}{7}x^7\left(2ace+b^2e+2bcd\right)+\frac{1}{5}x^5\left(2abe+2acd+b^2d\right)+\frac{1}{3}ax^3(ae+2bd)+\frac{1}{9}cx^9(2be+cd)+\frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^3)/3 + ((b^2*d + 2*a*c*d + 2*a*b*e)*
x^5)/5 + ((2*b*c*d + b^2*e + 2*a*c*e)*x^7)/7 + (c*(c*d + 2*b*e)*x
^9)/9 + (c^2*e*x^11)/11
```

Maple [A] time = 0.001, size = 91, normalized size = 1.

$$\frac{c^2ex^{11}}{11}+\frac{\left(2\,bce+c^2d\right)x^9}{9}+\frac{\left(2\,bcd+e\left(2\,ac+b^2\right)\right)x^7}{7}+\frac{\left(d\left(2\,ac+b^2\right)+2\,abe\right)x^5}{5}+\frac{\left(ea^2+2\,dab\right)x^3}{3}+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/11*c^2*e*x^11+1/9*(2*b*c*e+c^2*d)*x^9+1/7*(2*b*c*d+e*(2*a*c+b^2
))*x^7+1/5*(d*(2*a*c+b^2)+2*a*b*e)*x^5+1/3*(a^2*e+2*a*b*d)*x^3+a^
2*d*x
```

Maxima [A] time = 0.743316, size = 122, normalized size = 1.27

$$\frac{1}{11}\,c^{2}ex^{11} + \frac{1}{9}\left(c^{2}d + 2\,bce\right)x^{9} + \frac{1}{7}\left(2\,bcd + \left(b^{2} + 2\,ac\right)e\right)x^{7} + \frac{1}{5}\left(2\,abe + \left(b^{2} + 2\,ac\right)d\right)x^{5} + a^{2}dx + \frac{1}{3}\left(2\,abd + a^{2}e\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(e*x^2 + d),x, algorithm="maxima")
```

```
[Out] 1/11*c^2*e*x^11 + 1/9*(c^2*d + 2*b*c*e)*x^9 + 1/7*(2*b*c*d + (b^2
 + 2*a*c)*e)*x^7 + 1/5*(2*a*b*e + (b^2 + 2*a*c)*d)*x^5 + a^2*d*x
+ 1/3*(2*a*b*d + a^2*e)*x^3
```

Fricas [A] time = 0.238333, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}ec^{2} + \frac{1}{9}x^{9}dc^{2} + \frac{2}{9}x^{9}ecb + \frac{2}{7}x^{7}dcb + \frac{1}{7}x^{7}eb^{2} + \frac{2}{7}x^{7}eca + \frac{1}{5}x^{5}db^{2} + \frac{2}{5}x^{5}dca + \frac{2}{5}x^{5}eba + \frac{2}{3}x^{3}dba + \frac{1}{3}x^{3}ea^{2} + xda^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(e*x^2 + d),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e*c^2 + 1/9*x^9*d*c^2 + 2/9*x^9*e*c*b + 2/7*x^7*d*c*b +
 1/7*x^7*e*b^2 + 2/7*x^7*e*c*a + 1/5*x^5*d*b^2 + 2/5*x^5*d*c*a +
2/5*x^5*e*b*a + 2/3*x^3*d*b*a + 1/3*x^3*e*a^2 + x*d*a^2
```

Sympy [A] time = 0.152216, size = 107, normalized size = 1.11

$$a^{2}dx + \frac{c^{2}ex^{11}}{11} + x^{9}\left(\frac{2bce}{9} + \frac{c^{2}d}{9}\right) + x^{7}\left(\frac{2ace}{7} + \frac{b^{2}e}{7} + \frac{2bcd}{7}\right) + x^{5}\left(\frac{2abe}{5} + \frac{2acd}{5} + \frac{b^{2}d}{5}\right) + x^{3}\left(\frac{a^{2}e}{3} + \frac{2abd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d*x + c**2*e*x**11/11 + x**9*(2*b*c*e/9 + c**2*d/9) + x**7*(
2*a*c*e/7 + b**2*e/7 + 2*b*c*d/7) + x**5*(2*a*b*e/5 + 2*a*c*d/5 +
 b**2*d/5) + x**3*(a**2*e/3 + 2*a*b*d/3)
```

GIAC/XCAS [A] time = 0.269385, size = 143, normalized size = 1.49

$$\frac{1}{11}\,c^{2}x^{11}e + \frac{1}{9}\,c^{2}dx^{9} + \frac{2}{9}\,bcx^{9}e + \frac{2}{7}\,bcdx^{7} + \frac{1}{7}\,b^{2}x^{7}e + \frac{2}{7}\,acx^{7}e + \frac{1}{5}\,b^{2}dx^{5} + \frac{2}{5}\,acdx^{5} + \frac{2}{5}\,abx^{5}e + \frac{2}{3}\,abdx^{3} + \frac{1}{3}\,a^{2}x^{3}e + a^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(e*x^2 + d),x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11*e + 1/9*c^2*d*x^9 + 2/9*b*c*x^9*e + 2/7*b*c*d*x^7 +
 1/7*b^2*x^7*e + 2/7*a*c*x^7*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 +
2/5*a*b*x^5*e + 2/3*a*b*d*x^3 + 1/3*a^2*x^3*e + a^2*d*x
```

3.259 $\int \left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=49

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (
c^2*x^9)/9
```

Rubi [A] time = 0.0478887, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (
c^2*x^9)/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2abx^3}{3}+\frac{2bcx^7}{7}+\frac{c^2x^9}{9}+x^5\left(\frac{2ac}{5}+\frac{b^2}{5}\right)+\int a^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/
5) + Integral(a**2, x)
```

Mathematica [A] time = 0.00918063, size = 49, normalized size = 1.

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (
c^2*x^9)/9
```

Maple [A] time = 0.001, size = 42, normalized size = 0.9

$$a^2x+\frac{2\,abx^3}{3}+\frac{\left(2\,ac+b^2\right)x^5}{5}+\frac{2\,bcx^7}{7}+\frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2,x)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Maxima [A] time = 0.730107, size = 61, normalized size = 1.24

$$\frac{1}{9}\,c^2x^9 + \frac{2}{7}\,bcx^7 + \frac{1}{5}\,b^2x^5 + a^2x + \frac{2}{15}\,\left(3\,cx^5 + 5\,bx^3\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + a^2*x + 2/15*(3*c*x^5 +
 5*b*x^3)*a
```

Fricas [A] time = 0.237615, size = 1, normalized size = 0.02

$$\frac{1}{9}x^9c^2 + \frac{2}{7}x^7cb + \frac{1}{5}x^5b^2 + \frac{2}{5}x^5ca + \frac{2}{3}x^3ba + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^2 + 2/7*x^7*c*b + 1/5*x^5*b^2 + 2/5*x^5*c*a + 2/3*x^3*b
*a + x*a^2
```

Sympy [A] time = 0.123324, size = 48, normalized size = 0.98

$$a^2x + \frac{2abx^3}{3} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9} + x^5\left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/
5 + b**2/5)
```

GIAC/XCAS [A] time = 0.268892, size = 58, normalized size = 1.18

$$\frac{1}{9}\,c^2x^9 + \frac{2}{7}\,bcx^7 + \frac{1}{5}\,b^2x^5 + \frac{2}{5}\,acx^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x
^3 + a^2*x
```

3.260 $\int \frac{(a+bx^2+cx^4)^2}{d+ex^2} \, dx$

Optimal. Leaf size=143

$$\frac{x^3\left(-2ce(bd-ae)+b^2e^2+c^2d^2\right)}{3e^3}-\frac{x(cd-be)\left(cd^2-e(bd-2ae)\right)}{e^4}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)^2}{\sqrt{d}e^{9/2}}-\frac{cx^5(cd-2be)}{5e^2}+\frac{c^2x^7}{7e}$$

```
[Out] -(((c*d - b*e)*(c*d^2 - e*(b*d - 2*a*e))*x)/e^4) + ((c^2*d^2 + b^
2*e^2 - 2*c*e*(b*d - a*e))*x^3)/(3*e^3) - (c*(c*d - 2*b*e)*x^5)/(
5*e^2) + (c^2*x^7)/(7*e) + ((c*d^2 - b*d*e + a*e^2)^2*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(9/2))
```

Rubi [A] time = 0.297083, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{x^3\left(-2ce(bd-ae)+b^2e^2+c^2d^2\right)}{3e^3}-\frac{x(cd-be)\left(cd^2-e(bd-2ae)\right)}{e^4}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)^2}{\sqrt{d}e^{9/2}}-\frac{cx^5(cd-2be)}{5e^2}+\frac{c^2x^7}{7e}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2),x]
```


```
[Out] -(((c*d - b*e)*(c*d^2 - e*(b*d - 2*a*e))*x)/e^4) + ((c^2*d^2 + b^
2*e^2 - 2*c*e*(b*d - a*e))*x^3)/(3*e^3) - (c*(c*d - 2*b*e)*x^5)/(
5*e^2) + (c^2*x^7)/(7*e) + ((c*d^2 - b*d*e + a*e^2)^2*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(9/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^2x^7}{7e}+\frac{cx^5\left(2be-cd\right)}{5e^2}+\left(be-cd\right)\left(2ae^2-bde+cd^2\right)\int\frac{1}{e^4}\,dx+\frac{x^3\left(2ace^2+b^2e^2-2bcde+c^2d^2\right)}{3e^3}+\frac{\left(ae^2-bde+cd^2\right)^2\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2/(e*x**2+d),x)
```

```
[Out] c**2*x**7/(7*e) + c*x**5*(2*b*e - c*d)/(5*e**2) + (b*e - c*d)*(2*
a*e**2 - b*d*e + c*d**2)*Integral(e**(-4), x) + x**3*(2*a*c*e**2
+ b**2*e**2 - 2*b*c*d*e + c**2*d**2)/(3*e**3) + (a*e**2 - b*d*e +
 c*d**2)**2*atan(sqrt(e)*x/sqrt(d))/(sqrt(d)*e**(9/2))
```

Mathematica [A] time = 0.126169, size = 144, normalized size = 1.01

$$\frac{x^3\left(2ace^2+b^2e^2-2bcde+c^2d^2\right)}{3e^3}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)^2}{\sqrt{d}e^{9/2}}+\frac{x(be-cd)\left(2ae^2-bde+cd^2\right)}{e^4}+\frac{cx^5(2be-cd)}{5e^2}+\frac{c^2x^7}{7e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2),x]
```

```
[Out] ((-(c*d) + b*e)*(c*d^2 - b*d*e + 2*a*e^2)*x)/e^4 + ((c^2*d^2 - 2*
b*c*d*e + b^2*e^2 + 2*a*c*e^2)*x^3)/(3*e^3) + (c*(-(c*d) + 2*b*e)
*x^5)/(5*e^2) + (c^2*x^7)/(7*e) + ((c*d^2 - b*d*e + a*e^2)^2*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(9/2))
```

Maple [B] time = 0.005, size = 267, normalized size = 1.9

$$\frac{c^2x^7}{7\,e}+\frac{2\,x^5bc}{5\,e}-\frac{c^2dx^5}{5\,e^2}+\frac{2\,cx^3a}{3\,e}+\frac{b^2x^3}{3\,e}-\frac{2\,bx^3cd}{3\,e^2}+\frac{c^2x^3d^2}{3\,e^3}+2\,\frac{abx}{e}-2\,\frac{acdx}{e^2}-\frac{b^2dx}{e^2}+2\,\frac{bcd^2x}{e^3}-\frac{c^2d^3x}{e^4}+a^2\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-2\,\frac{dab}{e\sqrt{de}}\arctan\left(\frac{ex}{\sqrt{de}}\right)+2\,\frac{acd^2}{e^2\sqrt{de}}\arctan\left(\frac{ex}{\sqrt{de}}\right)+\frac{b^2d^2}{e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-2\,\frac{bcd^3}{e^3\sqrt{de}}\arctan\left(\frac{ex}{\sqrt{de}}\right)+\frac{c^2d^4}{e^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/(e*x^2+d),x)
```

```
[Out] 1/7*c^2*x^7/e+2/5/e*x^5*b*c-1/5*c^2*d*x^5/e^2+2/3*c/e*x^3*a+1/3/e
*x^3*b^2-2/3/e^2*x^3*b*c*d+1/3*c^2/e^3*x^3*d^2+2/e*a*b*x-2*c/e^2*
d*a*x-1/e^2*b^2*d*x+2/e^3*b*c*d^2*x-c^2/e^4*d^3*x+1/(d*e)^(1/2)*a
rctan(x*e/(d*e)^(1/2))*a^2-2/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)
)*a*b*d+2/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*c*d^2+1/e^2/(
d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b^2*d^2-2/e^3/(d*e)^(1/2)*arct
an(x*e/(d*e)^(1/2))*b*c*d^3+1/e^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1
/2))*c^2*d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/(e*x^2 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274675, size = 1, normalized size = 0.01

$$\left[\frac{105\left(c^2d^4-2\,bcd^3e-2\,abde^3+a^2e^4+\left(b^2+2\,ac\right)d^2e^2\right)\log\left(\frac{2\,dex+\left(ex^2-d\right)\sqrt{-de}}{ex^2+d}\right)+2\left(15\,c^2e^3x^7-21\left(c^2de^2-2\,bce^3\right)x^5+\cdots\right.}{210\,\sqrt{-de}e^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/(e*x^2 + d),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2
 + 2*a*c)*d^2*e^2)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2
+ d)) + 2*(15*c^2*e^3*x^7 - 21*(c^2*d*e^2 - 2*b*c*e^3)*x^5 + 35*(
```

```
c^2*d^2*e - 2*b*c*d*e^2 + (b^2 + 2*a*c)*e^3)*x^3 - 105*(c^2*d^3 -
 2*b*c*d^2*e - 2*a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x)*sqrt(-d*e))/(s
qrt(-d*e)*e^4), 1/105*(105*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 +
 a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*arctan(sqrt(d*e)*x/d) + (15*c^2
*e^3*x^7 - 21*(c^2*d*e^2 - 2*b*c*e^3)*x^5 + 35*(c^2*d^2*e - 2*b*c
*d*e^2 + (b^2 + 2*a*c)*e^3)*x^3 - 105*(c^2*d^3 - 2*b*c*d^2*e - 2*
a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x)*sqrt(d*e))/(sqrt(d*e)*e^4)]
```

Sympy [A] time = 4.83672, size = 366, normalized size = 2.56

$$\frac{c^2x^7}{7e} - \frac{\sqrt{-\frac{1}{de^9}}\left(ae^2-bde+cd^2\right)^2\log\left(-\frac{de^4\sqrt{-\frac{1}{de^9}}\left(ae^2-bde+cd^2\right)^2}{a^2e^4-2abde^3+2acd^2e^2+b^2d^2e^2-2bcd^3e+c^2d^4}+x\right)}{2} + \frac{\sqrt{-\frac{1}{de^9}}\left(ae^2-bde+cd^2\right)^2\log\left(\frac{de^4\sqrt{-\frac{1}{de^9}}\left(ae^2-bde+cd^2\right)^2}{a^2e^4-2abde^3+2acd^2e^2+b^2d^2e^2-2bcd^3e+c^2d^4}+x\right)}{2} + \frac{x^5\left(2bce-c^2d\right)}{5e^2} + \frac{x^3\left(2ace^2+b^2e^2-2bcde+c^2d^2\right)}{3e^3} + \frac{x\left(2abe^3-2acde^2-b^2de^2+2bcd^2e-c^2d^3\right)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/(e*x**2+d),x)
```

```
[Out] c**2*x**7/(7*e) - sqrt(-1/(d*e**9))*(a*e**2 - b*d*e + c*d**2)**2*
log(-d*e**4*sqrt(-1/(d*e**9))*(a*e**2 - b*d*e + c*d**2)**2/(a**2*
e**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d*
*3*e + c**2*d**4) + x)/2 + sqrt(-1/(d*e**9))*(a*e**2 - b*d*e + c*
d**2)**2*log(d*e**4*sqrt(-1/(d*e**9))*(a*e**2 - b*d*e + c*d**2)**
2/(a**2*e**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 -
2*b*c*d**3*e + c**2*d**4) + x)/2 + x**5*(2*b*c*e - c**2*d)/(5*e**
2) + x**3*(2*a*c*e**2 + b**2*e**2 - 2*b*c*d*e + c**2*d**2)/(3*e**
3) + x*(2*a*b*e**3 - 2*a*c*d*e**2 - b**2*d*e**2 + 2*b*c*d**2*e -
c**2*d**3)/e**4
```

GIAC/XCAS [A] time = 0.266511, size = 250, normalized size = 1.75

$$\frac{\left(c^2d^4 - 2\,bcd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,abde^3 + a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{\sqrt{d}} + \frac{1}{105}\left(15\,c^2x^7e^6 - 21\,c^2dx^5e^5 + 42\,bcx^5e^6 + 35\,c^2d^2x^3e^4 - 70\,bcdx^3e^5 - 105\,c^2d^3xe^3 + 35\,b^2x^3e^6 + 70\,acx^3e^6 + 210\,bcd^2xe^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/(e*x^2 + d),x, algorithm="giac")
```

```
[Out] (c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^
3 + a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt(d) + 1/105*(
15*c^2*x^7*e^6 - 21*c^2*d*x^5*e^5 + 42*b*c*x^5*e^6 + 35*c^2*d^2*x
^3*e^4 - 70*b*c*d*x^3*e^5 - 105*c^2*d^3*x*e^3 + 35*b^2*x^3*e^6 +
70*a*c*x^3*e^6 + 210*b*c*d^2*x*e^4 - 105*b^2*d*x*e^5 - 210*a*c*d*
x*e^5 + 210*a*b*x*e^6)*e^(-7)
```

3.261 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=166

$$\frac{x\left(-2ce(2bd-ae)+b^2e^2+3c^2d^2\right)}{e^4}+\frac{x\left(ae^2-bde+cd^2\right)^2}{2de^4\left(d+ex^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)\left(7cd^2-e(ae+3bd)\right)}{2d^{3/2}e^{9/2}}-\frac{2cx^3(cd-be)}{3e^3}+\frac{c^2x^5}{5e^2}$$

```
[Out] ((3*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d - a*e))*x)/e^4 - (2*c*(c*d -
 b*e)*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) + ((c*d^2 - b*d*e + a*e^2)
^2*x)/(2*d*e^4*(d + e*x^2)) - ((c*d^2 - b*d*e + a*e^2)*(7*c*d^2 -
 e*(3*b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2)
)
```

Rubi [A] time = 0.535022, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{x\left(-2ce(2bd-ae)+b^2e^2+3c^2d^2\right)}{e^4}+\frac{x\left(ae^2-bde+cd^2\right)^2}{2de^4\left(d+ex^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)\left(7cd^2-e(ae+3bd)\right)}{2d^{3/2}e^{9/2}}-\frac{2cx^3(cd-be)}{3e^3}+\frac{c^2x^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^2,x]
```

```
[Out] ((3*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d - a*e))*x)/e^4 - (2*c*(c*d -
 b*e)*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) + ((c*d^2 - b*d*e + a*e^2)
^2*x)/(2*d*e^4*(d + e*x^2)) - ((c*d^2 - b*d*e + a*e^2)*(7*c*d^2 -
 e*(3*b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2)
)
```

Rubi in Sympy [A] time = 155.121, size = 212, normalized size = 1.28

$$\frac{c^2x^7}{5e\left(d+ex^2\right)}+\frac{cx^3\left(10be-7cd\right)}{15e^3}+\frac{x\left(10ace^2+5b^2e^2-20bcde+14c^2d^2\right)}{5e^4}+\frac{x\left(5a^2e^4-10abde^3+10acd^2e^2+5b^2d^2e^2-10bcd^3e+7c^2d^4\right)}{10de^4\left(d+ex^2\right)}+\frac{\left(ae^2-bde+cd^2\right)\left(ae^2+3bde-7cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{\frac{3}{2}}e^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**2,x)
```

```
[Out] c**2*x**7/(5*e*(d + e*x**2)) + c*x**3*(10*b*e - 7*c*d)/(15*e**3)
+ x*(10*a*c*e**2 + 5*b**2*e**2 - 20*b*c*d*e + 14*c**2*d**2)/(5*e*
*4) + x*(5*a**2*e**4 - 10*a*b*d*e**3 + 10*a*c*d**2*e**2 + 5*b**2*
d**2*e**2 - 10*b*c*d**3*e + 7*c**2*d**4)/(10*d*e**4*(d + e*x**2))
 + (a*e**2 - b*d*e + c*d**2)*(a*e**2 + 3*b*d*e - 7*c*d**2)*atan(s
qrt(e)*x/sqrt(d))/(2*d**(3/2)*e**(9/2))
```

Mathematica [A] time = 0.186173, size = 183, normalized size = 1.1

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-e^2\left(a^2e^2+2abde-3b^2d^2\right)+2cd^2e(3ae-5bd)+7c^2d^4\right)}{2d^{3/2}e^{9/2}}+\frac{x\left(2ce(ae-2bd)+b^2e^2+3c^2d^2\right)}{e^4}+\frac{x\left(e(ae-bd)+cd^2\right)^2}{2de^4\left(d+ex^2\right)}+\frac{2cx^3(be-cd)}{3e^3}+\frac{c^2x^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^2,x]
```

```
[Out] ((3*c^2*d^2 + b^2*e^2 + 2*c*e*(-2*b*d + a*e))*x)/e^4 + (2*c*(-(c*
d) + b*e)*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) + ((c*d^2 + e*(-(b*d)
+ a*e))^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c^2*d^4 + 2*c*d^2*e*(-5*
b*d + 3*a*e) - e^2*(-3*b^2*d^2 + 2*a*b*d*e + a^2*e^2))*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

Maple [B] time = 0.015, size = 320, normalized size = 1.9

$$\frac{c^2x^5}{5e^2}+\frac{2bx^3c}{3e^2}-\frac{2c^2dx^3}{3e^3}+2\frac{acx}{e^2}+\frac{b^2x}{e^2}-4\frac{bcdx}{e^3}+3\frac{c^2d^2x}{e^4}+\frac{a^2x}{2d(ex^2+d)}-\frac{abx}{e(ex^2+d)}+\frac{adxc}{e^2(ex^2+d)}+\frac{dxb^2}{2e^2(ex^2+d)}-\frac{xbd^2c}{e^3(ex^2+d)}+\frac{d^3xc^2}{2e^4(ex^2+d)}+\frac{a^2}{2d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{ab}{e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-3\frac{acd}{e^2\sqrt{de}}\arctan\left(\frac{ex}{\sqrt{de}}\right)-\frac{3b^2d}{2e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+5\frac{bcd^2}{e^3\sqrt{de}}\arctan\left(\frac{ex}{\sqrt{de}}\right)-\frac{7c^2d^3}{2e^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x)
```

```
[Out] 1/5*c^2*x^5/e^2+2/3/e^2*x^3*b*c-2/3*c^2*d*x^3/e^3+2*c/e^2*a*x+1/e
^2*b^2*x-4/e^3*b*c*d*x+3*c^2/e^4*d^2*x+1/2/d*x/(e*x^2+d)*a^2-1/e*
x/(e*x^2+d)*a*b+1/e^2*d*x/(e*x^2+d)*a*c+1/2/e^2*d*x/(e*x^2+d)*b^2
-1/e^3*d^2*x/(e*x^2+d)*b*c+1/2/e^4*d^3*x/(e*x^2+d)*c^2+1/2/d/(d*e
)^(1/2)*arctan(x*e/(d*e)^(1/2))*a^2+1/e/(d*e)^(1/2)*arctan(x*e/(d
*e)^(1/2))*a*b-3/e^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*c-3/
2/e^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b^2+5/e^3*d^2/(d*e)^(
1/2)*arctan(x*e/(d*e)^(1/2))*b*c-7/2/e^4*d^3/(d*e)^(1/2)*arctan(x
*e/(d*e)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/(e*x^2 + d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284966, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(7\,c^2d^5-10\,bcd^4e-2\,abd^2e^3-a^2de^4+3\,\left(b^2+2\,ac\right)d^3e^2+\left(7\,c^2d^4e-10\,bcd^3e^2-2\,abde^4-a^2e^5+3\,\left(b^2+2\,ac\right)d^2e^3\right.\right.}{}\right.$$

$$-\frac{15\left(7\,c^2d^5-10\,bcd^4e-2\,abd^2e^3-a^2de^4+3\,\left(b^2+2\,ac\right)d^3e^2+\left(7\,c^2d^4e-10\,bcd^3e^2-2\,abde^4-a^2e^5+3\,\left(b^2+2\,ac\right)d^2e^3\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/(e*x^2 + d)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(7*c^2*d^5 - 10*b*c*d^4*e - 2*a*b*d^2*e^3 - a^2*d*e^4
+ 3*(b^2 + 2*a*c)*d^3*e^2 + (7*c^2*d^4*e - 10*b*c*d^3*e^2 - 2*a*b
*d*e^4 - a^2*e^5 + 3*(b^2 + 2*a*c)*d^2*e^3)*x^2)*log((2*d*e*x + (
e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d)) - 2*(6*c^2*d*e^3*x^7 - 2*(7*c
^2*d^2*e^2 - 10*b*c*d*e^3)*x^5 + 10*(7*c^2*d^3*e - 10*b*c*d^2*e^2
 + 3*(b^2 + 2*a*c)*d*e^3)*x^3 + 15*(7*c^2*d^4 - 10*b*c*d^3*e - 2*
a*b*d*e^3 + a^2*e^4 + 3*(b^2 + 2*a*c)*d^2*e^2)*x)*sqrt(-d*e))/((d
*e^5*x^2 + d^2*e^4)*sqrt(-d*e)), -1/30*(15*(7*c^2*d^5 - 10*b*c*d^
4*e - 2*a*b*d^2*e^3 - a^2*d*e^4 + 3*(b^2 + 2*a*c)*d^3*e^2 + (7*c^
2*d^4*e - 10*b*c*d^3*e^2 - 2*a*b*d*e^4 - a^2*e^5 + 3*(b^2 + 2*a*c
)*d^2*e^3)*x^2)*arctan(sqrt(d*e)*x/d) - (6*c^2*d*e^3*x^7 - 2*(7*c
^2*d^2*e^2 - 10*b*c*d*e^3)*x^5 + 10*(7*c^2*d^3*e - 10*b*c*d^2*e^2
 + 3*(b^2 + 2*a*c)*d*e^3)*x^3 + 15*(7*c^2*d^4 - 10*b*c*d^3*e - 2*
a*b*d*e^3 + a^2*e^4 + 3*(b^2 + 2*a*c)*d^2*e^2)*x)*sqrt(d*e))/((d*
e^5*x^2 + d^2*e^4)*sqrt(d*e))]
```

Sympy [A] time = 12.4832, size = 479, normalized size = 2.89

$$\frac{c^2x^5}{5e^2}+\frac{x\left(a^2e^4-2abde^3+2acd^2e^2+b^2d^2e^2-2bcd^3e+c^2d^4\right)}{2d^2e^4+2de^5x^2}$$
$$-\frac{\sqrt{-\frac{1}{d^3e^9}}\left(ae^2-bde+cd^2\right)\left(ae^2+3bde-7cd^2\right)\log\left(-\frac{d^2e^4\sqrt{-\frac{1}{d^3e^9}}\left(ae^2-bde+cd^2\right)\left(ae^2+3bde-7cd^2\right)}{a^2e^4+2abde^3-6acd^2e^2-3b^2d^2e^2+10bcd^3e-7c^2d^4}+x\right)}{4}$$
$$+\frac{\sqrt{-\frac{1}{d^3e^9}}\left(ae^2-bde+cd^2\right)\left(ae^2+3bde-7cd^2\right)\log\left(\frac{d^2e^4\sqrt{-\frac{1}{d^3e^9}}\left(ae^2-bde+cd^2\right)\left(ae^2+3bde-7cd^2\right)}{a^2e^4+2abde^3-6acd^2e^2-3b^2d^2e^2+10bcd^3e-7c^2d^4}+x\right)}{4}$$
$$+\frac{x^3\left(2bce-2c^2d\right)}{3e^3}+\frac{x\left(2ace^2+b^2e^2-4bcde+3c^2d^2\right)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**2,x)
```

```
[Out] c**2*x**5/(5*e**2) + x*(a**2*e**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**
2 + b**2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**4)/(2*d**2*e**4 + 2*d
*e**5*x**2) - sqrt(-1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2)*(a*e
**2 + 3*b*d*e - 7*c*d**2)*log(-d**2*e**4*sqrt(-1/(d**3*e**9))*(a*
e**2 - b*d*e + c*d**2)*(a*e**2 + 3*b*d*e - 7*c*d**2)/(a**2*e**4 +
 2*a*b*d*e**3 - 6*a*c*d**2*e**2 - 3*b**2*d**2*e**2 + 10*b*c*d**3*
e - 7*c**2*d**4) + x)/4 + sqrt(-1/(d**3*e**9))*(a*e**2 - b*d*e +
c*d**2)*(a*e**2 + 3*b*d*e - 7*c*d**2)*log(d**2*e**4*sqrt(-1/(d**3
*e**9))*(a*e**2 - b*d*e + c*d**2)*(a*e**2 + 3*b*d*e - 7*c*d**2)/(
a**2*e**4 + 2*a*b*d*e**3 - 6*a*c*d**2*e**2 - 3*b**2*d**2*e**2 + 1
0*b*c*d**3*e - 7*c**2*d**4) + x)/4 + x**3*(2*b*c*e - 2*c**2*d)/(3
*e**3) + x*(2*a*c*e**2 + b**2*e**2 - 4*b*c*d*e + 3*c**2*d**2)/e**
4
```

GIAC/XCAS [A] time = 0.265461, size = 279, normalized size = 1.68

$$\frac{1}{15}\left(3\,c^2x^5e^8 - 10\,c^2dx^3e^7 + 10\,bcx^3e^8 + 45\,c^2d^2xe^6 - 60\,bcdxe^7 + 15\,b^2xe^8 + 30\,acxe^8\right)e^{(-10)} - \frac{\left(7\,c^2d^4 - 10\,bcd^3e + 3\,b^2d^2e^2 + 6\,acd^2e^2 - 2\,abde^3 - a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{\left(c^2d^4x - 2\,bcd^3xe + b^2d^2xe^2 + 2\,acd^2xe^2 - 2\,abdxe^3 + a^2xe^4\right)e^{(-4)}}{2\,(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/(e*x^2 + d)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*c^2*x^5*e^8 - 10*c^2*d*x^3*e^7 + 10*b*c*x^3*e^8 + 45*c^2*
d^2*x*e^6 - 60*b*c*d*x*e^7 + 15*b^2*x*e^8 + 30*a*c*x*e^8)*e^(-10)
 - 1/2*(7*c^2*d^4 - 10*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2
- 2*a*b*d*e^3 - a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(3/
2) + 1/2*(c^2*d^4*x - 2*b*c*d^3*x*e + b^2*d^2*x*e^2 + 2*a*c*d^2*x
*e^2 - 2*a*b*d*x*e^3 + a^2*x*e^4)*e^(-4)/((x^2*e + d)*d)
```

3.262 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=201

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e^2\left(3a^2e^2+2abde+3b^2d^2\right)-6cd^2e(5bd-ae)+35c^2d^4\right)}{8d^{5/2}e^{9/2}} - \frac{x\left(-3ae^2-5bde+13cd^2\right)\left(ae^2-bde+cd^2\right)}{8d^2e^4\left(d+ex^2\right)} + \frac{x\left(ae^2-bde+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2} - \frac{cx(3cd-2be)}{e^4} + \frac{c^2x^3}{3e^3}$$

```
[Out] -((c*(3*c*d - 2*b*e)*x)/e^4) + (c^2*x^3)/(3*e^3) + ((c*d^2 - b*d*
e + a*e^2)^2*x)/(4*d*e^4*(d + e*x^2)^2) - ((13*c*d^2 - 5*b*d*e -
3*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(8*d^2*e^4*(d + e*x^2)) + ((3
5*c^2*d^4 - 6*c*d^2*e*(5*b*d - a*e) + e^2*(3*b^2*d^2 + 2*a*b*d*e
+ 3*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(9/2))
```

Rubi [A] time = 0.826522, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e^2\left(3a^2e^2+2abde+3b^2d^2\right)-6cd^2e(5bd-ae)+35c^2d^4\right)}{8d^{5/2}e^{9/2}} - \frac{x\left(-3ae^2-5bde+13cd^2\right)\left(ae^2-bde+cd^2\right)}{8d^2e^4\left(d+ex^2\right)} + \frac{x\left(ae^2-bde+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2} - \frac{cx(3cd-2be)}{e^4} + \frac{c^2x^3}{3e^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^3,x]
```

```
[Out] -((c*(3*c*d - 2*b*e)*x)/e^4) + (c^2*x^3)/(3*e^3) + ((c*d^2 - b*d*
e + a*e^2)^2*x)/(4*d*e^4*(d + e*x^2)^2) - ((13*c*d^2 - 5*b*d*e -
3*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(8*d^2*e^4*(d + e*x^2)) + ((3
5*c^2*d^4 - 6*c*d^2*e*(5*b*d - a*e) + e^2*(3*b^2*d^2 + 2*a*b*d*e
+ 3*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(9/2))
```

Rubi in Sympy [A] time = 170.48, size = 279, normalized size = 1.39

$$\frac{c^2x^7}{3e\left(d+ex^2\right)^2} + \frac{cx\left(6be-7cd\right)}{3e^4} + \frac{x\left(3a^2e^4-6abde^3+6acd^2e^2+3b^2d^2e^2-6bcd^3e+7c^2d^4\right)}{12de^4\left(d+ex^2\right)^2} + \frac{x\left(3a^2e^4+2abde^3-10acd^2e^2-5b^2d^2e^2+18bcd^3e-21c^2d^4\right)}{8d^2e^4\left(d+ex^2\right)} + \frac{\left(3a^2e^4+2abde^3+6acd^2e^2+3b^2d^2e^2-30bcd^3e+35c^2d^4\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{\frac{5}{2}}e^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**3,x)
```

```
[Out] c**2*x**7/(3*e*(d + e*x**2)**2) + c*x*(6*b*e - 7*c*d)/(3*e**4) +
x*(3*a**2*e**4 - 6*a*b*d*e**3 + 6*a*c*d**2*e**2 + 3*b**2*d**2*e**
2 - 6*b*c*d**3*e + 7*c**2*d**4)/(12*d*e**4*(d + e*x**2)**2) + x*(
3*a**2*e**4 + 2*a*b*d*e**3 - 10*a*c*d**2*e**2 - 5*b**2*d**2*e**2
+ 18*b*c*d**3*e - 21*c**2*d**4)/(8*d**2*e**4*(d + e*x**2)) + (3*a
**2*e**4 + 2*a*b*d*e**3 + 6*a*c*d**2*e**2 + 3*b**2*d**2*e**2 - 30
*b*c*d**3*e + 35*c**2*d**4)*atan(sqrt(e)*x/sqrt(d))/(8*d**(5/2)*e
**(9/2))
```

Mathematica [A] time = 0.213285, size = 217, normalized size = 1.08

$$-\frac{x\left(e^2\left(-3a^2e^2-2abde+5b^2d^2\right)-2cd^2e(9bd-5ae)+13c^2d^4\right)}{8d^2e^4\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e^2\left(3a^2e^2+2abde+3b^2d^2\right)+6cd^2e(ae-5bd)+35c^2d^4\right)}{8d^{5/2}e^{9/2}}+\frac{x\left(e(ae-bd)+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2}+\frac{cx(2be-3cd)}{e^4}+\frac{c^2x^3}{3e^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^3,x]
```

```
[Out] (c*(-3*c*d + 2*b*e)*x)/e^4 + (c^2*x^3)/(3*e^3) + ((c*d^2 + e*(-(b
*d) + a*e))^2*x)/(4*d*e^4*(d + e*x^2)^2) - ((13*c^2*d^4 - 2*c*d^2
*e*(9*b*d - 5*a*e) + e^2*(5*b^2*d^2 - 2*a*b*d*e - 3*a^2*e^2))*x)/
(8*d^2*e^4*(d + e*x^2)) + ((35*c^2*d^4 + 6*c*d^2*e*(-5*b*d + a*e)
 + e^2*(3*b^2*d^2 + 2*a*b*d*e + 3*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sq
rt[d]])/(8*d^(5/2)*e^(9/2))
```

Maple [B] time = 0.016, size = 402, normalized size = 2.

$$\frac{c^2x^3}{3\,e^3}+2\,\frac{bcx}{e^3}-3\,\frac{c^2dx}{e^4}+\frac{3\,a^2ex^3}{8\,(ex^2+d)^2\,d^2}+\frac{abx^3}{4\,(ex^2+d)^2\,d}-\frac{5\,x^3ac}{4\,e\,(ex^2+d)^2}-\frac{5\,b^2x^3}{8\,e\,(ex^2+d)^2}+\frac{9\,bx^3cd}{4\,e^2\,(ex^2+d)^2}-\frac{13\,d^2x^3c^2}{8\,e^3\,(ex^2+d)^2}+\frac{5\,a^2x}{8\,(ex^2+d)^2\,d}-\frac{abx}{4\,e\,(ex^2+d)^2}-\frac{3\,adxc}{4\,e^2\,(ex^2+d)^2}-\frac{3\,b^2dx}{8\,e^2\,(ex^2+d)^2}+\frac{7\,bcd^2x}{4\,e^3\,(ex^2+d)^2}-\frac{11\,d^3xc^2}{8\,e^4\,(ex^2+d)^2}+\frac{3\,a^2}{8\,d^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{ab}{4\,de}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{3\,ac}{4\,e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{3\,b^2}{8\,e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{15\,bcd}{4\,e^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{35\,c^2d^2}{8\,e^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x)
```

```
[Out] 1/3*c^2*x^3/e^3+2*c/e^3*b*x-3*c^2*d*x/e^4+3/8*e/(e*x^2+d)^2/d^2*x
^3*a^2+1/4/(e*x^2+d)^2/d*x^3*a*b-5/4/e/(e*x^2+d)^2*x^3*a*c-5/8/e/
(e*x^2+d)^2*x^3*b^2+9/4/e^2/(e*x^2+d)^2*x^3*b*c*d-13/8/e^3/(e*x^2
+d)^2*d^2*x^3*c^2+5/8/(e*x^2+d)^2/d*x*a^2-1/4/e/(e*x^2+d)^2*a*b*x
-3/4/e^2/(e*x^2+d)^2*d*x*a*c-3/8/e^2/(e*x^2+d)^2*b^2*d*x+7/4/e^3/
(e*x^2+d)^2*b*c*d^2*x-11/8/e^4/(e*x^2+d)^2*d^3*x*c^2+3/8/d^2/(d*e
)^(1/2)*arctan(x*e/(d*e)^(1/2))*a^2+1/4/e/d/(d*e)^(1/2)*arctan(x*
e/(d*e)^(1/2))*a*b+3/4/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*
c+3/8/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b^2-15/4/e^3*d/(d*e
)^(1/2)*arctan(x*e/(d*e)^(1/2))*b*c+35/8/e^4*d^2/(d*e)^(1/2)*arct
an(x*e/(d*e)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/(e*x^2 + d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.295759, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/(e*x^2 + d)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(35*c^2*d^6 - 30*b*c*d^5*e + 2*a*b*d^3*e^3 + 3*a^2*d^2*e
^4 + 3*(b^2 + 2*a*c)*d^4*e^2 + (35*c^2*d^4*e^2 - 30*b*c*d^3*e^3 +
 2*a*b*d*e^5 + 3*a^2*e^6 + 3*(b^2 + 2*a*c)*d^2*e^4)*x^4 + 2*(35*c
^2*d^5*e - 30*b*c*d^4*e^2 + 2*a*b*d^2*e^4 + 3*a^2*d*e^5 + 3*(b^2
+ 2*a*c)*d^3*e^3)*x^2)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*
x^2 + d)) + 2*(8*c^2*d^2*e^3*x^7 - 8*(7*c^2*d^3*e^2 - 6*b*c*d^2*e
^3)*x^5 - (175*c^2*d^4*e - 150*b*c*d^3*e^2 - 6*a*b*d*e^4 - 9*a^2*
e^5 + 15*(b^2 + 2*a*c)*d^2*e^3)*x^3 - 3*(35*c^2*d^5 - 30*b*c*d^4*
e + 2*a*b*d^2*e^3 - 5*a^2*d*e^4 + 3*(b^2 + 2*a*c)*d^3*e^2)*x)*sqr
t(-d*e))/((d^2*e^6*x^4 + 2*d^3*e^5*x^2 + d^4*e^4)*sqrt(-d*e)), 1/
24*(3*(35*c^2*d^6 - 30*b*c*d^5*e + 2*a*b*d^3*e^3 + 3*a^2*d^2*e^4
+ 3*(b^2 + 2*a*c)*d^4*e^2 + (35*c^2*d^4*e^2 - 30*b*c*d^3*e^3 + 2*
a*b*d*e^5 + 3*a^2*e^6 + 3*(b^2 + 2*a*c)*d^2*e^4)*x^4 + 2*(35*c^2*
d^5*e - 30*b*c*d^4*e^2 + 2*a*b*d^2*e^4 + 3*a^2*d*e^5 + 3*(b^2 + 2
*a*c)*d^3*e^3)*x^2)*arctan(sqrt(d*e)*x/d) + (8*c^2*d^2*e^3*x^7 -
8*(7*c^2*d^3*e^2 - 6*b*c*d^2*e^3)*x^5 - (175*c^2*d^4*e - 150*b*c*
d^3*e^2 - 6*a*b*d*e^4 - 9*a^2*e^5 + 15*(b^2 + 2*a*c)*d^2*e^3)*x^3
 - 3*(35*c^2*d^5 - 30*b*c*d^4*e + 2*a*b*d^2*e^3 - 5*a^2*d*e^4 + 3
*(b^2 + 2*a*c)*d^3*e^2)*x)*sqrt(d*e))/((d^2*e^6*x^4 + 2*d^3*e^5*x
^2 + d^4*e^4)*sqrt(d*e))]
```

Sympy [A] time = 53.1541, size = 396, normalized size = 1.97

$$\frac{c^2x^3}{3e^3} - \frac{\sqrt{-\frac{1}{d^5e^9}}\left(3a^2e^4 + 2abde^3 + 6acd^2e^2 + 3b^2d^2e^2 - 30bcd^3e + 35c^2d^4\right)\log\left(-d^3e^4\sqrt{-\frac{1}{d^5e^9}} + x\right)}{16}$$
$$+ \frac{\sqrt{-\frac{1}{d^5e^9}}\left(3a^2e^4 + 2abde^3 + 6acd^2e^2 + 3b^2d^2e^2 - 30bcd^3e + 35c^2d^4\right)\log\left(d^3e^4\sqrt{-\frac{1}{d^5e^9}} + x\right)}{16}$$
$$+ \frac{x^3\left(3a^2e^5 + 2abde^4 - 10acd^2e^3 - 5b^2d^2e^3 + 18bcd^3e^2 - 13c^2d^4e\right) + x\left(5a^2de^4 - 2abd^2e^3 - 6acd^3e^2 - 3b^2d^3e^2 + 14bcd^4e - \right.}{8d^4e^4 + 16d^3e^5x^2 + 8d^2e^6x^4}$$
$$+ \frac{x\left(2bce - 3c^2d\right)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**3,x)
```

```
[Out] c**2*x**3/(3*e**3) - sqrt(-1/(d**5*e**9))*(3*a**2*e**4 + 2*a*b*d*
e**3 + 6*a*c*d**2*e**2 + 3*b**2*d**2*e**2 - 30*b*c*d**3*e + 35*c*
*2*d**4)*log(-d**3*e**4*sqrt(-1/(d**5*e**9)) + x)/16 + sqrt(-1/(d
**5*e**9))*(3*a**2*e**4 + 2*a*b*d*e**3 + 6*a*c*d**2*e**2 + 3*b**2
*d**2*e**2 - 30*b*c*d**3*e + 35*c**2*d**4)*log(d**3*e**4*sqrt(-1/
(d**5*e**9)) + x)/16 + (x**3*(3*a**2*e**5 + 2*a*b*d*e**4 - 10*a*c
*d**2*e**3 - 5*b**2*d**2*e**3 + 18*b*c*d**3*e**2 - 13*c**2*d**4*e
) + x*(5*a**2*d*e**4 - 2*a*b*d**2*e**3 - 6*a*c*d**3*e**2 - 3*b**2
*d**3*e**2 + 14*b*c*d**4*e - 11*c**2*d**5))/(8*d**4*e**4 + 16*d**
3*e**5*x**2 + 8*d**2*e**6*x**4) + x*(2*b*c*e - 3*c**2*d)/e**4
```

GIAC/XCAS [A] time = 0.267756, size = 329, normalized size = 1.64

$$\frac{1}{3}\left(c^2x^3e^6 - 9\,c^2dxe^5 + 6\,bcxe^6\right)e^{(-9)} + \frac{\left(35\,c^2d^4 - 30\,bcd^3e + 3\,b^2d^2e^2 + 6\,acd^2e^2 + 2\,abde^3 + 3\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{8\,d^{\frac{5}{2}}} - \frac{\left(13\,c^2d^4x^3e - 18\,bcd^3x^3e^2 + 11\,c^2d^5x + 5\,b^2d^2x^3e^3 + 10\,acd^2x^3e^3 - 14\,bcd^4xe - 2\,abdx^3e^4 + 3\,b^2d^3xe^2 + 6\,acd^3xe^2 - 3\,a^2\right.}{8\,(x^2e + d)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/(e*x^2 + d)^3,x, algorithm="giac")
```

```
[Out] 1/3*(c^2*x^3*e^6 - 9*c^2*d*x*e^5 + 6*b*c*x*e^6)*e^(-9) + 1/8*(35*
c^2*d^4 - 30*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 + 2*a*b*d*
e^3 + 3*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(5/2) - 1/8
*(13*c^2*d^4*x^3*e - 18*b*c*d^3*x^3*e^2 + 11*c^2*d^5*x + 5*b^2*d^
2*x^3*e^3 + 10*a*c*d^2*x^3*e^3 - 14*b*c*d^4*x*e - 2*a*b*d*x^3*e^4
 + 3*b^2*d^3*x*e^2 + 6*a*c*d^3*x*e^2 - 3*a^2*x^3*e^5 + 2*a*b*d^2*
x*e^3 - 5*a^2*d*x*e^4)*e^(-4)/((x^2*e + d)^2*d^2)
```

3.263 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4}\,dx$

Optimal. Leaf size=250

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(ae+5bd)+35c^2d^4\right)}{16d^{7/2}e^{9/2}}+\frac{x\left(e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(11bd-ae)+29c^2d^4\right)}{16d^3e^4\left(d+ex^2\right)}-\frac{x\left(-5ae^2-7bde+19cd^2\right)\left(ae^2-bde+cd^2\right)}{24d^2e^4\left(d+ex^2\right)^2}+\frac{x\left(ae^2-bde+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}+\frac{c^2x}{e^4}$$

```
[Out] (c^2*x)/e^4 + ((c*d^2 - b*d*e + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^
3) - ((19*c*d^2 - 7*b*d*e - 5*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(
24*d^2*e^4*(d + e*x^2)^2) + ((29*c^2*d^4 - 2*c*d^2*e*(11*b*d - a*
e) + e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*x)/(16*d^3*e^4*(d + e
*x^2)) - ((35*c^2*d^4 - 2*c*d^2*e*(5*b*d + a*e) - e^2*(b^2*d^2 +
2*a*b*d*e + 5*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*
e^(9/2))
```

Rubi [A] time = 0.951392, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(ae+5bd)+35c^2d^4\right)}{16d^{7/2}e^{9/2}}+\frac{x\left(e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(11bd-ae)+29c^2d^4\right)}{16d^3e^4\left(d+ex^2\right)}-\frac{x\left(-5ae^2-7bde+19cd^2\right)\left(ae^2-bde+cd^2\right)}{24d^2e^4\left(d+ex^2\right)^2}+\frac{x\left(ae^2-bde+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}+\frac{c^2x}{e^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^4,x]
```


```
[Out] (c^2*x)/e^4 + ((c*d^2 - b*d*e + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^
3) - ((19*c*d^2 - 7*b*d*e - 5*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(
24*d^2*e^4*(d + e*x^2)^2) + ((29*c^2*d^4 - 2*c*d^2*e*(11*b*d - a*
e) + e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*x)/(16*d^3*e^4*(d + e
*x^2)) - ((35*c^2*d^4 - 2*c*d^2*e*(5*b*d + a*e) - e^2*(b^2*d^2 +
2*a*b*d*e + 5*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*
e^(9/2))
```

Rubi in Sympy [A] time = 162.457, size = 332, normalized size = 1.33

$$\frac{c^2x^7}{e\left(d+ex^2\right)^3}+\frac{x\left(a^2e^4-2abde^3+2acd^2e^2+b^2d^2e^2-2bcd^3e+7c^2d^4\right)}{6de^4\left(d+ex^2\right)^3}+\frac{x\left(5a^2e^4+2abde^3-14acd^2e^2-7b^2d^2e^2+26bcd^3e-91c^2d^4\right)}{24d^2e^4\left(d+ex^2\right)^2}+\frac{x\left(5a^2e^4+2abde^3+2acd^2e^2+b^2d^2e^2-22bcd^3e+77c^2d^4\right)}{16d^3e^4\left(d+ex^2\right)}+\frac{\left(5a^2e^4+2abde^3+2acd^2e^2+b^2d^2e^2+10bcd^3e-35c^2d^4\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{\frac{7}{2}}e^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**4,x)
```

```
[Out] c**2*x**7/(e*(d + e*x**2)**3) + x*(a**2*e**4 - 2*a*b*d*e**3 + 2*a
*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d**3*e + 7*c**2*d**4)/(6*d*
e**4*(d + e*x**2)**3) + x*(5*a**2*e**4 + 2*a*b*d*e**3 - 14*a*c*d*
*2*e**2 - 7*b**2*d**2*e**2 + 26*b*c*d**3*e - 91*c**2*d**4)/(24*d*
*2*e**4*(d + e*x**2)**2) + x*(5*a**2*e**4 + 2*a*b*d*e**3 + 2*a*c*
d**2*e**2 + b**2*d**2*e**2 - 22*b*c*d**3*e + 77*c**2*d**4)/(16*d*
*3*e**4*(d + e*x**2)) + (5*a**2*e**4 + 2*a*b*d*e**3 + 2*a*c*d**2*
e**2 + b**2*d**2*e**2 + 10*b*c*d**3*e - 35*c**2*d**4)*atan(sqrt(e
)*x/sqrt(d))/(16*d**(7/2)*e**(9/2))
```

Mathematica [A] time = 0.280701, size = 267, normalized size = 1.07

$$-\frac{x\left(e^2\left(-5a^2e^2-2abde+7b^2d^2\right)+2cd^2e(7ae-13bd)+19c^2d^4\right)}{24d^2e^4\left(d+ex^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(ae+5bd)+35c^2d^4\right)}{16d^{7/2}e^{9/2}}+\frac{x\left(e^2\left(5a^2e^2+2abde+b^2d^2\right)+2cd^2e(ae-11bd)+29c^2d^4\right)}{16d^3e^4\left(d+ex^2\right)}+\frac{x\left(e(ae-bd)+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}+\frac{c^2x}{e^4}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (c^2*x)/e^4 + ((c*d^2 + e*(-(b*d) + a*e))^2*x)/(6*d*e^4*(d + e*x^
2)^3) - ((19*c^2*d^4 + 2*c*d^2*e*(-13*b*d + 7*a*e) + e^2*(7*b^2*d
^2 - 2*a*b*d*e - 5*a^2*e^2))*x)/(24*d^2*e^4*(d + e*x^2)^2) + ((29
*c^2*d^4 + 2*c*d^2*e*(-11*b*d + a*e) + e^2*(b^2*d^2 + 2*a*b*d*e +
 5*a^2*e^2))*x)/(16*d^3*e^4*(d + e*x^2)) - ((35*c^2*d^4 - 2*c*d^2
*e*(5*b*d + a*e) - e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

Maple [B] time = 0.018, size = 506, normalized size = 2.

$$\frac{c^2x}{e^4}+\frac{b^2x^5}{16\left(ex^2+d\right)^3d}-\frac{b^2x^3}{6e\left(ex^2+d\right)^3}+\frac{11a^2x}{16\left(ex^2+d\right)^3d}+\frac{5a^2}{16d^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{5e^2x^5a^2}{16\left(ex^2+d\right)^3d^3}+\frac{acx^5}{8\left(ex^2+d\right)^3d}+\frac{abx^3}{3\left(ex^2+d\right)^3d}-\frac{adxc}{8e^2\left(ex^2+d\right)^3}+\frac{29dx^5c^2}{16e^2\left(ex^2+d\right)^3}+\frac{5a^2ex^3}{6\left(ex^2+d\right)^3d^2}+\frac{ac}{8e^2d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{35c^2d}{16e^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{x^3ac}{3e\left(ex^2+d\right)^3}+\frac{17d^2x^3c^2}{6e^3\left(ex^2+d\right)^3}+\frac{19d^3xc^2}{16e^4\left(ex^2+d\right)^3}+\frac{ex^5ab}{8\left(ex^2+d\right)^3d^2}-\frac{5bx^3cd}{3e^2\left(ex^2+d\right)^3}-\frac{5bcd^2x}{8e^3\left(ex^2+d\right)^3}+\frac{ab}{8d^2e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{11x^5bc}{8e\left(ex^2+d\right)^3}-\frac{abx}{8e\left(ex^2+d\right)^3}-\frac{b^2dx}{16e^2\left(ex^2+d\right)^3}+\frac{b^2}{16e^2d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{5bc}{8e^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x)
```

```
[Out] c^2*x/e^4+1/16/(e*x^2+d)^3/d*x^5*b^2-1/6/e/(e*x^2+d)^3*x^3*b^2+11
/16/(e*x^2+d)^3/d*x*a^2+5/16/d^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/
2))*a^2+5/16*e^2/(e*x^2+d)^3/d^3*x^5*a^2+1/8/(e*x^2+d)^3/d*x^5*a*
c+1/3/(e*x^2+d)^3/d*x^3*a*b-1/8/e^2/(e*x^2+d)^3*d*x*a*c+29/16/e^2
/(e*x^2+d)^3*d*x^5*c^2+5/6*e/(e*x^2+d)^3/d^2*x^3*a^2+1/8/e^2/d/(d
```

```
*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*c-35/16/e^4*d/(d*e)^(1/2)*arc
tan(x*e/(d*e)^(1/2))*c^2-1/3/e/(e*x^2+d)^3*x^3*a*c+17/6/e^3/(e*x^
2+d)^3*d^2*x^3*c^2+19/16/e^4/(e*x^2+d)^3*d^3*x*c^2+1/8*e/(e*x^2+d
)^3/d^2*x^5*a*b-5/3/e^2/(e*x^2+d)^3*x^3*b*c*d-5/8/e^3/(e*x^2+d)^3
*b*c*d^2*x+1/8/e/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*b-11/8
/e/(e*x^2+d)^3*x^5*b*c-1/8/e/(e*x^2+d)^3*a*b*x-1/16/e^2/(e*x^2+d)
^3*b^2*d*x+1/16/e^2/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b^2+5/8
/e^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/(e*x^2 + d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.299297, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/(e*x^2 + d)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(35*c^2*d^7 - 10*b*c*d^6*e - 2*a*b*d^4*e^3 - 5*a^2*d^3*
e^4 - (b^2 + 2*a*c)*d^5*e^2 + (35*c^2*d^4*e^3 - 10*b*c*d^3*e^4 -
2*a*b*d*e^6 - 5*a^2*e^7 - (b^2 + 2*a*c)*d^2*e^5)*x^6 + 3*(35*c^2*
d^5*e^2 - 10*b*c*d^4*e^3 - 2*a*b*d^2*e^5 - 5*a^2*d*e^6 - (b^2 + 2
*a*c)*d^3*e^4)*x^4 + 3*(35*c^2*d^6*e - 10*b*c*d^5*e^2 - 2*a*b*d^3
*e^4 - 5*a^2*d^2*e^5 - (b^2 + 2*a*c)*d^4*e^3)*x^2)*log((2*d*e*x +
 (e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d)) - 2*(48*c^2*d^3*e^3*x^7 + 3
*(77*c^2*d^4*e^2 - 22*b*c*d^3*e^3 + 2*a*b*d*e^5 + 5*a^2*e^6 + (b^
2 + 2*a*c)*d^2*e^4)*x^5 + 8*(35*c^2*d^5*e - 10*b*c*d^4*e^2 + 2*a*
b*d^2*e^4 + 5*a^2*d*e^5 - (b^2 + 2*a*c)*d^3*e^3)*x^3 + 3*(35*c^2*
d^6 - 10*b*c*d^5*e - 2*a*b*d^3*e^3 + 11*a^2*d^2*e^4 - (b^2 + 2*a*
c)*d^4*e^2)*x)*sqrt(-d*e))/((d^3*e^7*x^6 + 3*d^4*e^6*x^4 + 3*d^5*
e^5*x^2 + d^6*e^4)*sqrt(-d*e)), -1/48*(3*(35*c^2*d^7 - 10*b*c*d^6
*e - 2*a*b*d^4*e^3 - 5*a^2*d^3*e^4 - (b^2 + 2*a*c)*d^5*e^2 + (35*
c^2*d^4*e^3 - 10*b*c*d^3*e^4 - 2*a*b*d*e^6 - 5*a^2*e^7 - (b^2 + 2
*a*c)*d^2*e^5)*x^6 + 3*(35*c^2*d^5*e^2 - 10*b*c*d^4*e^3 - 2*a*b*d
^2*e^5 - 5*a^2*d*e^6 - (b^2 + 2*a*c)*d^3*e^4)*x^4 + 3*(35*c^2*d^6
*e - 10*b*c*d^5*e^2 - 2*a*b*d^3*e^4 - 5*a^2*d^2*e^5 - (b^2 + 2*a*
c)*d^4*e^3)*x^2)*arctan(sqrt(d*e)*x/d) - (48*c^2*d^3*e^3*x^7 + 3*
(77*c^2*d^4*e^2 - 22*b*c*d^3*e^3 + 2*a*b*d*e^5 + 5*a^2*e^6 + (b^2
 + 2*a*c)*d^2*e^4)*x^5 + 8*(35*c^2*d^5*e - 10*b*c*d^4*e^2 + 2*a*b
*d^2*e^4 + 5*a^2*d*e^5 - (b^2 + 2*a*c)*d^3*e^3)*x^3 + 3*(35*c^2*d
^6 - 10*b*c*d^5*e - 2*a*b*d^3*e^3 + 11*a^2*d^2*e^4 - (b^2 + 2*a*c
)*d^4*e^2)*x)*sqrt(d*e))/((d^3*e^7*x^6 + 3*d^4*e^6*x^4 + 3*d^5*e^
5*x^2 + d^6*e^4)*sqrt(d*e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.265931, size = 400, normalized size = 1.6

$$c^2 x e^{(-4)} - \frac{\left(35\,c^2 d^4 - 10\,bcd^3 e - b^2 d^2 e^2 - 2\,acd^2 e^2 - 2\,abde^3 - 5\,a^2 e^4\right) \arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{9}{2}\right)}}{16\,d^{\frac{7}{2}}} + \frac{\left(87\,c^2 d^4 x^5 e^2 - 66\,bcd^3 x^5 e^3 + 136\,c^2 d^5 x^3 e + 3\,b^2 d^2 x^5 e^4 + 6\,acd^2 x^5 e^4 - 80\,bcd^4 x^3 e^2 + 57\,c^2 d^6 x + 6\,abdx^5 e^5 - 8\,b^2 d^3 x^3 e^3 - \right.}{48\,(x^2 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/(e*x^2 + d)^4,x, algorithm="giac")
```

```
[Out] c^2*x*e^(-4) - 1/16*(35*c^2*d^4 - 10*b*c*d^3*e - b^2*d^2*e^2 - 2*
a*c*d^2*e^2 - 2*a*b*d*e^3 - 5*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*
e^(-9/2)/d^(7/2) + 1/48*(87*c^2*d^4*x^5*e^2 - 66*b*c*d^3*x^5*e^3
+ 136*c^2*d^5*x^3*e + 3*b^2*d^2*x^5*e^4 + 6*a*c*d^2*x^5*e^4 - 80*
b*c*d^4*x^3*e^2 + 57*c^2*d^6*x + 6*a*b*d*x^5*e^5 - 8*b^2*d^3*x^3*
e^3 - 16*a*c*d^3*x^3*e^3 - 30*b*c*d^5*x*e + 15*a^2*x^5*e^6 + 16*a
*b*d^2*x^3*e^4 - 3*b^2*d^4*x*e^2 - 6*a*c*d^4*x*e^2 + 40*a^2*d*x^3
*e^5 - 6*a*b*d^3*x*e^3 + 33*a^2*d^2*x*e^4)*e^(-4)/((x^2*e + d)^3*
d^3)
```

3.264 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx$

Optimal. Leaf size=317

$$-\frac{x\left(-e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(3ae+5bd)+93c^2d^4\right)}{128d^4e^4\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e^2\left(35a^2e^2+10abde+3b^2d^2\right)+2cd^2e(3ae+5bd)+35c^2d^4\right)}{128d^{9/2}e^{9/2}}+\frac{x\left(e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(59bd-3ae)+163c^2d^4\right)}{192d^3e^4\left(d+ex^2\right)^2}+\frac{x\left(ae^2-bde+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}-\frac{x\left(-7ae^2-9bde+25cd^2\right)\left(ae^2-bde+cd^2\right)}{48d^2e^4\left(d+ex^2\right)^3}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) - ((25*c*d^
2 - 9*b*d*e - 7*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(48*d^2*e^4*(d
+ e*x^2)^3) + ((163*c^2*d^4 - 2*c*d^2*e*(59*b*d - 3*a*e) + e^2*(3
*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(192*d^3*e^4*(d + e*x^2)^
2) - ((93*c^2*d^4 - 2*c*d^2*e*(5*b*d + 3*a*e) - e^2*(3*b^2*d^2 +
10*a*b*d*e + 35*a^2*e^2))*x)/(128*d^4*e^4*(d + e*x^2)) + ((35*c^2
*d^4 + 2*c*d^2*e*(5*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e +
35*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(128*d^(9/2)*e^(9/2))
```

Rubi [A] time = 1.23902, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{x\left(-e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(3ae+5bd)+93c^2d^4\right)}{128d^4e^4\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e^2\left(35a^2e^2+10abde+3b^2d^2\right)+2cd^2e(3ae+5bd)+35c^2d^4\right)}{128d^{9/2}e^{9/2}}+\frac{x\left(e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(59bd-3ae)+163c^2d^4\right)}{192d^3e^4\left(d+ex^2\right)^2}+\frac{x\left(ae^2-bde+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}-\frac{x\left(-7ae^2-9bde+25cd^2\right)\left(ae^2-bde+cd^2\right)}{48d^2e^4\left(d+ex^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^5,x]
```


```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) - ((25*c*d^
2 - 9*b*d*e - 7*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(48*d^2*e^4*(d
+ e*x^2)^3) + ((163*c^2*d^4 - 2*c*d^2*e*(59*b*d - 3*a*e) + e^2*(3
*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(192*d^3*e^4*(d + e*x^2)^
2) - ((93*c^2*d^4 - 2*c*d^2*e*(5*b*d + 3*a*e) - e^2*(3*b^2*d^2 +
10*a*b*d*e + 35*a^2*e^2))*x)/(128*d^4*e^4*(d + e*x^2)) + ((35*c^2
*d^4 + 2*c*d^2*e*(5*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e +
35*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(128*d^(9/2)*e^(9/2))
```

Rubi in Sympy [A] time = 173.066, size = 415, normalized size = 1.31

$$-\frac{c^2x^7}{e(d+ex^2)^4}+\frac{x\left(a^2e^4-2abde^3+2acd^2e^2+b^2d^2e^2-2bcd^3e-7c^2d^4\right)}{8de^4(d+ex^2)^4}+\frac{x\left(7a^2e^4+2abde^3-18acd^2e^2-9b^2d^2e^2+34bcd^3e+119c^2d^4\right)}{48d^2e^4(d+ex^2)^3}+\frac{x\left(35a^2e^4+10abde^3+6acd^2e^2+3b^2d^2e^2-118bcd^3e-413c^2d^4\right)}{192d^3e^4(d+ex^2)^2}+\frac{x\left(35a^2e^4+10abde^3+6acd^2e^2+3b^2d^2e^2+10bcd^3e+35c^2d^4\right)}{128d^4e^4(d+ex^2)}+\frac{\left(35a^2e^4+10abde^3+6acd^2e^2+3b^2d^2e^2+10bcd^3e+35c^2d^4\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{128d^{\frac{9}{2}}e^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**5,x)
```

```
[Out] -c**2*x**7/(e*(d + e*x**2)**4) + x*(a**2*e**4 - 2*a*b*d*e**3 + 2*
a*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d**3*e - 7*c**2*d**4)/(8*d
*e**4*(d + e*x**2)**4) + x*(7*a**2*e**4 + 2*a*b*d*e**3 - 18*a*c*d
**2*e**2 - 9*b**2*d**2*e**2 + 34*b*c*d**3*e + 119*c**2*d**4)/(48*
d**2*e**4*(d + e*x**2)**3) + x*(35*a**2*e**4 + 10*a*b*d*e**3 + 6*
a*c*d**2*e**2 + 3*b**2*d**2*e**2 - 118*b*c*d**3*e - 413*c**2*d**4
)/(192*d**3*e**4*(d + e*x**2)**2) + x*(35*a**2*e**4 + 10*a*b*d*e*
*3 + 6*a*c*d**2*e**2 + 3*b**2*d**2*e**2 + 10*b*c*d**3*e + 35*c**2
*d**4)/(128*d**4*e**4*(d + e*x**2)) + (35*a**2*e**4 + 10*a*b*d*e*
*3 + 6*a*c*d**2*e**2 + 3*b**2*d**2*e**2 + 10*b*c*d**3*e + 35*c**2
*d**4)*atan(sqrt(e)*x/sqrt(d))/(128*d**(9/2)*e**(9/2))
```

Mathematica [A] time = 0.441256, size = 345, normalized size = 1.09

$$-\frac{3\sqrt{d}\sqrt{e}x\left(-e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(3ae+5bd)+93c^2d^4\right)}{d+ex^2}+3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e^2\left(35a^2e^2+10abde+3b^2d^2\right)+2cd^2e(3ae+5bd)+\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((48*d^(7/2)*Sqrt[e]*(c*d^2 + e*(-(b*d) + a*e))^2*x)/(d + e*x^2)^
4 - (8*d^(5/2)*Sqrt[e]*(25*c^2*d^4 + 2*c*d^2*e*(-17*b*d + 9*a*e)
+ e^2*(9*b^2*d^2 - 2*a*b*d*e - 7*a^2*e^2))*x)/(d + e*x^2)^3 + (2*
d^(3/2)*Sqrt[e]*(163*c^2*d^4 + 2*c*d^2*e*(-59*b*d + 3*a*e) + e^2*
(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(d + e*x^2)^2 - (3*Sqrt
[d]*Sqrt[e]*(93*c^2*d^4 - 2*c*d^2*e*(5*b*d + 3*a*e) - e^2*(3*b^2*
d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(d + e*x^2) + 3*(35*c^2*d^4 +
2*c*d^2*e*(5*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*
e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(384*d^(9/2)*e^(9/2))
```

Maple [A] time = 0.016, size = 412, normalized size = 1.3

$$\frac{1}{(ex^2+d)^4}\left(\frac{\left(35\,a^2e^4+10\,dabe^3+6\,acd^2e^2+3\,b^2d^2e^2+10\,bcd^3e-93\,c^2d^4\right)x^7}{128\,d^4e}+\frac{\left(385\,a^2e^4+110\,dabe^3+66\,acd^2e^2+33\,b^2\right.}{384\,d^3e^2}\right.$$

$$+\frac{35\,a^2}{128\,d^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{5\,ab}{64\,d^3e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{3\,ac}{64\,d^2e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

$$+\frac{3\,b^2}{128\,d^2e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{5\,bc}{64\,de^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{35\,c^2}{128\,e^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x)
```

```
[Out] (1/128*(35*a^2*e^4+10*a*b*d*e^3+6*a*c*d^2*e^2+3*b^2*d^2*e^2+10*b*
c*d^3*e-93*c^2*d^4)/d^4/e*x^7+1/384*(385*a^2*e^4+110*a*b*d*e^3+66
*a*c*d^2*e^2+33*b^2*d^2*e^2-146*b*c*d^3*e-511*c^2*d^4)/d^3/e^2*x^
5+1/384*(511*a^2*e^4+146*a*b*d*e^3-66*a*c*d^2*e^2-33*b^2*d^2*e^2-
110*b*c*d^3*e-385*c^2*d^4)/d^2/e^3*x^3+1/128*(93*a^2*e^4-10*a*b*d
*e^3-6*a*c*d^2*e^2-3*b^2*d^2*e^2-10*b*c*d^3*e-35*c^2*d^4)/d/e^4*x
)/(e*x^2+d)^4+35/128/d^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a^2+
5/64/d^3/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*b+3/64/d^2/e^2/(
d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*c+3/128/d^2/e^2/(d*e)^(1/2)*
arctan(x*e/(d*e)^(1/2))*b^2+5/64/d/e^3/(d*e)^(1/2)*arctan(x*e/(d*
e)^(1/2))*b*c+35/128/e^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/(e*x^2 + d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.280959, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/(e*x^2 + d)^5,x, algorithm="fricas")
```

```
[Out] [1/768*(3*(35*c^2*d^8 + 10*b*c*d^7*e + 10*a*b*d^5*e^3 + 35*a^2*d^
4*e^4 + 3*(b^2 + 2*a*c)*d^6*e^2 + (35*c^2*d^4*e^4 + 10*b*c*d^3*e^
5 + 10*a*b*d*e^7 + 35*a^2*e^8 + 3*(b^2 + 2*a*c)*d^2*e^6)*x^8 + 4*
(35*c^2*d^5*e^3 + 10*b*c*d^4*e^4 + 10*a*b*d^2*e^6 + 35*a^2*d*e^7
+ 3*(b^2 + 2*a*c)*d^3*e^5)*x^6 + 6*(35*c^2*d^6*e^2 + 10*b*c*d^5*e
^3 + 10*a*b*d^3*e^5 + 35*a^2*d^2*e^6 + 3*(b^2 + 2*a*c)*d^4*e^4)*x
^4 + 4*(35*c^2*d^7*e + 10*b*c*d^6*e^2 + 10*a*b*d^4*e^4 + 35*a^2*d
^3*e^5 + 3*(b^2 + 2*a*c)*d^5*e^3)*x^2)*log((2*d*e*x + (e*x^2 - d)
*sqrt(-d*e))/(e*x^2 + d)) - 2*(3*(93*c^2*d^4*e^3 - 10*b*c*d^3*e^4
 - 10*a*b*d*e^6 - 35*a^2*e^7 - 3*(b^2 + 2*a*c)*d^2*e^5)*x^7 + (51
1*c^2*d^5*e^2 + 146*b*c*d^4*e^3 - 110*a*b*d^2*e^5 - 385*a^2*d*e^6
 - 33*(b^2 + 2*a*c)*d^3*e^4)*x^5 + (385*c^2*d^6*e + 110*b*c*d^5*e
^2 - 146*a*b*d^3*e^4 - 511*a^2*d^2*e^5 + 33*(b^2 + 2*a*c)*d^4*e^3
)*x^3 + 3*(35*c^2*d^7 + 10*b*c*d^6*e + 10*a*b*d^4*e^3 - 93*a^2*d^
3*e^4 + 3*(b^2 + 2*a*c)*d^5*e^2)*x)*sqrt(-d*e))/((d^4*e^8*x^8 + 4
*d^5*e^7*x^6 + 6*d^6*e^6*x^4 + 4*d^7*e^5*x^2 + d^8*e^4)*sqrt(-d*e
)), 1/384*(3*(35*c^2*d^8 + 10*b*c*d^7*e + 10*a*b*d^5*e^3 + 35*a^2
*d^4*e^4 + 3*(b^2 + 2*a*c)*d^6*e^2 + (35*c^2*d^4*e^4 + 10*b*c*d^3
*e^5 + 10*a*b*d*e^7 + 35*a^2*e^8 + 3*(b^2 + 2*a*c)*d^2*e^6)*x^8 +
 4*(35*c^2*d^5*e^3 + 10*b*c*d^4*e^4 + 10*a*b*d^2*e^6 + 35*a^2*d*e
^7 + 3*(b^2 + 2*a*c)*d^3*e^5)*x^6 + 6*(35*c^2*d^6*e^2 + 10*b*c*d^
5*e^3 + 10*a*b*d^3*e^5 + 35*a^2*d^2*e^6 + 3*(b^2 + 2*a*c)*d^4*e^4
)*x^4 + 4*(35*c^2*d^7*e + 10*b*c*d^6*e^2 + 10*a*b*d^4*e^4 + 35*a^
2*d^3*e^5 + 3*(b^2 + 2*a*c)*d^5*e^3)*x^2)*arctan(sqrt(d*e)*x/d) -
 (3*(93*c^2*d^4*e^3 - 10*b*c*d^3*e^4 - 10*a*b*d*e^6 - 35*a^2*e^7
- 3*(b^2 + 2*a*c)*d^2*e^5)*x^7 + (511*c^2*d^5*e^2 + 146*b*c*d^4*e
^3 - 110*a*b*d^2*e^5 - 385*a^2*d*e^6 - 33*(b^2 + 2*a*c)*d^3*e^4)*
x^5 + (385*c^2*d^6*e + 110*b*c*d^5*e^2 - 146*a*b*d^3*e^4 - 511*a^
```

```
2*d^2*e^5 + 33*(b^2 + 2*a*c)*d^4*e^3)*x^3 + 3*(35*c^2*d^7 + 10*b*
c*d^6*e + 10*a*b*d^4*e^3 - 93*a^2*d^3*e^4 + 3*(b^2 + 2*a*c)*d^5*e
^2)*x)*sqrt(d*e))/((d^4*e^8*x^8 + 4*d^5*e^7*x^6 + 6*d^6*e^6*x^4 +
 4*d^7*e^5*x^2 + d^8*e^4)*sqrt(d*e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**5,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.267793, size = 491, normalized size = 1.55

$$\frac{\left(35\,c^2d^4 + 10\,bcd^3e + 3\,b^2d^2e^2 + 6\,acd^2e^2 + 10\,abde^3 + 35\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{128\,d^{\frac{9}{2}}} - \frac{\left(279\,c^2d^4x^7e^3 - 30\,bcd^3x^7e^4 + 511\,c^2d^5x^5e^2 - 9\,b^2d^2x^7e^5 - 18\,acd^2x^7e^5 + 146\,bcd^4x^5e^3 + 385\,c^2d^6x^3e - 30\,abdx^7e^6 - 33\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/(e*x^2 + d)^5,x, algorithm="giac")
```

```
[Out] 1/128*(35*c^2*d^4 + 10*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2
+ 10*a*b*d*e^3 + 35*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d
^(9/2) - 1/384*(279*c^2*d^4*x^7*e^3 - 30*b*c*d^3*x^7*e^4 + 511*c^
2*d^5*x^5*e^2 - 9*b^2*d^2*x^7*e^5 - 18*a*c*d^2*x^7*e^5 + 146*b*c*
d^4*x^5*e^3 + 385*c^2*d^6*x^3*e - 30*a*b*d*x^7*e^6 - 33*b^2*d^3*x
^5*e^4 - 66*a*c*d^3*x^5*e^4 + 110*b*c*d^5*x^3*e^2 + 105*c^2*d^7*x
 - 105*a^2*x^7*e^7 - 110*a*b*d^2*x^5*e^5 + 33*b^2*d^4*x^3*e^3 + 6
6*a*c*d^4*x^3*e^3 + 30*b*c*d^6*x*e - 385*a^2*d*x^5*e^6 - 146*a*b*
d^3*x^3*e^4 + 9*b^2*d^5*x*e^2 + 18*a*c*d^5*x*e^2 - 511*a^2*d^2*x^
3*e^5 + 30*a*b*d^4*x*e^3 - 279*a^2*d^3*x*e^4)*e^(-4)/((x^2*e + d)
^4*d^4)
```

3.265 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3
*c*d^2 - e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e
^(5/2))
```

Rubi [A] time = 0.15356, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3
*c*d^2 - e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e
^(5/2))
```

Rubi in Sympy [A] time = 31.1555, size = 78, normalized size = 0.94

$$\frac{cx}{e^2}+\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)}+\frac{\left(ae^2+bde-3cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{\frac{3}{2}}e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d*e**2*(d + e*x**2)) +
(a*e**2 + b*d*e - 3*c*d**2)*atan(sqrt(e)*x/sqrt(d))/(2*d**(3/2)*e
**(5/2))
```

Mathematica [A] time = 0.101734, size = 88, normalized size = 1.06

$$\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-ae^2-bde+3cd^2\right)}{2d^{3/2}e^{5/2}}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - (
(3*c*d^2 - b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)
*e^(5/2))
```

Maple [A] time = 0., size = 118, normalized size = 1.4

$$\frac{cx}{e^2}+\frac{ax}{2\,d\,(ex^2+d)}-\frac{bx}{2\,e\,(ex^2+d)}+\frac{cdx}{2\,e^2\,(ex^2+d)}+\frac{a}{2\,d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{b}{2\,e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{3\,cd}{2\,e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] c*x/e^2+1/2/d*x/(e*x^2+d)*a-1/2/e*x/(e*x^2+d)*b+1/2/e^2*d*x/(e*x^
2+d)*c+1/2/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+1/2/e/(d*e)^(1
/2)*arctan(x*e/(d*e)^(1/2))*b-3/2/e^2*d/(d*e)^(1/2)*arctan(x*e/(d
*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.275941, size = 1, normalized size = 0.01

$$\left[-\frac{\left(3\,cd^3-bd^2e-ade^2+\left(3\,cd^2e-bde^2-ae^3\right)x^2\right)\log\left(\frac{2\,dex+(ex^2-d)\sqrt{-de}}{ex^2+d}\right)-2\left(2\,cdex^3+\left(3\,cd^2-bde+ae^2\right)x\right)\sqrt{-de}}{4\,(de^3x^2+d^2e^2)\sqrt{-de}},\; -\frac{\left(3\,cd^3-bd^2e-ade^2+\left(3\,cd^2e-bde^2-ae^3\right)x^2\right)\arctan\left(\frac{\sqrt{de}x}{d}\right)-\left(2\,cdex^3+\left(3\,cd^2-bde+ae^2\right)x\right)\sqrt{de}}{2\,(de^3x^2+d^2e^2)\sqrt{de}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^
3)*x^2)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d)) - 2*(
2*c*d*e*x^3 + (3*c*d^2 - b*d*e + a*e^2)*x)*sqrt(-d*e))/((d*e^3*x^
2 + d^2*e^2)*sqrt(-d*e)), -1/2*((3*c*d^3 - b*d^2*e - a*d*e^2 + (3
*c*d^2*e - b*d*e^2 - a*e^3)*x^2)*arctan(sqrt(d*e)*x/d) - (2*c*d*e
*x^3 + (3*c*d^2 - b*d*e + a*e^2)*x)*sqrt(d*e))/((d*e^3*x^2 + d^2*
e^2)*sqrt(d*e))]
```

Sympy [A] time = 3.69492, size = 153, normalized size = 1.84

$$\frac{cx}{e^2}+\frac{x\left(ae^2-bde+cd^2\right)}{2d^2e^2+2de^3x^2}-\frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2+bde-3cd^2\right)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2+bde-3cd^2\right)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x*
*2) - sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*
e**2*sqrt(-1/(d**3*e**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 +
 b*d*e - 3*c*d**2)*log(d**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4
```

GIAC/XCAS [A] time = 0.266333, size = 101, normalized size = 1.22

$$cxe^{(-2)} - \frac{\left(3\,cd^2 - bde - ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{\left(cd^2x - bdxe + axe^2\right)e^{(-2)}}{2\,(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(
d))*e^(-5/2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/(
(x^2*e + d)*d)
```

3.266 $\int \frac{a+x^2\left(b+cx^2\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3
*c*d^2 - e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e
^(5/2))
```

Rubi [A] time = 0.120728, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + x^2*(b + c*x^2))/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3
*c*d^2 - e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e
^(5/2))
```

Rubi in Sympy [A] time = 35.4924, size = 78, normalized size = 0.94

$$\frac{cx}{e^2}+\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)}+\frac{\left(ae^2+bde-3cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{\frac{3}{2}}e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+x**2*(c*x**2+b))/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d*e**2*(d + e*x**2)) +
(a*e**2 + b*d*e - 3*c*d**2)*atan(sqrt(e)*x/sqrt(d))/(2*d**(3/2)*e
**(5/2))
```

Mathematica [A] time = 0.0286442, size = 88, normalized size = 1.06

$$\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-ae^2-bde+3cd^2\right)}{2d^{3/2}e^{5/2}}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + x^2*(b + c*x^2))/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - (
(3*c*d^2 - b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)
*e^(5/2))
```

Maple [A] time = 0.007, size = 118, normalized size = 1.4

$$\frac{cx}{e^2}+\frac{ax}{2\,d\,(ex^2+d)}-\frac{bx}{2\,e\,(ex^2+d)}+\frac{cdx}{2\,e^2\,(ex^2+d)}+\frac{a}{2\,d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{b}{2\,e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{3\,cd}{2\,e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x)
```

```
[Out] c*x/e^2+1/2/d*x/(e*x^2+d)*a-1/2/e*x/(e*x^2+d)*b+1/2/e^2*d*x/(e*x^
2+d)*c+1/2/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+1/2/e/(d*e)^(1
/2)*arctan(x*e/(d*e)^(1/2))*b-3/2/e^2*d/(d*e)^(1/2)*arctan(x*e/(d
*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x^2 + b)*x^2 + a)/(e*x^2 + d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266277, size = 1, normalized size = 0.01

$$\left[-\frac{\left(3\,cd^3-bd^2e-ade^2+\left(3\,cd^2e-bde^2-ae^3\right)x^2\right)\log\left(\frac{2\,dex+\left(ex^2-d\right)\sqrt{-de}}{ex^2+d}\right)-2\left(2\,cdex^3+\left(3\,cd^2-bde+ae^2\right)x\right)\sqrt{-de}}{4\left(de^3x^2+d^2e^2\right)\sqrt{-de}},\ -\frac{\left(3\,cd^3-bd^2e-ade^2+\left(3\,cd^2e-bde^2-ae^3\right)x^2\right)\arctan\left(\frac{\sqrt{de}x}{d}\right)-\left(2\,cdex^3+\left(3\,cd^2-bde+ae^2\right)x\right)\sqrt{de}}{2\left(de^3x^2+d^2e^2\right)\sqrt{de}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x^2 + b)*x^2 + a)/(e*x^2 + d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^
3)*x^2)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d)) - 2*(
2*c*d*e*x^3 + (3*c*d^2 - b*d*e + a*e^2)*x)*sqrt(-d*e))/((d*e^3*x^
2 + d^2*e^2)*sqrt(-d*e)), -1/2*((3*c*d^3 - b*d^2*e - a*d*e^2 + (3
*c*d^2*e - b*d*e^2 - a*e^3)*x^2)*arctan(sqrt(d*e)*x/d) - (2*c*d*e
*x^3 + (3*c*d^2 - b*d*e + a*e^2)*x)*sqrt(d*e))/((d*e^3*x^2 + d^2*
e^2)*sqrt(d*e))]
```

Sympy [A] time = 3.71504, size = 153, normalized size = 1.84

$$\frac{cx}{e^2}+\frac{x\left(ae^2-bde+cd^2\right)}{2d^2e^2+2de^3x^2}-\frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2+bde-3cd^2\right)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2+bde-3cd^2\right)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+x**2*(c*x**2+b))/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x*
*2) - sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*
e**2*sqrt(-1/(d**3*e**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 +
 b*d*e - 3*c*d**2)*log(d**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4
```

GIAC/XCAS [A] time = 0.263886, size = 101, normalized size = 1.22

$$cxe^{(-2)} - \frac{\left(3\,cd^2 - bde - ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{\left(cd^2x - bdxe + axe^2\right)e^{(-2)}}{2\,(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x^2 + b)*x^2 + a)/(e*x^2 + d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(
d))*e^(-5/2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/(
(x^2*e + d)*d)
```

3.267 $\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=459

$$\frac{\left(\frac{2c^2e^2(a^2e^2+6abde+3b^2d^2)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4}{\sqrt{b^2-4ac}}+e(2cd-be)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\cdots}{\sqrt{b-\sqrt{b^2-\cdots}}}\right.}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(e(2cd-be)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)-\frac{2c^2e^2(a^2e^2+6abde+3b^2d^2)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cdots}}{\sqrt{\sqrt{b^2-\cdots}}}\right.}{\sqrt{2}c^{7/2}\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{e^2x\left(-ce(ae+4bd)+b^2e^2+6c^2d^2\right)}{c^3}+\frac{e^3x^3(4cd-be)}{3c^2}+\frac{e^4x^5}{5c}$$

```
[Out] (e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c
*d - b*e)*x^3)/(3*c^2) + (e^4*x^5)/(5*c) + ((e*(2*c*d - b*e)*(2*c
^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e)) + (2*c^4*d^4 + b^4*e^4 - 4*
b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*
b^2*d^2 + 6*a*b*d*e + a^2*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + ((e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2
- 2*c*e*(b*d + a*e)) - (2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d +
a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d
*e + a^2*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*
a*c]])
```

Rubi [A] time = 4.61514, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{\left(\frac{2c^2e^2(a^2e^2+6abde+3b^2d^2)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4}{\sqrt{b^2-4ac}}+e(2cd-be)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\cdots}{\sqrt{b-\sqrt{b^2-\cdots}}}\right.}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(e(2cd-be)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)-\frac{2c^2e^2(a^2e^2+6abde+3b^2d^2)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cdots}}{\sqrt{\sqrt{b^2-\cdots}}}\right.}{\sqrt{2}c^{7/2}\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{e^2x\left(-ce(ae+4bd)+b^2e^2+6c^2d^2\right)}{c^3}+\frac{e^3x^3(4cd-be)}{3c^2}+\frac{e^4x^5}{5c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c
*d - b*e)*x^3)/(3*c^2) + (e^4*x^5)/(5*c) + ((e*(2*c*d - b*e)*(2*c
^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e)) + (2*c^4*d^4 + b^4*e^4 - 4*
b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*
b^2*d^2 + 6*a*b*d*e + a^2*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + ((e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2
- 2*c*e*(b*d + a*e)) - (2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d +
a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d
*e + a^2*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*
a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.67091, size = 570, normalized size = 1.24

$$\frac{e^2x\left(-ce(ae+4bd)+b^2e^2+6c^2d^2\right)}{c^3}$$
$$+\frac{\left(4c^3d^2e\left(d\sqrt{b^2-4ac}-3ae-bd\right)+2c^2e^2\left(-3bd\left(d\sqrt{b^2-4ac}-2ae\right)+ae\left(ae-2d\sqrt{b^2-4ac}\right)+3b^2d^2\right)+2bce^3\left(2bd\sqrt{b}\right.\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4}}}$$
$$-\frac{\left(-4c^3d^2e\left(d\sqrt{b^2-4ac}+3ae+bd\right)+2c^2e^2\left(3bd\left(d\sqrt{b^2-4ac}+2ae\right)+ae\left(2d\sqrt{b^2-4ac}+ae\right)+3b^2d^2\right)-2bce^3\left(2b\left(d\sqrt{}\right.\right.\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+}}$$
$$+\frac{e^3x^3(4cd-be)}{3c^2}+\frac{e^4x^5}{5c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c
*d - b*e)*x^3)/(3*c^2) + (e^4*x^5)/(5*c) + ((2*c^4*d^4 + b^3*(b -
 Sqrt[b^2 - 4*a*c])*e^4 + 4*c^3*d^2*e*(-(b*d) + Sqrt[b^2 - 4*a*c]
*d - 3*a*e) + 2*b*c*e^3*(-2*b^2*d + 2*b*Sqrt[b^2 - 4*a*c]*d - 2*a
*b*e + a*Sqrt[b^2 - 4*a*c]*e) + 2*c^2*e^2*(3*b^2*d^2 - 3*b*d*(Sqr
t[b^2 - 4*a*c]*d - 2*a*e) + a*e*(-2*Sqrt[b^2 - 4*a*c]*d + a*e)))*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*c^4
*d^4 + b^3*(b + Sqrt[b^2 - 4*a*c])*e^4 - 4*c^3*d^2*e*(b*d + Sqrt[
b^2 - 4*a*c]*d + 3*a*e) - 2*b*c*e^3*(2*b^2*d + a*Sqrt[b^2 - 4*a*c
]*e + 2*b*(Sqrt[b^2 - 4*a*c]*d + a*e)) + 2*c^2*e^2*(3*b^2*d^2 + a
*e*(2*Sqrt[b^2 - 4*a*c]*d + a*e) + 3*b*d*(Sqrt[b^2 - 4*a*c]*d + 2
*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [B] time = 0.061, size = 1888, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^4/(c*x^4+b*x^2+a),x)
```

```
[Out] 2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d*e^3-1
/3*e^4/c^2*x^3*b+e^4/c^3*b^2*x+6*e^2/c*d^2*x-e^4/c^2*a*x-1/c/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*e^4-1/2/c^3/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*e^4+6/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*d^2*e^2+1/c^2*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*a*b*e^4-2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*d*e
^3+2/c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
```

```
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d*e^3-3/c*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*b*d^2*e^2+2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*d^3*e*b+2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*d^3*e*b+6/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
a*d^2*e^2-1/c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*e^4+2/c*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*a*d*e^3-2/c^2*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*b^2*d*e^3+3/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d^2*e^2-1
/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*e^4-1/2
/c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*e^4+2
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*d^3*e-3/c/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*b^2*d^2*e^2+2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d*e^3-3/c/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d^2*e^2+2/c^2/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2*e^4+2/c^2/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2*e^4-6/c/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*d*e^3-6/c/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*a*b*d*e^3+1/5*e^4*x^5/c-c/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d^4-c/(-4*a*c+b^2)^(1/2)*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*d^4-1/2/c^3*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*b^3*e^4+1/2/c^3*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*e^4+4/3*d
*e^3*x^3/c-4*e^3/c^2*b*d*x-2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d^3*
e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\,c^{2}e^{4}x^{5}+5\left(4\,c^{2}de^{3}-bce^{4}\right)x^{3}+15\left(6\,c^{2}d^{2}e^{2}-4\,bcde^{3}+\left(b^{2}-ac\right)e^{4}\right)x}{15\,c^{3}}+\frac{\int\frac{c^{3}d^{4}-6\,ac^{2}d^{2}e^{2}+4\,abcde^{3}-\left(ab^{2}-a^{2}c\right)e^{4}+\left(4\,c^{3}d^{3}e-6\,bc^{2}d^{2}e^{2}+4\left(b^{2}c-ac^{2}\right)de^{3}-\left(b^{3}-2\,abc\right)e^{4}\right)x^{2}}{cx^{4}+bx^{2}+a}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^4/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^2*e^4*x^5 + 5*(4*c^2*d*e^3 - b*c*e^4)*x^3 + 15*(6*c^2*d
^2*e^2 - 4*b*c*d*e^3 + (b^2 - a*c)*e^4)*x)/c^3 + integrate((c^3*d
^4 - 6*a*c^2*d^2*e^2 + 4*a*b*c*d*e^3 - (a*b^2 - a^2*c)*e^4 + (4*c
^3*d^3*e - 6*b*c^2*d^2*e^2 + 4*(b^2*c - a*c^2)*d*e^3 - (b^3 - 2*a
*b*c)*e^4)*x^2)/(c*x^4 + b*x^2 + a), x)/c^3
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^4/(c*x^4 + b*x^2 + a),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**4/(c*x**4+b*x**2+a),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 2.23661, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^4/(c*x^4 + b*x^2 + a),x, algorithm="giac")

[Out] Done
```

3.268 $\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=316

$$\frac{\left(e\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)+\frac{(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)-\frac{(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{e^2x(3cd-be)}{c^2}+\frac{e^3x^3}{3c}$$

```
[Out] (e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^3)/(3*c) + ((e*(3*c^2*d^2 + b^
2*e^2 - c*e*(3*b*d + a*e)) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 -
c*e*(b*d + 3*a*e)))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + ((e*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e)) - ((2
*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 2.27428, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{\left(e\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)+\frac{(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)-\frac{(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{e^2x(3cd-be)}{c^2}+\frac{e^3x^3}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^3)/(3*c) + ((e*(3*c^2*d^2 + b^
2*e^2 - c*e*(3*b*d + a*e)) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 -
c*e*(b*d + 3*a*e)))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + ((e*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e)) - ((2
*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-e^2(be-3cd)\int\frac{1}{c^2}\,dx+\frac{e^3x^3}{3c}-\frac{\sqrt{2}\left(-be\left(-ace^2+b^2e^2-3bcde+3c^2d^2\right)+2c\left(abe^3-3acde^2+c^2d^3\right)-e\sqrt{-4ac+b^2}\left(-ace^2+b^2e^2-3bcde+3c^2d^2\right)\right)\text{at}}{2c^{\frac{5}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\left(-be\left(-ace^2+b^2e^2-3bcde+3c^2d^2\right)+2c\left(abe^3-3acde^2+c^2d^3\right)+e\sqrt{-4ac+b^2}\left(-ace^2+b^2e^2-3bcde+3c^2d^2\right)\right)\text{ata}}{2c^{\frac{5}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**3/(c*x**4+b*x**2+a),x)
```

```
[Out] -e**2*(b*e - 3*c*d)*Integral(c**(-2), x) + e**3*x**3/(3*c) - sqrt
(2)*(-b*e*(-a*c*e**2 + b**2*e**2 - 3*b*c*d*e + 3*c**2*d**2) + 2*c
*(a*b*e**3 - 3*a*c*d*e**2 + c**2*d**3) - e*sqrt(-4*a*c + b**2)*(-
a*c*e**2 + b**2*e**2 - 3*b*c*d*e + 3*c**2*d**2))*atan(sqrt(2)*sqr
t(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*c**(5/2)*sqrt(b + sqrt(-
4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(2)*(-b*e*(-a*c*e**2 +
b**2*e**2 - 3*b*c*d*e + 3*c**2*d**2) + 2*c*(a*b*e**3 - 3*a*c*d*e*
*2 + c**2*d**3) + e*sqrt(-4*a*c + b**2)*(-a*c*e**2 + b**2*e**2 -
3*b*c*d*e + 3*c**2*d**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4
*a*c + b**2)))/(2*c**(5/2)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*
a*c + b**2))
```

Mathematica [A] time = 1.18753, size = 402, normalized size = 1.27

$$\frac{\frac{3\sqrt{2}\left(3c^2de\left(d\sqrt{b^2-4ac}-2ae-bd\right)+ce^2\left(-3bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+3abe+3b^2d\right)+b^2e^3\left(\sqrt{b^2-4ac}-b\right)+2c^3d^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\left(3c^2de\left(d\sqrt{b^2-4a\right.\right.}{}}{6c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] (6*Sqrt[c]*e^2*(3*c*d - b*e)*x + 2*c^(3/2)*e^3*x^3 + (3*Sqrt[2]*(
2*c^3*d^3 + b^2*(-b + Sqrt[b^2 - 4*a*c])*e^3 + 3*c^2*d*e*(-(b*d)
+ Sqrt[b^2 - 4*a*c]*d - 2*a*e) + c*e^2*(3*b^2*d - 3*b*Sqrt[b^2 -
4*a*c]*d + 3*a*b*e - a*Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(-2*c^3*d^3 + b^2*(b + Sqrt[b^2
- 4*a*c])*e^3 + 3*c^2*d*e*(b*d + Sqrt[b^2 - 4*a*c]*d + 2*a*e) - c
*e^2*(3*b^2*d + a*Sqrt[b^2 - 4*a*c]*e + 3*b*(Sqrt[b^2 - 4*a*c]*d
+ a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*c^(5/2))
```

Maple [B] time = 0.046, size = 1211, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^3/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*e^3*x^3/c-e^3/c^2*b*x+3*d*e^2*x/c-1/2/c*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*a*e^3+1/2/c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e^3-3/2/c*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*e^2*d*b+3/2*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*d^2*e-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
a*b*e^3+3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*d*e^2
+1/2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*e^3-
3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d*e^2+3
/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
```

```
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d^2*e-c/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d^3+1/2/c*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*a*e^3-1/2/c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b^2*e^3+3/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e^2*d*b-3/2*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*d^2*e-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*a*b*e^3+3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*a*d*e^2+1/2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*b^3*e^3-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*b^2*d*e^2+3/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*b*d^2*e-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ce^3x^3 + 3\left(3\,cde^2 - be^3\right)x}{3\,c^2} - \frac{-\int \frac{c^2d^3 - 3\,acde^2 + abe^3 + \left(3\,c^2d^2e - 3\,bcde^2 + \left(b^2 - ac\right)e^3\right)x^2}{cx^4 + bx^2 + a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/3*(c*e^3*x^3 + 3*(3*c*d*e^2 - b*e^3)*x)/c^2 - integrate(-(c^2*d
^3 - 3*a*c*d*e^2 + a*b*e^3 + (3*c^2*d^2*e - 3*b*c*d*e^2 + (b^2 -
a*c)*e^3)*x^2)/(c*x^4 + b*x^2 + a), x)/c^2
```

Fricas [A] time = 20.587, size = 12938, normalized size = 40.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c*e^3*x^3 + 3*sqrt(1/2)*c^2*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^
5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 1
5*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2
+ 2*a^3*c^3)*d*e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 + (a
*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*
b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3
*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 2
26*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7
+ 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*
(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*
a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^
10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)
*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3
 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))/(a*b^2*c^5 - 4*a^
2*c^6))*log(-2*(c^8*d^12 - 3*b*c^7*d^11*e + 3*(b^2*c^6 - 4*a*c^7)
*d^10*e^2 - (b^3*c^5 - 59*a*b*c^6)*d^9*e^3 - 9*(13*a*b^2*c^5 + 3*
a^2*c^6)*d^8*e^4 + 18*(7*a*b^3*c^4 + 5*a^2*b*c^5)*d^7*e^5 - 42*(2
*a*b^4*c^3 + 3*a^2*b^2*c^4)*d^6*e^6 + 18*(2*a*b^5*c^2 + 6*a^2*b^3
```

```
*c^3 - a^3*b*c^4)*d^5*e^7 - 9*(a*b^6*c + 7*a^2*b^4*c^2 - 2*a^3*b^
2*c^3 - 3*a^4*c^4)*d^4*e^8 + (a*b^7 + 21*a^2*b^5*c + 10*a^3*b^3*c
^2 - 55*a^4*b*c^3)*d^3*e^9 - 3*(a^2*b^6 + 4*a^3*b^4*c - 9*a^4*b^2
*c^2 - 4*a^5*c^3)*d^2*e^10 + 3*(a^3*b^5 - a^4*b^3*c - 3*a^5*b*c^2
)*d*e^11 - (a^4*b^4 - 3*a^5*b^2*c + a^6*c^2)*e^12)*x + sqrt(1/2)*
((b^2*c^7 - 4*a*c^8)*d^9 - 18*(a*b^2*c^6 - 4*a^2*c^7)*d^7*e^2 + 2
1*(a*b^3*c^5 - 4*a^2*b*c^6)*d^6*e^3 - 15*(a*b^4*c^4 - 8*a^2*b^2*c
^5 + 16*a^3*c^6)*d^5*e^4 + 3*(2*a*b^5*c^3 - 37*a^2*b^3*c^4 + 116*
a^3*b*c^5)*d^4*e^5 - (a*b^6*c^2 - 72*a^2*b^4*c^3 + 318*a^3*b^2*c^
4 - 184*a^4*c^5)*d^3*e^6 - 3*(11*a^2*b^5*c^2 - 61*a^3*b^3*c^3 + 6
8*a^4*b*c^4)*d^2*e^7 + 3*(3*a^2*b^6*c - 19*a^3*b^4*c^2 + 29*a^4*b
^2*c^3 - 4*a^5*c^4)*d*e^8 - (a^2*b^7 - 7*a^3*b^5*c + 13*a^4*b^3*c
^2 - 4*a^5*b*c^3)*e^9 - ((a*b^3*c^7 - 4*a^2*b*c^8)*d^3 - 6*(a^2*b
^2*c^7 - 4*a^3*c^8)*d^2*e + 3*(a^2*b^3*c^6 - 4*a^3*b*c^7)*d*e^2 -
 (a^2*b^4*c^5 - 6*a^3*b^2*c^6 + 8*a^4*c^7)*e^3)*sqrt((c^10*d^12 -
 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^
2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4
*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^
5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 +
 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a
^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b
^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*
a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a
^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3
*c^11)))*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 -
 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2*b*c^3
)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^
5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 + (a*b^2*c^5 - 4*a^2*c^6)*sqrt
((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^
2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e
^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(
13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*
a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b
^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c
^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3
*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3
*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2
*c^10 - 4*a^3*c^11)))/(a*b^2*c^5 - 4*a^2*c^6))) - 3*sqrt(1/2)*c^2
*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b
^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^
4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 - 5*a^
2*b^3*c + 5*a^3*b*c^2)*e^6 + (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d
^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 -
17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(
a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b
^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*
c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 +
 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*
a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2
 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c +
 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 -
4*a^3*c^11)))/(a*b^2*c^5 - 4*a^2*c^6))*log(-2*(c^8*d^12 - 3*b*c^7
*d^11*e + 3*(b^2*c^6 - 4*a*c^7)*d^10*e^2 - (b^3*c^5 - 59*a*b*c^6)
*d^9*e^3 - 9*(13*a*b^2*c^5 + 3*a^2*c^6)*d^8*e^4 + 18*(7*a*b^3*c^4
 + 5*a^2*b*c^5)*d^7*e^5 - 42*(2*a*b^4*c^3 + 3*a^2*b^2*c^4)*d^6*e^
6 + 18*(2*a*b^5*c^2 + 6*a^2*b^3*c^3 - a^3*b*c^4)*d^5*e^7 - 9*(a*b
^6*c + 7*a^2*b^4*c^2 - 2*a^3*b^2*c^3 - 3*a^4*c^4)*d^4*e^8 + (a*b^
7 + 21*a^2*b^5*c + 10*a^3*b^3*c^2 - 55*a^4*b*c^3)*d^3*e^9 - 3*(a^
2*b^6 + 4*a^3*b^4*c - 9*a^4*b^2*c^2 - 4*a^5*c^3)*d^2*e^10 + 3*(a^
3*b^5 - a^4*b^3*c - 3*a^5*b*c^2)*d*e^11 - (a^4*b^4 - 3*a^5*b^2*c
+ a^6*c^2)*e^12)*x - sqrt(1/2)*((b^2*c^7 - 4*a*c^8)*d^9 - 18*(a*b
^2*c^6 - 4*a^2*c^7)*d^7*e^2 + 21*(a*b^3*c^5 - 4*a^2*b*c^6)*d^6*e^
3 - 15*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^4 + 3*(2*a*
b^5*c^3 - 37*a^2*b^3*c^4 + 116*a^3*b*c^5)*d^4*e^5 - (a*b^6*c^2 -
72*a^2*b^4*c^3 + 318*a^3*b^2*c^4 - 184*a^4*c^5)*d^3*e^6 - 3*(11*a
^2*b^5*c^2 - 61*a^3*b^3*c^3 + 68*a^4*b*c^4)*d^2*e^7 + 3*(3*a^2*b^
6*c - 19*a^3*b^4*c^2 + 29*a^4*b^2*c^3 - 4*a^5*c^4)*d*e^8 - (a^2*b
^7 - 7*a^3*b^5*c + 13*a^4*b^3*c^2 - 4*a^5*b*c^3)*e^9 - ((a*b^3*c^
7 - 4*a^2*b*c^8)*d^3 - 6*(a^2*b^2*c^7 - 4*a^3*c^8)*d^2*e + 3*(a^2
*b^3*c^6 - 4*a^3*b*c^7)*d*e^2 - (a^2*b^4*c^5 - 6*a^3*b^2*c^6 + 8*
a^4*c^7)*e^3)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^
9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 5
```

```
2*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c
^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33
*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*
b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*
c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*
(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11
+ (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c
^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))*sqrt(-(b*c^5*d^6 - 12*a*c
^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^
3 + 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2
*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6
 + (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 +
40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(
a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^
6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5
*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8
- 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6
*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d
^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b
*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^
2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))/(a*b^2*c^5 -
 4*a^2*c^6))) + 3*sqrt(1/2)*c^2*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^5*e
 + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(
a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2
*a^3*c^3)*d*e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 - (a*b^
2*c^5 - 4*a^2*c^6)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c
^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^
6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*
a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 1
5*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11
*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2
*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10
- 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*
e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 +
a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))/(a*b^2*c^5 - 4*a^2*c
^6))*log(-2*(c^8*d^12 - 3*b*c^7*d^11*e + 3*(b^2*c^6 - 4*a*c^7)*d^
10*e^2 - (b^3*c^5 - 59*a*b*c^6)*d^9*e^3 - 9*(13*a*b^2*c^5 + 3*a^2
*c^6)*d^8*e^4 + 18*(7*a*b^3*c^4 + 5*a^2*b*c^5)*d^7*e^5 - 42*(2*a*
b^4*c^3 + 3*a^2*b^2*c^4)*d^6*e^6 + 18*(2*a*b^5*c^2 + 6*a^2*b^3*c^
3 - a^3*b*c^4)*d^5*e^7 - 9*(a*b^6*c + 7*a^2*b^4*c^2 - 2*a^3*b^2*c
^3 - 3*a^4*c^4)*d^4*e^8 + (a*b^7 + 21*a^2*b^5*c + 10*a^3*b^3*c^2
- 55*a^4*b*c^3)*d^3*e^9 - 3*(a^2*b^6 + 4*a^3*b^4*c - 9*a^4*b^2*c^
2 - 4*a^5*c^3)*d^2*e^10 + 3*(a^3*b^5 - a^4*b^3*c - 3*a^5*b*c^2)*d
*e^11 - (a^4*b^4 - 3*a^5*b^2*c + a^6*c^2)*e^12)*x + sqrt(1/2)*((b
^2*c^7 - 4*a*c^8)*d^9 - 18*(a*b^2*c^6 - 4*a^2*c^7)*d^7*e^2 + 21*(
a*b^3*c^5 - 4*a^2*b*c^6)*d^6*e^3 - 15*(a*b^4*c^4 - 8*a^2*b^2*c^5
+ 16*a^3*c^6)*d^5*e^4 + 3*(2*a*b^5*c^3 - 37*a^2*b^3*c^4 + 116*a^3
*b*c^5)*d^4*e^5 - (a*b^6*c^2 - 72*a^2*b^4*c^3 + 318*a^3*b^2*c^4 -
 184*a^4*c^5)*d^3*e^6 - 3*(11*a^2*b^5*c^2 - 61*a^3*b^3*c^3 + 68*a
^4*b*c^4)*d^2*e^7 + 3*(3*a^2*b^6*c - 19*a^3*b^4*c^2 + 29*a^4*b^2*
c^3 - 4*a^5*c^4)*d*e^8 - (a^2*b^7 - 7*a^3*b^5*c + 13*a^4*b^3*c^2
- 4*a^5*b*c^3)*e^9 + ((a*b^3*c^7 - 4*a^2*b*c^8)*d^3 - 6*(a^2*b^2*
c^7 - 4*a^3*c^8)*d^2*e + 3*(a^2*b^3*c^6 - 4*a^3*b*c^7)*d*e^2 - (a
^2*b^4*c^5 - 6*a^3*b^2*c^6 + 8*a^4*c^7)*e^3)*sqrt((c^10*d^12 - 30
*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c
^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^
5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 -
 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17
*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*
b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*
c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4
*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*
b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^
11)))*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20
*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d
^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 -
 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 - (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c
^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c
^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5
- 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*
a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3
*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*
```

```
c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3
+ 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^
5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^
6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^
10 - 4*a^3*c^11)))/(a*b^2*c^5 - 4*a^2*c^6))) - 3*sqrt(1/2)*c^2*sq
rt(-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*
c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 -
 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 - 5*a^2*b
^3*c + 5*a^3*b*c^2)*e^6 - (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d^12
 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*
a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b
^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*
c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5
 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20
*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4
*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 +
7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11
*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a
^3*c^11)))/(a*b^2*c^5 - 4*a^2*c^6))*log(-2*(c^8*d^12 - 3*b*c^7*d^
11*e + 3*(b^2*c^6 - 4*a*c^7)*d^10*e^2 - (b^3*c^5 - 59*a*b*c^6)*d^
9*e^3 - 9*(13*a*b^2*c^5 + 3*a^2*c^6)*d^8*e^4 + 18*(7*a*b^3*c^4 +
5*a^2*b*c^5)*d^7*e^5 - 42*(2*a*b^4*c^3 + 3*a^2*b^2*c^4)*d^6*e^6 +
 18*(2*a*b^5*c^2 + 6*a^2*b^3*c^3 - a^3*b*c^4)*d^5*e^7 - 9*(a*b^6*
c + 7*a^2*b^4*c^2 - 2*a^3*b^2*c^3 - 3*a^4*c^4)*d^4*e^8 + (a*b^7 +
 21*a^2*b^5*c + 10*a^3*b^3*c^2 - 55*a^4*b*c^3)*d^3*e^9 - 3*(a^2*b
^6 + 4*a^3*b^4*c - 9*a^4*b^2*c^2 - 4*a^5*c^3)*d^2*e^10 + 3*(a^3*b
^5 - a^4*b^3*c - 3*a^5*b*c^2)*d*e^11 - (a^4*b^4 - 3*a^5*b^2*c + a
^6*c^2)*e^12)*x - sqrt(1/2)*((b^2*c^7 - 4*a*c^8)*d^9 - 18*(a*b^2*
c^6 - 4*a^2*c^7)*d^7*e^2 + 21*(a*b^3*c^5 - 4*a^2*b*c^6)*d^6*e^3 -
 15*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^4 + 3*(2*a*b^5
*c^3 - 37*a^2*b^3*c^4 + 116*a^3*b*c^5)*d^4*e^5 - (a*b^6*c^2 - 72*
a^2*b^4*c^3 + 318*a^3*b^2*c^4 - 184*a^4*c^5)*d^3*e^6 - 3*(11*a^2*
b^5*c^2 - 61*a^3*b^3*c^3 + 68*a^4*b*c^4)*d^2*e^7 + 3*(3*a^2*b^6*c
 - 19*a^3*b^4*c^2 + 29*a^4*b^2*c^3 - 4*a^5*c^4)*d*e^8 - (a^2*b^7
- 7*a^3*b^5*c + 13*a^4*b^3*c^2 - 4*a^5*b*c^3)*e^9 + ((a*b^3*c^7 -
 4*a^2*b*c^8)*d^3 - 6*(a^2*b^2*c^7 - 4*a^3*c^8)*d^2*e + 3*(a^2*b^
3*c^6 - 4*a^3*b*c^7)*d*e^2 - (a^2*b^4*c^5 - 6*a^3*b^2*c^6 + 8*a^4
*c^7)*e^3)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e
^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a
^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)
*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^
2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5
*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2
 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^
2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (
a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)
*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))*sqrt(-(b*c^5*d^6 - 12*a*c^5*
d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 +
 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^
2 + 2*a^3*c^3)*d*e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 -
(a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*
a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b
^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 +
 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^
7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 2
0*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(1
1*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*
e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^
4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c
^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))/(a*b^2*c^5 - 4*
a^2*c^6))) + 6*(3*c*d*e^2 - b*e^3)*x)/c^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.74218, size = 1, normalized size = 0.

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.269 $\int \frac{(d+ex^2)^2}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=238

$$\frac{\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}+e(2cd-be)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e(2cd-be)-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{e^2x}{c}$$

```
[Out] (e^2*x)/c + ((e*(2*c*d - b*e) + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d
 + a*e))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + ((e*(2*c*d - b*e) - (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.23784, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}+e(2cd-be)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e(2cd-be)-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{e^2x}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*x)/c + ((e*(2*c*d - b*e) + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d
 + a*e))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + ((e*(2*c*d - b*e) - (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2\int\frac{1}{c}\,dx+\frac{\sqrt{2}\left(-be\left(be-2cd\right)+2c\left(ae^2-cd^2\right)-e\sqrt{-4ac+b^2}\left(be-2cd\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\left(-be\left(be-2cd\right)+2c\left(ae^2-cd^2\right)+e\sqrt{-4ac+b^2}\left(be-2cd\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2/(c*x**4+b*x**2+a),x)
```

```
[Out] e**2*Integral(1/c, x) + sqrt(2)*(-b*e*(b*e - 2*c*d) + 2*c*(a*e**2
 - c*d**2) - e*sqrt(-4*a*c + b**2)*(b*e - 2*c*d))*atan(sqrt(2)*sq
rt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*c**(3/2)*sqrt(b + sqrt(
```

```
-4*a*c + b**2))*sqrt(-4*a*c + b**2)) - sqrt(2)*(-b*e*(b*e - 2*c*d
) + 2*c*(a*e**2 - c*d**2) + e*sqrt(-4*a*c + b**2)*(b*e - 2*c*d))*
atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*c**(3/2)
*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.632238, size = 269, normalized size = 1.13

$$\frac{\frac{\sqrt{2}\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*e^2*x + (Sqrt[2]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c]
)*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(2*c^2*d^2 + b*(b + Sqrt[b^2 -
 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c
]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2))
```

Maple [B] time = 0.034, size = 695, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^2/(c*x^4+b*x^2+a),x)
```

```
[Out] e^2*x/c-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e^2+2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*d*e+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*a*e^2-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2
*e^2+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d*e-c/(-
4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d^2+1/2/c*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*b*e^2-2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d*e+
1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*e^2-1/2/c/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e^2+1/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d*e-c/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^2x}{c}-\frac{-\int\frac{cd^2-ae^2+(2\,cde-be^2)\,x^2}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] e^2*x/c - integrate(-(c*d^2 - a*e^2 + (2*c*d*e - b*e^2)*x^2)/(c*x
^4 + b*x^2 + a), x)/c
```

Fricas [A] time = 1.92252, size = 6332, normalized size = 26.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*e^2*x - sqrt(1/2)*c*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a
*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b
*c)*e^4 + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^
2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*
a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2
*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8
)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c^5
*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d
^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2
*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(
a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)*x + sqrt(1/2)*(
(b^2*c^4 - 4*a*c^5)*d^6 - 7*(a*b^2*c^3 - 4*a^2*c^4)*d^4*e^2 + 4*(
a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^3 - (a*b^4*c - 11*a^2*b^2*c^2 + 28
*a^3*c^3)*d^2*e^4 - 4*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^5 + (a^2*b^4
- 5*a^3*b^2*c + 4*a^4*c^2)*e^6 - ((a*b^3*c^4 - 4*a^2*b*c^5)*d^2 -
 4*(a^2*b^2*c^4 - 4*a^3*c^5)*d*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*e^
2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*
b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2
*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^
2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))*s
qrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c
- 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c^3 - 4*a^2
*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a
^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*
b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 +
(a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7))
)/(a*b^2*c^3 - 4*a^2*c^4))) + sqrt(1/2)*c*sqrt(-(b*c^3*d^4 - 8*a*
c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (
a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 -
12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(
a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d
^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c
 + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c
^4))*log(2*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c
^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(
a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2
)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)*
x - sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^6 - 7*(a*b^2*c^3 - 4*a^2*c^4
)*d^4*e^2 + 4*(a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^3 - (a*b^4*c - 11*a
^2*b^2*c^2 + 28*a^3*c^3)*d^2*e^4 - 4*(a^2*b^3*c - 4*a^3*b*c^2)*d*
e^5 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*e^6 - ((a*b^3*c^4 - 4*a
^2*b*c^5)*d^2 - 4*(a^2*b^2*c^4 - 4*a^3*c^5)*d*e + (a^2*b^3*c^3 -
4*a^3*b*c^4)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^
5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4
 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c
^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 -
 4*a^3*c^7)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^
2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*
b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4
*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*
e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*
b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^
```

```
6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) - sqrt(1/2)*c*sqrt(-(b
*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2
*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^2*c^3 - 4*a^2*c^4)*s
qrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^
3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2
 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^
4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^
2*c^3 - 4*a^2*c^4))*log(2*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d
^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3
)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^
2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2
 - a^4*c)*e^8)*x + sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^6 - 7*(a*b^2*
c^3 - 4*a^2*c^4)*d^4*e^2 + 4*(a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^3 -
(a*b^4*c - 11*a^2*b^2*c^2 + 28*a^3*c^3)*d^2*e^4 - 4*(a^2*b^3*c -
4*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*e^6 + ((
a*b^3*c^4 - 4*a^2*b*c^5)*d^2 - 4*(a^2*b^2*c^4 - 4*a^3*c^5)*d*e +
(a^2*b^3*c^3 - 4*a^3*b*c^4)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2
 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a
^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*
b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)
/(a^2*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6
*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2
*b*c)*e^4 - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*
e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 1
9*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a
^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e
^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) + sqrt(
1/2)*c*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(
a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^2*c^3
 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^
3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4
*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*
d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a
^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c^5*d^8 - 2*b*c^4*d^7*e
 + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*
c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a
*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d
*e^7 - (a^3*b^2 - a^4*c)*e^8)*x - sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*
d^6 - 7*(a*b^2*c^3 - 4*a^2*c^4)*d^4*e^2 + 4*(a*b^3*c^2 - 4*a^2*b*
c^3)*d^3*e^3 - (a*b^4*c - 11*a^2*b^2*c^2 + 28*a^3*c^3)*d^2*e^4 -
4*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^
4*c^2)*e^6 + ((a*b^3*c^4 - 4*a^2*b*c^5)*d^2 - 4*(a^2*b^2*c^4 - 4*
a^3*c^5)*d*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*e^2)*sqrt((c^6*d^8 - 1
2*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a
*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^
2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c
+ a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^4 - 8*
a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 +
 (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8
- 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2
*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)
*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2
*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2
*c^4))))/c
```

Sympy [A] time = 123.774, size = 920, normalized size = 3.87

$$\text{RootSum}\left(t^4\left(256a^3c^5 - 128a^2b^2c^4 + 16ab^4c^3\right) + t^2\left(48a^3bc^2e^4 - 128a^3c^3de^3 - 28a^2b^3ce^4 + 96a^2b^2c^2de^3 - 96a^2bc^3d^2e^2 + 1\right.\right.$$

$$+ \frac{e^2x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**2/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**5 - 128*a**2*b**2*c**4 + 16*a*b**4*c**
```

```
3) + _t**2*(48*a**3*b*c**2*e**4 - 128*a**3*c**3*d*e**3 - 28*a**2*
b**3*c*e**4 + 96*a**2*b**2*c**2*d*e**3 - 96*a**2*b*c**3*d**2*e**2
 + 128*a**2*c**4*d**3*e + 4*a*b**5*e**4 - 16*a*b**4*c*d*e**3 + 24
*a*b**3*c**2*d**2*e**2 - 32*a*b**2*c**3*d**3*e - 16*a*b*c**4*d**4
 + 4*b**3*c**3*d**4) + a**4*e**8 - 4*a**3*b*d*e**7 + 4*a**3*c*d**
2*e**6 + 6*a**2*b**2*d**2*e**6 - 12*a**2*b*c*d**3*e**5 + 6*a**2*c
**2*d**4*e**4 - 4*a*b**3*d**3*e**5 + 12*a*b**2*c*d**4*e**4 - 12*a
*b*c**2*d**5*e**3 + 4*a*c**3*d**6*e**2 + b**4*d**4*e**4 - 4*b**3*
c*d**5*e**3 + 6*b**2*c**2*d**6*e**2 - 4*b*c**3*d**7*e + c**4*d**8
, Lambda(_t, _t*log(x + (32*_t**3*a**3*b*c**4*e**2 - 128*_t**3*a*
*3*c**5*d*e - 8*_t**3*a**2*b**3*c**3*e**2 + 32*_t**3*a**2*b**2*c*
*4*d*e + 32*_t**3*a**2*b*c**5*d**2 - 8*_t**3*a*b**3*c**4*d**2 - 4
*_t*a**4*c**2*e**6 + 8*_t*a**3*b**2*c*e**6 - 36*_t*a**3*b*c**2*d*
e**5 + 60*_t*a**3*c**3*d**2*e**4 - 2*_t*a**2*b**4*e**6 + 12*_t*a*
*2*b**3*c*d*e**5 - 30*_t*a**2*b**2*c**2*d**2*e**4 + 40*_t*a**2*b*
c**3*d**3*e**3 - 60*_t*a**2*c**4*d**4*e**2 + 12*_t*a*b*c**4*d**5*
e + 4*_t*a*c**5*d**6 - 2*_t*b**2*c**4*d**6)/(a**4*c*e**8 - a**3*b
**2*e**8 + 2*a**3*b*c*d*e**7 - 4*a**3*c**2*d**2*e**6 + 2*a**2*b**
3*d*e**7 - 9*a**2*b**2*c*d**2*e**6 + 18*a**2*b*c**2*d**3*e**5 - 1
0*a**2*c**3*d**4*e**4 - a*b**4*d**2*e**6 + 6*a*b**3*c*d**3*e**5 -
 15*a*b**2*c**2*d**4*e**4 + 14*a*b*c**3*d**5*e**3 - 4*a*c**4*d**6
*e**2 + b**2*c**3*d**6*e**2 - 2*b*c**4*d**7*e + c**5*d**8)))) + e
**2*x/c
```

GIAC/XCAS [A] time = 1.24386, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.270 $\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=174

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqr
t[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.42486, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqr
t[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 32.053, size = 185, normalized size = 1.06

$$\frac{\sqrt{2}\left(be-2cd+e\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\left(be-2cd-e\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] sqrt(2)*(b*e - 2*c*d + e*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c
)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*sqrt(c)*sqrt(b + sqrt(-4*a*
c + b**2))*sqrt(-4*a*c + b**2)) - sqrt(2)*(b*e - 2*c*d - e*sqrt(-
4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2
)))/(2*sqrt(c)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.268763, size = 172, normalized size = 0.99

$$\frac{\frac{\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(d + e*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((-2
*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*S
qrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.024, size = 328, normalized size = 1.9

$$\frac{\sqrt{2}e}{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{b\sqrt{2}e}{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$-c\sqrt{2}d\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$-\frac{\sqrt{2}e}{2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{b\sqrt{2}e}{2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$-c\sqrt{2}dArtanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*b*e-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*d-1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
h(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+1/2/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e-c/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 0.310926, size = 2059, normalized size = 11.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4
*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 -
 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(c^2*d^4 - b*c*d^3*e
+ a*b*d*e^3 - a^2*e^4)*x + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*
b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c
- 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2
*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2
*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2
*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*sqrt(
-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2
*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^
2*c - 4*a^2*c^2))*log(-2*(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e
^4)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^
2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqr
t((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))
*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqr
t((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))
/(a*b^2*c - 4*a^2*c^2))) + 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d
*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^
2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*l
og(-2*(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x + sqrt(1/2)*(
(b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*b^3*c - 4*a
^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*
d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4
*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*
d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c
^2))) - 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b
^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b
^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(c^2*d^4 - b*
c*d^3*e + a*b*d*e^3 - a^2*e^4)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d
^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^
2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/
(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2
- (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/
(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2)))
```

Sympy [A] time = 16.1684, size = 314, normalized size = 1.8

RootSum $\left(t^4\left(256a^3c^3 - 128a^2b^2c^2 + 16ab^4c\right) + t^2\left(-16a^2bce^2 + 64a^2c^2de + 4ab^3e^2 - 16ab^2cde - 16abc^2d^2 + 4b^3cd^2\right) + a^2\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**3 - 128*a**2*b**2*c**2 + 16*a*b**4*c)
+ _t**2*(-16*a**2*b*c*e**2 + 64*a**2*c**2*d*e + 4*a*b**3*e**2 - 1
6*a*b**2*c*d*e - 16*a*b*c**2*d**2 + 4*b**3*c*d**2) + a**2*e**4 -
2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d**3*e +
c**2*d**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*c**2*e - 16*_t**3
```

```
*a**2*b**2*c*e - 32*_t**3*a**2*b*c**2*d + 8*_t**3*a*b**3*c*d - 2*
_t*a**2*b*e**3 + 12*_t*a**2*c*d*e**2 - 6*_t*a*b*c*d**2*e - 4*_t*a
*c**2*d**3 + 2*_t*b**2*c*d**3)/(a**2*e**4 - a*b*d*e**3 + b*c*d**3
*e - c**2*d**4))))
```

GIAC/XCAS [A] time = 0.808676, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.271 $\int \frac{1}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[
2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.209277, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[
2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 19.6128, size = 138, normalized size = 0.92

$$-\frac{\sqrt{2}\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**2+a),x)
```

```
[Out] -sqrt(2)*sqrt(c)*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b*
*2)))/(sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(
2)*sqrt(c)*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/
(sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.161404, size = 129, normalized size = 0.86

$$\frac{\sqrt{2}\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
4*a*c]]]/Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[
b^2 - 4*a*c]
```

Maple [A] time = 0.002, size = 116, normalized size = 0.8

$$-c\sqrt{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$-c\sqrt{2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^2+a),x)
```

```
[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-c/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 0.266291, size = 828, normalized size = 5.52

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,cx+\sqrt{\frac{1}{2}}\left(b^2-4\,ac-\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,cx-\sqrt{\frac{1}{2}}\left(b^2-4\,ac-\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,cx+\sqrt{\frac{1}{2}}\left(b^2-4\,ac+\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,cx-\sqrt{\frac{1}{2}}\left(b^2-4\,ac+\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*
c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^
3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2
*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*
sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*
a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/
sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^
2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a*b
^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c
*x + sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 -
4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b - (a*b^2 - 4*a^2*c)/s
qrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*
(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(
-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c
)))
```

Sympy [A] time = 3.03658, size = 87, normalized size = 0.58

$$\operatorname{RootSum}\left(t^{4}\left(256a^{3}c^{2}-128a^{2}b^{2}c+16ab^{4}\right)+t^{2}\left(-16abc+4b^{3}\right)+c,\left(t\mapsto t\log\left(x+\frac{32t^{3}a^{2}bc-8t^{3}ab^{3}+4tac-2tb^{2}}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t*
*2*(-16*a*b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2
*b*c - 8*_t**3*a*b**3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

GIAC/XCAS [A] time = 0.368573, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.272 $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=254

$$-\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\sqrt{c}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2-bde+cd^2)}$$

```
[Out] -((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b
^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))) - (Sqrt[c]*(e + (2*c*d - b
*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*
e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d
^2 - b*d*e + a*e^2))
```

Rubi [A] time = 1.19177, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\sqrt{c}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b
^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))) - (Sqrt[c]*(e + (2*c*d - b
*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*
e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d
^2 - b*d*e + a*e^2))
```

Rubi in Sympy [A] time = 93.1741, size = 255, normalized size = 1.

$$\frac{\sqrt{2}\sqrt{c}\left(be-2cd-e\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\,(ae^2-bde+cd^2)}-\frac{\sqrt{2}\sqrt{c}\left(be-2cd+e\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\,(ae^2-bde+cd^2)}+\frac{e^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2-bde+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] sqrt(2)*sqrt(c)*(b*e - 2*c*d - e*sqrt(-4*a*c + b**2))*atan(sqrt(2
)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*sqrt(b + sqrt(-4*a*
c + b**2))*sqrt(-4*a*c + b**2)*(a*e**2 - b*d*e + c*d**2)) - sqrt(
2)*sqrt(c)*(b*e - 2*c*d + e*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqr
t(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*sqrt(b - sqrt(-4*a*c + b
**2))*sqrt(-4*a*c + b**2)*(a*e**2 - b*d*e + c*d**2)) + e**(3/2)*a
tan(sqrt(e)*x/sqrt(d))/(sqrt(d)*(a*e**2 - b*d*e + c*d**2))
```

Mathematica [A] time = 0.517167, size = 274, normalized size = 1.08

$$\frac{\sqrt{c}\left(e\sqrt{b^2-4ac}+be-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(-ae^2+bde-cd^2\right)}+\frac{\sqrt{c}\left(e\sqrt{b^2-4ac}-be+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\left(-ae^2+bde-cd^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[c]*(-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (Sqrt[c
]*(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (e^(3/2)*ArcTan
[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Maple [B] time = 0.027, size = 480, normalized size = 1.9

$$-\frac{c\sqrt{2}e}{2\,ae^2-2\,bde+2\,cd^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}+\frac{c\sqrt{2}be}{2\,ae^2-2\,bde+2\,cd^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}-\frac{c^2\sqrt{2}d}{ae^2-bde+cd^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}+\frac{c\sqrt{2}e}{2\,ae^2-2\,bde+2\,cd^2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}+\frac{c\sqrt{2}be}{2\,ae^2-2\,bde+2\,cd^2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}-\frac{c^2\sqrt{2}d}{ae^2-bde+cd^2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}+\frac{e^2}{ae^2-bde+cd^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+1/2/(a*
e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b*e-1/(a*e^2-b*d*e+c*d^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*d+1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*e+1/2/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*b*e-1/(a*e^2-b*d*e+c*d^2)*c^2/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d+e^2/(a*e^2-b*d*e+c*d^2)
/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.273 $\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=429

$$\frac{\sqrt{c}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)^2}-\frac{\sqrt{c}\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)^2}+\frac{e^2x}{2d\,(d+ex^2)\,(ae^2-bde+cd^2)}+\frac{e^{3/2}(2cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2-bde+cd^2)^2}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\,(ae^2-bde+cd^2)}$$

```
[Out] (e^2*x)/(2*d*(c*d^2 - b*d*e + a*e^2)*(d + e*x^2)) + (Sqrt[c]*(2*c
^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 -
4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(
c*d^2 - b*d*e + a*e^2)^2) - (Sqrt[c]*(2*c^2*d^2 + b*(b - Sqrt[b^2
 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b
^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2
) + (e^(3/2)*(2*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*
(c*d^2 - b*d*e + a*e^2)^2) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(2*d^(3/2)*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 2.81315, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\sqrt{c}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)^2}-\frac{\sqrt{c}\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)^2}+\frac{e^2x}{2d\,(d+ex^2)\,(ae^2-bde+cd^2)}+\frac{e^{3/2}(2cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2-bde+cd^2)^2}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)),x]
```


```
[Out] (e^2*x)/(2*d*(c*d^2 - b*d*e + a*e^2)*(d + e*x^2)) + (Sqrt[c]*(2*c
^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 -
4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(
c*d^2 - b*d*e + a*e^2)^2) - (Sqrt[c]*(2*c^2*d^2 + b*(b - Sqrt[b^2
 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b
^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2
) + (e^(3/2)*(2*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*
(c*d^2 - b*d*e + a*e^2)^2) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(2*d^(3/2)*(c*d^2 - b*d*e + a*e^2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.658, size = 354, normalized size = 0.83

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}-b\right)-2c^2d^2\right)\tan^{-1}\left(\cdots\right.}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\cdots}{2\left(e(ae-bd)+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((e^2*(c*d^2 + e*(-(b*d) + a*e))*x)/(d*(d + e*x^2)) + (Sqrt[2]*Sq
rt[c]*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + S
qrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]
) + (Sqrt[2]*Sqrt[c]*(-2*c^2*d^2 + b*(-b + Sqrt[b^2 - 4*a*c])*e^2
 + 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + S
qrt[b^2 - 4*a*c]]) + (e^(3/2)*(5*c*d^2 + e*(-3*b*d + a*e))*ArcTan
[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2))/(2*(c*d^2 + e*(-(b*d) + a*e))^2)
```

Maple [B] time = 0.037, size = 1141, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e^2-1/
(a*e^2-b*d*e+c*d^2)^2*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d*e+1/(a*e
^2-b*d*e+c*d^2)^2*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*a*e^2-1/2/(a*e^2-b*d*e+c*d^2)^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*b^2*e^2+1/(a*e^2-b*d*e+c*d^2)^2*c^2/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d*e-1/(a*e^2-b*d*e+c*d
^2)^2*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d^2-1/2
/(a*e^2-b*d*e+c*d^2)^2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e^2+1/
(a*e^2-b*d*e+c*d^2)^2*c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d*e+1/(
a*e^2-b*d*e+c*d^2)^2*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*a*e^2-1/2/(a*e^2-b*d*e+c*d^2)^2*c/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e^2+1/(a*e^2-b*d*e+c*d^2)^2*c^
2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d*e-1/(a*e
^2-b*d*e+c*d^2)^2*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
```

```
^(1/2))*d^2+1/2*e^4/(a*e^2-b*d*e+c*d^2)^2/d*x/(e*x^2+d)*a-1/2*e^3
/(a*e^2-b*d*e+c*d^2)^2*x/(e*x^2+d)*b+1/2*e^2/(a*e^2-b*d*e+c*d^2)^
2*d*x/(e*x^2+d)*c+1/2*e^4/(a*e^2-b*d*e+c*d^2)^2/d/(d*e)^(1/2)*arc
tan(x*e/(d*e)^(1/2))*a-3/2*e^3/(a*e^2-b*d*e+c*d^2)^2/(d*e)^(1/2)*
arctan(x*e/(d*e)^(1/2))*b+5/2*e^2/(a*e^2-b*d*e+c*d^2)^2*d/(d*e)^(
1/2)*arctan(x*e/(d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 17.0261, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2),x, algorithm="giac")

[Out] Done
```

3.274 $\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=563

$$\frac{x\left(c\left(-\frac{abe(ae^2+3cd^2)}{c}-2ad\left(cd^2-3ae^2\right)+b^2d^3\right)-x^2\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$-\frac{\left(ab^3e^3-b^2\left(ae^3\sqrt{b^2-4ac}-3acde^2+c^2d^3\right)+6ac\left(ae^2+cd^2\right)\left(e\sqrt{b^2-4ac}+2cd\right)-bc\left(cd^2\left(d\sqrt{b^2-4ac}+12ae\right)+ae^2\right.\right.}{2\sqrt{2}ac^{3/2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

$$+\frac{\left(ab^3e^3-b^2\left(-ae^3\sqrt{b^2-4ac}-3acde^2+c^2d^3\right)+6ac\left(ae^2+cd^2\right)\left(2cd-e\sqrt{b^2-4ac}\right)+bc\left(cd^2\left(d\sqrt{b^2-4ac}-12ae\right)+ae\right.\right.}{2\sqrt{2}ac^{3/2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (x*(c*(b^2*d^3 - 2*a*d*(c*d^2 - 3*a*e^2) - (a*b*e*(3*c*d^2 + a*e^
2))/c) - (a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 +
3*a*e^2))*x^2))/(2*a*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((a*b
^3*e^3 + 6*a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) - b^
2*(c^2*d^3 - 3*a*c*d*e^2 + a*Sqrt[b^2 - 4*a*c]*e^3) - b*c*(a*e^2*
(3*Sqrt[b^2 - 4*a*c]*d + 8*a*e) + c*d^2*(Sqrt[b^2 - 4*a*c]*d + 12
*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*a*c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + ((a*b^3*e^3 + 6*a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(c*d^2 +
 a*e^2) - b^2*(c^2*d^3 - 3*a*c*d*e^2 - a*Sqrt[b^2 - 4*a*c]*e^3) +
 b*c*(c*d^2*(Sqrt[b^2 - 4*a*c]*d - 12*a*e) + a*e^2*(3*Sqrt[b^2 -
4*a*c]*d - 8*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*a*c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqr
t[b^2 - 4*a*c]])
```

Rubi [A] time = 7.33284, antiderivative size = 563, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{x\left(-x^2\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)-abe\left(ae^2+3cd^2\right)-2acd\left(cd^2-3ae^2\right)+b^2cd^3\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$-\frac{\left(ab^3e^3-b^2\left(ae^3\sqrt{b^2-4ac}-3acde^2+c^2d^3\right)+6ac\left(ae^2+cd^2\right)\left(e\sqrt{b^2-4ac}+2cd\right)-bc\left(cd^2\left(d\sqrt{b^2-4ac}+12ae\right)+ae^2\right.\right.}{2\sqrt{2}ac^{3/2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

$$+\frac{\left(ab^3e^3-b^2\left(-ae^3\sqrt{b^2-4ac}-3acde^2+c^2d^3\right)+6ac\left(ae^2+cd^2\right)\left(2cd-e\sqrt{b^2-4ac}\right)+bc\left(cd^2\left(d\sqrt{b^2-4ac}-12ae\right)+ae\right.\right.}{2\sqrt{2}ac^{3/2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*c*d^3 - 2*a*c*d*(c*d^2 - 3*a*e^2) - a*b*e*(3*c*d^2 + a*e^
2) - (a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*
e^2))*x^2))/(2*a*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((a*b^3*e
^3 + 6*a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) - b^2*(c
^2*d^3 - 3*a*c*d*e^2 + a*Sqrt[b^2 - 4*a*c]*e^3) - b*c*(a*e^2*(3*S
qrt[b^2 - 4*a*c]*d + 8*a*e) + c*d^2*(Sqrt[b^2 - 4*a*c]*d + 12*a*e
)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*a*c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + ((a*b^3*e^3 + 6*a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e
^2) - b^2*(c^2*d^3 - 3*a*c*d*e^2 - a*Sqrt[b^2 - 4*a*c]*e^3) + b*c
*(c*d^2*(Sqrt[b^2 - 4*a*c]*d - 12*a*e) + a*e^2*(3*Sqrt[b^2 - 4*a*
c]*d - 8*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(2*Sqrt[2]*a*c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^
2 - 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 3.57093, size = 540, normalized size = 0.96

$$\frac{\frac{2\sqrt{c}x\left(b\left(-a^2e^3-3acde\left(d-ex^2\right)+c^2d^3x^2\right)+b^2\left(cd^3-ae^3x^2\right)+2ac\left(ae^2\left(3d+ex^2\right)-cd^2\left(d+3ex^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(-ab^3e^3+b^2\left(ae^3\sqrt{b^2-4ac}-3acde^2+c^2d^3\right)-6ac\left(ae^2+c\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*Sqrt[c]*x*(b^2*(c*d^3 - a*e^3*x^2) + b*(-(a^2*e^3) + c^2*d^3*
x^2 - 3*a*c*d*e*(d - e*x^2)) + 2*a*c*(a*e^2*(3*d + e*x^2) - c*d^2
*(d + 3*e*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*
(-(a*b^3*e^3) - 6*a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^
2) + b^2*(c^2*d^3 - 3*a*c*d*e^2 + a*Sqrt[b^2 - 4*a*c]*e^3) + b*c*
(a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 8*a*e) + c*d^2*(Sqrt[b^2 - 4*a*c]
*d + 12*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[
2]*(a*b^3*e^3 + 6*a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^
2) + b^2*(-(c^2*d^3) + 3*a*c*d*e^2 + a*Sqrt[b^2 - 4*a*c]*e^3) + b
*c*(c*d^2*(Sqrt[b^2 - 4*a*c]*d - 12*a*e) + a*e^2*(3*Sqrt[b^2 - 4*
a*c]*d - 8*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a
*c^(3/2))
```

Maple [B] time = 0.204, size = 8504, normalized size = 15.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(bc^2d^3-6\,ac^2d^2e+3\,abcde^2-\left(ab^2-2\,a^2c\right)e^3\right)x^3-\left(3\,abcd^2e-6\,a^2cde^2+a^2be^3-\left(b^2c-2\,ac^2\right)d^3\right)x}{2\left(a^2b^2c-4\,a^3c^2+\left(ab^2c^2-4\,a^2c^3\right)x^4+\left(ab^3c-4\,a^2bc^2\right)x^2\right)}-\frac{-\int\frac{3\,abcd^2e-6\,a^2cde^2+a^2be^3+\left(b^2c-6\,ac^2\right)d^3+\left(bc^2d^3-6\,ac^2d^2e+3\,abcde^2+\left(ab^2-6\,a^2c\right)e^3\right)x^2}{cx^4+bx^2+a}\,dx}{2\left(ab^2c-4\,a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^3/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*
c)*e^3)*x^3 - (3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c
 - 2*a*c^2)*d^3)*x)/(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c
^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2) - 1/2*integrate(-(3*a*b*c*
d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 + (b^2*c - 6*a*c^2)*d^3 + (b*c^
2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 + (a*b^2 - 6*a^2*c)*e^3)*x^
2)/(c*x^4 + b*x^2 + a), x)/(a*b^2*c - 4*a^2*c^2)
```

Fricas [A] time = 44.4893, size = 16358, normalized size = 29.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^3/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^
2*c)*e^3)*x^3 - sqrt(1/2)*(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4
*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2)*sqrt(-((b^5*c^3 - 15
*a*b^3*c^4 + 60*a^2*b*c^5)*d^6 + 6*(a*b^4*c^3 - 6*a^2*b^2*c^4 - 2
4*a^3*c^5)*d^5*e - 3*(3*a^2*b^3*c^3 - 92*a^3*b*c^4)*d^4*e^2 - 8*(
11*a^3*b^2*c^3 + 36*a^4*c^4)*d^3*e^3 - 3*(3*a^3*b^3*c^2 - 92*a^4*
b*c^3)*d^2*e^4 + 6*(a^3*b^4*c - 6*a^4*b^2*c^2 - 24*a^5*c^3)*d*e^5
 + (a^3*b^5 - 15*a^4*b^3*c + 60*a^5*b*c^2)*e^6 + (a^3*b^6*c^3 - 1
2*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6)*sqrt(-(108*a^3*b*c^6
*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a
^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*
c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4
 + 12*(a^3*b^3*c^4 - 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*
a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12*(a^4*b^3*c^3 - 18*a^5*b*c
^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e^8 - 18*(a^6*b^2
*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*d*e^11
- (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*e^12)/(a^6*b^6*c^6 - 12*a
^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))/(a^3*b^6*c^3 - 12*a^4
*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6))*log(-((5*b^4*c^6 - 81*a*
b^2*c^7 + 324*a^2*c^8)*d^12 - 3*(3*b^5*c^5 - 65*a*b^3*c^6 + 324*a
^2*b*c^7)*d^11*e + 3*(b^6*c^4 - 42*a*b^4*c^5 + 252*a^2*b^2*c^6 +
432*a^3*c^7)*d^10*e^2 + (b^7*c^3 + 3*a*b^5*c^4 + 33*a^2*b^3*c^5 -
 2916*a^3*b*c^6)*d^9*e^3 + 9*(a*b^6*c^3 - 15*a^2*b^4*c^4 + 195*a^
3*b^2*c^5 + 180*a^4*c^6)*d^8*e^4 - 162*(a^3*b^3*c^4 + 12*a^4*b*c^
5)*d^7*e^5 + 162*(a^4*b^3*c^3 + 12*a^5*b*c^4)*d^5*e^7 - 9*(a^3*b^
6*c - 15*a^4*b^4*c^2 + 195*a^5*b^2*c^3 + 180*a^6*c^4)*d^4*e^8 - (
a^3*b^7 + 3*a^4*b^5*c + 33*a^5*b^3*c^2 - 2916*a^6*b*c^3)*d^3*e^9
- 3*(a^4*b^6 - 42*a^5*b^4*c + 252*a^6*b^2*c^2 + 432*a^7*c^3)*d^2*
e^10 + 3*(3*a^5*b^5 - 65*a^6*b^3*c + 324*a^7*b*c^2)*d*e^11 - (5*a
^6*b^4 - 81*a^7*b^2*c + 324*a^8*c^2)*e^12)*x + 1/2*sqrt(1/2)*((b^
8*c^4 - 23*a*b^6*c^5 + 190*a^2*b^4*c^6 - 672*a^3*b^2*c^7 + 864*a^
4*c^8)*d^9 + 9*(a*b^7*c^4 - 15*a^2*b^5*c^5 + 72*a^3*b^3*c^6 - 112
*a^4*b*c^7)*d^8*e + 3*(a^2*b^6*c^4 + 28*a^3*b^4*c^5 - 272*a^4*b^2
*c^6 + 576*a^5*c^7)*d^7*e^2 + (a^2*b^7*c^3 - 80*a^3*b^5*c^4 + 592
*a^4*b^3*c^5 - 1152*a^5*b*c^6)*d^6*e^3 + 15*(a^3*b^6*c^3 - 8*a^4*
b^4*c^4 + 16*a^5*b^2*c^5)*d^5*e^4 - 6*(a^3*b^7*c^2 - 17*a^4*b^5*c
^3 + 88*a^5*b^3*c^4 - 144*a^6*b*c^5)*d^4*e^5 - (a^3*b^8*c - 5*a^4
*b^6*c^2 + 100*a^5*b^4*c^3 - 816*a^6*b^2*c^4 + 1728*a^7*c^5)*d^3*
e^6 - 3*(a^4*b^7*c - 32*a^5*b^5*c^2 + 208*a^6*b^3*c^3 - 384*a^7*b
*c^4)*d^2*e^7 - 54*(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4)*d*e
^8 - (a^5*b^7 - 17*a^6*b^5*c + 88*a^7*b^3*c^2 - 144*a^8*b*c^3)*e^
9 - ((a^3*b^9*c^4 - 20*a^4*b^7*c^5 + 144*a^5*b^5*c^6 - 448*a^6*b^
3*c^7 + 512*a^7*b*c^8)*d^3 + 3*(a^4*b^8*c^4 - 8*a^5*b^6*c^5 + 128
*a^7*b^2*c^7 - 256*a^8*c^8)*d^2*e - 12*(a^5*b^7*c^4 - 12*a^6*b^5*
c^5 + 48*a^7*b^3*c^6 - 64*a^8*b*c^7)*d*e^2 - (a^5*b^8*c^3 - 24*a^
6*b^6*c^4 + 192*a^7*b^4*c^5 - 640*a^8*b^2*c^6 + 768*a^9*c^7)*e^3)
*sqrt(-(108*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6
- 18*a*b^2*c^7 + 81*a^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*
d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5
```

 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b*c^5)*d^7*e^5 +
 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12*(a^4
*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*
d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c -
 9*a^7*b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*e^12
)/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))*
sqrt(-((b^5*c^3 - 15*a*b^3*c^4 + 60*a^2*b*c^5)*d^6 + 6*(a*b^4*c^3
 - 6*a^2*b^2*c^4 - 24*a^3*c^5)*d^5*e - 3*(3*a^2*b^3*c^3 - 92*a^3*
b*c^4)*d^4*e^2 - 8*(11*a^3*b^2*c^3 + 36*a^4*c^4)*d^3*e^3 - 3*(3*a
^3*b^3*c^2 - 92*a^4*b*c^3)*d^2*e^4 + 6*(a^3*b^4*c - 6*a^4*b^2*c^2
 - 24*a^5*c^3)*d*e^5 + (a^3*b^5 - 15*a^4*b^3*c + 60*a^5*b*c^2)*e^
6 + (a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6)*
sqrt(-(108*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6 -
 18*a*b^2*c^7 + 81*a^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d
^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5
- 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b*c^5)*d^7*e^5 +
2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12*(a^4*
b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d
^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c -
9*a^7*b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*e^12)
/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))/(
a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6))) + s
qrt(1/2)*(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (
a*b^3*c - 4*a^2*b*c^2)*x^2)*sqrt(-((b^5*c^3 - 15*a*b^3*c^4 + 60*a
^2*b*c^5)*d^6 + 6*(a*b^4*c^3 - 6*a^2*b^2*c^4 - 24*a^3*c^5)*d^5*e
- 3*(3*a^2*b^3*c^3 - 92*a^3*b*c^4)*d^4*e^2 - 8*(11*a^3*b^2*c^3 +
36*a^4*c^4)*d^3*e^3 - 3*(3*a^3*b^3*c^2 - 92*a^4*b*c^3)*d^2*e^4 +
6*(a^3*b^4*c - 6*a^4*b^2*c^2 - 24*a^5*c^3)*d*e^5 + (a^3*b^5 - 15*
a^4*b^3*c + 60*a^5*b*c^2)*e^6 + (a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 4
8*a^5*b^2*c^5 - 64*a^6*c^6)*sqrt(-(108*a^3*b*c^6*d^9*e^3 + 108*a^
6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d^12 - 12
*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*
d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^
4 - 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162
*a^5*c^5)*d^6*e^6 + 12*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(
2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)
*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*d*e^11 - (a^6*b^4 - 18*a
^7*b^2*c + 81*a^8*c^2)*e^12)/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 + 48*a
^8*b^2*c^8 - 64*a^9*c^9)))/(a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5
*b^2*c^5 - 64*a^6*c^6))*log(-((5*b^4*c^6 - 81*a*b^2*c^7 + 324*a^2
*c^8)*d^12 - 3*(3*b^5*c^5 - 65*a*b^3*c^6 + 324*a^2*b*c^7)*d^11*e
+ 3*(b^6*c^4 - 42*a*b^4*c^5 + 252*a^2*b^2*c^6 + 432*a^3*c^7)*d^10
*e^2 + (b^7*c^3 + 3*a*b^5*c^4 + 33*a^2*b^3*c^5 - 2916*a^3*b*c^6)*
d^9*e^3 + 9*(a*b^6*c^3 - 15*a^2*b^4*c^4 + 195*a^3*b^2*c^5 + 180*a
^4*c^6)*d^8*e^4 - 162*(a^3*b^3*c^4 + 12*a^4*b*c^5)*d^7*e^5 + 162*
(a^4*b^3*c^3 + 12*a^5*b*c^4)*d^5*e^7 - 9*(a^3*b^6*c - 15*a^4*b^4*
c^2 + 195*a^5*b^2*c^3 + 180*a^6*c^4)*d^4*e^8 - (a^3*b^7 + 3*a^4*b
^5*c + 33*a^5*b^3*c^2 - 2916*a^6*b*c^3)*d^3*e^9 - 3*(a^4*b^6 - 42
*a^5*b^4*c + 252*a^6*b^2*c^2 + 432*a^7*c^3)*d^2*e^10 + 3*(3*a^5*b
^5 - 65*a^6*b^3*c + 324*a^7*b*c^2)*d*e^11 - (5*a^6*b^4 - 81*a^7*b
^2*c + 324*a^8*c^2)*e^12)*x - 1/2*sqrt(1/2)*((b^8*c^4 - 23*a*b^6*
c^5 + 190*a^2*b^4*c^6 - 672*a^3*b^2*c^7 + 864*a^4*c^8)*d^9 + 9*(a
*b^7*c^4 - 15*a^2*b^5*c^5 + 72*a^3*b^3*c^6 - 112*a^4*b*c^7)*d^8*e
 + 3*(a^2*b^6*c^4 + 28*a^3*b^4*c^5 - 272*a^4*b^2*c^6 + 576*a^5*c^
7)*d^7*e^2 + (a^2*b^7*c^3 - 80*a^3*b^5*c^4 + 592*a^4*b^3*c^5 - 11
52*a^5*b*c^6)*d^6*e^3 + 15*(a^3*b^6*c^3 - 8*a^4*b^4*c^4 + 16*a^5*
b^2*c^5)*d^5*e^4 - 6*(a^3*b^7*c^2 - 17*a^4*b^5*c^3 + 88*a^5*b^3*c
^4 - 144*a^6*b*c^5)*d^4*e^5 - (a^3*b^8*c - 5*a^4*b^6*c^2 + 100*a^
5*b^4*c^3 - 816*a^6*b^2*c^4 + 1728*a^7*c^5)*d^3*e^6 - 3*(a^4*b^7*
c - 32*a^5*b^5*c^2 + 208*a^6*b^3*c^3 - 384*a^7*b*c^4)*d^2*e^7 - 5
4*(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4)*d*e^8 - (a^5*b^7 - 1
7*a^6*b^5*c + 88*a^7*b^3*c^2 - 144*a^8*b*c^3)*e^9 - ((a^3*b^9*c^4
 - 20*a^4*b^7*c^5 + 144*a^5*b^5*c^6 - 448*a^6*b^3*c^7 + 512*a^7*b
*c^8)*d^3 + 3*(a^4*b^8*c^4 - 8*a^5*b^6*c^5 + 128*a^7*b^2*c^7 - 25
6*a^8*c^8)*d^2*e - 12*(a^5*b^7*c^4 - 12*a^6*b^5*c^5 + 48*a^7*b^3*
c^6 - 64*a^8*b*c^7)*d*e^2 - (a^5*b^8*c^3 - 24*a^6*b^6*c^4 + 192*a
^7*b^4*c^5 - 640*a^8*b^2*c^6 + 768*a^9*c^7)*e^3)*sqrt(-(108*a^3*b
*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 +
81*a^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*
b^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8
e^4 + 12(a^3*b^3*c^4 - 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 +
 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12*(a^4*b^3*c^3 - 18*a^5

*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e^8 - 18*(a^6
*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*d*e
^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*e^12)/(a^6*b^6*c^6 -
12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))*sqrt(-((b^5*c^3 -
15*a*b^3*c^4 + 60*a^2*b*c^5)*d^6 + 6*(a*b^4*c^3 - 6*a^2*b^2*c^4
- 24*a^3*c^5)*d^5*e - 3*(3*a^2*b^3*c^3 - 92*a^3*b*c^4)*d^4*e^2 -
8*(11*a^3*b^2*c^3 + 36*a^4*c^4)*d^3*e^3 - 3*(3*a^3*b^3*c^2 - 92*a
^4*b*c^3)*d^2*e^4 + 6*(a^3*b^4*c - 6*a^4*b^2*c^2 - 24*a^5*c^3)*d*
e^5 + (a^3*b^5 - 15*a^4*b^3*c + 60*a^5*b*c^2)*e^6 + (a^3*b^6*c^3
- 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6)*sqrt(-(108*a^3*b*
c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 8
1*a^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b
^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*
e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 +
18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12*(a^4*b^3*c^3 - 18*a^5*
b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e^8 - 18*(a^6*
b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*d*e^
11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*e^12)/(a^6*b^6*c^6 - 1
2*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))/(a^3*b^6*c^3 - 12*
a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6))) - sqrt(1/2)*(a^2*b^2
*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b
*c^2)*x^2)*sqrt(-((b^5*c^3 - 15*a*b^3*c^4 + 60*a^2*b*c^5)*d^6 + 6
*(a*b^4*c^3 - 6*a^2*b^2*c^4 - 24*a^3*c^5)*d^5*e - 3*(3*a^2*b^3*c^
3 - 92*a^3*b*c^4)*d^4*e^2 - 8*(11*a^3*b^2*c^3 + 36*a^4*c^4)*d^3*e
^3 - 3*(3*a^3*b^3*c^2 - 92*a^4*b*c^3)*d^2*e^4 + 6*(a^3*b^4*c - 6*
a^4*b^2*c^2 - 24*a^5*c^3)*d*e^5 + (a^3*b^5 - 15*a^4*b^3*c + 60*a^
5*b*c^2)*e^6 - (a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 6
4*a^6*c^6)*sqrt(-(108*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 -
(b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a
^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a
^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b*c^5)
*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6
+ 12*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9
*a^6*c^4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a
^6*b^3*c - 9*a^7*b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8
*c^2)*e^12)/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a
^9*c^9)))/(a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6
*c^6))*log(-((5*b^4*c^6 - 81*a*b^2*c^7 + 324*a^2*c^8)*d^12 - 3*(3
*b^5*c^5 - 65*a*b^3*c^6 + 324*a^2*b*c^7)*d^11*e + 3*(b^6*c^4 - 42
*a*b^4*c^5 + 252*a^2*b^2*c^6 + 432*a^3*c^7)*d^10*e^2 + (b^7*c^3 +
3*a*b^5*c^4 + 33*a^2*b^3*c^5 - 2916*a^3*b*c^6)*d^9*e^3 + 9*(a*b^
6*c^3 - 15*a^2*b^4*c^4 + 195*a^3*b^2*c^5 + 180*a^4*c^6)*d^8*e^4 -
162*(a^3*b^3*c^4 + 12*a^4*b*c^5)*d^7*e^5 + 162*(a^4*b^3*c^3 + 12
*a^5*b*c^4)*d^5*e^7 - 9*(a^3*b^6*c - 15*a^4*b^4*c^2 + 195*a^5*b^2
*c^3 + 180*a^6*c^4)*d^4*e^8 - (a^3*b^7 + 3*a^4*b^5*c + 33*a^5*b^3
*c^2 - 2916*a^6*b*c^3)*d^3*e^9 - 3*(a^4*b^6 - 42*a^5*b^4*c + 252*
a^6*b^2*c^2 + 432*a^7*c^3)*d^2*e^10 + 3*(3*a^5*b^5 - 65*a^6*b^3*c
+ 324*a^7*b*c^2)*d*e^11 - (5*a^6*b^4 - 81*a^7*b^2*c + 324*a^8*c^
2)*e^12)*x + 1/2*sqrt(1/2)*((b^8*c^4 - 23*a*b^6*c^5 + 190*a^2*b^4
*c^6 - 672*a^3*b^2*c^7 + 864*a^4*c^8)*d^9 + 9*(a*b^7*c^4 - 15*a^2
*b^5*c^5 + 72*a^3*b^3*c^6 - 112*a^4*b*c^7)*d^8*e + 3*(a^2*b^6*c^4
+ 28*a^3*b^4*c^5 - 272*a^4*b^2*c^6 + 576*a^5*c^7)*d^7*e^2 + (a^2
*b^7*c^3 - 80*a^3*b^5*c^4 + 592*a^4*b^3*c^5 - 1152*a^5*b*c^6)*d^6
e^3 + 15(a^3*b^6*c^3 - 8*a^4*b^4*c^4 + 16*a^5*b^2*c^5)*d^5*e^4
- 6*(a^3*b^7*c^2 - 17*a^4*b^5*c^3 + 88*a^5*b^3*c^4 - 144*a^6*b*c^
5)*d^4*e^5 - (a^3*b^8*c - 5*a^4*b^6*c^2 + 100*a^5*b^4*c^3 - 816*a
^6*b^2*c^4 + 1728*a^7*c^5)*d^3*e^6 - 3*(a^4*b^7*c - 32*a^5*b^5*c^
2 + 208*a^6*b^3*c^3 - 384*a^7*b*c^4)*d^2*e^7 - 54*(a^6*b^4*c^2 -
8*a^7*b^2*c^3 + 16*a^8*c^4)*d*e^8 - (a^5*b^7 - 17*a^6*b^5*c + 88*
a^7*b^3*c^2 - 144*a^8*b*c^3)*e^9 + ((a^3*b^9*c^4 - 20*a^4*b^7*c^5
+ 144*a^5*b^5*c^6 - 448*a^6*b^3*c^7 + 512*a^7*b*c^8)*d^3 + 3*(a^
4*b^8*c^4 - 8*a^5*b^6*c^5 + 128*a^7*b^2*c^7 - 256*a^8*c^8)*d^2*e
- 12*(a^5*b^7*c^4 - 12*a^6*b^5*c^5 + 48*a^7*b^3*c^6 - 64*a^8*b*c^
7)*d*e^2 - (a^5*b^8*c^3 - 24*a^6*b^6*c^4 + 192*a^7*b^4*c^5 - 640*
a^8*b^2*c^6 + 768*a^9*c^7)*e^3)*sqrt(-(108*a^3*b*c^6*d^9*e^3 + 10
8*a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d^12
- 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c
^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^
3*c^4 - 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 +
162*a^5*c^5)*d^6*e^6 + 12*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 -
9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*
c^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*d*e^11 - (a^6*b^4 -

18*a^7*b^2*c + 81*a^8*c^2)*e^12)/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 +
48*a^8*b^2*c^8 - 64*a^9*c^9)))*sqrt(-((b^5*c^3 - 15*a*b^3*c^4 + 6
0*a^2*b*c^5)*d^6 + 6*(a*b^4*c^3 - 6*a^2*b^2*c^4 - 24*a^3*c^5)*d^5
e - 3(3*a^2*b^3*c^3 - 92*a^3*b*c^4)*d^4*e^2 - 8*(11*a^3*b^2*c^3
 + 36*a^4*c^4)*d^3*e^3 - 3*(3*a^3*b^3*c^2 - 92*a^4*b*c^3)*d^2*e^4
 + 6*(a^3*b^4*c - 6*a^4*b^2*c^2 - 24*a^5*c^3)*d*e^5 + (a^3*b^5 -
15*a^4*b^3*c + 60*a^5*b*c^2)*e^6 - (a^3*b^6*c^3 - 12*a^4*b^4*c^4
+ 48*a^5*b^2*c^5 - 64*a^6*c^6)*sqrt(-(108*a^3*b*c^6*d^9*e^3 + 108
*a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d^12 -
 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^
7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3
*c^4 - 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 +
162*a^5*c^5)*d^6*e^6 + 12*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 -
9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c
^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*d*e^11 - (a^6*b^4 - 1
8*a^7*b^2*c + 81*a^8*c^2)*e^12)/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 + 4
8*a^8*b^2*c^8 - 64*a^9*c^9)))/(a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*
a^5*b^2*c^5 - 64*a^6*c^6))) + sqrt(1/2)*(a^2*b^2*c - 4*a^3*c^2 +
(a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2)*sqrt(-
((b^5*c^3 - 15*a*b^3*c^4 + 60*a^2*b*c^5)*d^6 + 6*(a*b^4*c^3 - 6*a
^2*b^2*c^4 - 24*a^3*c^5)*d^5*e - 3*(3*a^2*b^3*c^3 - 92*a^3*b*c^4)
*d^4*e^2 - 8*(11*a^3*b^2*c^3 + 36*a^4*c^4)*d^3*e^3 - 3*(3*a^3*b^3
*c^2 - 92*a^4*b*c^3)*d^2*e^4 + 6*(a^3*b^4*c - 6*a^4*b^2*c^2 - 24*
a^5*c^3)*d*e^5 + (a^3*b^5 - 15*a^4*b^3*c + 60*a^5*b*c^2)*e^6 - (a
^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6)*sqrt(-
(108*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*
b^2*c^7 + 81*a^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e
- 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^
4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3
*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12*(a^4*b^3*c^
3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e^8
 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*
b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*e^12)/(a^6*
b^6*c^6 - 12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))/(a^3*b^
6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6))*log(-((5*b
^4*c^6 - 81*a*b^2*c^7 + 324*a^2*c^8)*d^12 - 3*(3*b^5*c^5 - 65*a*b
^3*c^6 + 324*a^2*b*c^7)*d^11*e + 3*(b^6*c^4 - 42*a*b^4*c^5 + 252*
a^2*b^2*c^6 + 432*a^3*c^7)*d^10*e^2 + (b^7*c^3 + 3*a*b^5*c^4 + 33
*a^2*b^3*c^5 - 2916*a^3*b*c^6)*d^9*e^3 + 9*(a*b^6*c^3 - 15*a^2*b^
4*c^4 + 195*a^3*b^2*c^5 + 180*a^4*c^6)*d^8*e^4 - 162*(a^3*b^3*c^4
 + 12*a^4*b*c^5)*d^7*e^5 + 162*(a^4*b^3*c^3 + 12*a^5*b*c^4)*d^5*e
^7 - 9*(a^3*b^6*c - 15*a^4*b^4*c^2 + 195*a^5*b^2*c^3 + 180*a^6*c^
4)*d^4*e^8 - (a^3*b^7 + 3*a^4*b^5*c + 33*a^5*b^3*c^2 - 2916*a^6*b
*c^3)*d^3*e^9 - 3*(a^4*b^6 - 42*a^5*b^4*c + 252*a^6*b^2*c^2 + 432
*a^7*c^3)*d^2*e^10 + 3*(3*a^5*b^5 - 65*a^6*b^3*c + 324*a^7*b*c^2)
*d*e^11 - (5*a^6*b^4 - 81*a^7*b^2*c + 324*a^8*c^2)*e^12)*x - 1/2*
sqrt(1/2)*((b^8*c^4 - 23*a*b^6*c^5 + 190*a^2*b^4*c^6 - 672*a^3*b^
2*c^7 + 864*a^4*c^8)*d^9 + 9*(a*b^7*c^4 - 15*a^2*b^5*c^5 + 72*a^3
*b^3*c^6 - 112*a^4*b*c^7)*d^8*e + 3*(a^2*b^6*c^4 + 28*a^3*b^4*c^5
 - 272*a^4*b^2*c^6 + 576*a^5*c^7)*d^7*e^2 + (a^2*b^7*c^3 - 80*a^3
*b^5*c^4 + 592*a^4*b^3*c^5 - 1152*a^5*b*c^6)*d^6*e^3 + 15*(a^3*b^
6*c^3 - 8*a^4*b^4*c^4 + 16*a^5*b^2*c^5)*d^5*e^4 - 6*(a^3*b^7*c^2
- 17*a^4*b^5*c^3 + 88*a^5*b^3*c^4 - 144*a^6*b*c^5)*d^4*e^5 - (a^3
*b^8*c - 5*a^4*b^6*c^2 + 100*a^5*b^4*c^3 - 816*a^6*b^2*c^4 + 1728
*a^7*c^5)*d^3*e^6 - 3*(a^4*b^7*c - 32*a^5*b^5*c^2 + 208*a^6*b^3*c
^3 - 384*a^7*b*c^4)*d^2*e^7 - 54*(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 1
6*a^8*c^4)*d*e^8 - (a^5*b^7 - 17*a^6*b^5*c + 88*a^7*b^3*c^2 - 144
*a^8*b*c^3)*e^9 + ((a^3*b^9*c^4 - 20*a^4*b^7*c^5 + 144*a^5*b^5*c^
6 - 448*a^6*b^3*c^7 + 512*a^7*b*c^8)*d^3 + 3*(a^4*b^8*c^4 - 8*a^5
*b^6*c^5 + 128*a^7*b^2*c^7 - 256*a^8*c^8)*d^2*e - 12*(a^5*b^7*c^4
 - 12*a^6*b^5*c^5 + 48*a^7*b^3*c^6 - 64*a^8*b*c^7)*d*e^2 - (a^5*b
^8*c^3 - 24*a^6*b^6*c^4 + 192*a^7*b^4*c^5 - 640*a^8*b^2*c^6 + 768
*a^9*c^7)*e^3)*sqrt(-(108*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e
^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d^12 - 12*(a*b^3*c^6 -
 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*
(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b*
c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6
e^6 + 12(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3
 - 9*a^6*c^4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 1
2*(a^6*b^3*c - 9*a^7*b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81
*a^8*c^2)*e^12)/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 -
64*a^9*c^9)))*sqrt(-((b^5*c^3 - 15*a*b^3*c^4 + 60*a^2*b*c^5)*d^6

```
+ 6*(a*b^4*c^3 - 6*a^2*b^2*c^4 - 24*a^3*c^5)*d^5*e - 3*(3*a^2*b^3
*c^3 - 92*a^3*b*c^4)*d^4*e^2 - 8*(11*a^3*b^2*c^3 + 36*a^4*c^4)*d^
3*e^3 - 3*(3*a^3*b^3*c^2 - 92*a^4*b*c^3)*d^2*e^4 + 6*(a^3*b^4*c -
 6*a^4*b^2*c^2 - 24*a^5*c^3)*d*e^5 + (a^3*b^5 - 15*a^4*b^3*c + 60
*a^5*b*c^2)*e^6 - (a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5
- 64*a^6*c^6)*sqrt(-(108*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^
9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d^12 - 12*(a*b^3*c^6 -
9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(
2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b*c
^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*
e^6 + 12*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3
- 9*a^6*c^4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 12
*(a^6*b^3*c - 9*a^7*b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*
a^8*c^2)*e^12)/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 6
4*a^9*c^9)))/(a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*
a^6*c^6))) - 2*(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*
c - 2*a*c^2)*d^3)*x)/(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*
c^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**3/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.275 $\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=386

$$\frac{x\left(x^2\left(abe^2-4acde+bcd^2\right)-2abde-2a\left(cd^2-ae^2\right)+b^2d^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(\frac{b^2\left(cd^2-ae^2\right)+8abcde-4ac\left(ae^2+3cd^2\right)}{\sqrt{b^2-4ac}}+abe^2-4acde+bcd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(-\frac{b^2\left(cd^2-ae^2\right)+8abcde-4ac\left(ae^2+3cd^2\right)}{\sqrt{b^2-4ac}}+abe^2-4acde+bcd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*
d*e + a*b*e^2)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((
b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2
) - 4*a*c*(3*c*d^2 + a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2
 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d^2 - 4*a*c*d*e +
a*b*e^2 - (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^2 + a
*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqr
t[b^2 - 4*a*c]])
```

Rubi [A] time = 4.46892, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{x\left(x^2\left(abe^2-4acde+bcd^2\right)-2abde-2a\left(cd^2-ae^2\right)+b^2d^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(\frac{b^2\left(cd^2-ae^2\right)+8abcde-4ac\left(ae^2+3cd^2\right)}{\sqrt{b^2-4ac}}+abe^2-4acde+bcd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(-\frac{b^2\left(cd^2-ae^2\right)+8abcde-4ac\left(ae^2+3cd^2\right)}{\sqrt{b^2-4ac}}+abe^2-4acde+bcd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*
d*e + a*b*e^2)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((
b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2
) - 4*a*c*(3*c*d^2 + a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2
 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d^2 - 4*a*c*d*e +
a*b*e^2 - (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^2 + a
*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqr
t[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 151.364, size = 415, normalized size = 1.08

$$\frac{x\left(2a^2e^2-2abde-2acd^2+b^2d^2+x^2\left(abe^2-4acde+bcd^2\right)\right)}{2a\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(b\left(abe^2-4acde+bcd^2\right)+2c\left(2a^2e^2-2abde+6acd^2-b^2d^2\right)+\sqrt{-4ac+b^2}\left(abe^2-4acde+bcd^2\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+}}}\right.}{4a\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{\sqrt{2}\left(b\left(abe^2-4acde+bcd^2\right)+2c\left(2a^2e^2-2abde+6acd^2-b^2d^2\right)-\sqrt{-4ac+b^2}\left(abe^2-4acde+bcd^2\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac}}}\right.}{4a\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] x*(2*a**2*e**2 - 2*a*b*d*e - 2*a*c*d**2 + b**2*d**2 + x**2*(a*b*e
**2 - 4*a*c*d*e + b*c*d**2))/(2*a*(-4*a*c + b**2)*(a + b*x**2 + c
*x**4)) + sqrt(2)*(b*(a*b*e**2 - 4*a*c*d*e + b*c*d**2) + 2*c*(2*a
**2*e**2 - 2*a*b*d*e + 6*a*c*d**2 - b**2*d**2) + sqrt(-4*a*c + b*
*2)*(a*b*e**2 - 4*a*c*d*e + b*c*d**2))*atan(sqrt(2)*sqrt(c)*x/sqr
t(b + sqrt(-4*a*c + b**2)))/(4*a*sqrt(c)*sqrt(b + sqrt(-4*a*c + b
**2))*(-4*a*c + b**2)**(3/2)) - sqrt(2)*(b*(a*b*e**2 - 4*a*c*d*e
+ b*c*d**2) + 2*c*(2*a**2*e**2 - 2*a*b*d*e + 6*a*c*d**2 - b**2*d*
*2) - sqrt(-4*a*c + b**2)*(a*b*e**2 - 4*a*c*d*e + b*c*d**2))*atan
(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(4*a*sqrt(c)*sq
rt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2))
```

Mathematica [A] time = 2.54715, size = 415, normalized size = 1.08

$$\frac{\frac{2x\left(2a^2e^2+abe\left(ex^2-2d\right)-2acd\left(d+2ex^2\right)+b^2d^2+bcd^2x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(b^2\left(cd^2-ae^2\right)-4ac\left(e\left(d\sqrt{b^2-4ac}+ae\right)+3cd^2\right)+b\left(cd\left(d\sqrt{b^2-4ac}+8ae\right)+ae^2\sqrt{b^2-4ac}\right)\right)\tan^{-}}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{4a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(b^2*d^2 + 2*a^2*e^2 + b*c*d^2*x^2 + a*b*e*(-2*d + e*x^2) -
 2*a*c*d*(d + 2*e*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (S
qrt[2]*(b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^2 + e*(Sqrt[b^2 - 4*a*
c]*d + a*e)) + b*(a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c
]*d + 8*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[2]*(b^2*(-(c*d^2) + a*e^2) + b*(a*Sqrt[b^2 - 4*a*c]*e^2 +
 c*d*(Sqrt[b^2 - 4*a*c]*d - 8*a*e)) + 4*a*c*(3*c*d^2 + e*(-(Sqrt[
b^2 - 4*a*c]*d) + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 -
 4*a*c]]))/(4*a)
```

Maple [B] time = 0.18, size = 5421, normalized size = 14.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(bcd^2-4\,acde+abe^2)\,x^3-(2\,abde-2\,a^2e^2-(b^2-2\,ac)\,d^2)\,x}{2\,((ab^2c-4\,a^2c^2)x^4+a^2b^2-4\,a^3c+(ab^3-4\,a^2bc)x^2)}+\frac{\int\frac{2\,abde-2\,a^2e^2+(b^2-6\,ac)\,d^2+(bcd^2-4\,acde+abe^2)\,x^2}{cx^4+bx^2+a}\,dx}{2\,(ab^2-4\,a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c*d^2 - 4*a*c*d*e + a*b*e^2)*x^3 - (2*a*b*d*e - 2*a^2*e^2
 - (b^2 - 2*a*c)*d^2)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4
*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((2*a*b*d*e - 2*
a^2*e^2 + (b^2 - 6*a*c)*d^2 + (b*c*d^2 - 4*a*c*d*e + a*b*e^2)*x^2
)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [A] time = 5.84578, size = 9906, normalized size = 25.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*c*d^2 - 4*a*c*d*e + a*b*e^2)*x^3 + sqrt(1/2)*((a*b^2*c
- 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*s
qrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*
a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^
2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*c
)*e^4 + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4
)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e
^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b
^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^
2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4
*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2
 + 48*a^5*b^2*c^3 - 64*a^6*c^4))*log(((5*b^4*c^3 - 81*a*b^2*c^4 +
 324*a^2*c^5)*d^8 - 2*(3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*
d^7*e + (b^6*c - 51*a*b^4*c^2 + 336*a^2*b^2*c^3 + 432*a^3*c^4)*d^
6*e^2 + 2*(3*a*b^5*c - 27*a^2*b^3*c^2 - 244*a^3*b*c^3)*d^5*e^3 +
(3*a^2*b^4*c + 150*a^3*b^2*c^2 + 152*a^4*c^3)*d^4*e^4 - 10*(a^3*b
^3*c + 12*a^4*b*c^2)*d^3*e^5 - (a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c
^2)*d^2*e^6 - 2*(a^4*b^3 + 12*a^5*b*c)*d*e^7 + (3*a^5*b^2 + 4*a^6
*c)*e^8)*x + 1/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c
^3 - 672*a^3*b^2*c^4 + 864*a^4*c^5)*d^6 + 6*(a*b^7*c - 15*a^2*b^5
*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*c^4)*d^5*e + 2*(2*a^2*b^6*c - a
^3*b^4*c^2 - 88*a^4*b^2*c^3 + 240*a^5*c^4)*d^4*e^2 - 12*(a^3*b^5*
c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^3 - (a^3*b^6 - 18*a^4*b^4
*c + 96*a^5*b^2*c^2 - 160*a^6*c^3)*d^2*e^4 - 2*(a^4*b^5 - 8*a^5*b
^3*c + 16*a^6*b*c^2)*d*e^5 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^
2)*e^6 - ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6
*b^3*c^4 + 512*a^7*b*c^5)*d^2 + 2*(a^4*b^8*c - 8*a^5*b^6*c^2 + 12
8*a^7*b^2*c^4 - 256*a^8*c^5)*d*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2
+ 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^2)*sqrt(-(16*a^3*b*c^2*d^5*e^3
 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*
a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e -
 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2
)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^
9*c^5)))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*
b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3
```

```
*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3 +
12*a^4*b*c)*e^4 + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 -
64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a
^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^
8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c
^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 -
12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a
^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))) - sqrt(1/2)*((a*b^2*c
 - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*
sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6
*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d
^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*
c)*e^4 + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^
4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*
e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*
b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e
^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^
4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^
2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))*log(((5*b^4*c^3 - 81*a*b^2*c^4
+ 324*a^2*c^5)*d^8 - 2*(3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)
*d^7*e + (b^6*c - 51*a*b^4*c^2 + 336*a^2*b^2*c^3 + 432*a^3*c^4)*d
^6*e^2 + 2*(3*a*b^5*c - 27*a^2*b^3*c^2 - 244*a^3*b*c^3)*d^5*e^3 +
 (3*a^2*b^4*c + 150*a^3*b^2*c^2 + 152*a^4*c^3)*d^4*e^4 - 10*(a^3*
b^3*c + 12*a^4*b*c^2)*d^3*e^5 - (a^3*b^4 - 24*a^4*b^2*c - 48*a^5*
c^2)*d^2*e^6 - 2*(a^4*b^3 + 12*a^5*b*c)*d*e^7 + (3*a^5*b^2 + 4*a^
6*c)*e^8)*x - 1/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*
c^3 - 672*a^3*b^2*c^4 + 864*a^4*c^5)*d^6 + 6*(a*b^7*c - 15*a^2*b^
5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*c^4)*d^5*e + 2*(2*a^2*b^6*c -
a^3*b^4*c^2 - 88*a^4*b^2*c^3 + 240*a^5*c^4)*d^4*e^2 - 12*(a^3*b^5
*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^3 - (a^3*b^6 - 18*a^4*b^
4*c + 96*a^5*b^2*c^2 - 160*a^6*c^3)*d^2*e^4 - 2*(a^4*b^5 - 8*a^5*
b^3*c + 16*a^6*b*c^2)*d*e^5 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c
^2)*e^6 - ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^
6*b^3*c^4 + 512*a^7*b*c^5)*d^2 + 2*(a^4*b^8*c - 8*a^5*b^6*c^2 + 1
28*a^7*b^2*c^4 - 256*a^8*c^5)*d*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2
 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^2)*sqrt(-(16*a^3*b*c^2*d^5*e^
3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18
*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e
- 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^
2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a
^9*c^5)))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a
*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^
3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3 +
 12*a^4*b*c)*e^4 + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 -
 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*
a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d
^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*
c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 -
 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*
a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))) + sqrt(1/2)*((a*b^2*
c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c -
6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*
d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b
*c)*e^4 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c
^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2
*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a
*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*
e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b
^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c
^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))*log(((5*b^4*c^3 - 81*a*b^2*c^4
 + 324*a^2*c^5)*d^8 - 2*(3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4
)*d^7*e + (b^6*c - 51*a*b^4*c^2 + 336*a^2*b^2*c^3 + 432*a^3*c^4)*
d^6*e^2 + 2*(3*a*b^5*c - 27*a^2*b^3*c^2 - 244*a^3*b*c^3)*d^5*e^3
+ (3*a^2*b^4*c + 150*a^3*b^2*c^2 + 152*a^4*c^3)*d^4*e^4 - 10*(a^3
*b^3*c + 12*a^4*b*c^2)*d^3*e^5 - (a^3*b^4 - 24*a^4*b^2*c - 48*a^5
*c^2)*d^2*e^6 - 2*(a^4*b^3 + 12*a^5*b*c)*d*e^7 + (3*a^5*b^2 + 4*a
^6*c)*e^8)*x + 1/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4
*c^3 - 672*a^3*b^2*c^4 + 864*a^4*c^5)*d^6 + 6*(a*b^7*c - 15*a^2*b
^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*c^4)*d^5*e + 2*(2*a^2*b^6*c -
 a^3*b^4*c^2 - 88*a^4*b^2*c^3 + 240*a^5*c^4)*d^4*e^2 - 12*(a^3*b^
5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^3 - (a^3*b^6 - 18*a^4*b
```

```
^4*c + 96*a^5*b^2*c^2 - 160*a^6*c^3)*d^2*e^4 - 2*(a^4*b^5 - 8*a^5
*b^3*c + 16*a^6*b*c^2)*d*e^5 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*
c^2)*e^6 + ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a
^6*b^3*c^4 + 512*a^7*b*c^5)*d^2 + 2*(a^4*b^8*c - 8*a^5*b^6*c^2 +
128*a^7*b^2*c^4 - 256*a^8*c^5)*d*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^
2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^2)*sqrt(-(16*a^3*b*c^2*d^5*e
^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 1
8*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e
 - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c
^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*
a^9*c^5)))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(
a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a
^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3
+ 12*a^4*b*c)*e^4 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3
- 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4
*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*
d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3
*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2
- 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12
*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))) - sqrt(1/2)*((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2
)*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c -
 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)
*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*
b*c)*e^4 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*
c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^
2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(
a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6
*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*
b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*
c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))*log(((5*b^4*c^3 - 81*a*b^2*c^
4 + 324*a^2*c^5)*d^8 - 2*(3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^
4)*d^7*e + (b^6*c - 51*a*b^4*c^2 + 336*a^2*b^2*c^3 + 432*a^3*c^4)
*d^6*e^2 + 2*(3*a*b^5*c - 27*a^2*b^3*c^2 - 244*a^3*b*c^3)*d^5*e^3
 + (3*a^2*b^4*c + 150*a^3*b^2*c^2 + 152*a^4*c^3)*d^4*e^4 - 10*(a^
3*b^3*c + 12*a^4*b*c^2)*d^3*e^5 - (a^3*b^4 - 24*a^4*b^2*c - 48*a^
5*c^2)*d^2*e^6 - 2*(a^4*b^3 + 12*a^5*b*c)*d*e^7 + (3*a^5*b^2 + 4*
a^6*c)*e^8)*x - 1/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^
4*c^3 - 672*a^3*b^2*c^4 + 864*a^4*c^5)*d^6 + 6*(a*b^7*c - 15*a^2*
b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*c^4)*d^5*e + 2*(2*a^2*b^6*c
- a^3*b^4*c^2 - 88*a^4*b^2*c^3 + 240*a^5*c^4)*d^4*e^2 - 12*(a^3*b
^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^3 - (a^3*b^6 - 18*a^4*
b^4*c + 96*a^5*b^2*c^2 - 160*a^6*c^3)*d^2*e^4 - 2*(a^4*b^5 - 8*a^
5*b^3*c + 16*a^6*b*c^2)*d*e^5 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7
*c^2)*e^6 + ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*
a^6*b^3*c^4 + 512*a^7*b*c^5)*d^2 + 2*(a^4*b^8*c - 8*a^5*b^6*c^2 +
 128*a^7*b^2*c^4 - 256*a^8*c^5)*d*e - 4*(a^5*b^7*c - 12*a^6*b^5*c
^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^2)*sqrt(-(16*a^3*b*c^2*d^5*
e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 -
18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*
e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*
c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64
*a^9*c^5)))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*
(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*
a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3
 + 12*a^4*b*c)*e^4 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3
 - 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 -
4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)
*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^
3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2
 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 1
2*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))) - 2*(2*a*b*d*e - 2
*a^2*e^2 - (b^2 - 2*a*c)*d^2)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2
*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.276 $\int \frac{d+ex^2}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=293

$$\frac{x\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^2))/(2*a*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(b*d - 2*a*e + (b^2*d - 12*
a*c*d + 4*a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*e - (b^2*d - 12*a*c*d +
4*a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2
- 4*a*c]])
```

Rubi [A] time = 1.44407, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{x\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^2))/(2*a*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(b*d - 2*a*e + (b^2*d - 12*
a*c*d + 4*a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*e - (b^2*d - 12*a*c*d +
4*a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2
- 4*a*c]])
```

Rubi in Sympy [A] time = 168.52, size = 279, normalized size = 0.95

$$-\frac{\sqrt{2}\sqrt{c}\left(4abe-12acd+b^2d+\sqrt{-4ac+b^2}\,(2ae-bd)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4a\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\sqrt{2}\sqrt{c}\left(4abe-12acd+b^2d-\sqrt{-4ac+b^2}\,(2ae-bd)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4a\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{x\left(-abe-2acd+b^2d-cx^2\,(2ae-bd)\right)}{2a\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -sqrt(2)*sqrt(c)*(4*a*b*e - 12*a*c*d + b**2*d + sqrt(-4*a*c + b**
2)*(2*a*e - b*d))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b
**2)))/(4*a*sqrt(b + sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2))
 + sqrt(2)*sqrt(c)*(4*a*b*e - 12*a*c*d + b**2*d - sqrt(-4*a*c + b
**2)*(2*a*e - b*d))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c +
 b**2)))/(4*a*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2
)) + x*(-a*b*e - 2*a*c*d + b**2*d - c*x**2*(2*a*e - b*d))/(2*a*(-
4*a*c + b**2)*(a + b*x**2 + c*x**4))
```

Mathematica [A] time = 1.50795, size = 310, normalized size = 1.06

$$\frac{\frac{2x\left(b\left(cdx^2-ae\right)-2ac\left(d+ex^2\right)+b^2d\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(b\left(d\sqrt{b^2-4ac}+4ae\right)-2a\left(e\sqrt{b^2-4ac}+6cd\right)+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(bd\sqrt{b^2-4ac}-2ae\sqrt{b^2-4ac}-4\ldots\right.}{(b^2-4ac)^{3\ldots}}}{4a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(b^2*d + b*(-(a*e) + c*d*x^2) - 2*a*c*(d + e*x^2)))/((b^2 -
 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[
b^2 - 4*a*c]*d + 4*a*e) - 2*a*(6*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*
c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d
) + 12*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - 4*a*b*e - 2*a*Sqrt[b^2 - 4
*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [B] time = 0.1, size = 1761, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/4/(4*a*c-b^2)*(-4*a*c+b^2)^(1/2)/a*x/(x^2+1/2/c*(-4*a*c+b^2)^(1
/2)+1/2*b/c)*d+1/2/(4*a*c-b^2)*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/
2*b/c)*e-1/4/(4*a*c-b^2)/a*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/
c)*b*d-12*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*d*a-8*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)/(
4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d+3/4*c/(4*a*c-b^2)
/(-4*a*c+b^2)^(1/2)/a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
b^4*d+2*c^2/(4*a*c-b^2)*a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*e+3/2*c/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b^2*e-c^2/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b*d-3/4*c/(4*a*c-b^2)/a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*b^3*d+4*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*a/(4*a*c+3*b^2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*b*e+3*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)
/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*e-1/4/(4*a*c-b^2)
```

```
*(-4*a*c+b^2)^(1/2)/a*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*d+
1/2/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*e-1/4/(4
*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*b*d-12*c^3/(
4*a*c-b^2)/(-4*a*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*d*a-8*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)/(4*a*c+3*b^2)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d+3/4*c/(4*a*c-b^2)/(-4*a*c+
b^2)^(1/2)/a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*d-
2*c^2/(4*a*c-b^2)*a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*e-3/2*c/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b^2*e+c^2/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*b*d+3/4*c/(4*a*c-b^2)/a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*b^3*d+4*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*a/(4*a*c+3*b
^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e+3*c/(4*a*c-b^2)/(-4*a*c+b
^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(bcd-2\,ace)x^3-\left(abe-\left(b^2-2\,ac\right)d\right)x}{2\left((ab^2c-4\,a^2c^2)x^4+a^2b^2-4\,a^3c+(ab^3-4\,a^2bc)x^2\right)}-\frac{-\int\frac{abe+(bcd-2\,ace)x^2+\left(b^2-6\,ac\right)d}{cx^4+bx^2+a}\,dx}{2\left(ab^2-4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c*d - 2*a*c*e)*x^3 - (a*b*e - (b^2 - 2*a*c)*d)*x)/((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2
) - 1/2*integrate(-(a*b*e + (b*c*d - 2*a*c*e)*x^2 + (b^2 - 6*a*c)
*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [A] time = 0.995099, size = 6174, normalized size = 21.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*c*d - 2*a*c*e)*x^3 - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x
^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5 - 1
5*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c
^2)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4
 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6
*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^
2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
64*a^6*c^3))*log(-((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d^4 -
 (3*b^5*c - 65*a*b^3*c^2 + 324*a^2*b*c^3)*d^3*e - 3*(3*a*b^4*c -
28*a^2*b^2*c^2)*d^2*e^2 - (9*a^2*b^3*c - 20*a^3*b*c^2)*d*e^3 - (3
*a^3*b^2*c + 4*a^4*c^2)*e^4)*x + 1/2*sqrt(1/2)*((b^8 - 23*a*b^6*c
 + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4)*d^3 + 3*(a*b^
7 - 15*a^2*b^5*c + 72*a^3*b^3*c^2 - 112*a^4*b*c^3)*d^2*e + 3*(a^2
*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 - 32*a^5*c^3)*d*e^2 + (a^3*b
^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 - ((a^3*b^9 - 20*a^4*b^7*c +
```

```
144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*d + (a^4*b^8
- 8*a^5*b^6*c + 128*a^7*b^2*c^3 - 256*a^8*c^4)*e)*sqrt((4*a^3*b*d
*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 -
 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*
a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-((b^5 - 15*a*b^3
*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*e
 + (a^2*b^3 + 12*a^3*b*c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*
b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a
*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b
^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*
c^3))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c
 + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b*c
^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 1
2*a^3*b*c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^
6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2
*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d
^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/
(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log(-((5*
b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d^4 - (3*b^5*c - 65*a*b^3*c
^2 + 324*a^2*b*c^3)*d^3*e - 3*(3*a*b^4*c - 28*a^2*b^2*c^2)*d^2*e^
2 - (9*a^2*b^3*c - 20*a^3*b*c^2)*d*e^3 - (3*a^3*b^2*c + 4*a^4*c^2
)*e^4)*x - 1/2*sqrt(1/2)*((b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 6
72*a^3*b^2*c^3 + 864*a^4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*b^5*c + 72*
a^3*b^3*c^2 - 112*a^4*b*c^3)*d^2*e + 3*(a^2*b^6 - 10*a^3*b^4*c +
32*a^4*b^2*c^2 - 32*a^5*c^3)*d*e^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*
a^5*b*c^2)*e^3 - ((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448
*a^6*b^3*c^3 + 512*a^7*b*c^4)*d + (a^4*b^8 - 8*a^5*b^6*c + 128*a^
7*b^2*c^3 - 256*a^8*c^4)*e)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4
- 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*
(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2
*c^2 - 64*a^9*c^3)))*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2
 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b
*c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*
sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d
^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)
/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^
6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a
*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)
*x^2)*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6
*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 - (a^3*
b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d
*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 -
 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*
a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log(-((5*b^4*c^2 - 81*a*b^2*c^3
 + 324*a^2*c^4)*d^4 - (3*b^5*c - 65*a*b^3*c^2 + 324*a^2*b*c^3)*d^
3*e - 3*(3*a*b^4*c - 28*a^2*b^2*c^2)*d^2*e^2 - (9*a^2*b^3*c - 20*
a^3*b*c^2)*d*e^3 - (3*a^3*b^2*c + 4*a^4*c^2)*e^4)*x + 1/2*sqrt(1/
2)*((b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a
^4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*b^5*c + 72*a^3*b^3*c^2 - 112*a^4*
b*c^3)*d^2*e + 3*(a^2*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 - 32*a^
5*c^3)*d*e^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 + ((a^3
*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7
*b*c^4)*d + (a^4*b^8 - 8*a^5*b^6*c + 128*a^7*b^2*c^3 - 256*a^8*c^
4)*e)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*
c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^
2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*s
qrt(-((b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^
2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 - (a^3*b^6 - 1
2*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 +
a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*
b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4
*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*
a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^
4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5 - 15
*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^
2)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 - (a^3*b^6 - 12*a^4*b^4*c + 4
8*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4
- 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*
(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2
*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 6
```

```
4*a^6*c^3))*log(-((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d^4 -
(3*b^5*c - 65*a*b^3*c^2 + 324*a^2*b*c^3)*d^3*e - 3*(3*a*b^4*c - 2
8*a^2*b^2*c^2)*d^2*e^2 - (9*a^2*b^3*c - 20*a^3*b*c^2)*d*e^3 - (3*
a^3*b^2*c + 4*a^4*c^2)*e^4)*x - 1/2*sqrt(1/2)*((b^8 - 23*a*b^6*c
+ 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4)*d^3 + 3*(a*b^7
 - 15*a^2*b^5*c + 72*a^3*b^3*c^2 - 112*a^4*b*c^3)*d^2*e + 3*(a^2*
b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 - 32*a^5*c^3)*d*e^2 + (a^3*b^
5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 + ((a^3*b^9 - 20*a^4*b^7*c +
144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*d + (a^4*b^8 -
 8*a^5*b^6*c + 128*a^7*b^2*c^3 - 256*a^8*c^4)*e)*sqrt((4*a^3*b*d*
e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 -
9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a
^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-((b^5 - 15*a*b^3*
c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*e
+ (a^2*b^3 + 12*a^3*b*c)*e^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*
b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^
2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c
^3))) - 2*(a*b*e - (b^2 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4
 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

Sympy [A] time = 143.205, size = 1180, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (x**3*(2*a*c*e - b*c*d) + x*(a*b*e + 2*a*c*d - b**2*d))/(8*a**3*c
 - 2*a**2*b**2 + x**4*(8*a**2*c**2 - 2*a*b**2*c) + x**2*(8*a**2*b
*c - 2*a*b**3)) + RootSum(_t**4*(1048576*a**9*c**6 - 1572864*a**8
*b**2*c**5 + 983040*a**7*b**4*c**4 - 327680*a**6*b**6*c**3 + 6144
0*a**5*b**8*c**2 - 6144*a**4*b**10*c + 256*a**3*b**12) + _t**2*(-
12288*a**6*b*c**4*e**2 + 49152*a**6*c**5*d*e + 8192*a**5*b**3*c**
3*e**2 - 24576*a**5*b**2*c**4*d*e - 61440*a**5*b*c**5*d**2 - 1536
*a**4*b**5*c**2*e**2 - 2048*a**4*b**4*c**3*d*e + 61440*a**4*b**3*
c**4*d**2 + 3072*a**3*b**6*c**2*d*e - 24064*a**3*b**5*c**3*d**2 +
 16*a**2*b**9*e**2 - 576*a**2*b**8*c*d*e + 4608*a**2*b**7*c**2*d*
*2 + 32*a*b**10*d*e - 432*a*b**9*c*d**2 + 16*b**11*d**2) + 16*a**
4*c**3*e**4 + 24*a**3*b**2*c**2*e**4 - 224*a**3*b*c**3*d*e**3 + 2
88*a**3*c**4*d**2*e**2 + 9*a**2*b**4*c*e**4 - 144*a**2*b**3*c**2*
d*e**3 + 960*a**2*b**2*c**3*d**2*e**2 - 2016*a**2*b*c**4*d**3*e +
 1296*a**2*c**5*d**4 + 18*a*b**5*c*d*e**3 - 198*a*b**4*c**2*d**2*
e**2 + 496*a*b**3*c**3*d**3*e - 360*a*b**2*c**4*d**4 + 9*b**6*c*d
**2*e**2 - 30*b**5*c**2*d**3*e + 25*b**4*c**3*d**4, Lambda(_t, _t
*log(x + (16384*_t**3*a**8*c**4*e - 8192*_t**3*a**7*b**2*c**3*e -
 32768*_t**3*a**7*b*c**4*d + 28672*_t**3*a**6*b**3*c**3*d + 512*_
t**3*a**5*b**6*c*e - 9216*_t**3*a**5*b**5*c**2*d - 64*_t**3*a**4*
b**8*e + 1280*_t**3*a**4*b**7*c*d - 64*_t**3*a**3*b**9*d - 128*_t
*a**5*b*c**2*e**3 + 576*_t*a**5*c**3*d*e**2 - 16*_t*a**4*b**3*c*e
**3 + 192*_t*a**4*b**2*c**2*d*e**2 - 576*_t*a**4*b*c**3*d**2*e -
1728*_t*a**4*c**4*d**3 - 4*_t*a**3*b**5*e**3 + 60*_t*a**3*b**4*c*
d*e**2 - 528*_t*a**3*b**3*c**2*d**2*e + 2304*_t*a**3*b**2*c**3*d*
*3 - 12*_t*a**2*b**6*d*e**2 + 168*_t*a**2*b**5*c*d**2*e - 740*_t*
a**2*b**4*c**2*d**3 - 12*_t*a*b**7*d**2*e + 92*_t*a*b**6*c*d**3 -
 4*_t*b**8*d**3)/(4*a**4*c**2*e**4 + 3*a**3*b**2*c*e**4 - 20*a**3
*b*c**2*d*e**3 + 9*a**2*b**3*c*d*e**3 - 84*a**2*b**2*c**2*d**2*e*
*2 + 324*a**2*b*c**3*d**3*e - 324*a**2*c**4*d**4 + 9*a*b**4*c*d**
2*e**2 - 65*a*b**3*c**2*d**3*e + 81*a*b**2*c**3*d**4 + 3*b**5*c*d
**3*e - 5*b**4*c**2*d**4))))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.277 $\int \frac{1}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=252

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) + (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*
a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c
- b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]])
```

Rubi [A] time = 0.89651, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) + (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*
a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c
- b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]])
```

Rubi in Sympy [A] time = 61.7599, size = 230, normalized size = 0.91

$$-\frac{\sqrt{2}\sqrt{c}\left(-12ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4a\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\sqrt{2}\sqrt{c}\left(-12ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4a\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -sqrt(2)*sqrt(c)*(-12*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atan(sq
rt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(4*a*sqrt(b + sqrt
(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2)) + sqrt(2)*sqrt(c)*(-12*a
*c + b**2 + b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b
- sqrt(-4*a*c + b**2)))/(4*a*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*
c + b**2)**(3/2)) + x*(-2*a*c + b**2 + b*c*x**2)/(2*a*(-4*a*c + b
**2)*(a + b*x**2 + c*x**4))
```

Mathematica [A] time = 0.817889, size = 243, normalized size = 0.96

$$\frac{\frac{2x\left(-2ac+b^2+bcx^2\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+12ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] ((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) + (Sqrt[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^
(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*
a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/(4*a)
```

Maple [B] time = 0.003, size = 733, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/4/(4*a*c-b^2)/a*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*b-c/(
-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2
*b/c)+1/4/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2/c*(-4*a*c+b
^2)^(1/2)+1/2*b/c)*b^2-1/4*c/(4*a*c-b^2)/a*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*b^2-1/4/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b
^2)^(1/2))*b+c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2/
c*(-4*a*c+b^2)^(1/2))-1/4/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*x/(x^2
+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*b^2+1/4*c/(4*a*c-b^2)/a*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b
^2)/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bcx^3 + \left(b^2 - 2\,ac\right)x}{2\left((ab^2c - 4\,a^2c^2)x^4 + a^2b^2 - 4\,a^3c + (ab^3 - 4\,a^2bc)x^2\right)} + \frac{\int \frac{bcx^2+b^2-6\,ac}{cx^4+bx^2+a}\,dx}{2\left(ab^2 - 4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(-2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*
b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2
 + b^2 - 6*a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [A] time = 0.321271, size = 3117, normalized size = 12.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(-2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 -
 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*
a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3
)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4
)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3
*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^
2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*
a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c
 + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c
^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3
*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*
((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a
^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c +
81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((
5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 -
23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 - (a
^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a
^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*
b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c +
60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*
c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^
5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4
+ a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*
b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a
^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c
 + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 +
324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^
2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7*c + 144
*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*
b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64
*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 1
2*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c
 + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*
c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) -
 sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^
3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3
*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18
*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*
c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1
/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a
^4*c^4 + (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*
c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^
6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15
```

```
*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^
2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 1
2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^
4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + 2*(b^2 - 2*a*c)*x)/((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2
)
```

Sympy [A] time = 15.2992, size = 394, normalized size = 1.56

$$-\frac{bcx^3 + x\left(-2ac + b^2\right)}{8a^3c - 2a^2b^2 + x^4\left(8a^2c^2 - 2ab^2c\right) + x^2\left(8a^2bc - 2ab^3\right)}$$
$$+\mathrm{RootSum}\left(t^4\left(1048576a^9c^6 - 1572864a^8b^2c^5 + 983040a^7b^4c^4 - 327680a^6b^6c^3 + 61440a^5b^8c^2 - 6144a^4b^{10}c + 256a^3b^{12}\right) + t\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -(b*c*x**3 + x*(-2*a*c + b**2))/(8*a**3*c - 2*a**2*b**2 + x**4*(8
*a**2*c**2 - 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + RootSu
m(_t**4*(1048576*a**9*c**6 - 1572864*a**8*b**2*c**5 + 983040*a**7
*b**4*c**4 - 327680*a**6*b**6*c**3 + 61440*a**5*b**8*c**2 - 6144*
a**4*b**10*c + 256*a**3*b**12) + _t**2*(-61440*a**5*b*c**5 + 6144
0*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**2*b**7*c**2 - 4
32*a*b**9*c + 16*b**11) + 1296*a**2*c**5 - 360*a*b**2*c**4 + 25*b
**4*c**3, Lambda(_t, _t*log(x + (32768*_t**3*a**7*b*c**4 - 28672*
_t**3*a**6*b**3*c**3 + 9216*_t**3*a**5*b**5*c**2 - 1280*_t**3*a**
4*b**7*c + 64*_t**3*a**3*b**9 + 1728*_t*a**4*c**4 - 2304*_t*a**3*
b**2*c**3 + 740*_t*a**2*b**4*c**2 - 92*_t*a*b**6*c + 4*_t*b**8)/(
324*a**2*c**4 - 81*a*b**2*c**3 + 5*b**4*c**2))))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(-2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.278 $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=660

$$-\frac{\sqrt{c}e^2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)^2}-\frac{\sqrt{c}e^2\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)^2}+\frac{x\left(cx^2\left(2ace+b^2(-e)+bcd\right)+3abce-2ac^2d+b^3(-e)+b^2cd\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\left(\frac{8abce-12ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+2ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\left(-\frac{8abce-12ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+2ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2-bde+cd^2\right)^2}$$

```
[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e +
2*a*c*e)*x^2))/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*
x^2 + c*x^4)) - (Sqrt[c]*e^2*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[
2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (Sqrt
[c]*(b*c*d - b^2*e + 2*a*c*e + (b^2*c*d - 12*a*c^2*d - b^3*e + 8*
a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2
- 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*e^2*(e + (2*c*d - b
*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*
e + a*e^2)^2) + (Sqrt[c]*(b*c*d - b^2*e + 2*a*c*e - (b^2*c*d - 12
*a*c^2*d - b^3*e + 8*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*
c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^2
)
```

Rubi [A] time = 6.51583, antiderivative size = 660, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{\sqrt{c}e^2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)^2}-\frac{\sqrt{c}e^2\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)^2}+\frac{x\left(cx^2\left(2ace+b^2(-e)+bcd\right)+3abce-2ac^2d+b^3(-e)+b^2cd\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\left(\frac{8abce-12ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+2ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\left(-\frac{8abce-12ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+2ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e +
2*a*c*e)*x^2))/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*
x^2 + c*x^4)) - (Sqrt[c]*e^2*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[
2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (Sqrt
[c]*(b*c*d - b^2*e + 2*a*c*e + (b^2*c*d - 12*a*c^2*d - b^3*e + 8*
a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2
- 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*e^2*(e + (2*c*d - b
*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*
e + a*e^2)^2) + (Sqrt[c]*(b*c*d - b^2*e + 2*a*c*e - (b^2*c*d - 12
*a*c^2*d - b^3*e + 8*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*
c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^2
)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 6.28336, size = 708, normalized size = 1.07

$$\frac{\frac{2x\left(e(ae-bd)+cd^2\right)\left(-bc\left(3ae+cdx^2\right)+2ac^2\left(d-ex^2\right)+b^3e+b^2c\left(ex^2-d\right)\right)}{a(4ac-b^2)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(b^2\left(-cde\left(2d\sqrt{b^2-4ac}+3ae\right)-3ae^3\sqrt{b^2-4ac}+c^2d^3\right)+2ac\left(cde\left(d\sqrt{b^2-4ac}-\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((2*(c*d^2 + e*(-(b*d) + a*e))*x*(b^3*e - b*c*(3*a*e + c*d*x^2) +
 2*a*c^2*(d - e*x^2) + b^2*c*(-d + e*x^2)))/(a*(-b^2 + 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^4*d*e^2 + 2*a*c*(-6*c^2*d
^3 + 5*a*Sqrt[b^2 - 4*a*c]*e^3 + c*d*e*(Sqrt[b^2 - 4*a*c]*d - 14*
a*e)) + b^3*e*(-2*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d - 3*a*e)) + b^2*
(c^2*d^3 - 3*a*Sqrt[b^2 - 4*a*c]*e^3 - c*d*e*(2*Sqrt[b^2 - 4*a*c]
*d + 3*a*e)) + b*c*(a*e^2*(-(Sqrt[b^2 - 4*a*c]*d) + 16*a*e) + c*d
^2*(Sqrt[b^2 - 4*a*c]*d + 20*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-(b^4*d*e^2) - b^2*(c^2*d^3 +
3*a*Sqrt[b^2 - 4*a*c]*e^3 + c*d*e*(2*Sqrt[b^2 - 4*a*c]*d - 3*a*e)
) + b^3*e*(2*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + 2*a*c*(6*
c^2*d^3 + 5*a*Sqrt[b^2 - 4*a*c]*e^3 + c*d*e*(Sqrt[b^2 - 4*a*c]*d
+ 14*a*e)) + b*c*(c*d^2*(Sqrt[b^2 - 4*a*c]*d - 20*a*e) - a*e^2*(S
qrt[b^2 - 4*a*c]*d + 16*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 -
 4*a*c]]) + (4*e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d])/(4*(
c*d^2 + e*(-(b*d) + a*e))^2)
```

Maple [B] time = 0.153, size = 10749, normalized size = 16.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)^2*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)^2*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^2*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.279 $\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=1087

result too large to display

```
[Out] (e^4*x)/(2*d*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x^2)) - (x*(2*b^3*c
*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*a*c^
2*(c*d^2 - a*e^2) + c*(2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - b*c*
(c*d^2 - 3*a*e^2))*x^2))/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*e^2*(3*c^2*d^2 + b*(
b + Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d + 2*Sqrt[b^2 - 4*a*c]*d +
 a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*
e^2)^3) + (Sqrt[c]*(b^4*e^2 - b^3*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e)
 - 4*a*c^2*(3*c*d^2 - e*(Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + b^2*c*(c
*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 9*a*e)) - b*c*(3*a*Sqrt[b^2 - 4
*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 16*a*e)))*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*
c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) -
 (Sqrt[2]*Sqrt[c]*e^2*(3*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2
- c*e*(3*b*d - 2*Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^3) - (Sqrt[c]*(b^4*e^2
 - b^3*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + b*c*(3*a*Sqrt[b^2 - 4*a*
c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 16*a*e)) + b^2*c*(c*d^2 + e*(
2*Sqrt[b^2 - 4*a*c]*d - 9*a*e)) - 4*a*c^2*(3*c*d^2 + e*(Sqrt[b^2
- 4*a*c]*d - 3*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
- 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (2*e^(7/2)*(2*c*d - b*e)*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^3) +
 (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 - b*d*e
+ a*e^2)^2)
```

Rubi [A] time = 24.4454, antiderivative size = 1088, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{xe^4}{2d(cd^2-bed+ae^2)^2(ex^2+d)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}(cd^2-bed+ae^2)^2}+\frac{2(2cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{\sqrt{d}(cd^2-bed+ae^2)^3}$$

$$+\frac{\sqrt{2}\sqrt{c}\left(3c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-ce\left(3bd+2\sqrt{b^2-4ac}d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)e^2}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}(cd^2-bed+ae^2)^3}$$

$$-\frac{\sqrt{2}\sqrt{c}\left(3c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-ce\left(3bd-2\sqrt{b^2-4ac}d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)e^2}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}(cd^2-bed+ae^2)^3}$$

$$+\frac{\sqrt{c}\left(e^2b^4-e\left(2cd-\sqrt{b^2-4ac}e\right)b^3+c\left(cd^2-e\left(2\sqrt{b^2-4ac}d+9ae\right)\right)b^2-c\left(3a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d+16ae\right)\right)\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}(cd^2-bed+ae^2)^2}$$

$$-\frac{\sqrt{c}\left(e^2b^4-e\left(2cd+\sqrt{b^2-4ac}e\right)b^3+c\left(cd^2+e\left(2\sqrt{b^2-4ac}d-9ae\right)\right)b^2+c\left(3a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-16ae\right)\right)\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}(cd^2-e(bd-ae))^2}$$

$$-\frac{x\left(-e^2b^4+2cdeb^3-c\left(cd^2-4ae^2\right)b^2-6ac^2deb+c\left(-e^2b^3+2cdeb^2-c\left(cd^2-3ae^2\right)b-4ac^2de\right)x^2+2ac^2\left(cd^2-ae^2\right)\right)}{2a(b^2-4ac)(cd^2-bed+ae^2)^2(cx^4+bx^2+a)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (e^4*x)/(2*d*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x^2)) - (x*(2*b^3*c
*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*a*c^
```

```
2*(c*d^2 - a*e^2) + c*(2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - b*c*
(c*d^2 - 3*a*e^2))*x^2))/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*e^2*(3*c^2*d^2 + b*(
b + Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d + 2*Sqrt[b^2 - 4*a*c]*d +
 a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*
e^2)^3) + (Sqrt[c]*(b^4*e^2 - b^3*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e)
 - 4*a*c^2*(3*c*d^2 - e*(Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + b^2*c*(c
*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 9*a*e)) - b*c*(3*a*Sqrt[b^2 - 4
*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 16*a*e)))*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*
c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) -
 (Sqrt[2]*Sqrt[c]*e^2*(3*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2
- c*e*(3*b*d - 2*Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^3) - (Sqrt[c]*(b^4*e^2
 - b^3*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + b*c*(3*a*Sqrt[b^2 - 4*a*
c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 16*a*e)) + b^2*c*(c*d^2 + e*(
2*Sqrt[b^2 - 4*a*c]*d - 9*a*e)) - 4*a*c^2*(3*c*d^2 + e*(Sqrt[b^2
- 4*a*c]*d - 3*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
- 4*a*c]]*(c*d^2 - e*(b*d - a*e))^2) + (2*e^(7/2)*(2*c*d - b*e)*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^3) +
 (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 - b*d*e
+ a*e^2)^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 8.0096, size = 1235, normalized size = 1.14

$$\frac{xe^4}{2d\left(cd^2-bed+ae^2\right)^2\left(ex^2+d\right)}+\frac{\left(9cd^2-5bed+ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}\left(cd^2-bed+ae^2\right)^3}$$

$$+\frac{\sqrt{c}\left(de^3b^5-5ae^4b^4+\sqrt{b^2-4ac}de^3b^4-3cd^2e^2b^4-5a\sqrt{b^2-4ac}e^4b^3+5acde^3b^3-3c\sqrt{b^2-4ac}d^2e^2b^3+3c^2d^3eb^3-c^3d^4\ldots\right.}{}$$

$$+\frac{\sqrt{c}\left(-de^3b^5+5ae^4b^4+\sqrt{b^2-4ac}de^3b^4+3cd^2e^2b^4-5a\sqrt{b^2-4ac}e^4b^3-5acde^3b^3-3c\sqrt{b^2-4ac}d^2e^2b^3-3c^2d^3eb^3+c^3d\ldots\right.}{}$$

$$+\frac{-e^2xb^4-ce^2x^3b^3+2cdexb^3+2c^2dex^3b^2-c^2d^2xb^2+4ace^2xb^2-c^3d^2x^3b+3ac^2e^2x^3b-6ac^2dexb-4ac^3dex^3+2ac^3d^2x\ldots}{2a\left(4ac-b^2\right)\left(cd^2-bed+ae^2\right)^2\left(cx^4+bx^2+a\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (e^4*x)/(2*d*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x^2)) + (-(b^2*c^2*
d^2*x) + 2*a*c^3*d^2*x + 2*b^3*c*d*e*x - 6*a*b*c^2*d*e*x - b^4*e^
2*x + 4*a*b^2*c*e^2*x - 2*a^2*c^2*e^2*x - b*c^3*d^2*x^3 + 2*b^2*c
^2*d*e*x^3 - 4*a*c^3*d*e*x^3 - b^3*c*e^2*x^3 + 3*a*b*c^2*e^2*x^3)
/(2*a*(-b^2 + 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x^2 + c*x^4
)) + (Sqrt[c]*(-(b^2*c^3*d^4) + 12*a*c^4*d^4 - b*c^3*Sqrt[b^2 - 4
```

```
*a*c]*d^4 + 3*b^3*c^2*d^3*e - 28*a*b*c^3*d^3*e + 3*b^2*c^2*Sqrt[b
^2 - 4*a*c]*d^3*e - 4*a*c^3*Sqrt[b^2 - 4*a*c]*d^3*e - 3*b^4*c*d^2
*e^2 + 12*a*b^2*c^2*d^2*e^2 + 48*a^2*c^3*d^2*e^2 - 3*b^3*c*Sqrt[b
^2 - 4*a*c]*d^2*e^2 + 6*a*b*c^2*Sqrt[b^2 - 4*a*c]*d^2*e^2 + b^5*d
*e^3 + 5*a*b^3*c*d*e^3 - 52*a^2*b*c^2*d*e^3 + b^4*Sqrt[b^2 - 4*a*
c]*d*e^3 + 7*a*b^2*c*Sqrt[b^2 - 4*a*c]*d*e^3 - 36*a^2*c^2*Sqrt[b^
2 - 4*a*c]*d*e^3 - 5*a*b^4*e^4 + 29*a^2*b^2*c*e^4 - 28*a^3*c^2*e^
4 - 5*a*b^3*Sqrt[b^2 - 4*a*c]*e^4 + 19*a^2*b*c*Sqrt[b^2 - 4*a*c]*
e^4)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*
Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^
2) + b*d*e - a*e^2)^3) + (Sqrt[c]*(b^2*c^3*d^4 - 12*a*c^4*d^4 - b
*c^3*Sqrt[b^2 - 4*a*c]*d^4 - 3*b^3*c^2*d^3*e + 28*a*b*c^3*d^3*e +
 3*b^2*c^2*Sqrt[b^2 - 4*a*c]*d^3*e - 4*a*c^3*Sqrt[b^2 - 4*a*c]*d^
3*e + 3*b^4*c*d^2*e^2 - 12*a*b^2*c^2*d^2*e^2 - 48*a^2*c^3*d^2*e^2
 - 3*b^3*c*Sqrt[b^2 - 4*a*c]*d^2*e^2 + 6*a*b*c^2*Sqrt[b^2 - 4*a*c
]*d^2*e^2 - b^5*d*e^3 - 5*a*b^3*c*d*e^3 + 52*a^2*b*c^2*d*e^3 + b^
4*Sqrt[b^2 - 4*a*c]*d*e^3 + 7*a*b^2*c*Sqrt[b^2 - 4*a*c]*d*e^3 - 3
6*a^2*c^2*Sqrt[b^2 - 4*a*c]*d*e^3 + 5*a*b^4*e^4 - 29*a^2*b^2*c*e^
4 + 28*a^3*c^2*e^4 - 5*a*b^3*Sqrt[b^2 - 4*a*c]*e^4 + 19*a^2*b*c*S
qrt[b^2 - 4*a*c]*e^4)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
- 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)^3) + (e^(7/2)*(9*c*d^2 - 5*b
*d*e + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 - b*
d*e + a*e^2)^3)
```

Maple [B] time = 0.193, size = 14860, normalized size = 13.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)^2*(e*x^2 + d)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)^2*(e*x^2 + d)^2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^2*(e*x^2 + d)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.280 $\int \left(d+ex^2\right)^{5/2}\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=215

$$\frac{x\left(d+ex^2\right)^{5/2}\left(80ae^2-10bde+3cd^2\right)}{480e^2}+\frac{dx\left(d+ex^2\right)^{3/2}\left(80ae^2-10bde+3cd^2\right)}{384e^2}+\frac{d^2x\sqrt{d+ex^2}\left(80ae^2-10bde+3cd^2\right)}{256e^2}+\frac{d^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(80ae^2-10bde+3cd^2\right)}{256e^{5/2}}-\frac{x\left(d+ex^2\right)^{7/2}(3cd-10be)}{80e^2}+\frac{cx^3\left(d+ex^2\right)^{7/2}}{10e}$$

```
[Out] (d^2*(3*c*d^2 - 10*b*d*e + 80*a*e^2)*x*Sqrt[d + e*x^2])/(256*e^2)
 + (d*(3*c*d^2 - 10*b*d*e + 80*a*e^2)*x*(d + e*x^2)^(3/2))/(384*e
^2) + ((3*c*d^2 - 10*b*d*e + 80*a*e^2)*x*(d + e*x^2)^(5/2))/(480*
e^2) - ((3*c*d - 10*b*e)*x*(d + e*x^2)^(7/2))/(80*e^2) + (c*x^3*(
d + e*x^2)^(7/2))/(10*e) + (d^3*(3*c*d^2 - 10*b*d*e + 80*a*e^2)*A
rcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(256*e^(5/2))
```

Rubi [A] time = 0.302854, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{x\left(d+ex^2\right)^{5/2}\left(80ae^2-10bde+3cd^2\right)}{480e^2}+\frac{dx\left(d+ex^2\right)^{3/2}\left(80ae^2-10bde+3cd^2\right)}{384e^2}+\frac{d^2x\sqrt{d+ex^2}\left(80ae^2-10bde+3cd^2\right)}{256e^2}+\frac{d^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(80ae^2-10bde+3cd^2\right)}{256e^{5/2}}-\frac{x\left(d+ex^2\right)^{7/2}(3cd-10be)}{80e^2}+\frac{cx^3\left(d+ex^2\right)^{7/2}}{10e}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (d^2*(3*c*d^2 - 10*b*d*e + 80*a*e^2)*x*Sqrt[d + e*x^2])/(256*e^2)
 + (d*(3*c*d^2 - 10*b*d*e + 80*a*e^2)*x*(d + e*x^2)^(3/2))/(384*e
^2) + ((3*c*d^2 - 10*b*d*e + 80*a*e^2)*x*(d + e*x^2)^(5/2))/(480*
e^2) - ((3*c*d - 10*b*e)*x*(d + e*x^2)^(7/2))/(80*e^2) + (c*x^3*(
d + e*x^2)^(7/2))/(10*e) + (d^3*(3*c*d^2 - 10*b*d*e + 80*a*e^2)*A
rcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(256*e^(5/2))
```

Rubi in Sympy [A] time = 27.2144, size = 211, normalized size = 0.98

$$\frac{cx^3\left(d+ex^2\right)^{\frac{7}{2}}}{10e}+\frac{d^3\left(80ae^2-10bde+3cd^2\right)\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{256e^{\frac{5}{2}}}+\frac{d^2x\sqrt{d+ex^2}\left(80ae^2-10bde+3cd^2\right)}{256e^2}+\frac{dx\left(d+ex^2\right)^{\frac{3}{2}}\left(80ae^2-10bde+3cd^2\right)}{384e^2}+\frac{x\left(d+ex^2\right)^{\frac{7}{2}}(10be-3cd)}{80e^2}+\frac{x\left(d+ex^2\right)^{\frac{5}{2}}\left(80ae^2-10bde+3cd^2\right)}{480e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**(5/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] c*x**3*(d + e*x**2)**(7/2)/(10*e) + d**3*(80*a*e**2 - 10*b*d*e +
3*c*d**2)*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(256*e**(5/2)) + d**2
```

```
*x*sqrt(d + e*x**2)*(80*a*e**2 - 10*b*d*e + 3*c*d**2)/(256*e**2)
+ d*x*(d + e*x**2)**(3/2)*(80*a*e**2 - 10*b*d*e + 3*c*d**2)/(384*
e**2) + x*(d + e*x**2)**(7/2)*(10*b*e - 3*c*d)/(80*e**2) + x*(d +
 e*x**2)**(5/2)*(80*a*e**2 - 10*b*d*e + 3*c*d**2)/(480*e**2)
```

Mathematica [A] time = 0.383082, size = 177, normalized size = 0.82

$$\sqrt{d+ex^2}\left(\frac{1}{480}x^5\left(80ae^2+170bde+93cd^2\right)+\frac{dx^3\left(208ae^2+118bde+3cd^2\right)}{384e}-\frac{d^2x\left(-176ae^2-10bde+3cd^2\right)}{256e^2}+\frac{1}{80}ex^7(10be+21cd)+\frac{1}{10}ce^2x^9\right)+\frac{d^3\log\left(\sqrt{e}\sqrt{d+ex^2}+ex\right)\left(80ae^2-10bde+3cd^2\right)}{256e^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] Sqrt[d + e*x^2]*(-(d^2*(3*c*d^2 - 10*b*d*e - 176*a*e^2)*x)/(256*e
^2) + (d*(3*c*d^2 + 118*b*d*e + 208*a*e^2)*x^3)/(384*e) + ((93*c*
d^2 + 170*b*d*e + 80*a*e^2)*x^5)/480 + (e*(21*c*d + 10*b*e)*x^7)/
80 + (c*e^2*x^9)/10) + (d^3*(3*c*d^2 - 10*b*d*e + 80*a*e^2)*Log[e
*x + Sqrt[e]*Sqrt[d + e*x^2]])/(256*e^(5/2))
```

Maple [A] time = 0.014, size = 283, normalized size = 1.3

$$\frac{ax}{6}\left(ex^2+d\right)^{\frac{5}{2}}+\frac{5\,adx}{24}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{5\,ad^2x}{16}\sqrt{ex^2+d}+\frac{5\,ad^3}{16}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)\frac{1}{\sqrt{e}}+\frac{bx}{8\,e}\left(ex^2+d\right)^{\frac{7}{2}}-\frac{bdx}{48\,e}\left(ex^2+d\right)^{\frac{5}{2}}-\frac{5\,xbd^2}{192\,e}\left(ex^2+d\right)^{\frac{3}{2}}-\frac{5\,d^3bx}{128\,e}\sqrt{ex^2+d}-\frac{5\,d^4b}{128}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{3}{2}}+\frac{cx^3}{10\,e}\left(ex^2+d\right)^{\frac{7}{2}}-\frac{3\,cdx}{80\,e^2}\left(ex^2+d\right)^{\frac{7}{2}}+\frac{cd^2x}{160\,e^2}\left(ex^2+d\right)^{\frac{5}{2}}+\frac{cd^3x}{128\,e^2}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{3\,cd^4x}{256\,e^2}\sqrt{ex^2+d}+\frac{3\,cd^5}{256}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/6*a*x*(e*x^2+d)^(5/2)+5/24*a*d*x*(e*x^2+d)^(3/2)+5/16*a*d^2*x*(
e*x^2+d)^(1/2)+5/16*a*d^3/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+1
/8*b*x*(e*x^2+d)^(7/2)/e-1/48*b*d/e*x*(e*x^2+d)^(5/2)-5/192*b*d^2
/e*x*(e*x^2+d)^(3/2)-5/128*b*d^3/e*x*(e*x^2+d)^(1/2)-5/128*b*d^4/
e^(3/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+1/10*c*x^3*(e*x^2+d)^(7/2)/
e-3/80*c*d/e^2*x*(e*x^2+d)^(7/2)+1/160*c*d^2/e^2*x*(e*x^2+d)^(5/2
)+1/128*c*d^3/e^2*x*(e*x^2+d)^(3/2)+3/256*c*d^4/e^2*x*(e*x^2+d)^(
1/2)+3/256*c*d^5/e^(5/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.494495, size = 1, normalized size = 0.

$$\left[\frac{2\left(384\,ce^4x^9+48\left(21\,cde^3+10\,be^4\right)x^7+8\left(93\,cd^2e^2+170\,bde^3+80\,ae^4\right)x^5+10\left(3\,cd^3e+118\,bd^2e^2+208\,ade^3\right)x^3-15\left(3\right.\right.}{7680\,e^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(2*(384*c*e^4*x^9 + 48*(21*c*d*e^3 + 10*b*e^4)*x^7 + 8*(9
3*c*d^2*e^2 + 170*b*d*e^3 + 80*a*e^4)*x^5 + 10*(3*c*d^3*e + 118*b
*d^2*e^2 + 208*a*d*e^3)*x^3 - 15*(3*c*d^4 - 10*b*d^3*e - 176*a*d^
2*e^2)*x)*sqrt(e*x^2 + d)*sqrt(e) + 15*(3*c*d^5 - 10*b*d^4*e + 80
*a*d^3*e^2)*log(-2*sqrt(e*x^2 + d)*e*x - (2*e*x^2 + d)*sqrt(e)))/
e^(5/2), 1/3840*((384*c*e^4*x^9 + 48*(21*c*d*e^3 + 10*b*e^4)*x^7
+ 8*(93*c*d^2*e^2 + 170*b*d*e^3 + 80*a*e^4)*x^5 + 10*(3*c*d^3*e +
 118*b*d^2*e^2 + 208*a*d*e^3)*x^3 - 15*(3*c*d^4 - 10*b*d^3*e - 17
6*a*d^2*e^2)*x)*sqrt(e*x^2 + d)*sqrt(-e) + 15*(3*c*d^5 - 10*b*d^4
*e + 80*a*d^3*e^2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(sqrt(-e)*
e^2)]
```

Sympy [A] time = 166.516, size = 505, normalized size = 2.35

$$\frac{ad^{\frac{5}{2}}x\sqrt{1+\frac{ex^2}{d}}}{2}+\frac{3ad^{\frac{5}{2}}x}{16\sqrt{1+\frac{ex^2}{d}}}+\frac{35ad^{\frac{3}{2}}ex^3}{48\sqrt{1+\frac{ex^2}{d}}}+\frac{17a\sqrt{d}e^2x^5}{24\sqrt{1+\frac{ex^2}{d}}}+\frac{5ad^3\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16\sqrt{e}}$$
$$+\frac{ae^3x^7}{6\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}+\frac{5bd^{\frac{7}{2}}x}{128e\sqrt{1+\frac{ex^2}{d}}}+\frac{133bd^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{ex^2}{d}}}+\frac{127bd^{\frac{3}{2}}ex^5}{192\sqrt{1+\frac{ex^2}{d}}}+\frac{23b\sqrt{d}e^2x^7}{48\sqrt{1+\frac{ex^2}{d}}}$$
$$-\frac{5bd^4\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{128e^{\frac{3}{2}}}+\frac{be^3x^9}{8\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}-\frac{3cd^{\frac{9}{2}}x}{256e^2\sqrt{1+\frac{ex^2}{d}}}-\frac{cd^{\frac{7}{2}}x^3}{256e\sqrt{1+\frac{ex^2}{d}}}$$
$$+\frac{129cd^{\frac{5}{2}}x^5}{640\sqrt{1+\frac{ex^2}{d}}}+\frac{73cd^{\frac{3}{2}}ex^7}{160\sqrt{1+\frac{ex^2}{d}}}+\frac{29c\sqrt{d}e^2x^9}{80\sqrt{1+\frac{ex^2}{d}}}+\frac{3cd^5\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{256e^{\frac{5}{2}}}+\frac{ce^3x^{11}}{10\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**(5/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**(5/2)*x*sqrt(1 + e*x**2/d)/2 + 3*a*d**(5/2)*x/(16*sqrt(1 + e
*x**2/d)) + 35*a*d**(3/2)*e*x**3/(48*sqrt(1 + e*x**2/d)) + 17*a*s
qrt(d)*e**2*x**5/(24*sqrt(1 + e*x**2/d)) + 5*a*d**3*asinh(sqrt(e)
*x/sqrt(d))/(16*sqrt(e)) + a*e**3*x**7/(6*sqrt(d)*sqrt(1 + e*x**2
/d)) + 5*b*d**(7/2)*x/(128*e*sqrt(1 + e*x**2/d)) + 133*b*d**(5/2)
*x**3/(384*sqrt(1 + e*x**2/d)) + 127*b*d**(3/2)*e*x**5/(192*sqrt(
1 + e*x**2/d)) + 23*b*sqrt(d)*e**2*x**7/(48*sqrt(1 + e*x**2/d)) -
 5*b*d**4*asinh(sqrt(e)*x/sqrt(d))/(128*e**(3/2)) + b*e**3*x**9/(
8*sqrt(d)*sqrt(1 + e*x**2/d)) - 3*c*d**(9/2)*x/(256*e**2*sqrt(1 +
 e*x**2/d)) - c*d**(7/2)*x**3/(256*e*sqrt(1 + e*x**2/d)) + 129*c*
d**(5/2)*x**5/(640*sqrt(1 + e*x**2/d)) + 73*c*d**(3/2)*e*x**7/(16
0*sqrt(1 + e*x**2/d)) + 29*c*sqrt(d)*e**2*x**9/(80*sqrt(1 + e*x**
2/d)) + 3*c*d**5*asinh(sqrt(e)*x/sqrt(d))/(256*e**(5/2)) + c*e**3
*x**11/(10*sqrt(d)*sqrt(1 + e*x**2/d))
```

GIAC/XCAS [A] time = 0.287158, size = 243, normalized size = 1.13

$$-\frac{1}{256}\left(3\,cd^5 - 10\,bd^4e + 80\,ad^3e^2\right)e^{\left(-\frac{5}{2}\right)}\ln\left(\left|-xe^{\frac{1}{2}} + \sqrt{x^2e + d}\right|\right) + \frac{1}{3840}\left(2\left(4\left(6\left(8\,cx^2e^2 + \left(21\,cde^9 + 10\,be^{10}\right)e^{(-8)}\right)x^2 + \left(93\,cd^2e^8 + 170\,bde^9 + 80\,ae^{10}\right)e^{(-8)}\right)x^2 + 5\left(3\,cd^3e^7 + 118\,bd^2e^8 + 2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(5/2),x, algorithm="giac")
```

```
[Out] -1/256*(3*c*d^5 - 10*b*d^4*e + 80*a*d^3*e^2)*e^(-5/2)*ln(abs(-x*e
^(1/2) + sqrt(x^2*e + d))) + 1/3840*(2*(4*(6*(8*c*x^2*e^2 + (21*c
*d*e^9 + 10*b*e^10)*e^(-8))*x^2 + (93*c*d^2*e^8 + 170*b*d*e^9 + 8
0*a*e^10)*e^(-8))*x^2 + 5*(3*c*d^3*e^7 + 118*b*d^2*e^8 + 208*a*d*
e^9)*e^(-8))*x^2 - 15*(3*c*d^4*e^6 - 10*b*d^3*e^7 - 176*a*d^2*e^8
)*e^(-8))*sqrt(x^2*e + d)*x
```

3.281 $\int \left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=175

$$\frac{x\left(d+ex^2\right)^{3/2}\left(48ae^2-8bde+3cd^2\right)}{192e^2}+\frac{dx\sqrt{d+ex^2}\left(48ae^2-8bde+3cd^2\right)}{128e^2}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(48ae^2-8bde+3cd^2\right)}{128e^{5/2}}-\frac{x\left(d+ex^2\right)^{5/2}(3cd-8be)}{48e^2}+\frac{cx^3\left(d+ex^2\right)^{5/2}}{8e}$$

```
[Out] (d*(3*c*d^2 - 8*b*d*e + 48*a*e^2)*x*Sqrt[d + e*x^2])/(128*e^2) +
((3*c*d^2 - 8*b*d*e + 48*a*e^2)*x*(d + e*x^2)^(3/2))/(192*e^2) -
((3*c*d - 8*b*e)*x*(d + e*x^2)^(5/2))/(48*e^2) + (c*x^3*(d + e*x^
2)^(5/2))/(8*e) + (d^2*(3*c*d^2 - 8*b*d*e + 48*a*e^2)*ArcTanh[(Sq
rt[e]*x)/Sqrt[d + e*x^2]])/(128*e^(5/2))
```

Rubi [A] time = 0.244707, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{x\left(d+ex^2\right)^{3/2}\left(48ae^2-8bde+3cd^2\right)}{192e^2}+\frac{dx\sqrt{d+ex^2}\left(48ae^2-8bde+3cd^2\right)}{128e^2}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(48ae^2-8bde+3cd^2\right)}{128e^{5/2}}-\frac{x\left(d+ex^2\right)^{5/2}(3cd-8be)}{48e^2}+\frac{cx^3\left(d+ex^2\right)^{5/2}}{8e}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (d*(3*c*d^2 - 8*b*d*e + 48*a*e^2)*x*Sqrt[d + e*x^2])/(128*e^2) +
((3*c*d^2 - 8*b*d*e + 48*a*e^2)*x*(d + e*x^2)^(3/2))/(192*e^2) -
((3*c*d - 8*b*e)*x*(d + e*x^2)^(5/2))/(48*e^2) + (c*x^3*(d + e*x^
2)^(5/2))/(8*e) + (d^2*(3*c*d^2 - 8*b*d*e + 48*a*e^2)*ArcTanh[(Sq
rt[e]*x)/Sqrt[d + e*x^2]])/(128*e^(5/2))
```

Rubi in Sympy [A] time = 23.3841, size = 170, normalized size = 0.97

$$\frac{cx^3\left(d+ex^2\right)^{\frac{5}{2}}}{8e}+\frac{d^2\left(48ae^2-8bde+3cd^2\right)\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{128e^{\frac{5}{2}}}+\frac{dx\sqrt{d+ex^2}\left(48ae^2-8bde+3cd^2\right)}{128e^2}+\frac{x\left(d+ex^2\right)^{\frac{5}{2}}(8be-3cd)}{48e^2}+\frac{x\left(d+ex^2\right)^{\frac{3}{2}}\left(48ae^2-8bde+3cd^2\right)}{192e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**(3/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] c*x**3*(d + e*x**2)**(5/2)/(8*e) + d**2*(48*a*e**2 - 8*b*d*e + 3*
c*d**2)*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(128*e**(5/2)) + d*x*sq
rt(d + e*x**2)*(48*a*e**2 - 8*b*d*e + 3*c*d**2)/(128*e**2) + x*(d
 + e*x**2)**(5/2)*(8*b*e - 3*c*d)/(48*e**2) + x*(d + e*x**2)**(3/
2)*(48*a*e**2 - 8*b*d*e + 3*c*d**2)/(192*e**2)
```

Mathematica [A] time = 0.197433, size = 146, normalized size = 0.83

$$\sqrt{d+ex^2}\left(\frac{x^3\left(48ae^2+56bde+3cd^2\right)}{192e}-\frac{dx\left(-80ae^2-8bde+3cd^2\right)}{128e^2}+\frac{1}{48}x^5(8be+9cd)+\frac{1}{8}cex^7\right)+\frac{d^2\log\left(\sqrt{e}\sqrt{d+ex^2}+ex\right)\left(48ae^2-8bde+3cd^2\right)}{128e^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] Sqrt[d + e*x^2]*(-(d*(3*c*d^2 - 8*b*d*e - 80*a*e^2)*x)/(128*e^2)
+ ((3*c*d^2 + 56*b*d*e + 48*a*e^2)*x^3)/(192*e) + ((9*c*d + 8*b*e
)*x^5)/48 + (c*e*x^7)/8) + (d^2*(3*c*d^2 - 8*b*d*e + 48*a*e^2)*Lo
g[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(128*e^(5/2))
```

Maple [A] time = 0.014, size = 229, normalized size = 1.3

$$\frac{ax}{4}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{3\,adx}{8}\sqrt{ex^2+d}+\frac{3\,ad^2}{8}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)\frac{1}{\sqrt{e}}+\frac{bx}{6\,e}\left(ex^2+d\right)^{\frac{5}{2}}-\frac{bdx}{24\,e}\left(ex^2+d\right)^{\frac{3}{2}}-\frac{xbd^2}{16\,e}\sqrt{ex^2+d}-\frac{bd^3}{16}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{3}{2}}+\frac{cx^3}{8\,e}\left(ex^2+d\right)^{\frac{5}{2}}-\frac{cdx}{16\,e^2}\left(ex^2+d\right)^{\frac{5}{2}}+\frac{cd^2x}{64\,e^2}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{3\,cd^3x}{128\,e^2}\sqrt{ex^2+d}+\frac{3\,cd^4}{128}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*a*x*(e*x^2+d)^(3/2)+3/8*a*d*x*(e*x^2+d)^(1/2)+3/8*a*d^2/e^(1/
2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+1/6*b*x*(e*x^2+d)^(5/2)/e-1/24*b
*d/e*x*(e*x^2+d)^(3/2)-1/16*b*d^2/e*x*(e*x^2+d)^(1/2)-1/16*b*d^3/
e^(3/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+1/8*c*x^3*(e*x^2+d)^(5/2)/e
-1/16*c*d/e^2*x*(e*x^2+d)^(5/2)+1/64*c*d^2/e^2*x*(e*x^2+d)^(3/2)+
3/128*c*d^3/e^2*x*(e*x^2+d)^(1/2)+3/128*c*d^4/e^(5/2)*ln(x*e^(1/2
)+(e*x^2+d)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.364653, size = 1, normalized size = 0.01

$$\left[\frac{2\left(48\,ce^3x^7+8\left(9\,cde^2+8\,be^3\right)x^5+2\left(3\,cd^2e+56\,bde^2+48\,ae^3\right)x^3-3\left(3\,cd^3-8\,bd^2e-80\,ade^2\right)x\right)\sqrt{ex^2+d}\sqrt{e}+3\left(3\,cd\right.}{768\,e^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/768*(2*(48*c*e^3*x^7 + 8*(9*c*d*e^2 + 8*b*e^3)*x^5 + 2*(3*c*d^
2*e + 56*b*d*e^2 + 48*a*e^3)*x^3 - 3*(3*c*d^3 - 8*b*d^2*e - 80*a*
d*e^2)*x)*sqrt(e*x^2 + d)*sqrt(e) + 3*(3*c*d^4 - 8*b*d^3*e + 48*a
*d^2*e^2)*log(-2*sqrt(e*x^2 + d)*e*x - (2*e*x^2 + d)*sqrt(e)))/e^
(5/2), 1/384*((48*c*e^3*x^7 + 8*(9*c*d*e^2 + 8*b*e^3)*x^5 + 2*(3*
c*d^2*e + 56*b*d*e^2 + 48*a*e^3)*x^3 - 3*(3*c*d^3 - 8*b*d^2*e - 8
0*a*d*e^2)*x)*sqrt(e*x^2 + d)*sqrt(-e) + 3*(3*c*d^4 - 8*b*d^3*e +
 48*a*d^2*e^2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(sqrt(-e)*e^2)
]
```

Sympy [A] time = 85.9207, size = 413, normalized size = 2.36

$$\frac{ad^{\frac{3}{2}}x\sqrt{1+\frac{ex^2}{d}}}{2}+\frac{ad^{\frac{3}{2}}x}{8\sqrt{1+\frac{ex^2}{d}}}+\frac{3a\sqrt{d}ex^3}{8\sqrt{1+\frac{ex^2}{d}}}+\frac{3ad^2\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{e}}+\frac{ae^2x^5}{4\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}+\frac{bd^{\frac{5}{2}}x}{16e\sqrt{1+\frac{ex^2}{d}}}+\frac{17bd^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{ex^2}{d}}}+\frac{11b\sqrt{d}ex^5}{24\sqrt{1+\frac{ex^2}{d}}}-\frac{bd^3\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16e^{\frac{3}{2}}}+\frac{be^2x^7}{6\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}-\frac{3cd^{\frac{7}{2}}x}{128e^2\sqrt{1+\frac{ex^2}{d}}}-\frac{cd^{\frac{5}{2}}x^3}{128e\sqrt{1+\frac{ex^2}{d}}}+\frac{13cd^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{ex^2}{d}}}+\frac{5c\sqrt{d}ex^7}{16\sqrt{1+\frac{ex^2}{d}}}+\frac{3cd^4\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{128e^{\frac{5}{2}}}+\frac{ce^2x^9}{8\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**(3/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**(3/2)*x*sqrt(1 + e*x**2/d)/2 + a*d**(3/2)*x/(8*sqrt(1 + e*x*
*2/d)) + 3*a*sqrt(d)*e*x**3/(8*sqrt(1 + e*x**2/d)) + 3*a*d**2*asi
nh(sqrt(e)*x/sqrt(d))/(8*sqrt(e)) + a*e**2*x**5/(4*sqrt(d)*sqrt(1
 + e*x**2/d)) + b*d**(5/2)*x/(16*e*sqrt(1 + e*x**2/d)) + 17*b*d**
(3/2)*x**3/(48*sqrt(1 + e*x**2/d)) + 11*b*sqrt(d)*e*x**5/(24*sqrt
(1 + e*x**2/d)) - b*d**3*asinh(sqrt(e)*x/sqrt(d))/(16*e**(3/2)) +
 b*e**2*x**7/(6*sqrt(d)*sqrt(1 + e*x**2/d)) - 3*c*d**(7/2)*x/(128
*e**2*sqrt(1 + e*x**2/d)) - c*d**(5/2)*x**3/(128*e*sqrt(1 + e*x**
2/d)) + 13*c*d**(3/2)*x**5/(64*sqrt(1 + e*x**2/d)) + 5*c*sqrt(d)*
e*x**7/(16*sqrt(1 + e*x**2/d)) + 3*c*d**4*asinh(sqrt(e)*x/sqrt(d)
)/(128*e**(5/2)) + c*e**2*x**9/(8*sqrt(d)*sqrt(1 + e*x**2/d))
```

GIAC/XCAS [A] time = 0.269201, size = 196, normalized size = 1.12

$$-\frac{1}{128}\left(3\,cd^4-8\,bd^3e+48\,ad^2e^2\right)e^{\left(-\frac{5}{2}\right)}\ln\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)+\frac{1}{384}\left(2\left(4\left(6\,cx^2e+\left(9\,cde^6+8\,be^7\right)e^{(-6)}\right)x^2+\left(3\,cd^2e^5+56\,bde^6+48\,ae^7\right)e^{(-6)}\right)x^2-3\left(3\,cd^3e^4-8\,bd^2e^5-80\,ade^6\right)e^{(-6)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2),x, algorithm="giac")
```

```
[Out] -1/128*(3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*e^(-5/2)*ln(abs(-x*e^
(1/2) + sqrt(x^2*e + d))) + 1/384*(2*(4*(6*c*x^2*e + (9*c*d*e^6 +
 8*b*e^7)*e^(-6))*x^2 + (3*c*d^2*e^5 + 56*b*d*e^6 + 48*a*e^7)*e^(
-6))*x^2 - 3*(3*c*d^3*e^4 - 8*b*d^2*e^5 - 80*a*d*e^6)*e^(-6))*sqr
t(x^2*e + d)*x
```

3.282 $\int \sqrt{d+ex^2}\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=132

$$\frac{x\sqrt{d+ex^2}\left(8ae^2-2bde+cd^2\right)}{16e^2}+\frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(8ae^2-2bde+cd^2\right)}{16e^{5/2}}-\frac{x\left(d+ex^2\right)^{3/2}(cd-2be)}{8e^2}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

```
[Out] ((c*d^2 - 2*b*d*e + 8*a*e^2)*x*Sqrt[d + e*x^2])/(16*e^2) - ((c*d
- 2*b*e)*x*(d + e*x^2)^(3/2))/(8*e^2) + (c*x^3*(d + e*x^2)^(3/2))
/(6*e) + (d*(c*d^2 - 2*b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[
d + e*x^2]])/(16*e^(5/2))
```

Rubi [A] time = 0.211031, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{x\sqrt{d+ex^2}\left(8ae^2-2bde+cd^2\right)}{16e^2}+\frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(8ae^2-2bde+cd^2\right)}{16e^{5/2}}-\frac{x\left(d+ex^2\right)^{3/2}(cd-2be)}{8e^2}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d^2 - 2*b*d*e + 8*a*e^2)*x*Sqrt[d + e*x^2])/(16*e^2) - ((c*d
- 2*b*e)*x*(d + e*x^2)^(3/2))/(8*e^2) + (c*x^3*(d + e*x^2)^(3/2))
/(6*e) + (d*(c*d^2 - 2*b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[
d + e*x^2]])/(16*e^(5/2))
```

Rubi in Sympy [A] time = 19.8546, size = 124, normalized size = 0.94

$$\frac{cx^3\left(d+ex^2\right)^{\frac{3}{2}}}{6e}+\frac{d\left(8ae^2-2bde+cd^2\right)\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{16e^{\frac{5}{2}}}+\frac{x\left(d+ex^2\right)^{\frac{3}{2}}(2be-cd)}{8e^2}+\frac{x\sqrt{d+ex^2}\left(8ae^2-2bde+cd^2\right)}{16e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**(1/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] c*x**3*(d + e*x**2)**(3/2)/(6*e) + d*(8*a*e**2 - 2*b*d*e + c*d**2
)*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(16*e**(5/2)) + x*(d + e*x**2
)**(3/2)*(2*b*e - c*d)/(8*e**2) + x*sqrt(d + e*x**2)*(8*a*e**2 -
2*b*d*e + c*d**2)/(16*e**2)
```

Mathematica [A] time = 0.126588, size = 112, normalized size = 0.85

$$\frac{3d\log\left(\sqrt{e}\sqrt{d+ex^2}+ex\right)\left(8ae^2-2bde+cd^2\right)+\sqrt{e}x\sqrt{d+ex^2}\left(6e\left(4ae+b\left(d+2ex^2\right)\right)+c\left(-3d^2+2dex^2+8e^2x^4\right)\right)}{48e^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(c*(-3*d^2 + 2*d*e*x^2 + 8*e^2*x^4) +
6*e*(4*a*e + b*(d + 2*e*x^2))) + 3*d*(c*d^2 - 2*b*d*e + 8*a*e^2)*
Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(48*e^(5/2))
```

Maple [A] time = 0.012, size = 175, normalized size = 1.3

$$\frac{ax}{2}\sqrt{ex^2+d}+\frac{ad}{2}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)\frac{1}{\sqrt{e}}+\frac{bx}{4e}\left(ex^2+d\right)^{\frac{3}{2}}-\frac{bdx}{8e}\sqrt{ex^2+d}-\frac{bd^2}{8}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{3}{2}}+\frac{cx^3}{6e}\left(ex^2+d\right)^{\frac{3}{2}}-\frac{cdx}{8e^2}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{cd^2x}{16e^2}\sqrt{ex^2+d}+\frac{cd^3}{16}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*a*x*(e*x^2+d)^(1/2)+1/2*a*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1
/2))+1/4*b*x*(e*x^2+d)^(3/2)/e-1/8*b*d/e*x*(e*x^2+d)^(1/2)-1/8*b*
d^2/e^(3/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+1/6*c*x^3*(e*x^2+d)^(3/
2)/e-1/8*c*d/e^2*x*(e*x^2+d)^(3/2)+1/16*c*d^2/e^2*x*(e*x^2+d)^(1/
2)+1/16*c*d^3/e^(5/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.328305, size = 1, normalized size = 0.01

$$\left[\frac{2\left(8\,ce^2x^5+2\left(cde+6\,be^2\right)x^3-3\left(cd^2-2\,bde-8\,ae^2\right)x\right)\sqrt{ex^2+d}\sqrt{e}+3\left(cd^3-2\,bd^2e+8\,ade^2\right)\log\left(-2\sqrt{ex^2+d}ex-\left(\right.\right.}{96\,e^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d),x, algorithm="fricas")
```

```
[Out] [1/96*(2*(8*c*e^2*x^5 + 2*(c*d*e + 6*b*e^2)*x^3 - 3*(c*d^2 - 2*b*
d*e - 8*a*e^2)*x)*sqrt(e*x^2 + d)*sqrt(e) + 3*(c*d^3 - 2*b*d^2*e
+ 8*a*d*e^2)*log(-2*sqrt(e*x^2 + d)*e*x - (2*e*x^2 + d)*sqrt(e)))
/e^(5/2), 1/48*((8*c*e^2*x^5 + 2*(c*d*e + 6*b*e^2)*x^3 - 3*(c*d^2
 - 2*b*d*e - 8*a*e^2)*x)*sqrt(e*x^2 + d)*sqrt(-e) + 3*(c*d^3 - 2*
b*d^2*e + 8*a*d*e^2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(sqrt(-e
)*e^2)]
```

Sympy [A] time = 33.4551, size = 272, normalized size = 2.06

$$\frac{a\sqrt{d}x\sqrt{1+\frac{ex^2}{d}}}{2}+\frac{ad\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{e}}+\frac{bd^{\frac{3}{2}}x}{8e\sqrt{1+\frac{ex^2}{d}}}+\frac{3b\sqrt{d}x^3}{8\sqrt{1+\frac{ex^2}{d}}}-\frac{bd^2\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8e^{\frac{3}{2}}}+\frac{bex^5}{4\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}-\frac{cd^{\frac{5}{2}}x}{16e^2\sqrt{1+\frac{ex^2}{d}}}-\frac{cd^{\frac{3}{2}}x^3}{48e\sqrt{1+\frac{ex^2}{d}}}+\frac{5c\sqrt{d}x^5}{24\sqrt{1+\frac{ex^2}{d}}}+\frac{cd^3\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16e^{\frac{5}{2}}}+\frac{cex^7}{6\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**(1/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*sqrt(d)*x*sqrt(1 + e*x**2/d)/2 + a*d*asinh(sqrt(e)*x/sqrt(d))/(
2*sqrt(e)) + b*d**(3/2)*x/(8*e*sqrt(1 + e*x**2/d)) + 3*b*sqrt(d)*
x**3/(8*sqrt(1 + e*x**2/d)) - b*d**2*asinh(sqrt(e)*x/sqrt(d))/(8*
e**(3/2)) + b*e*x**5/(4*sqrt(d)*sqrt(1 + e*x**2/d)) - c*d**(5/2)*
x/(16*e**2*sqrt(1 + e*x**2/d)) - c*d**(3/2)*x**3/(48*e*sqrt(1 + e
*x**2/d)) + 5*c*sqrt(d)*x**5/(24*sqrt(1 + e*x**2/d)) + c*d**3*asi
nh(sqrt(e)*x/sqrt(d))/(16*e**(5/2)) + c*e*x**7/(6*sqrt(d)*sqrt(1
+ e*x**2/d))
```

GIAC/XCAS [A] time = 0.266153, size = 143, normalized size = 1.08

$$-\frac{1}{16}\left(cd^3-2\,bd^2e+8\,ade^2\right)e^{\left(-\frac{5}{2}\right)}\ln\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)+\frac{1}{48}\left(2\left(4\,cx^2+\left(cde^3+6\,be^4\right)e^{(-4)}\right)x^2-3\left(cd^2e^2-2\,bde^3-8\,ae^4\right)e^{(-4)}\right)\sqrt{x^2e+d}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d),x, algorithm="giac")
```

```
[Out] -1/16*(c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*e^(-5/2)*ln(abs(-x*e^(1/2)
+ sqrt(x^2*e + d))) + 1/48*(2*(4*c*x^2 + (c*d*e^3 + 6*b*e^4)*e^(-
4))*x^2 - 3*(c*d^2*e^2 - 2*b*d*e^3 - 8*a*e^4)*e^(-4))*sqrt(x^2*e
+ d)*x
```

3.283 $\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=97

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(8ae^2-4bde+3cd^2\right)}{8e^{5/2}}-\frac{x\sqrt{d+ex^2}(3cd-4be)}{8e^2}+\frac{cx^3\sqrt{d+ex^2}}{4e}$$

```
[Out] -((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/(8*e^2) + (c*x^3*Sqrt[d + e*
x^2])/(4*e) + ((3*c*d^2 - 4*b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/
Sqrt[d + e*x^2]])/(8*e^(5/2))
```

Rubi [A] time = 0.13211, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(8ae^2-4bde+3cd^2\right)}{8e^{5/2}}-\frac{x\sqrt{d+ex^2}(3cd-4be)}{8e^2}+\frac{cx^3\sqrt{d+ex^2}}{4e}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/Sqrt[d + e*x^2],x]
```

```
[Out] -((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/(8*e^2) + (c*x^3*Sqrt[d + e*
x^2])/(4*e) + ((3*c*d^2 - 4*b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/
Sqrt[d + e*x^2]])/(8*e^(5/2))
```

Rubi in Sympy [A] time = 16.893, size = 90, normalized size = 0.93

$$\frac{cx^3\sqrt{d+ex^2}}{4e}+\frac{x\sqrt{d+ex^2}\,(4be-3cd)}{8e^2}+\frac{\left(8ae^2-4bde+3cd^2\right)\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] c*x**3*sqrt(d + e*x**2)/(4*e) + x*sqrt(d + e*x**2)*(4*b*e - 3*c*d
)/(8*e**2) + (8*a*e**2 - 4*b*d*e + 3*c*d**2)*atanh(sqrt(e)*x/sqrt
(d + e*x**2))/(8*e**(5/2))
```

Mathematica [A] time = 0.0928843, size = 85, normalized size = 0.88

$$\frac{\log\left(\sqrt{e}\sqrt{d+ex^2}+ex\right)\left(8ae^2-4bde+3cd^2\right)+\sqrt{e}x\sqrt{d+ex^2}\left(4be-3cd+2cex^2\right)}{8e^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(-3*c*d + 4*b*e + 2*c*e*x^2) + (3*c*d^
2 - 4*b*d*e + 8*a*e^2)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(8*e^(
5/2))
```

Maple [A] time = 0.011, size = 122, normalized size = 1.3

$$a\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)\frac{1}{\sqrt{e}}+\frac{bx}{2\,e}\sqrt{ex^2+d}-\frac{bd}{2}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{3}{2}}+\frac{cx^3}{4\,e}\sqrt{ex^2+d}-\frac{3\,cdx}{8\,e^2}\sqrt{ex^2+d}+\frac{3\,cd^2}{8}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] a*ln(x*e^(1/2)+(e*x^2+d)^(1/2))/e^(1/2)+1/2*b*x/e*(e*x^2+d)^(1/2)
-1/2*b*d/e^(3/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+1/4*c*x^3*(e*x^2+d
)^(1/2)/e-3/8*c*d/e^2*x*(e*x^2+d)^(1/2)+3/8*c*d^2/e^(5/2)*ln(x*e^
(1/2)+(e*x^2+d)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/sqrt(e*x^2 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.2953, size = 1, normalized size = 0.01

$$\left[\frac{2\left(2\,cex^3-(3\,cd-4\,be)x\right)\sqrt{ex^2+d}\sqrt{e}+\left(3\,cd^2-4\,bde+8\,ae^2\right)\log\left(-2\sqrt{ex^2+d}\,ex-\left(2\,ex^2+d\right)\sqrt{e}\right)}{16\,e^{\frac{5}{2}}},\ \frac{(2\,cex^3-(3\,cd-4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/sqrt(e*x^2 + d),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(2*c*e*x^3 - (3*c*d - 4*b*e)*x)*sqrt(e*x^2 + d)*sqrt(e)
+ (3*c*d^2 - 4*b*d*e + 8*a*e^2)*log(-2*sqrt(e*x^2 + d)*e*x - (2*e
*x^2 + d)*sqrt(e)))/e^(5/2), 1/8*((2*c*e*x^3 - (3*c*d - 4*b*e)*x)
*sqrt(e*x^2 + d)*sqrt(-e) + (3*c*d^2 - 4*b*d*e + 8*a*e^2)*arctan(
sqrt(-e)*x/sqrt(e*x^2 + d)))/(sqrt(-e)*e^2)]
```

Sympy [A] time = 19.5, size = 230, normalized size = 2.37

$$a\left(\begin{cases}\frac{\sqrt{-\frac{d}{e}}\operatorname{asin}\left(x\sqrt{-\frac{e}{d}}\right)}{\sqrt{d}} & \text{for}\ d>0\wedge e<0\\ \frac{\sqrt{\frac{d}{e}}\operatorname{asinh}\left(x\sqrt{\frac{e}{d}}\right)}{\sqrt{d}} & \text{for}\ d>0\wedge e>0\\ \frac{\sqrt{-\frac{d}{e}}\operatorname{acosh}\left(x\sqrt{-\frac{e}{d}}\right)}{\sqrt{-d}} & \text{for}\ e>0\wedge d<0\end{cases}\right)+\frac{b\sqrt{d}x\sqrt{1+\frac{ex^2}{d}}}{2e}-\frac{bd\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{\frac{3}{2}}}-\frac{3cd^{\frac{3}{2}}x}{8e^2\sqrt{1+\frac{ex^2}{d}}}-\frac{c\sqrt{d}x^3}{8e\sqrt{1+\frac{ex^2}{d}}}+\frac{3cd^2\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8e^{\frac{5}{2}}}+\frac{cx^5}{4\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] a*Piecewise((sqrt(-d/e)*asin(x*sqrt(-e/d))/sqrt(d), (d > 0) & (e
< 0)), (sqrt(d/e)*asinh(x*sqrt(e/d))/sqrt(d), (d > 0) & (e > 0)),
 (sqrt(-d/e)*acosh(x*sqrt(-e/d))/sqrt(-d), (e > 0) & (d < 0))) +
b*sqrt(d)*x*sqrt(1 + e*x**2/d)/(2*e) - b*d*asinh(sqrt(e)*x/sqrt(d
))/(2*e**(3/2)) - 3*c*d**(3/2)*x/(8*e**2*sqrt(1 + e*x**2/d)) - c*
sqrt(d)*x**3/(8*e*sqrt(1 + e*x**2/d)) + 3*c*d**2*asinh(sqrt(e)*x/
sqrt(d))/(8*e**(5/2)) + c*x**5/(4*sqrt(d)*sqrt(1 + e*x**2/d))
```

GIAC/XCAS [A] time = 0.268649, size = 107, normalized size = 1.1

$$-\frac{1}{8}\left(3\,cd^2 - 4\,bde + 8\,ae^2\right)e^{\left(-\frac{5}{2}\right)}\ln\left(\left|-xe^{\frac{1}{2}} + \sqrt{x^2e + d}\right|\right) + \frac{1}{8}\left(2\,cx^2e^{(-1)} - \left(3\,cde - 4\,be^2\right)e^{(-3)}\right)\sqrt{x^2e + d}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/sqrt(e*x^2 + d),x, algorithm="giac")
```

```
[Out] -1/8*(3*c*d^2 - 4*b*d*e + 8*a*e^2)*e^(-5/2)*ln(abs(-x*e^(1/2) + s
qrt(x^2*e + d))) + 1/8*(2*c*x^2*e^(-1) - (3*c*d*e - 4*b*e^2)*e^(-
3))*sqrt(x^2*e + d)*x
```

3.284 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{d\sqrt{d+ex^2}}-\frac{(3cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{5/2}}+\frac{cx\sqrt{d+ex^2}}{2e^2}$$

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(d*Sqrt[d + e*x^2]) + (c*x*Sqrt[d +
 e*x^2])/(2*e^2) - ((3*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
e*x^2]])/(2*e^(5/2))
```

Rubi [A] time = 0.152003, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{d\sqrt{d+ex^2}}-\frac{(3cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{5/2}}+\frac{cx\sqrt{d+ex^2}}{2e^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2),x]
```

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(d*Sqrt[d + e*x^2]) + (c*x*Sqrt[d +
 e*x^2])/(2*e^2) - ((3*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
e*x^2]])/(2*e^(5/2))
```

Rubi in Sympy [A] time = 19.5912, size = 85, normalized size = 0.96

$$\frac{cx\sqrt{d+ex^2}}{2e^2}+\frac{(2be-3cd)\,\mathrm{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{\frac{5}{2}}}+\frac{x\left(ae^2-bde+cd^2\right)}{de^2\sqrt{d+ex^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(3/2),x)
```

```
[Out] c*x*sqrt(d + e*x**2)/(2*e**2) + (2*b*e - 3*c*d)*atanh(sqrt(e)*x/s
qrt(d + e*x**2))/(2*e**(5/2)) + x*(a*e**2 - b*d*e + c*d**2)/(d*e*
*2*sqrt(d + e*x**2))
```

Mathematica [A] time = 0.152854, size = 87, normalized size = 0.98

$$\frac{\frac{\sqrt{e}x\left(2e(ae-bd)+cd\left(3d+ex^2\right)\right)}{d\sqrt{d+ex^2}}+(2be-3cd)\log\left(\sqrt{e}\sqrt{d+ex^2}+ex\right)}{2e^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2),x]
```

```
[Out] ((Sqrt[e]*x*(2*e*(-(b*d) + a*e) + c*d*(3*d + e*x^2)))/(d*Sqrt[d +
 e*x^2]) + (-3*c*d + 2*b*e)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(
2*e^(5/2))
```

Maple [A] time = 0.011, size = 112, normalized size = 1.3

$$\frac{ax}{d}\frac{1}{\sqrt{ex^2+d}}-\frac{bx}{e}\frac{1}{\sqrt{ex^2+d}}+b\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{3}{2}}+\frac{cx^3}{2e}\frac{1}{\sqrt{ex^2+d}}+\frac{3cdx}{2e^2}\frac{1}{\sqrt{ex^2+d}}-\frac{3cd}{2}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x)
```

```
[Out] a*x/d/(e*x^2+d)^(1/2)-b*x/e/(e*x^2+d)^(1/2)+b/e^(3/2)*ln(x*e^(1/2
)+(e*x^2+d)^(1/2))+1/2*c*x^3/e/(e*x^2+d)^(1/2)+3/2*c*d/e^2*x/(e*x
^2+d)^(1/2)-3/2*c*d/e^(5/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.298928, size = 1, normalized size = 0.01

$$\left[\frac{2\left(cdex^3+\left(3cd^2-2bde+2ae^2\right)x\right)\sqrt{ex^2+d}\sqrt{e}-\left(3cd^3-2bd^2e+\left(3cd^2e-2bde^2\right)x^2\right)\log\left(-2\sqrt{ex^2+d}ex-\left(2ex^2+\right.\right.}{4\left(de^3x^2+d^2e^2\right)\sqrt{e}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(c*d*e*x^3 + (3*c*d^2 - 2*b*d*e + 2*a*e^2)*x)*sqrt(e*x^2
+ d)*sqrt(e) - (3*c*d^3 - 2*b*d^2*e + (3*c*d^2*e - 2*b*d*e^2)*x^2
)*log(-2*sqrt(e*x^2 + d)*e*x - (2*e*x^2 + d)*sqrt(e)))/((d*e^3*x^
2 + d^2*e^2)*sqrt(e)), 1/2*((c*d*e*x^3 + (3*c*d^2 - 2*b*d*e + 2*a
*e^2)*x)*sqrt(e*x^2 + d)*sqrt(-e) - (3*c*d^3 - 2*b*d^2*e + (3*c*d
^2*e - 2*b*d*e^2)*x^2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/((d*e^
3*x^2 + d^2*e^2)*sqrt(-e))]
```

Sympy [A] time = 22.1744, size = 134, normalized size = 1.51

$$\frac{ax}{d^{\frac{3}{2}}\sqrt{1+\frac{ex^2}{d}}}+b\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{\frac{3}{2}}}-\frac{x}{\sqrt{d}e\sqrt{1+\frac{ex^2}{d}}}\right)+c\left(\frac{3\sqrt{d}x}{2e^2\sqrt{1+\frac{ex^2}{d}}}-\frac{3d\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{d}e\sqrt{1+\frac{ex^2}{d}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(3/2),x)
```

```
[Out] a*x/(d**(3/2)*sqrt(1 + e*x**2/d)) + b*(asinh(sqrt(e)*x/sqrt(d))/e
**(3/2) - x/(sqrt(d)*e*sqrt(1 + e*x**2/d))) + c*(3*sqrt(d)*x/(2*e
**2*sqrt(1 + e*x**2/d)) - 3*d*asinh(sqrt(e)*x/sqrt(d))/(2*e**(5/2
)) + x**3/(2*sqrt(d)*e*sqrt(1 + e*x**2/d)))
```

GIAC/XCAS [A] time = 0.26993, size = 108, normalized size = 1.21

$$\frac{1}{2}\,(3\,cd-2\,be)e^{\left(-\frac{5}{2}\right)}\ln\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)+\frac{\left(cx^2e^{(-1)}+\frac{\left(3\,cd^2e-2\,bde^2+2\,ae^3\right)e^{(-3)}}{d}\right)x}{2\,\sqrt{x^2e+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(3*c*d - 2*b*e)*e^(-5/2)*ln(abs(-x*e^(1/2) + sqrt(x^2*e + d))
) + 1/2*(c*x^2*e^(-1) + (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*e^(-3)/
d)*x/sqrt(x^2*e + d)
```

3.285 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{x\left(4cd^2-e(2ae+bd)\right)}{3d^2e^2\sqrt{d+ex^2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{c\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}$$

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(3*d*(d + e*x^2)^(3/2)) - ((4*c*d^2
 - e*(b*d + 2*a*e))*x)/(3*d^2*e^2*Sqrt[d + e*x^2]) + (c*ArcTanh[(
Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(5/2)
```

Rubi [A] time = 0.148578, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{x\left(4cd^2-e(2ae+bd)\right)}{3d^2e^2\sqrt{d+ex^2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{c\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2),x]
```

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(3*d*(d + e*x^2)^(3/2)) - ((4*c*d^2
 - e*(b*d + 2*a*e))*x)/(3*d^2*e^2*Sqrt[d + e*x^2]) + (c*ArcTanh[(
Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(5/2)
```

Rubi in Sympy [A] time = 28.7816, size = 97, normalized size = 0.96

$$\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{\frac{5}{2}}}+\frac{x\left(ae^2-bde+cd^2\right)}{3de^2\left(d+ex^2\right)^{\frac{3}{2}}}+\frac{x\left(2ae^2+bde-4cd^2\right)}{3d^2e^2\sqrt{d+ex^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(5/2),x)
```

```
[Out] c*atanh(sqrt(e)*x/sqrt(d + e*x**2))/e**(5/2) + x*(a*e**2 - b*d*e
+ c*d**2)/(3*d*e**2*(d + e*x**2)**(3/2)) + x*(2*a*e**2 + b*d*e -
4*c*d**2)/(3*d**2*e**2*sqrt(d + e*x**2))
```

Mathematica [A] time = 0.243741, size = 91, normalized size = 0.9

$$\frac{x\left(e^2\left(3ad+2aex^2+bdx^2\right)-cd^2\left(3d+4ex^2\right)\right)}{3d^2e^2\left(d+ex^2\right)^{3/2}}+\frac{c\log\left(\sqrt{e}\sqrt{d+ex^2}+ex\right)}{e^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2),x]
```

```
[Out] (x*(-(c*d^2*(3*d + 4*e*x^2)) + e^2*(3*a*d + b*d*x^2 + 2*a*e*x^2))
)/(3*d^2*e^2*(d + e*x^2)^(3/2)) + (c*Log[e*x + Sqrt[e]*Sqrt[d + e
*x^2]])/e^(5/2)
```

Maple [A] time = 0.011, size = 124, normalized size = 1.2

$$\frac{ax}{3\,d}\left(ex^{2}+d\right)^{-\frac{3}{2}}+\frac{2\,ax}{3\,d^{2}}\frac{1}{\sqrt{ex^{2}+d}}-\frac{bx}{3\,e}\left(ex^{2}+d\right)^{-\frac{3}{2}}+\frac{bx}{3\,de}\frac{1}{\sqrt{ex^{2}+d}}-\frac{cx^{3}}{3\,e}\left(ex^{2}+d\right)^{-\frac{3}{2}}-\frac{cx}{e^{2}}\frac{1}{\sqrt{ex^{2}+d}}+c\ln\left(x\sqrt{e}+\sqrt{ex^{2}+d}\right)e^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a*x/d/(e*x^2+d)^(3/2)+2/3*a/d^2*x/(e*x^2+d)^(1/2)-1/3*b/e*x/(
e*x^2+d)^(3/2)+1/3*b/d/e*x/(e*x^2+d)^(1/2)-1/3*c*x^3/e/(e*x^2+d)^
(3/2)-c/e^2*x/(e*x^2+d)^(1/2)+c/e^(5/2)*ln(x*e^(1/2)+(e*x^2+d)^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.305514, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(\left(4\,cd^{2}e-bde^{2}-2\,ae^{3}\right)x^{3}+3\left(cd^{3}-ade^{2}\right)x\right)\sqrt{ex^{2}+d}\sqrt{e}-3\left(cd^{2}e^{2}x^{4}+2\,cd^{3}ex^{2}+cd^{4}\right)\log\left(-2\sqrt{ex^{2}+d}ex-\left(2\,ex\right.\right.}{6\left(d^{2}e^{4}x^{4}+2\,d^{3}e^{3}x^{2}+d^{4}e^{2}\right)\sqrt{e}},\right.$$
$$\left.-\frac{\left(\left(4\,cd^{2}e-bde^{2}-2\,ae^{3}\right)x^{3}+3\left(cd^{3}-ade^{2}\right)x\right)\sqrt{ex^{2}+d}\sqrt{-e}-3\left(cd^{2}e^{2}x^{4}+2\,cd^{3}ex^{2}+cd^{4}\right)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^{2}+d}}\right)}{3\left(d^{2}e^{4}x^{4}+2\,d^{3}e^{3}x^{2}+d^{4}e^{2}\right)\sqrt{-e}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*((4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3 + 3*(c*d^3 - a*d*e^
2)*x)*sqrt(e*x^2 + d)*sqrt(e) - 3*(c*d^2*e^2*x^4 + 2*c*d^3*e*x^2
+ c*d^4)*log(-2*sqrt(e*x^2 + d)*e*x - (2*e*x^2 + d)*sqrt(e)))/((d
^2*e^4*x^4 + 2*d^3*e^3*x^2 + d^4*e^2)*sqrt(e)), -1/3*(((4*c*d^2*e
 - b*d*e^2 - 2*a*e^3)*x^3 + 3*(c*d^3 - a*d*e^2)*x)*sqrt(e*x^2 + d
)*sqrt(-e) - 3*(c*d^2*e^2*x^4 + 2*c*d^3*e*x^2 + c*d^4)*arctan(sqr
t(-e)*x/sqrt(e*x^2 + d)))/((d^2*e^4*x^4 + 2*d^3*e^3*x^2 + d^4*e^2
)*sqrt(-e))]
```

Sympy [A] time = 55.0696, size = 450, normalized size = 4.46

$$a\left(\frac{3dx}{3d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}}+\frac{2ex^3}{3d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}}\right)+\frac{bx^3}{3d^{\frac{5}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{3}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}}+c\left(\frac{3d^{\frac{39}{2}}e^{11}\sqrt{1+\frac{ex^2}{d}}\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}}+\frac{3d^{\frac{37}{2}}e^{12}x^2\sqrt{1+\frac{ex^2}{d}}\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}}-\frac{3d^{19}e^{\frac{23}{2}}x}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}}-\frac{4d^{18}e^{\frac{25}{2}}x^3}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(5/2),x)
```

```
[Out] a*(3*d*x/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(
1 + e*x**2/d)) + 2*e*x**3/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(
5/2)*e*x**2*sqrt(1 + e*x**2/d))) + b*x**3/(3*d**(5/2)*sqrt(1 + e*
x**2/d) + 3*d**(3/2)*e*x**2*sqrt(1 + e*x**2/d)) + c*(3*d**(39/2)*
e**11*sqrt(1 + e*x**2/d)*asinh(sqrt(e)*x/sqrt(d))/(3*d**(39/2)*e*
*(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*x**2*sqrt(1 +
e*x**2/d)) + 3*d**(37/2)*e**12*x**2*sqrt(1 + e*x**2/d)*asinh(sqrt
(e)*x/sqrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(
37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)) - 3*d**19*e**(23/2)*x/(3
*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*x
**2*sqrt(1 + e*x**2/d)) - 4*d**18*e**(25/2)*x**3/(3*d**(39/2)*e**
(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*x**2*sqrt(1 + e
*x**2/d)))
```

GIAC/XCAS [A] time = 0.271994, size = 119, normalized size = 1.18

$$-ce^{\left(-\frac{5}{2}\right)}\ln\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)-\frac{\left(\frac{\left(4cd^2e^2-bde^3-2ae^4\right)x^2e^{(-3)}}{d^2}+\frac{3\left(cd^3e-ade^3\right)e^{(-3)}}{d^2}\right)x}{3\left(x^2e+d\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^(5/2),x, algorithm="giac")
```

```
[Out] -c*e^(-5/2)*ln(abs(-x*e^(1/2) + sqrt(x^2*e + d))) - 1/3*((4*c*d^2
*e^2 - b*d*e^3 - 2*a*e^4)*x^2*e^(-3)/d^2 + 3*(c*d^3*e - a*d*e^3)*
e^(-3)/d^2)*x/(x^2*e + d)^(3/2)
```

3.286 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{7/2}}\,dx$

Optimal. Leaf size=86

$$\frac{x^5\left(2e(4ae+bd)+3cd^2\right)}{15d^3\left(d+ex^2\right)^{5/2}}+\frac{x^3(4ae+bd)}{3d^2\left(d+ex^2\right)^{5/2}}+\frac{ax}{d\left(d+ex^2\right)^{5/2}}$$

```
[Out] (a*x)/(d*(d + e*x^2)^(5/2)) + ((b*d + 4*a*e)*x^3)/(3*d^2*(d + e*x
^2)^(5/2)) + ((3*c*d^2 + 2*e*(b*d + 4*a*e))*x^5)/(15*d^3*(d + e*x
^2)^(5/2))
```

Rubi [A] time = 0.192974, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{x^5\left(2e(4ae+bd)+3cd^2\right)}{15d^3\left(d+ex^2\right)^{5/2}}+\frac{x^3(4ae+bd)}{3d^2\left(d+ex^2\right)^{5/2}}+\frac{ax}{d\left(d+ex^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(7/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(5/2)) + ((b*d + 4*a*e)*x^3)/(3*d^2*(d + e*x
^2)^(5/2)) + ((3*c*d^2 + 2*e*(b*d + 4*a*e))*x^5)/(15*d^3*(d + e*x
^2)^(5/2))
```

Rubi in Sympy [A] time = 28.7167, size = 119, normalized size = 1.38

$$\frac{x\left(ae^2-bde+cd^2\right)}{5de^2\left(d+ex^2\right)^{\frac{5}{2}}}+\frac{x\left(4ae^2+bde-cd^2+5cdex^2\right)}{15d^2e^2\left(d+ex^2\right)^{\frac{3}{2}}}+\frac{2x\left(4ae^2+bde-cd^2\right)}{15d^3e^2\sqrt{d+ex^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(7/2),x)
```

```
[Out] x*(a*e**2 - b*d*e + c*d**2)/(5*d*e**2*(d + e*x**2)**(5/2)) + x*(4
*a*e**2 + b*d*e - c*d**2 + 5*c*d*e*x**2)/(15*d**2*e**2*(d + e*x**
2)**(3/2)) + 2*x*(4*a*e**2 + b*d*e - c*d**2)/(15*d**3*e**2*sqrt(d
 + e*x**2))
```

Mathematica [A] time = 0.0850195, size = 67, normalized size = 0.78

$$\frac{a\left(15d^2x+20dex^3+8e^2x^5\right)+dx^3\left(5bd+2bex^2+3cdx^2\right)}{15d^3\left(d+ex^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(7/2),x]
```

```
[Out] (d*x^3*(5*b*d + 3*c*d*x^2 + 2*b*e*x^2) + a*(15*d^2*x + 20*d*e*x^3
 + 8*e^2*x^5))/(15*d^3*(d + e*x^2)^(5/2))
```

Maple [A] time = 0.006, size = 66, normalized size = 0.8

$$\frac{x\left(8\,ae^{2}x^{4}+2\,bdex^{4}+3\,cd^{2}x^{4}+20\,adex^{2}+5\,bd^{2}x^{2}+15\,ad^{2}\right)}{15\,d^{3}}\left(ex^{2}+d\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x)
```

```
[Out] 1/15*x*(8*a*e^2*x^4+2*b*d*e*x^4+3*c*d^2*x^4+20*a*d*e*x^2+5*b*d^2*
x^2+15*a*d^2)/(e*x^2+d)^(5/2)/d^3
```

Maxima [A] time = 0.764345, size = 234, normalized size = 2.72

$$-\frac{cx^{3}}{2\,(ex^{2}+d)^{\frac{5}{2}}e}+\frac{8\,ax}{15\,\sqrt{ex^{2}+d}d^{3}}+\frac{4\,ax}{15\,(ex^{2}+d)^{\frac{3}{2}}d^{2}}+\frac{ax}{5\,(ex^{2}+d)^{\frac{5}{2}}d}+\frac{cx}{10\,(ex^{2}+d)^{\frac{3}{2}}e^{2}}+\frac{cx}{5\,\sqrt{ex^{2}+d}de^{2}}-\frac{3\,cdx}{10\,(ex^{2}+d)^{\frac{5}{2}}e^{2}}-\frac{bx}{5\,(ex^{2}+d)^{\frac{5}{2}}e}+\frac{2\,bx}{15\,\sqrt{ex^{2}+d}d^{2}e}+\frac{bx}{15\,(ex^{2}+d)^{\frac{3}{2}}de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*c*x^3/((e*x^2 + d)^(5/2)*e) + 8/15*a*x/(sqrt(e*x^2 + d)*d^3)
 + 4/15*a*x/((e*x^2 + d)^(3/2)*d^2) + 1/5*a*x/((e*x^2 + d)^(5/2)*
d) + 1/10*c*x/((e*x^2 + d)^(3/2)*e^2) + 1/5*c*x/(sqrt(e*x^2 + d)*
d*e^2) - 3/10*c*d*x/((e*x^2 + d)^(5/2)*e^2) - 1/5*b*x/((e*x^2 + d
)^(5/2)*e) + 2/15*b*x/(sqrt(e*x^2 + d)*d^2*e) + 1/15*b*x/((e*x^2
+ d)^(3/2)*d*e)
```

Fricas [A] time = 0.308491, size = 126, normalized size = 1.47

$$\frac{\left(\left(3\,cd^{2}+2\,bde+8\,ae^{2}\right)x^{5}+15\,ad^{2}x+5\left(bd^{2}+4\,ade\right)x^{3}\right)\sqrt{ex^{2}+d}}{15\,(d^{3}e^{3}x^{6}+3\,d^{4}e^{2}x^{4}+3\,d^{5}ex^{2}+d^{6})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*((3*c*d^2 + 2*b*d*e + 8*a*e^2)*x^5 + 15*a*d^2*x + 5*(b*d^2 +
 4*a*d*e)*x^3)*sqrt(e*x^2 + d)/(d^3*e^3*x^6 + 3*d^4*e^2*x^4 + 3*d
^5*e*x^2 + d^6)
```

Sympy [A] time = 160.57, size = 639, normalized size = 7.43

$$a\left(\frac{15d^5x}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}}+\frac{35d^4ex^3}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}}+\frac{28d^3e^2x^5}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}}+\frac{8d^2e^3x^7}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}}\right)+b\left(\frac{5dx^3}{15d^{\frac{9}{2}}\sqrt{1+\frac{ex^2}{d}}+30d^{\frac{7}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{5}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}}+\frac{2ex^5}{15d^{\frac{9}{2}}\sqrt{1+\frac{ex^2}{d}}+30d^{\frac{7}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{5}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}}\right)+\frac{cx^5}{5d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+10d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+5d^{\frac{3}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(7/2),x)
```

```
[Out] a*(15*d**5*x/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x*
*2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d)
 + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + 35*d**4*e*x**3/(1
5*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x
**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)
*e**3*x**6*sqrt(1 + e*x**2/d)) + 28*d**3*e**2*x**5/(15*d**(17/2)*
sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*
d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6*s
qrt(1 + e*x**2/d)) + 8*d**2*e**3*x**7/(15*d**(17/2)*sqrt(1 + e*x*
*2/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**
2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**
2/d))) + b*(5*d*x**3/(15*d**(9/2)*sqrt(1 + e*x**2/d) + 30*d**(7/2
)*e*x**2*sqrt(1 + e*x**2/d) + 15*d**(5/2)*e**2*x**4*sqrt(1 + e*x*
*2/d)) + 2*e*x**5/(15*d**(9/2)*sqrt(1 + e*x**2/d) + 30*d**(7/2)*e
*x**2*sqrt(1 + e*x**2/d) + 15*d**(5/2)*e**2*x**4*sqrt(1 + e*x**2/
d))) + c*x**5/(5*d**(7/2)*sqrt(1 + e*x**2/d) + 10*d**(5/2)*e*x**2
*sqrt(1 + e*x**2/d) + 5*d**(3/2)*e**2*x**4*sqrt(1 + e*x**2/d))
```

GIAC/XCAS [A] time = 0.270656, size = 101, normalized size = 1.17

$$\frac{\left(x^2\left(\frac{\left(3\,cd^2e^2+2\,bde^3+8\,ae^4\right)x^2e^{(-2)}}{d^3}+\frac{5\left(bd^2e^2+4\,ade^3\right)e^{(-2)}}{d^3}\right)+\frac{15\,a}{d}\right)x}{15\left(x^2e+d\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(x^2*((3*c*d^2*e^2 + 2*b*d*e^3 + 8*a*e^4)*x^2*e^(-2)/d^3 + 5
*(b*d^2*e^2 + 4*a*d*e^3)*e^(-2)/d^3) + 15*a/d)*x/(x^2*e + d)^(5/2
)
```

3.287 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}}\,dx$

Optimal. Leaf size=126

$$\frac{2ex^7\left(4e(6ae+bd)+3cd^2\right)}{105d^4\left(d+ex^2\right)^{7/2}}+\frac{x^5\left(4e(6ae+bd)+3cd^2\right)}{15d^3\left(d+ex^2\right)^{7/2}}+\frac{x^3(6ae+bd)}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{ax}{d\left(d+ex^2\right)^{7/2}}$$

```
[Out] (a*x)/(d*(d + e*x^2)^(7/2)) + ((b*d + 6*a*e)*x^3)/(3*d^2*(d + e*x
^2)^(7/2)) + ((3*c*d^2 + 4*e*(b*d + 6*a*e))*x^5)/(15*d^3*(d + e*x
^2)^(7/2)) + (2*e*(3*c*d^2 + 4*e*(b*d + 6*a*e))*x^7)/(105*d^4*(d
+ e*x^2)^(7/2))
```

Rubi [A] time = 0.267988, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{2ex^7\left(4e(6ae+bd)+3cd^2\right)}{105d^4\left(d+ex^2\right)^{7/2}}+\frac{x^5\left(4e(6ae+bd)+3cd^2\right)}{15d^3\left(d+ex^2\right)^{7/2}}+\frac{x^3(6ae+bd)}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{ax}{d\left(d+ex^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(7/2)) + ((b*d + 6*a*e)*x^3)/(3*d^2*(d + e*x
^2)^(7/2)) + ((3*c*d^2 + 4*e*(b*d + 6*a*e))*x^5)/(15*d^3*(d + e*x
^2)^(7/2)) + (2*e*(3*c*d^2 + 4*e*(b*d + 6*a*e))*x^7)/(105*d^4*(d
+ e*x^2)^(7/2))
```

Rubi in Sympy [A] time = 31.5851, size = 155, normalized size = 1.23

$$\frac{x\left(ae^2-bde+cd^2\right)}{7de^2\left(d+ex^2\right)^{\frac{7}{2}}}+\frac{x\left(6ae^2+bde-8cd^2\right)}{35d^2e^2\left(d+ex^2\right)^{\frac{5}{2}}}+\frac{x\left(24ae^2+4bde+3cd^2\right)}{105d^3e^2\left(d+ex^2\right)^{\frac{3}{2}}}+\frac{2x\left(24ae^2+4bde+3cd^2\right)}{105d^4e^2\sqrt{d+ex^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(9/2),x)
```

```
[Out] x*(a*e**2 - b*d*e + c*d**2)/(7*d*e**2*(d + e*x**2)**(7/2)) + x*(6
*a*e**2 + b*d*e - 8*c*d**2)/(35*d**2*e**2*(d + e*x**2)**(5/2)) +
x*(24*a*e**2 + 4*b*d*e + 3*c*d**2)/(105*d**3*e**2*(d + e*x**2)**(
3/2)) + 2*x*(24*a*e**2 + 4*b*d*e + 3*c*d**2)/(105*d**4*e**2*sqrt(
d + e*x**2))
```

Mathematica [A] time = 0.109725, size = 101, normalized size = 0.8

$$\frac{3a\left(35d^3x+70d^2ex^3+56de^2x^5+16e^3x^7\right)+dx^3\left(b\left(35d^2+28dex^2+8e^2x^4\right)+3cdx^2\left(7d+2ex^2\right)\right)}{105d^4\left(d+ex^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2),x]
```

```
[Out] (3*a*(35*d^3*x + 70*d^2*e*x^3 + 56*d*e^2*x^5 + 16*e^3*x^7) + d*x^
3*(3*c*d*x^2*(7*d + 2*e*x^2) + b*(35*d^2 + 28*d*e*x^2 + 8*e^2*x^4
```

```
)))/(105*d^4*(d + e*x^2)^(7/2))
```

Maple [A] time = 0.007, size = 100, normalized size = 0.8

$$\frac{x\left(48\,ae^{3}x^{6}+8\,bde^{2}x^{6}+6\,cd^{2}ex^{6}+168\,ade^{2}x^{4}+28\,bd^{2}ex^{4}+21\,cd^{3}x^{4}+210\,ad^{2}ex^{2}+35\,bd^{3}x^{2}+105\,ad^{3}\right)}{105\,d^{4}}\left(ex^{2}+d\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x)
```

```
[Out] 1/105*x*(48*a*e^3*x^6+8*b*d*e^2*x^6+6*c*d^2*e*x^6+168*a*d*e^2*x^4
+28*b*d^2*e*x^4+21*c*d^3*x^4+210*a*d^2*e*x^2+35*b*d^3*x^2+105*a*d
^3)/(e*x^2+d)^(7/2)/d^4
```

Maxima [A] time = 0.756594, size = 306, normalized size = 2.43

$$-\frac{cx^{3}}{4\,(ex^{2}+d)^{\frac{7}{2}}e}+\frac{16\,ax}{35\,\sqrt{ex^{2}+d}d^{4}}+\frac{8\,ax}{35\,(ex^{2}+d)^{\frac{3}{2}}d^{3}}+\frac{6\,ax}{35\,(ex^{2}+d)^{\frac{5}{2}}d^{2}}+\frac{ax}{7\,(ex^{2}+d)^{\frac{7}{2}}d}+\frac{3\,cx}{140\,(ex^{2}+d)^{\frac{5}{2}}e^{2}}+\frac{2\,cx}{35\,\sqrt{ex^{2}+d}d^{2}e^{2}}+\frac{cx}{35\,(ex^{2}+d)^{\frac{3}{2}}de^{2}}-\frac{3\,cdx}{28\,(ex^{2}+d)^{\frac{7}{2}}e^{2}}-\frac{bx}{7\,(ex^{2}+d)^{\frac{7}{2}}e}+\frac{8\,bx}{105\,\sqrt{ex^{2}+d}d^{3}e}+\frac{4\,bx}{105\,(ex^{2}+d)^{\frac{3}{2}}d^{2}e}+\frac{bx}{35\,(ex^{2}+d)^{\frac{5}{2}}de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*c*x^3/((e*x^2 + d)^(7/2)*e) + 16/35*a*x/(sqrt(e*x^2 + d)*d^4
) + 8/35*a*x/((e*x^2 + d)^(3/2)*d^3) + 6/35*a*x/((e*x^2 + d)^(5/2
)*d^2) + 1/7*a*x/((e*x^2 + d)^(7/2)*d) + 3/140*c*x/((e*x^2 + d)^(
5/2)*e^2) + 2/35*c*x/(sqrt(e*x^2 + d)*d^2*e^2) + 1/35*c*x/((e*x^2
 + d)^(3/2)*d*e^2) - 3/28*c*d*x/((e*x^2 + d)^(7/2)*e^2) - 1/7*b*x
/((e*x^2 + d)^(7/2)*e) + 8/105*b*x/(sqrt(e*x^2 + d)*d^3*e) + 4/10
5*b*x/((e*x^2 + d)^(3/2)*d^2*e) + 1/35*b*x/((e*x^2 + d)^(5/2)*d*e
)
```

Fricas [A] time = 0.402601, size = 184, normalized size = 1.46

$$\frac{\left(2\left(3\,cd^{2}e+4\,bde^{2}+24\,ae^{3}\right)x^{7}+7\left(3\,cd^{3}+4\,bd^{2}e+24\,ade^{2}\right)x^{5}+105\,ad^{3}x+35\left(bd^{3}+6\,ad^{2}e\right)x^{3}\right)\sqrt{ex^{2}+d}}{105\,(d^{4}e^{4}x^{8}+4\,d^{5}e^{3}x^{6}+6\,d^{6}e^{2}x^{4}+4\,d^{7}ex^{2}+d^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*c*d^2*e + 4*b*d*e^2 + 24*a*e^3)*x^7 + 7*(3*c*d^3 + 4*
b*d^2*e + 24*a*d*e^2)*x^5 + 105*a*d^3*x + 35*(b*d^3 + 6*a*d^2*e)*
x^3)*sqrt(e*x^2 + d)/(d^4*e^4*x^8 + 4*d^5*e^3*x^6 + 6*d^6*e^2*x^4
 + 4*d^7*e*x^2 + d^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.269935, size = 153, normalized size = 1.21

$$\frac{\left(\left(x^2\left(\frac{2\left(3\,cd^2e^4+4\,bde^5+24\,ae^6\right)x^2e^{(-3)}}{d^4}+\frac{7\left(3\,cd^3e^3+4\,bd^2e^4+24\,ade^5\right)e^{(-3)}}{d^4}\right)+\frac{35\left(bd^3e^3+6\,ad^2e^4\right)e^{(-3)}}{d^4}\right)x^2+\frac{105\,a}{d}\right)x}{105\left(x^2e+d\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*(2*(3*c*d^2*e^4 + 4*b*d*e^5 + 24*a*e^6)*x^2*e^(-3)/d^
4 + 7*(3*c*d^3*e^3 + 4*b*d^2*e^4 + 24*a*d*e^5)*e^(-3)/d^4) + 35*(
b*d^3*e^3 + 6*a*d^2*e^4)*e^(-3)/d^4)*x^2 + 105*a/d)*x/(x^2*e + d)
^(7/2)
```

3.288 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}}\,dx$

Optimal. Leaf size=165

$$\frac{8e^2x^9\left(2e(8ae+bd)+cd^2\right)}{315d^5\left(d+ex^2\right)^{9/2}}+\frac{4ex^7\left(2e(8ae+bd)+cd^2\right)}{35d^4\left(d+ex^2\right)^{9/2}}+\frac{x^5\left(2e(8ae+bd)+cd^2\right)}{5d^3\left(d+ex^2\right)^{9/2}}+\frac{x^3(8ae+bd)}{3d^2\left(d+ex^2\right)^{9/2}}+\frac{ax}{d\left(d+ex^2\right)^{9/2}}$$

```
[Out] (a*x)/(d*(d + e*x^2)^(9/2)) + ((b*d + 8*a*e)*x^3)/(3*d^2*(d + e*x
^2)^(9/2)) + ((c*d^2 + 2*e*(b*d + 8*a*e))*x^5)/(5*d^3*(d + e*x^2)
^(9/2)) + (4*e*(c*d^2 + 2*e*(b*d + 8*a*e))*x^7)/(35*d^4*(d + e*x^
2)^(9/2)) + (8*e^2*(c*d^2 + 2*e*(b*d + 8*a*e))*x^9)/(315*d^5*(d +
 e*x^2)^(9/2))
```

Rubi [A] time = 0.390302, antiderivative size = 164, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{x^5\left(\frac{2e(8ae+bd)}{d^2}+c\right)}{5d\left(d+ex^2\right)^{9/2}}+\frac{8e^2x^9\left(2e(8ae+bd)+cd^2\right)}{315d^5\left(d+ex^2\right)^{9/2}}+\frac{4ex^7\left(2e(8ae+bd)+cd^2\right)}{35d^4\left(d+ex^2\right)^{9/2}}+\frac{x^3(8ae+bd)}{3d^2\left(d+ex^2\right)^{9/2}}+\frac{ax}{d\left(d+ex^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(11/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(9/2)) + ((b*d + 8*a*e)*x^3)/(3*d^2*(d + e*x
^2)^(9/2)) + ((c + (2*e*(b*d + 8*a*e))/d^2)*x^5)/(5*d*(d + e*x^2)
^(9/2)) + (4*e*(c*d^2 + 2*e*(b*d + 8*a*e))*x^7)/(35*d^4*(d + e*x^
2)^(9/2)) + (8*e^2*(c*d^2 + 2*e*(b*d + 8*a*e))*x^9)/(315*d^5*(d +
 e*x^2)^(9/2))
```

Rubi in Sympy [A] time = 35.1959, size = 192, normalized size = 1.16

$$\frac{x\left(ae^2-bde+cd^2\right)}{9de^2\left(d+ex^2\right)^{\frac{9}{2}}}+\frac{x\left(8ae^2+bde-10cd^2\right)}{63d^2e^2\left(d+ex^2\right)^{\frac{7}{2}}}+\frac{x\left(16ae^2+2bde+cd^2\right)}{105d^3e^2\left(d+ex^2\right)^{\frac{5}{2}}}+\frac{4x\left(16ae^2+2bde+cd^2\right)}{315d^4e^2\left(d+ex^2\right)^{\frac{3}{2}}}+\frac{8x\left(16ae^2+2bde+cd^2\right)}{315d^5e^2\sqrt{d+ex^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(11/2),x)
```

```
[Out] x*(a*e**2 - b*d*e + c*d**2)/(9*d*e**2*(d + e*x**2)**(9/2)) + x*(8
*a*e**2 + b*d*e - 10*c*d**2)/(63*d**2*e**2*(d + e*x**2)**(7/2)) +
 x*(16*a*e**2 + 2*b*d*e + c*d**2)/(105*d**3*e**2*(d + e*x**2)**(5
/2)) + 4*x*(16*a*e**2 + 2*b*d*e + c*d**2)/(315*d**4*e**2*(d + e*x
**2)**(3/2)) + 8*x*(16*a*e**2 + 2*b*d*e + c*d**2)/(315*d**5*e**2*
sqrt(d + e*x**2))
```

Mathematica [A] time = 0.149372, size = 132, normalized size = 0.8

$$\frac{a\left(315d^4x+840d^3ex^3+1008d^2e^2x^5+576de^3x^7+128e^4x^9\right)+dx^3\left(b\left(105d^3+126d^2ex^2+72de^2x^4+16e^3x^6\right)+cdx^2\left(63d^2\right.\right.}{315d^5\left(d+ex^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(11/2),x]
```

```
[Out] (a*(315*d^4*x + 840*d^3*e*x^3 + 1008*d^2*e^2*x^5 + 576*d*e^3*x^7
+ 128*e^4*x^9) + d*x^3*(c*d*x^2*(63*d^2 + 36*d*e*x^2 + 8*e^2*x^4)
 + b*(105*d^3 + 126*d^2*e*x^2 + 72*d*e^2*x^4 + 16*e^3*x^6)))/(315
*d^5*(d + e*x^2)^(9/2))
```

Maple [A] time = 0.008, size = 136, normalized size = 0.8

$$\frac{x\left(128\,ae^{4}x^{8}+16\,bde^{3}x^{8}+8\,cd^{2}e^{2}x^{8}+576\,ade^{3}x^{6}+72\,bd^{2}e^{2}x^{6}+36\,cd^{3}ex^{6}+1008\,ad^{2}e^{2}x^{4}+126\,bd^{3}ex^{4}+63\,cd^{4}x^{4}+840\right.}{315\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x)
```

```
[Out] 1/315*x*(128*a*e^4*x^8+16*b*d*e^3*x^8+8*c*d^2*e^2*x^8+576*a*d*e^3
*x^6+72*b*d^2*e^2*x^6+36*c*d^3*e*x^6+1008*a*d^2*e^2*x^4+126*b*d^3
*e*x^4+63*c*d^4*x^4+840*a*d^3*e*x^2+105*b*d^4*x^2+315*a*d^4)/(e*x
^2+d)^(9/2)/d^5
```

Maxima [A] time = 0.746915, size = 379, normalized size = 2.3

$$-\frac{cx^{3}}{6\,(ex^{2}+d)^{\frac{9}{2}}e}+\frac{128\,ax}{315\,\sqrt{ex^{2}+d}d^{5}}+\frac{64\,ax}{315\,(ex^{2}+d)^{\frac{3}{2}}d^{4}}+\frac{16\,ax}{105\,(ex^{2}+d)^{\frac{5}{2}}d^{3}}+\frac{8\,ax}{63\,(ex^{2}+d)^{\frac{7}{2}}d^{2}}+\frac{ax}{9\,(ex^{2}+d)^{\frac{9}{2}}d}+\frac{cx}{126\,(ex^{2}+d)^{\frac{7}{2}}e^{2}}+\frac{8\,cx}{315\,\sqrt{ex^{2}+d}d^{3}e^{2}}+\frac{4\,cx}{315\,(ex^{2}+d)^{\frac{3}{2}}d^{2}e^{2}}+\frac{cx}{105\,(ex^{2}+d)^{\frac{5}{2}}de^{2}}-\frac{cdx}{18\,(ex^{2}+d)^{\frac{9}{2}}e^{2}}-\frac{bx}{9\,(ex^{2}+d)^{\frac{9}{2}}e}+\frac{16\,bx}{315\,\sqrt{ex^{2}+d}d^{4}e}+\frac{8\,bx}{315\,(ex^{2}+d)^{\frac{3}{2}}d^{3}e}+\frac{2\,bx}{105\,(ex^{2}+d)^{\frac{5}{2}}d^{2}e}+\frac{bx}{63\,(ex^{2}+d)^{\frac{7}{2}}de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/6*c*x^3/((e*x^2 + d)^(9/2)*e) + 128/315*a*x/(sqrt(e*x^2 + d)*d
^5) + 64/315*a*x/((e*x^2 + d)^(3/2)*d^4) + 16/105*a*x/((e*x^2 + d
)^(5/2)*d^3) + 8/63*a*x/((e*x^2 + d)^(7/2)*d^2) + 1/9*a*x/((e*x^2
 + d)^(9/2)*d) + 1/126*c*x/((e*x^2 + d)^(7/2)*e^2) + 8/315*c*x/(s
qrt(e*x^2 + d)*d^3*e^2) + 4/315*c*x/((e*x^2 + d)^(3/2)*d^2*e^2) +
 1/105*c*x/((e*x^2 + d)^(5/2)*d*e^2) - 1/18*c*d*x/((e*x^2 + d)^(9
/2)*e^2) - 1/9*b*x/((e*x^2 + d)^(9/2)*e) + 16/315*b*x/(sqrt(e*x^2
 + d)*d^4*e) + 8/315*b*x/((e*x^2 + d)^(3/2)*d^3*e) + 2/105*b*x/((
e*x^2 + d)^(5/2)*d^2*e) + 1/63*b*x/((e*x^2 + d)^(7/2)*d*e)
```

Fricas [A] time = 0.555679, size = 239, normalized size = 1.45

$$\frac{\left(8\left(cd^{2}e^{2}+2\,bde^{3}+16\,ae^{4}\right)x^{9}+36\left(cd^{3}e+2\,bd^{2}e^{2}+16\,ade^{3}\right)x^{7}+315\,ad^{4}x+63\left(cd^{4}+2\,bd^{3}e+16\,ad^{2}e^{2}\right)x^{5}+105\left(bd^{4}+\right.\right.}{315\left(d^{5}e^{5}x^{10}+5\,d^{6}e^{4}x^{8}+10\,d^{7}e^{3}x^{6}+10\,d^{8}e^{2}x^{4}+5\,d^{9}ex^{2}+d^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(8*(c*d^2*e^2 + 2*b*d*e^3 + 16*a*e^4)*x^9 + 36*(c*d^3*e + 2
*b*d^2*e^2 + 16*a*d*e^3)*x^7 + 315*a*d^4*x + 63*(c*d^4 + 2*b*d^3*
e + 16*a*d^2*e^2)*x^5 + 105*(b*d^4 + 8*a*d^3*e)*x^3)*sqrt(e*x^2 +
 d)/(d^5*e^5*x^10 + 5*d^6*e^4*x^8 + 10*d^7*e^3*x^6 + 10*d^8*e^2*x
^4 + 5*d^9*e*x^2 + d^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(11/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.271193, size = 200, normalized size = 1.21

$$\frac{(((4\,x^{2}(\frac{2\,(cd^{2}e^{6}+2\,bde^{7}+16\,ae^{8})\,x^{2}e^{(-4)}}{d^{5}}+\frac{9\,(cd^{3}e^{5}+2\,bd^{2}e^{6}+16\,ade^{7})\,e^{(-4)}}{d^{5}})+\frac{63\,(cd^{4}e^{4}+2\,bd^{3}e^{5}+16\,ad^{2}e^{6})\,e^{(-4)}}{d^{5}})x^{2}+\frac{105\,(bd^{4}e^{4}+8\,ad^{3}e^{5})\,e^{(-4)}}{d^{5}})x}{315\,(x^{2}e+d)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^(11/2),x, algorithm="giac")
```

```
[Out] 1/315*(((4*x^2*(2*(c*d^2*e^6 + 2*b*d*e^7 + 16*a*e^8)*x^2*e^(-4)/d
^5 + 9*(c*d^3*e^5 + 2*b*d^2*e^6 + 16*a*d*e^7)*e^(-4)/d^5) + 63*(c
*d^4*e^4 + 2*b*d^3*e^5 + 16*a*d^2*e^6)*e^(-4)/d^5)*x^2 + 105*(b*d
^4*e^4 + 8*a*d^3*e^5)*e^(-4)/d^5)*x^2 + 315*a/d)*x/(x^2*e + d)^(9
/2)
```

3.289 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}\,dx$

Optimal. Leaf size=210

$$\frac{16e^3x^{11}\left(8e(10ae+bd)+3cd^2\right)}{3465d^6\left(d+ex^2\right)^{11/2}}+\frac{8e^2x^9\left(8e(10ae+bd)+3cd^2\right)}{315d^5\left(d+ex^2\right)^{11/2}}+\frac{2ex^7\left(8e(10ae+bd)+3cd^2\right)}{35d^4\left(d+ex^2\right)^{11/2}}+\frac{x^5\left(8e(10ae+bd)+3cd^2\right)}{15d^3\left(d+ex^2\right)^{11/2}}+\frac{x^3(10ae+bd)}{3d^2\left(d+ex^2\right)^{11/2}}+\frac{ax}{d\left(d+ex^2\right)^{11/2}}$$

```
[Out] (a*x)/(d*(d + e*x^2)^(11/2)) + ((b*d + 10*a*e)*x^3)/(3*d^2*(d + e
*x^2)^(11/2)) + ((3*c*d^2 + 8*e*(b*d + 10*a*e))*x^5)/(15*d^3*(d +
 e*x^2)^(11/2)) + (2*e*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^7)/(35*d^
4*(d + e*x^2)^(11/2)) + (8*e^2*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^9
)/(315*d^5*(d + e*x^2)^(11/2)) + (16*e^3*(3*c*d^2 + 8*e*(b*d + 10
*a*e))*x^11)/(3465*d^6*(d + e*x^2)^(11/2))
```

Rubi [A] time = 0.437505, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{16e^3x^{11}\left(8e(10ae+bd)+3cd^2\right)}{3465d^6\left(d+ex^2\right)^{11/2}}+\frac{8e^2x^9\left(8e(10ae+bd)+3cd^2\right)}{315d^5\left(d+ex^2\right)^{11/2}}+\frac{2ex^7\left(8e(10ae+bd)+3cd^2\right)}{35d^4\left(d+ex^2\right)^{11/2}}+\frac{x^5\left(8e(10ae+bd)+3cd^2\right)}{15d^3\left(d+ex^2\right)^{11/2}}+\frac{x^3(10ae+bd)}{3d^2\left(d+ex^2\right)^{11/2}}+\frac{ax}{d\left(d+ex^2\right)^{11/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(13/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(11/2)) + ((b*d + 10*a*e)*x^3)/(3*d^2*(d + e
*x^2)^(11/2)) + ((3*c*d^2 + 8*e*(b*d + 10*a*e))*x^5)/(15*d^3*(d +
 e*x^2)^(11/2)) + (2*e*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^7)/(35*d^
4*(d + e*x^2)^(11/2)) + (8*e^2*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^9
)/(315*d^5*(d + e*x^2)^(11/2)) + (16*e^3*(3*c*d^2 + 8*e*(b*d + 10
*a*e))*x^11)/(3465*d^6*(d + e*x^2)^(11/2))
```

Rubi in Sympy [A] time = 39.3151, size = 240, normalized size = 1.14

$$\frac{x\left(ae^2-bde+cd^2\right)}{11de^2\left(d+ex^2\right)^{\frac{11}{2}}}+\frac{x\left(10ae^2+bde-12cd^2\right)}{99d^2e^2\left(d+ex^2\right)^{\frac{9}{2}}}+\frac{x\left(80ae^2+8bde+3cd^2\right)}{693d^3e^2\left(d+ex^2\right)^{\frac{7}{2}}}+\frac{2x\left(80ae^2+8bde+3cd^2\right)}{1155d^4e^2\left(d+ex^2\right)^{\frac{5}{2}}}+\frac{8x\left(80ae^2+8bde+3cd^2\right)}{3465d^5e^2\left(d+ex^2\right)^{\frac{3}{2}}}+\frac{16x\left(80ae^2+8bde+3cd^2\right)}{3465d^6e^2\sqrt{d+ex^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(13/2),x)
```

```
[Out] x*(a*e**2 - b*d*e + c*d**2)/(11*d*e**2*(d + e*x**2)**(11/2)) + x*
(10*a*e**2 + b*d*e - 12*c*d**2)/(99*d**2*e**2*(d + e*x**2)**(9/2)
) + x*(80*a*e**2 + 8*b*d*e + 3*c*d**2)/(693*d**3*e**2*(d + e*x**2
)**(7/2)) + 2*x*(80*a*e**2 + 8*b*d*e + 3*c*d**2)/(1155*d**4*e**2*
(d + e*x**2)**(5/2)) + 8*x*(80*a*e**2 + 8*b*d*e + 3*c*d**2)/(3465
*d**5*e**2*(d + e*x**2)**(3/2)) + 16*x*(80*a*e**2 + 8*b*d*e + 3*c
*d**2)/(3465*d**6*e**2*sqrt(d + e*x**2))
```

Mathematica [A] time = 0.157361, size = 167, normalized size = 0.8

$$\frac{5a\left(693d^5x+2310d^4ex^3+3696d^3e^2x^5+3168d^2e^3x^7+1408de^4x^9+256e^5x^{11}\right)+dx^3\left(b\left(1155d^4+1848d^3ex^2+1584d^2e^2x^4\right.\right.}{3465d^6\left(d+ex^2\right)^{11/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(13/2),x]
```

```
[Out] (5*a*(693*d^5*x + 2310*d^4*e*x^3 + 3696*d^3*e^2*x^5 + 3168*d^2*e^
3*x^7 + 1408*d*e^4*x^9 + 256*e^5*x^11) + d*x^3*(3*c*d*x^2*(231*d^
3 + 198*d^2*e*x^2 + 88*d*e^2*x^4 + 16*e^3*x^6) + b*(1155*d^4 + 18
48*d^3*e*x^2 + 1584*d^2*e^2*x^4 + 704*d*e^3*x^6 + 128*e^4*x^8)))/
(3465*d^6*(d + e*x^2)^(11/2))
```

Maple [A] time = 0.009, size = 172, normalized size = 0.8

$$\frac{x\left(1280\,ae^5x^{10}+128\,bde^4x^{10}+48\,cd^2e^3x^{10}+7040\,ade^4x^8+704\,bd^2e^3x^8+264\,cd^3e^2x^8+15840\,ad^2e^3x^6+1584\,bd^3e^2x^6+5\right.}{3465\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x)
```

```
[Out] 1/3465*x*(1280*a*e^5*x^10+128*b*d*e^4*x^10+48*c*d^2*e^3*x^10+7040
*a*d*e^4*x^8+704*b*d^2*e^3*x^8+264*c*d^3*e^2*x^8+15840*a*d^2*e^3*
x^6+1584*b*d^3*e^2*x^6+594*c*d^4*e*x^6+18480*a*d^3*e^2*x^4+1848*b
*d^4*e*x^4+693*c*d^5*x^4+11550*a*d^4*e*x^2+1155*b*d^5*x^2+3465*a*
d^5)/(e*x^2+d)^(11/2)/d^6
```

Maxima [A] time = 0.739303, size = 452, normalized size = 2.15

$$-\frac{cx^3}{8\left(ex^2+d\right)^{\frac{11}{2}}e}+\frac{256\,ax}{693\,\sqrt{ex^2+d}d^6}+\frac{128\,ax}{693\left(ex^2+d\right)^{\frac{3}{2}}d^5}+\frac{32\,ax}{231\left(ex^2+d\right)^{\frac{5}{2}}d^4}+\frac{80\,ax}{693\left(ex^2+d\right)^{\frac{7}{2}}d^3}$$
$$+\frac{10\,ax}{99\left(ex^2+d\right)^{\frac{9}{2}}d^2}+\frac{ax}{11\left(ex^2+d\right)^{\frac{11}{2}}d}+\frac{cx}{264\left(ex^2+d\right)^{\frac{9}{2}}e^2}+\frac{16\,cx}{1155\,\sqrt{ex^2+d}d^4e^2}+\frac{8\,cx}{1155\left(ex^2+d\right)^{\frac{3}{2}}d^3e^2}$$
$$+\frac{2\,cx}{385\left(ex^2+d\right)^{\frac{5}{2}}d^2e^2}+\frac{cx}{231\left(ex^2+d\right)^{\frac{7}{2}}de^2}-\frac{3\,cdx}{88\left(ex^2+d\right)^{\frac{11}{2}}e^2}-\frac{bx}{11\left(ex^2+d\right)^{\frac{11}{2}}e}+\frac{128\,bx}{3465\,\sqrt{ex^2+d}d^5e}$$
$$+\frac{64\,bx}{3465\left(ex^2+d\right)^{\frac{3}{2}}d^4e}+\frac{16\,bx}{1155\left(ex^2+d\right)^{\frac{5}{2}}d^3e}+\frac{8\,bx}{693\left(ex^2+d\right)^{\frac{7}{2}}d^2e}+\frac{bx}{99\left(ex^2+d\right)^{\frac{9}{2}}de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^(13/2),x, algorithm="maxima")
```

```
[Out] -1/8*c*x^3/((e*x^2 + d)^(11/2)*e) + 256/693*a*x/(sqrt(e*x^2 + d)*
d^6) + 128/693*a*x/((e*x^2 + d)^(3/2)*d^5) + 32/231*a*x/((e*x^2 +
 d)^(5/2)*d^4) + 80/693*a*x/((e*x^2 + d)^(7/2)*d^3) + 10/99*a*x/(
(e*x^2 + d)^(9/2)*d^2) + 1/11*a*x/((e*x^2 + d)^(11/2)*d) + 1/264*
c*x/((e*x^2 + d)^(9/2)*e^2) + 16/1155*c*x/(sqrt(e*x^2 + d)*d^4*e^
2) + 8/1155*c*x/((e*x^2 + d)^(3/2)*d^3*e^2) + 2/385*c*x/((e*x^2 +
 d)^(5/2)*d^2*e^2) + 1/231*c*x/((e*x^2 + d)^(7/2)*d*e^2) - 3/88*c
*d*x/((e*x^2 + d)^(11/2)*e^2) - 1/11*b*x/((e*x^2 + d)^(11/2)*e) +
 128/3465*b*x/(sqrt(e*x^2 + d)*d^5*e) + 64/3465*b*x/((e*x^2 + d)^
(3/2)*d^4*e) + 16/1155*b*x/((e*x^2 + d)^(5/2)*d^3*e) + 8/693*b*x/
((e*x^2 + d)^(7/2)*d^2*e) + 1/99*b*x/((e*x^2 + d)^(9/2)*d*e)
```

Fricas [A] time = 0.845007, size = 302, normalized size = 1.44

$$\frac{\left(16\left(3\,cd^2e^3+8\,bde^4+80\,ae^5\right)x^{11}+88\left(3\,cd^3e^2+8\,bd^2e^3+80\,ade^4\right)x^9+198\left(3\,cd^4e+8\,bd^3e^2+80\,ad^2e^3\right)x^7+3465\,ad^5x+\cdots}{3465\left(d^6e^6x^{12}+6\,d^7e^5x^{10}+15\,d^8e^4x^8+20\,d^9e^3x^6+15\,d^{10}e^2x^4+6\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(16*(3*c*d^2*e^3 + 8*b*d*e^4 + 80*a*e^5)*x^11 + 88*(3*c*d^
3*e^2 + 8*b*d^2*e^3 + 80*a*d*e^4)*x^9 + 198*(3*c*d^4*e + 8*b*d^3*
e^2 + 80*a*d^2*e^3)*x^7 + 3465*a*d^5*x + 231*(3*c*d^5 + 8*b*d^4*e
 + 80*a*d^3*e^2)*x^5 + 1155*(b*d^5 + 10*a*d^4*e)*x^3)*sqrt(e*x^2
+ d)/(d^6*e^6*x^12 + 6*d^7*e^5*x^10 + 15*d^8*e^4*x^8 + 20*d^9*e^3
*x^6 + 15*d^10*e^2*x^4 + 6*d^11*e*x^2 + d^12)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(13/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.271024, size = 255, normalized size = 1.21

$$\frac{\left(\left(\left(2\left(4\,x^2\left(\frac{2\left(3\,cd^2e^8+8\,bde^9+80\,ae^{10}\right)x^2e^{(-5)}}{d^6}+\frac{11\left(3\,cd^3e^7+8\,bd^2e^8+80\,ade^9\right)e^{(-5)}}{d^6}\right)+\frac{99\left(3\,cd^4e^6+8\,bd^3e^7+80\,ad^2e^8\right)e^{(-5)}}{d^6}\right)x^2+\frac{231\left(3\,cd^5e^5+8\,bd^4\cdots\right.}{d\cdots}\cdots}{3465\left(x^2e+d\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^(13/2),x, algorithm="giac")
```

```
[Out] 1/3465*(((2*(4*x^2*(2*(3*c*d^2*e^8 + 8*b*d*e^9 + 80*a*e^10)*x^2*e
^(-5)/d^6 + 11*(3*c*d^3*e^7 + 8*b*d^2*e^8 + 80*a*d*e^9)*e^(-5)/d^
6) + 99*(3*c*d^4*e^6 + 8*b*d^3*e^7 + 80*a*d^2*e^8)*e^(-5)/d^6)*x^
2 + 231*(3*c*d^5*e^5 + 8*b*d^4*e^6 + 80*a*d^3*e^7)*e^(-5)/d^6)*x^
2 + 1155*(b*d^5*e^5 + 10*a*d^4*e^6)*e^(-5)/d^6)*x^2 + 3465*a/d)*x
/(x^2*e + d)^(11/2)
```

3.290 $\int \left(7+5x^2\right)^3 \sqrt{2+3x^2+x^4}\,dx$

Optimal. Leaf size=193

$$\frac{275}{7}\left(x^4+3x^2+2\right)^{3/2}x+\frac{1}{21}\left(757x^2+2608\right)\sqrt{x^4+3x^2+2}\,x+\frac{577\left(x^2+2\right)x}{3\sqrt{x^4+3x^2+2}}+\frac{2945\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{21\sqrt{x^4+3x^2+2}}-\frac{577\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{125}{9}\left(x^4+3x^2+2\right)^{3/2}x^3$$

```
[Out] (577*x*(2 + x^2))/(3*Sqrt[2 + 3*x^2 + x^4]) + (x*(2608 + 757*x^2)
*Sqrt[2 + 3*x^2 + x^4])/21 + (275*x*(2 + 3*x^2 + x^4)^(3/2))/7 +
(125*x^3*(2 + 3*x^2 + x^4)^(3/2))/9 - (577*Sqrt[2]*(1 + x^2)*Sqrt
[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^
2 + x^4]) + (2945*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ell
ipticF[ArcTan[x], 1/2])/(21*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.207848, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{275}{7}\left(x^4+3x^2+2\right)^{3/2}x+\frac{1}{21}\left(757x^2+2608\right)\sqrt{x^4+3x^2+2}\,x+\frac{577\left(x^2+2\right)x}{3\sqrt{x^4+3x^2+2}}+\frac{2945\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{21\sqrt{x^4+3x^2+2}}-\frac{577\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{125}{9}\left(x^4+3x^2+2\right)^{3/2}x^3$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (577*x*(2 + x^2))/(3*Sqrt[2 + 3*x^2 + x^4]) + (x*(2608 + 757*x^2)
*Sqrt[2 + 3*x^2 + x^4])/21 + (275*x*(2 + 3*x^2 + x^4)^(3/2))/7 +
(125*x^3*(2 + 3*x^2 + x^4)^(3/2))/9 - (577*Sqrt[2]*(1 + x^2)*Sqrt
[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^
2 + x^4]) + (2945*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ell
ipticF[ArcTan[x], 1/2])/(21*Sqrt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 36.0919, size = 182, normalized size = 0.94

$$\frac{125x^3\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{9}+\frac{577x\left(2x^2+4\right)}{6\sqrt{x^4+3x^2+2}}+\frac{x\left(\frac{3785x^2}{7}+\frac{13040}{7}\right)\sqrt{x^4+3x^2+2}}{15}+\frac{275x\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{7}-\frac{577\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)E\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{12\sqrt{x^4+3x^2+2}}+\frac{2945\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{84\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**3*(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] 125*x**3*(x**4 + 3*x**2 + 2)**(3/2)/9 + 577*x*(2*x**2 + 4)/(6*sqr
t(x**4 + 3*x**2 + 2)) + x*(3785*x**2/7 + 13040/7)*sqrt(x**4 + 3*x
**2 + 2)/15 + 275*x*(x**4 + 3*x**2 + 2)**(3/2)/7 - 577*sqrt((2*x*
```

```
*2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_e(atan(x), 1/2)/(12*sqr
t(x**4 + 3*x**2 + 2)) + 2945*sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**
2 + 4)*elliptic_f(atan(x), 1/2)/(84*sqrt(x**4 + 3*x**2 + 2))
```

Mathematica [C] time = 0.0936792, size = 119, normalized size = 0.62

$$\frac{875x^{11} + 7725x^9 + 28496x^7 + 57312x^5 + 61214x^3 - 5553i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 12117i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\right.}{63\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (25548*x + 61214*x^3 + 57312*x^5 + 28496*x^7 + 7725*x^9 + 875*x^1
1 - (12117*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/S
qrt[2]], 2] - (5553*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*Ar
cSinh[x/Sqrt[2]], 2])/(63*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.029, size = 172, normalized size = 0.9

$$\frac{4258\,x}{21}\sqrt{x^4+3\,x^2+2} - \frac{2945\,i}{21}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}} + \frac{577\,i}{6}\sqrt{2}\left(EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right) - EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}} + \frac{11446\,x^3}{63}\sqrt{x^4+3\,x^2+2} + \frac{1700\,x^5}{21}\sqrt{x^4+3\,x^2+2} + \frac{125\,x^7}{9}\sqrt{x^4+3\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 4258/21*x*(x^4+3*x^2+2)^(1/2)-2945/21*I*2^(1/2)*(2*x^2+4)^(1/2)*(
x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2
))+577/6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1
/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x
,2^(1/2)))+11446/63*x^3*(x^4+3*x^2+2)^(1/2)+1700/21*x^5*(x^4+3*x^
2+2)^(1/2)+125/9*x^7*(x^4+3*x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(125\,x^6+525\,x^4+735\,x^2+343\right)\sqrt{x^4+3\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3,x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(x^4 + 3*x^2 + 2
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^2+1)(x^2+2)}\left(5x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**3*(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3, x)
```

3.291 $\int (7+5x^2)^2 \sqrt{2+3x^2+x^4}\,dx$

Optimal. Leaf size=168

$$\frac{25}{7}x\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{21}x\left(114x^2+407\right)\sqrt{x^4+3x^2+2}+\frac{31x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}+\frac{472\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{21\sqrt{x^4+3x^2+2}}-\frac{31\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

```
[Out] (31*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(407 + 114*x^2)*Sqrt[
2 + 3*x^2 + x^4])/21 + (25*x*(2 + 3*x^2 + x^4)^(3/2))/7 - (31*Sqr
t[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2
])/Sqrt[2 + 3*x^2 + x^4] + (472*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/
(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(21*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.162065, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{25}{7}x\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{21}x\left(114x^2+407\right)\sqrt{x^4+3x^2+2}+\frac{31x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}+\frac{472\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{21\sqrt{x^4+3x^2+2}}-\frac{31\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (31*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(407 + 114*x^2)*Sqrt[
2 + 3*x^2 + x^4])/21 + (25*x*(2 + 3*x^2 + x^4)^(3/2))/7 - (31*Sqr
t[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2
])/Sqrt[2 + 3*x^2 + x^4] + (472*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/
(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(21*Sqrt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 26.8322, size = 162, normalized size = 0.96

$$\frac{31x\left(2x^2+4\right)}{2\sqrt{x^4+3x^2+2}}+\frac{x\left(\frac{570x^2}{7}+\frac{2035}{7}\right)\sqrt{x^4+3x^2+2}}{15}+\frac{25x\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{7}-\frac{31\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)E\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+3x^2+2}}+\frac{118\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)F\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{21\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**2*(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] 31*x*(2*x**2 + 4)/(2*sqrt(x**4 + 3*x**2 + 2)) + x*(570*x**2/7 + 2
035/7)*sqrt(x**4 + 3*x**2 + 2)/15 + 25*x*(x**4 + 3*x**2 + 2)**(3/
2)/7 - 31*sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_e(a
tan(x), 1/2)/(4*sqrt(x**4 + 3*x**2 + 2)) + 118*sqrt((2*x**2 + 4)/
(x**2 + 1))*(4*x**2 + 4)*elliptic_f(atan(x), 1/2)/(21*sqrt(x**4 +
 3*x**2 + 2))
```

Mathematica [C] time = 0.0764958, size = 114, normalized size = 0.68

$$\frac{75x^9+564x^7+1724x^5+2349x^3-293i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-651i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+1}{21\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (1114*x + 2349*x^3 + 1724*x^5 + 564*x^7 + 75*x^9 - (651*I)*Sqrt[1
 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (293*I
)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])
/(21*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.011, size = 155, normalized size = 0.9

$$\frac{557\,x}{21}\sqrt{x^4+3\,x^2+2}-\frac{472\,i}{21}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}} +\frac{31\,i}{2}\sqrt{2}\left(EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)-EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}} +\frac{113\,x^3}{7}\sqrt{x^4+3\,x^2+2}+\frac{25\,x^5}{7}\sqrt{x^4+3\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 557/21*x*(x^4+3*x^2+2)^(1/2)-472/21*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^
2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))
+31/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)
*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^
(1/2)))+113/7*x^3*(x^4+3*x^2+2)^(1/2)+25/7*x^5*(x^4+3*x^2+2)^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(25\,x^4+70\,x^2+49\right)\sqrt{x^4+3\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2,x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^2+1)(x^2+2)}\left(5x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**2*(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2, x)
```

3.292 $\int \left(7+5x^2\right) \sqrt{2+3x^2+x^4}\,dx$

Optimal. Leaf size=149

$$\frac{5x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}+\frac{1}{3}x\left(3x^2+10\right)\sqrt{x^4+3x^2+2}+\frac{11\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

```
[Out] (5*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(10 + 3*x^2)*Sqrt[2 +
3*x^2 + x^4])/3 - (5*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*
EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (11*Sqrt[2]*(1
 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*S
qrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.114905, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{5x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}+\frac{1}{3}x\left(3x^2+10\right)\sqrt{x^4+3x^2+2}+\frac{11\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (5*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(10 + 3*x^2)*Sqrt[2 +
3*x^2 + x^4])/3 - (5*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*
EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (11*Sqrt[2]*(1
 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*S
qrt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 17.9323, size = 139, normalized size = 0.93

$$\frac{5x\left(2x^2+4\right)}{2\sqrt{x^4+3x^2+2}}+\frac{x\left(15x^2+50\right)\sqrt{x^4+3x^2+2}}{15}-\frac{5\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)E\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{4\sqrt{x^4+3x^2+2}}+\frac{11\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{12\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)*(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] 5*x*(2*x**2 + 4)/(2*sqrt(x**4 + 3*x**2 + 2)) + x*(15*x**2 + 50)*s
qrt(x**4 + 3*x**2 + 2)/15 - 5*sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x*
*2 + 4)*elliptic_e(atan(x), 1/2)/(4*sqrt(x**4 + 3*x**2 + 2)) + 11
*sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_f(atan(x), 1
/2)/(12*sqrt(x**4 + 3*x**2 + 2))
```

Mathematica [C] time = 0.0671692, size = 109, normalized size = 0.73

$$\frac{3x^7+19x^5+36x^3-7i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-15i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+20x}{3\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (20*x + 36*x^3 + 19*x^5 + 3*x^7 - (15*I)*Sqrt[1 + x^2]*Sqrt[2 + x
^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt
[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 +
 x^4])
```

Maple [C] time = 0.009, size = 137, normalized size = 0.9

$$\frac{10\,x}{3}\sqrt{x^4+3\,x^2+2}-\frac{11\,i}{3}\sqrt{2}\mathit{EllipticF}\left(\frac{i}{2}\sqrt{2}x,\,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}+\frac{5\,i}{2}\sqrt{2}\left(\mathit{EllipticF}\left(\frac{i}{2}\sqrt{2}x,\,\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{i}{2}\sqrt{2}x,\,\sqrt{2}\right)\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}+x^3\sqrt{x^4+3\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 10/3*x*(x^4+3*x^2+2)^(1/2)-11/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)
^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+5/2
*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(Ell
ipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)
))+x^3*(x^4+3*x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\left(x^2+1\right)\left(x^2+2\right)}\left(5x^2+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)*(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 2}\left(5x^2 + 7\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7), x)
```

3.293 $\int \sqrt{2+3x^2+x^4}\,dx$

Optimal. Leaf size=141

$$\frac{1}{3}\sqrt{x^4+3x^2+2}x+\frac{(x^2+2)x}{\sqrt{x^4+3x^2+2}}+\frac{2\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

```
[Out] (x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*Sqrt[2 + 3*x^2 + x^4])/3
 - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[
x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (2*Sqrt[2]*(1 + x^2)*Sqrt[(2 +
x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^
4])
```

Rubi [A] time = 0.0993112, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{1}{3}\sqrt{x^4+3x^2+2}x+\frac{(x^2+2)x}{\sqrt{x^4+3x^2+2}}+\frac{2\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*Sqrt[2 + 3*x^2 + x^4])/3
 - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[
x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (2*Sqrt[2]*(1 + x^2)*Sqrt[(2 +
x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^
4])
```

Rubi in Sympy [A] time = 15.3214, size = 128, normalized size = 0.91

$$\frac{x(2x^2+4)}{2\sqrt{x^4+3x^2+2}}+\frac{x\sqrt{x^4+3x^2+2}}{3}-\frac{\sqrt{\frac{2x^2+4}{x^2+1}}(4x^2+4)E\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+3x^2+2}}+\frac{\sqrt{\frac{2x^2+4}{x^2+1}}(4x^2+4)F\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{6\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+3*x**2+2)**(1/2),x)
```

```
[Out] x*(2*x**2 + 4)/(2*sqrt(x**4 + 3*x**2 + 2)) + x*sqrt(x**4 + 3*x**2
 + 2)/3 - sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_e(a
tan(x), 1/2)/(4*sqrt(x**4 + 3*x**2 + 2)) + sqrt((2*x**2 + 4)/(x**
2 + 1))*(4*x**2 + 4)*elliptic_f(atan(x), 1/2)/(6*sqrt(x**4 + 3*x*
*2 + 2))
```

Mathematica [C] time = 0.0596253, size = 102, normalized size = 0.72

$$\frac{x^5+3x^3-i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-3i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+2x}{3\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (2*x + 3*x^3 + x^5 - (3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[
I*ArcSinh[x/Sqrt[2]], 2] - I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Elliptic
F[I*ArcSinh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.005, size = 121, normalized size = 0.9

$$\frac{x}{3}\sqrt{x^4+3x^2+2}-\frac{2i}{3}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}+\frac{i}{2}\sqrt{2}\left(EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)-EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 1/3*x*(x^4+3*x^2+2)^(1/2)-2/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(
1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/2*I
*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(Ellip
ticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^4+3x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{x^4+3x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^4+3x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**4 + 3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2), x)
```

3.294 $\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx$

Optimal. Leaf size=178

$$\frac{x\left(x^2+2\right)}{5\sqrt{x^4+3x^2+2}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{2x^2+2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] (x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt
[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^
2 + x^4]) + ((1 + x^2)*Sqrt[(2 + x^2)/(2 + 2*x^2)]*EllipticF[ArcT
an[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x
^2)/(1 + x^2)]*EllipticPi[2/7, ArcTan[x], 1/2])/(35*Sqrt[2]*Sqrt[
2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.332071, antiderivative size = 232, normalized size of antiderivative = 1.3, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{x\left(x^2+2\right)}{5\sqrt{x^4+3x^2+2}}+\frac{4\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{25\sqrt{x^4+3x^2+2}}-\frac{3\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{25\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}}+\frac{3\left(x^2+2\right)\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] (x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt
[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^
2 + x^4]) - (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcT
an[x], 1/2])/(25*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (4*Sqrt[2]*(1 +
 x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(25*Sq
rt[2 + 3*x^2 + x^4]) + (3*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/
2])/(35*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [C] time = 0.0876833, size = 90, normalized size = 0.51

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\left(21F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+35E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-6\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{175\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] ((-I/175)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(35*EllipticE[I*ArcSinh[x/S
qrt[2]], 2] + 21*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - 6*EllipticP
i[10/7, I*ArcSinh[x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] time = 0.036, size = 138, normalized size = 0.8

$$-\frac{3i}{50}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}-\frac{i}{10}\sqrt{2}EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}+\frac{6i}{175}\sqrt{2}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}EllipticPi\left(\frac{i}{2}\sqrt{2}x,\frac{10}{7},\sqrt{2}\right)\frac{1}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x)
```

```
[Out] -3/50*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)
*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-1/10*I*2^(1/2)*(2*x^2+4)^(1/2
)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(
1/2))+6/175*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+
2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+3x^2+2}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+3x^2+2}}{5x^2+7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2+1)(x^2+2)}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7), x)
```

GIAC/XCAS [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 3\,x^2 + 2}}{5\,x^2 + 7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)
```

3.295 $\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2}\,dx$

Optimal. Leaf size=209

$$\frac{\sqrt{x^4+3x^2+2}\,x}{14(5x^2+7)}-\frac{(x^2+2)\,x}{70\sqrt{x^4+3x^2+2}}+\frac{3(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{140\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{(x^2+2)\,\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{980\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

```
[Out] -(x*(2 + x^2))/(70*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x
^4])/(14*(7 + 5*x^2)) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elli
pticE[ArcTan[x], 1/2])/(35*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (3*(1
 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(140
*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - ((2 + x^2)*EllipticPi[2/7, ArcT
an[x], 1/2])/(980*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^
2 + x^4])
```

Rubi [A] time = 0.336011, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{\sqrt{x^4+3x^2+2}\,x}{14(5x^2+7)}-\frac{(x^2+2)\,x}{70\sqrt{x^4+3x^2+2}}+\frac{3(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{140\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{(x^2+2)\,\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{980\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] -(x*(2 + x^2))/(70*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x
^4])/(14*(7 + 5*x^2)) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elli
pticE[ArcTan[x], 1/2])/(35*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (3*(1
 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(140
*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - ((2 + x^2)*EllipticPi[2/7, ArcT
an[x], 1/2])/(980*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^
2 + x^4])
```

Rubi in Sympy [A] time = 52.7469, size = 173, normalized size = 0.83

$$\frac{x\sqrt{x^4+3x^2+2}}{70x^2+98}-\frac{x\sqrt{x^4+3x^2+2}}{70(x^2+1)}+\frac{\sqrt{x^4+3x^2+2}E\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{70\sqrt{\frac{\frac{x^2}{2}+1}{x^2+1}}(x^2+1)}+\frac{3\sqrt{x^4+3x^2+2}F\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{280\sqrt{\frac{\frac{x^2}{2}+1}{x^2+1}}(x^2+1)}-\frac{\sqrt{x^4+3x^2+2}\left(\frac{2}{7};\operatorname{atan}(x)|\frac{1}{2}\right)}{1960\sqrt{\frac{\frac{x^2}{2}+1}{x^2+1}}(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7)**2,x)
```

```
[Out] x*sqrt(x**4 + 3*x**2 + 2)/(70*x**2 + 98) - x*sqrt(x**4 + 3*x**2 +
 2)/(70*(x**2 + 1)) + sqrt(x**4 + 3*x**2 + 2)*elliptic_e(atan(x),
```

```
1/2)/(70*sqrt((x**2/2 + 1)/(x**2 + 1))*(x**2 + 1)) + 3*sqrt(x**4
+ 3*x**2 + 2)*elliptic_f(atan(x), 1/2)/(280*sqrt((x**2/2 + 1)/(x
**2 + 1))*(x**2 + 1)) - sqrt(x**4 + 3*x**2 + 2)*elliptic_pi(2/7,
atan(x), 1/2)/(1960*sqrt((x**2/2 + 1)/(x**2 + 1))*(x**2 + 1))
```

Mathematica [C] time = 0.209846, size = 208, normalized size = 1.

$$\frac{175x^5 + 525x^3 - 84i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) + 35i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 5}{2450\left(5x^2+7\right)\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (350*x + 525*x^3 + 175*x^5 + (35*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(
7 + 5*x^2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (84*I)*Sqrt[1 + x
^2]*Sqrt[2 + x^2]*(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]
- (7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/
Sqrt[2]], 2] - (5*I)*x^2*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[1
0/7, I*ArcSinh[x/Sqrt[2]], 2])/(2450*(7 + 5*x^2)*Sqrt[2 + 3*x^2 +
 x^4])
```

Maple [C] time = 0.027, size = 162, normalized size = 0.8

$$\frac{x}{70\,x^2+98}\sqrt{x^4+3\,x^2+2} - \frac{3\,i}{175}\sqrt{2}\mathit{EllipticF}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}} + \frac{i}{140}\sqrt{2}\mathit{EllipticE}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}} - \frac{i}{2450}\sqrt{2}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}\mathit{EllipticPi}\left(\frac{i}{2}\sqrt{2}x,\frac{10}{7},\sqrt{2}\right)\frac{1}{\sqrt{x^4+3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x)
```

```
[Out] 1/14*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-3/175*I*2^(1/2)*(2*x^2+4)^(1
/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2
^(1/2))+1/140*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+
2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))-1/2450*I*2^(1/2)*(1+1
/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*
2^(1/2)*x,10/7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+3\,x^2+2}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}}{25\,x^4+70\,x^2+49},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{(x^2+1)\,(x^2+2)}}{(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^4+3\,x^2+2}}{(5\,x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^2, x)
```

3.296 $\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^3}\,dx$

Optimal. Leaf size=237

$$\frac{11\sqrt{x^4+3x^2+2}\,x}{2352\,(5x^2+7)}+\frac{\sqrt{x^4+3x^2+2}\,x}{28\,(5x^2+7)^2}-\frac{11\,(x^2+2)\,x}{11760\sqrt{x^4+3x^2+2}}+\frac{81\,(x^2+1)\,\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{7840\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{11\,(x^2+1)\,\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5880\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{1201\,(x^2+2)\,\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{164640\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

```
[Out] (-11*x*(2 + x^2))/(11760*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x
^2 + x^4])/(28*(7 + 5*x^2)^2) + (11*x*Sqrt[2 + 3*x^2 + x^4])/(235
2*(7 + 5*x^2)) + (11*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elliptic
E[ArcTan[x], 1/2])/(5880*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (81*(1
+ x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(7840
*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (1201*(2 + x^2)*EllipticPi[2/7,
 ArcTan[x], 1/2])/(164640*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[
2 + 3*x^2 + x^4])
```

Rubi [A] time = 1.15798, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$

$$\frac{11\sqrt{x^4+3x^2+2}\,x}{2352\,(5x^2+7)}+\frac{\sqrt{x^4+3x^2+2}\,x}{28\,(5x^2+7)^2}-\frac{11\,(x^2+2)\,x}{11760\sqrt{x^4+3x^2+2}}+\frac{81\,(x^2+1)\,\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{7840\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{11\,(x^2+1)\,\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5880\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{1201\,(x^2+2)\,\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{164640\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (-11*x*(2 + x^2))/(11760*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x
^2 + x^4])/(28*(7 + 5*x^2)^2) + (11*x*Sqrt[2 + 3*x^2 + x^4])/(235
2*(7 + 5*x^2)) + (11*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elliptic
E[ArcTan[x], 1/2])/(5880*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (81*(1
+ x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(7840
*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (1201*(2 + x^2)*EllipticPi[2/7,
 ArcTan[x], 1/2])/(164640*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[
2 + 3*x^2 + x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7)**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.30031, size = 174, normalized size = 0.73

$$\frac{-434i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+385i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-1201i\sqrt{x^2+1}\sqrt{x^2+2}\left(\frac{10}{7};i\sinh^{-1}\right.}{411600\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] ((14700*x*(2 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + (1925*x*(2 + 3*x^2 +
 x^4))/(7 + 5*x^2) + (385*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Elliptic
E[I*ArcSinh[x/Sqrt[2]], 2] - (434*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*
EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (1201*I)*Sqrt[1 + x^2]*Sqrt[
2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(411600*Sqrt[
2 + 3*x^2 + x^4])
```

Maple [C] time = 0.029, size = 186, normalized size = 0.8

$$\frac{x}{28\left(5x^2+7\right)^2}\sqrt{x^4+3x^2+2}+\frac{11x}{11760x^2+16464}\sqrt{x^4+3x^2+2}$$
$$-\frac{31i}{58800}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}$$
$$+\frac{11i}{23520}\sqrt{2}EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}$$
$$-\frac{1201i}{411600}\sqrt{2}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}EllipticPi\left(\frac{i}{2}\sqrt{2}x,\frac{10}{7},\sqrt{2}\right)\frac{1}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x)
```

```
[Out] 1/28*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2+11/2352*x*(x^4+3*x^2+2)^(1
/2)/(5*x^2+7)-31/58800*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x
^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+11/23520*I*2
^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Elliptic
E(1/2*I*2^(1/2)*x,2^(1/2))-1201/411600*I*2^(1/2)*(1+1/2*x^2)^(1/2
)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10
/7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^4+3x^2+2}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+3x^2+2}}{125x^6+525x^4+735x^2+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^6 + 525*x^4 + 735*x^2 + 343
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2+1)(x^2+2)}}{(5x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7)**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+3\,x^2+2}}{(5\,x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^3, x)
```

3.297 $\int \left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=219

$$\frac{3825}{143}\left(x^4+3x^2+2\right)^{5/2}x+\frac{\left(65345x^2+208212\right)\left(x^4+3x^2+2\right)^{3/2}x}{3003}+\frac{\left(297911x^2+1032541\right)\sqrt{x^4+3x^2+2}\,x}{5005}+\frac{20884\left(x^2+2\right)x}{65\sqrt{x^4+3x^2+2}}+\frac{1171349\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5005\sqrt{x^4+3x^2+2}}-\frac{20884\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{65\sqrt{x^4+3x^2+2}}+\frac{125}{13}\left(x^4+3x^2+2\right)^{5/2}x^3$$

```
[Out] (20884*x*(2 + x^2))/(65*Sqrt[2 + 3*x^2 + x^4]) + (x*(1032541 + 29
7911*x^2)*Sqrt[2 + 3*x^2 + x^4])/5005 + (x*(208212 + 65345*x^2)*(
2 + 3*x^2 + x^4)^(3/2))/3003 + (3825*x*(2 + 3*x^2 + x^4)^(5/2))/1
43 + (125*x^3*(2 + 3*x^2 + x^4)^(5/2))/13 - (20884*Sqrt[2]*(1 + x
^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(65*Sqrt
[2 + 3*x^2 + x^4]) + (1171349*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticF[ArcTan[x], 1/2])/(5005*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.241824, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{3825}{143}\left(x^4+3x^2+2\right)^{5/2}x+\frac{\left(65345x^2+208212\right)\left(x^4+3x^2+2\right)^{3/2}x}{3003}+\frac{\left(297911x^2+1032541\right)\sqrt{x^4+3x^2+2}\,x}{5005}+\frac{20884\left(x^2+2\right)x}{65\sqrt{x^4+3x^2+2}}+\frac{1171349\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5005\sqrt{x^4+3x^2+2}}-\frac{20884\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{65\sqrt{x^4+3x^2+2}}+\frac{125}{13}\left(x^4+3x^2+2\right)^{5/2}x^3$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (20884*x*(2 + x^2))/(65*Sqrt[2 + 3*x^2 + x^4]) + (x*(1032541 + 29
7911*x^2)*Sqrt[2 + 3*x^2 + x^4])/5005 + (x*(208212 + 65345*x^2)*(
2 + 3*x^2 + x^4)^(3/2))/3003 + (3825*x*(2 + 3*x^2 + x^4)^(5/2))/1
43 + (125*x^3*(2 + 3*x^2 + x^4)^(5/2))/13 - (20884*Sqrt[2]*(1 + x
^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(65*Sqrt
[2 + 3*x^2 + x^4]) + (1171349*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticF[ArcTan[x], 1/2])/(5005*Sqrt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 42.2697, size = 209, normalized size = 0.95

$$\frac{125x^3\left(x^4+3x^2+2\right)^{\frac{5}{2}}}{13}+\frac{10442x\left(2x^2+4\right)}{65\sqrt{x^4+3x^2+2}}+\frac{x\left(\frac{196035x^2}{143}+\frac{624636}{143}\right)\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{63}+\frac{x\left(\frac{2681199x^2}{143}+\frac{9292869}{143}\right)\sqrt{x^4+3x^2+2}}{315}+\frac{3825x\left(x^4+3x^2+2\right)^{\frac{5}{2}}}{143}-\frac{5221\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)E\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{65\sqrt{x^4+3x^2+2}}+\frac{1171349\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{20020\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**3*(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] 125*x**3*(x**4 + 3*x**2 + 2)**(5/2)/13 + 10442*x*(2*x**2 + 4)/(65
*sqrt(x**4 + 3*x**2 + 2)) + x*(196035*x**2/143 + 624636/143)*(x**
4 + 3*x**2 + 2)**(3/2)/63 + x*(2681199*x**2/143 + 9292869/143)*sq
rt(x**4 + 3*x**2 + 2)/315 + 3825*x*(x**4 + 3*x**2 + 2)**(5/2)/143
 - 5221*sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_e(ata
n(x), 1/2)/(65*sqrt(x**4 + 3*x**2 + 2)) + 1171349*sqrt((2*x**2 +
4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_f(atan(x), 1/2)/(20020*sqrt(
x**4 + 3*x**2 + 2))
```

Mathematica [C]   time = 0.087309, size = 129, normalized size = 0.59

$$\frac{144375x^{15} + 1701000x^{13} + 8705725x^{11} + 25350660x^9 + 46218643x^7 + 54938052x^5 + 40493455x^3 - 2203890i\sqrt{x^2+1}\sqrt{x^2+}}{15015\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (13572486*x + 40493455*x^3 + 54938052*x^5 + 46218643*x^7 + 253506
60*x^9 + 8705725*x^11 + 1701000*x^13 + 144375*x^15 - (4824204*I)*
Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] -
(2203890*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqr
t[2]], 2])/(15015*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C]   time = 0.026, size = 206, normalized size = 0.9

$$\frac{598324x^5}{1001}\sqrt{x^4+3x^2+2} + \frac{10067363x^3}{15015}\sqrt{x^4+3x^2+2} + \frac{2262081x}{5005}\sqrt{x^4+3x^2+2} - \frac{1171349i}{5005}\sqrt{2}\mathit{EllipticF}\left(\frac{i}{2}\sqrt{2}x, \sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}} + \frac{10442i}{65}\sqrt{2}\left(\mathit{EllipticF}\left(\frac{i}{2}\sqrt{2}x, \sqrt{2}\right) - \mathit{EllipticE}\left(\frac{i}{2}\sqrt{2}x, \sqrt{2}\right)\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}} + \frac{131810x^7}{429}\sqrt{x^4+3x^2+2} + \frac{12075x^9}{143}\sqrt{x^4+3x^2+2} + \frac{125x^{11}}{13}\sqrt{x^4+3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] 598324/1001*x^5*(x^4+3*x^2+2)^(1/2)+10067363/15015*x^3*(x^4+3*x^2
+2)^(1/2)+2262081/5005*x*(x^4+3*x^2+2)^(1/2)-1171349/5005*I*2^(1/
2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/
2*I*2^(1/2)*x,2^(1/2))+10442/65*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)
^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-El
lipticE(1/2*I*2^(1/2)*x,2^(1/2)))+131810/429*x^7*(x^4+3*x^2+2)^(1
/2)+12075/143*x^9*(x^4+3*x^2+2)^(1/2)+125/13*x^11*(x^4+3*x^2+2)^(
1/2)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 2)^{\frac{3}{2}} (5x^2 + 7)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(125\,x^{10} + 900\,x^{8} + 2560\,x^{6} + 3598\,x^{4} + 2499\,x^{2} + 686\right)\sqrt{x^{4} + 3\,x^{2} + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3,x, algorithm="fricas")
```

```
[Out] integral((125*x^10 + 900*x^8 + 2560*x^6 + 3598*x^4 + 2499*x^2 + 6
86)*sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(x^{2} + 1\right)\left(x^{2} + 2\right)\right)^{\frac{3}{2}} \left(5x^{2} + 7\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**3*(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^{4} + 3\,x^{2} + 2\right)^{\frac{3}{2}} \left(5\,x^{2} + 7\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

3.298 $\int (7+5x^2)^2 (2+3x^2+x^4)^{3/2} \, dx$

Optimal. Leaf size=198

$$\frac{25}{11}x\left(x^4+3x^2+2\right)^{5/2}+\frac{1}{693}x\left(2240x^2+7281\right)\left(x^4+3x^2+2\right)^{3/2}+\frac{x\left(10643x^2+36783\right)\sqrt{x^4+3x^2+2}}{1155}+\frac{742x\left(x^2+2\right)}{15\sqrt{x^4+3x^2+2}}+\frac{13879\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{385\sqrt{x^4+3x^2+2}}-\frac{742\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{15\sqrt{x^4+3x^2+2}}$$

```
[Out] (742*x*(2 + x^2))/(15*Sqrt[2 + 3*x^2 + x^4]) + (x*(36783 + 10643*
x^2)*Sqrt[2 + 3*x^2 + x^4])/1155 + (x*(7281 + 2240*x^2)*(2 + 3*x^
2 + x^4)^(3/2))/693 + (25*x*(2 + 3*x^2 + x^4)^(5/2))/11 - (742*Sq
rt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/
2])/(15*Sqrt[2 + 3*x^2 + x^4]) + (13879*Sqrt[2]*(1 + x^2)*Sqrt[(2
 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(385*Sqrt[2 + 3*x^2
 + x^4])
```

Rubi [A] time = 0.195864, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{25}{11}x\left(x^4+3x^2+2\right)^{5/2}+\frac{1}{693}x\left(2240x^2+7281\right)\left(x^4+3x^2+2\right)^{3/2}+\frac{x\left(10643x^2+36783\right)\sqrt{x^4+3x^2+2}}{1155}+\frac{742x\left(x^2+2\right)}{15\sqrt{x^4+3x^2+2}}+\frac{13879\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{385\sqrt{x^4+3x^2+2}}-\frac{742\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{15\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (742*x*(2 + x^2))/(15*Sqrt[2 + 3*x^2 + x^4]) + (x*(36783 + 10643*
x^2)*Sqrt[2 + 3*x^2 + x^4])/1155 + (x*(7281 + 2240*x^2)*(2 + 3*x^
2 + x^4)^(3/2))/693 + (25*x*(2 + 3*x^2 + x^4)^(5/2))/11 - (742*Sq
rt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/
2])/(15*Sqrt[2 + 3*x^2 + x^4]) + (13879*Sqrt[2]*(1 + x^2)*Sqrt[(2
 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(385*Sqrt[2 + 3*x^2
 + x^4])
```

Rubi in Sympy [A] time = 33.077, size = 189, normalized size = 0.95

$$\frac{371x\left(2x^2+4\right)}{15\sqrt{x^4+3x^2+2}}+\frac{x\left(\frac{2240x^2}{11}+\frac{7281}{11}\right)\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{63}+\frac{x\left(\frac{31929x^2}{11}+\frac{110349}{11}\right)\sqrt{x^4+3x^2+2}}{315}+\frac{25x\left(x^4+3x^2+2\right)^{\frac{5}{2}}}{11}-\frac{371\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)E\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{30\sqrt{x^4+3x^2+2}}+\frac{13879\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)F\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{1540\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**2*(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] 371*x*(2*x**2 + 4)/(15*sqrt(x**4 + 3*x**2 + 2)) + x*(2240*x**2/11
 + 7281/11)*(x**4 + 3*x**2 + 2)**(3/2)/63 + x*(31929*x**2/11 + 11
```

```
0349/11)*sqrt(x**4 + 3*x**2 + 2)/315 + 25*x*(x**4 + 3*x**2 + 2)**
(5/2)/11 - 371*sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*ellipti
c_e(atan(x), 1/2)/(30*sqrt(x**4 + 3*x**2 + 2)) + 13879*sqrt((2*x*
*2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_f(atan(x), 1/2)/(1540*s
qrt(x**4 + 3*x**2 + 2))
```

Mathematica [C] time = 0.0832228, size = 124, normalized size = 0.63

$$\frac{7875x^{13} + 82075x^{11} + 363480x^9 + 892084x^7 + 1333551x^5 + 1160065x^3 - 78420i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 1714}{3465\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (429318*x + 1160065*x^3 + 1333551*x^5 + 892084*x^7 + 363480*x^9 +
 82075*x^11 + 7875*x^13 - (171402*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*
EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (78420*I)*Sqrt[1 + x^2]*Sqrt
[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(3465*Sqrt[2 + 3*x^
2 + x^4])
```

Maple [C] time = 0.011, size = 189, normalized size = 1.

$$\frac{11492\,x^5}{231}\sqrt{x^4+3\,x^2+2} + \frac{258044\,x^3}{3465}\sqrt{x^4+3\,x^2+2} + \frac{23851\,x}{385}\sqrt{x^4+3\,x^2+2} - \frac{13879\,i}{385}\sqrt{2}\mathit{EllipticF}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}} + \frac{371\,i}{15}\sqrt{2}\left(\mathit{EllipticF}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right) - \mathit{EllipticE}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}} + \frac{1670\,x^7}{99}\sqrt{x^4+3\,x^2+2} + \frac{25\,x^9}{11}\sqrt{x^4+3\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] 11492/231*x^5*(x^4+3*x^2+2)^(1/2)+258044/3465*x^3*(x^4+3*x^2+2)^(
1/2)+23851/385*x*(x^4+3*x^2+2)^(1/2)-13879/385*I*2^(1/2)*(2*x^2+4
)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)
*x,2^(1/2))+371/15*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3
*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I
*2^(1/2)*x,2^(1/2)))+1670/99*x^7*(x^4+3*x^2+2)^(1/2)+25/11*x^9*(x
^4+3*x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4 + 3\,x^2 + 2\right)^{\frac{3}{2}}\left(5\,x^2 + 7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(25\,x^8+145\,x^6+309\,x^4+287\,x^2+98\right)\sqrt{x^4+3\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2,x, algorithm="fricas")
```

```
[Out] integral((25*x^8 + 145*x^6 + 309*x^4 + 287*x^2 + 98)*sqrt(x^4 + 3
*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**2*(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

3.299 $\int (7+5x^2) (2+3x^2+x^4)^{3/2} \, dx$

Optimal. Leaf size=179

$$\frac{1}{63}x\left(35x^2+108\right)\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{105}x\left(149x^2+519\right)\sqrt{x^4+3x^2+2}+\frac{116x\left(x^2+2\right)}{15\sqrt{x^4+3x^2+2}}+\frac{197\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{x^4+3x^2+2}}-\frac{116\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{15\sqrt{x^4+3x^2+2}}$$

```
[Out] (116*x*(2 + x^2))/(15*Sqrt[2 + 3*x^2 + x^4]) + (x*(519 + 149*x^2)
*Sqrt[2 + 3*x^2 + x^4])/105 + (x*(108 + 35*x^2)*(2 + 3*x^2 + x^4)
^(3/2))/63 - (116*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ell
ipticE[ArcTan[x], 1/2])/(15*Sqrt[2 + 3*x^2 + x^4]) + (197*Sqrt[2]
*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(
35*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.150632, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{1}{63}x\left(35x^2+108\right)\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{105}x\left(149x^2+519\right)\sqrt{x^4+3x^2+2}+\frac{116x\left(x^2+2\right)}{15\sqrt{x^4+3x^2+2}}+\frac{197\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{x^4+3x^2+2}}-\frac{116\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{15\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (116*x*(2 + x^2))/(15*Sqrt[2 + 3*x^2 + x^4]) + (x*(519 + 149*x^2)
*Sqrt[2 + 3*x^2 + x^4])/105 + (x*(108 + 35*x^2)*(2 + 3*x^2 + x^4)
^(3/2))/63 - (116*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ell
ipticE[ArcTan[x], 1/2])/(15*Sqrt[2 + 3*x^2 + x^4]) + (197*Sqrt[2]
*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(
35*Sqrt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 23.3048, size = 163, normalized size = 0.91

$$\frac{58x\left(2x^2+4\right)}{15\sqrt{x^4+3x^2+2}}+\frac{x\left(35x^2+108\right)\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{63}+\frac{x\left(447x^2+1557\right)\sqrt{x^4+3x^2+2}}{315}-\frac{29\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)E\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{15\sqrt{x^4+3x^2+2}}+\frac{197\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)F\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{140\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)*(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] 58*x*(2*x**2 + 4)/(15*sqrt(x**4 + 3*x**2 + 2)) + x*(35*x**2 + 108
)*(x**4 + 3*x**2 + 2)**(3/2)/63 + x*(447*x**2 + 1557)*sqrt(x**4 +
 3*x**2 + 2)/315 - 29*sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*
elliptic_e(atan(x), 1/2)/(15*sqrt(x**4 + 3*x**2 + 2)) + 197*sqrt(
(2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_f(atan(x), 1/2)/(1
40*sqrt(x**4 + 3*x**2 + 2))
```

Mathematica [C] time = 0.0748744, size = 119, normalized size = 0.66

$$\frac{175x^{11}+1590x^9+5962x^7+12018x^5+12745x^3-1110i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-2436i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\right.}{315\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (5274*x + 12745*x^3 + 12018*x^5 + 5962*x^7 + 1590*x^9 + 175*x^11
- (2436*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt
[2]], 2] - (1110*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSi
nh[x/Sqrt[2]], 2])/(315*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.008, size = 172, normalized size = 1.

$$\frac{71x^5}{21}\sqrt{x^4+3x^2+2}+\frac{2417x^3}{315}\sqrt{x^4+3x^2+2}+\frac{293x}{35}\sqrt{x^4+3x^2+2} - \frac{197i}{35}\sqrt{2}\mathit{EllipticF}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}} + \frac{58i}{15}\sqrt{2}\left(\mathit{EllipticF}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}} + \frac{5x^7}{9}\sqrt{x^4+3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] 71/21*x^5*(x^4+3*x^2+2)^(1/2)+2417/315*x^3*(x^4+3*x^2+2)^(1/2)+29
3/35*x*(x^4+3*x^2+2)^(1/2)-197/35*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+5
8/15*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*
(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(
1/2)))+5/9*x^7*(x^4+3*x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^4+3x^2+2\right)^{\frac{3}{2}}\left(5x^2+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(5x^6+22x^4+31x^2+14\right)\sqrt{x^4+3x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7),x, algorithm="fricas")
```

```
[Out] integral((5*x^6 + 22*x^4 + 31*x^2 + 14)*sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(x^{2}+1\right)\left(x^{2}+2\right)\right)^{\frac{3}{2}}\left(5x^{2}+7\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)*(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^{4}+3\,x^{2}+2\right)^{\frac{3}{2}}\left(5\,x^{2}+7\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

3.300 $\int \left(2+3x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=172

$$\frac{1}{7}x\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{35}x\left(9x^2+29\right)\sqrt{x^4+3x^2+2}+\frac{6x\left(x^2+2\right)}{5\sqrt{x^4+3x^2+2}}+\frac{31\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{x^4+3x^2+2}}-\frac{6\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}}$$

```
[Out] (6*x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) + (x*(29 + 9*x^2)*Sqrt[
2 + 3*x^2 + x^4])/35 + (x*(2 + 3*x^2 + x^4)^(3/2))/7 - (6*Sqrt[2]
*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(
5*Sqrt[2 + 3*x^2 + x^4]) + (31*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(
1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(35*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.13493, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\frac{1}{7}x\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{35}x\left(9x^2+29\right)\sqrt{x^4+3x^2+2}+\frac{6x\left(x^2+2\right)}{5\sqrt{x^4+3x^2+2}}+\frac{31\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{x^4+3x^2+2}}-\frac{6\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (6*x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) + (x*(29 + 9*x^2)*Sqrt[
2 + 3*x^2 + x^4])/35 + (x*(2 + 3*x^2 + x^4)^(3/2))/7 - (6*Sqrt[2]
*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(
5*Sqrt[2 + 3*x^2 + x^4]) + (31*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(
1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(35*Sqrt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 21.1195, size = 156, normalized size = 0.91

$$\frac{3x\left(2x^2+4\right)}{5\sqrt{x^4+3x^2+2}}+\frac{x\left(9x^2+29\right)\sqrt{x^4+3x^2+2}}{35}+\frac{x\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{7}-\frac{3\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)E\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{10\sqrt{x^4+3x^2+2}}+\frac{31\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)F\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{140\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+3*x**2+2)**(3/2),x)
```

```
[Out] 3*x*(2*x**2 + 4)/(5*sqrt(x**4 + 3*x**2 + 2)) + x*(9*x**2 + 29)*sq
rt(x**4 + 3*x**2 + 2)/35 + x*(x**4 + 3*x**2 + 2)**(3/2)/7 - 3*sqr
t((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_e(atan(x), 1/2)/
(10*sqrt(x**4 + 3*x**2 + 2)) + 31*sqrt((2*x**2 + 4)/(x**2 + 1))*(
4*x**2 + 4)*elliptic_f(atan(x), 1/2)/(140*sqrt(x**4 + 3*x**2 + 2)
)
```

Mathematica [C] time = 0.0665449, size = 114, normalized size = 0.66

$$\frac{5x^9+39x^7+121x^5+165x^3-20i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-42i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+78x}{35\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (78*x + 165*x^3 + 121*x^5 + 39*x^7 + 5*x^9 - (42*I)*Sqrt[1 + x^2]
*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (20*I)*Sqrt[1
 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(35*Sqr
t[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.005, size = 155, normalized size = 0.9

$$\frac{x^5}{7}\sqrt{x^4+3x^2+2}+\frac{24x^3}{35}\sqrt{x^4+3x^2+2}+\frac{39x}{35}\sqrt{x^4+3x^2+2} - \frac{31i}{35}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}} + \frac{3i}{5}\sqrt{2}\left(EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)-EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+3*x^2+2)^(3/2),x)
```

```
[Out] 1/7*x^5*(x^4+3*x^2+2)^(1/2)+24/35*x^3*(x^4+3*x^2+2)^(1/2)+39/35*x
*(x^4+3*x^2+2)^(1/2)-31/35*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2
)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+3/5*I*2^
(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(Elliptic
F(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^4+3x^2+2\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(x^4+3x^2+2\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 2)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+3x^2+2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((x**4 + 3*x**2 + 2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2), x)
```

3.301 $\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2}\,dx$

Optimal. Leaf size=207

$$\frac{24x\left(x^2+2\right)}{125\sqrt{x^4+3x^2+2}}+\frac{1}{75}x\left(3x^2+11\right)\sqrt{x^4+3x^2+2}+\frac{56\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{375\sqrt{x^4+3x^2+2}}-\frac{24\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{125\sqrt{x^4+3x^2+2}}-\frac{9\sqrt{2}\left(x^2+2\right)\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{875\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

```
[Out] (24*x*(2 + x^2))/(125*Sqrt[2 + 3*x^2 + x^4]) + (x*(11 + 3*x^2)*Sq
rt[2 + 3*x^2 + x^4])/75 - (24*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticE[ArcTan[x], 1/2])/(125*Sqrt[2 + 3*x^2 + x^4]) +
 (56*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan
[x], 1/2])/(375*Sqrt[2 + 3*x^2 + x^4]) - (9*Sqrt[2]*(2 + x^2)*Ell
ipticPi[2/7, ArcTan[x], 1/2])/(875*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt
[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.488555, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{24x\left(x^2+2\right)}{125\sqrt{x^4+3x^2+2}}+\frac{1}{75}x\left(3x^2+11\right)\sqrt{x^4+3x^2+2}+\frac{56\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{375\sqrt{x^4+3x^2+2}}-\frac{24\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{125\sqrt{x^4+3x^2+2}}-\frac{9\sqrt{2}\left(x^2+2\right)\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{875\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (24*x*(2 + x^2))/(125*Sqrt[2 + 3*x^2 + x^4]) + (x*(11 + 3*x^2)*Sq
rt[2 + 3*x^2 + x^4])/75 - (24*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticE[ArcTan[x], 1/2])/(125*Sqrt[2 + 3*x^2 + x^4]) +
 (56*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan
[x], 1/2])/(375*Sqrt[2 + 3*x^2 + x^4]) - (9*Sqrt[2]*(2 + x^2)*Ell
ipticPi[2/7, ArcTan[x], 1/2])/(875*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt
[2 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 63.7172, size = 214, normalized size = 1.03

$$\frac{x^3\sqrt{x^4+3x^2+2}}{25}+\frac{12x\left(2x^2+4\right)}{125\sqrt{x^4+3x^2+2}}+\frac{11x\sqrt{x^4+3x^2+2}}{75}-\frac{6\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)E\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{125\sqrt{x^4+3x^2+2}}+\frac{47\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{1500\sqrt{x^4+3x^2+2}}+\frac{48\sqrt{2}\sqrt{x^4+3x^2+2}\left(-\frac{3}{7};\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|-1\right)}{875\sqrt{\frac{2x^2+2}{x^2+2}}\left(2x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7),x)
```

```
[Out] x**3*sqrt(x**4 + 3*x**2 + 2)/25 + 12*x*(2*x**2 + 4)/(125*sqrt(x**
4 + 3*x**2 + 2)) + 11*x*sqrt(x**4 + 3*x**2 + 2)/75 - 6*sqrt((2*x*
*2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_e(atan(x), 1/2)/(125*sq
```

```
rt(x**4 + 3*x**2 + 2)) + 47*sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2
 + 4)*elliptic_f(atan(x), 1/2)/(1500*sqrt(x**4 + 3*x**2 + 2)) + 4
8*sqrt(2)*sqrt(x**4 + 3*x**2 + 2)*elliptic_pi(-3/7, atan(sqrt(2)*
x/2), -1)/(875*sqrt((2*x**2 + 2)/(x**2 + 2))*(2*x**2 + 4))
```

Mathematica [C] time = 0.132934, size = 148, normalized size = 0.71

$$\frac{525x^7 + 3500x^5 + 6825x^3 - 1022i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 2520i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 108i\sqrt{}}{13125\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (3850*x + 6825*x^3 + 3500*x^5 + 525*x^7 - (2520*I)*Sqrt[1 + x^2]*
Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (1022*I)*Sqrt[
1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (108*
I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[
2]], 2])/(13125*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.021, size = 170, normalized size = 0.8

$$\frac{x^3}{25}\sqrt{x^4+3x^2+2}+\frac{11x}{75}\sqrt{x^4+3x^2+2} - \frac{73i}{1875}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}} - \frac{12i}{125}\sqrt{2}EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}} - \frac{36i}{4375}\sqrt{2}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}EllipticPi\left(\frac{i}{2}\sqrt{2}x,\frac{10}{7},\sqrt{2}\right)\frac{1}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x)
```

```
[Out] 1/25*x^3*(x^4+3*x^2+2)^(1/2)+11/75*x*(x^4+3*x^2+2)^(1/2)-73/1875*
I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Ellip
ticF(1/2*I*2^(1/2)*x,2^(1/2))-12/125*I*2^(1/2)*(2*x^2+4)^(1/2)*(x
^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2)
)-36/4375*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)
^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{5\,x^2+7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7),x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

3.302 $\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx$

Optimal. Leaf size=222

$$-\frac{3\sqrt{x^4+3x^2+2}\,x}{175\left(5x^2+7\right)}+\frac{1}{75}\sqrt{x^4+3x^2+2}\,x+\frac{9\left(x^2+2\right)x}{175\sqrt{x^4+3x^2+2}}+\frac{59\left(x^2+1\right)\sqrt{\frac{x^2+2}{2x^2+2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{1050\sqrt{x^4+3x^2+2}}-\frac{9\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{175\sqrt{x^4+3x^2+2}}+\frac{9\left(x^2+1\right)\sqrt{\frac{x^2+2}{2x^2+2}}\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2450\sqrt{x^4+3x^2+2}}$$

```
[Out] (9*x*(2 + x^2))/(175*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 +
 x^4])/75 - (3*x*Sqrt[2 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) - (9*Sq
rt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/
2])/(175*Sqrt[2 + 3*x^2 + x^4]) + (59*(1 + x^2)*Sqrt[(2 + x^2)/(2
 + 2*x^2)]*EllipticF[ArcTan[x], 1/2])/(1050*Sqrt[2 + 3*x^2 + x^4]
) + (9*(1 + x^2)*Sqrt[(2 + x^2)/(2 + 2*x^2)]*EllipticPi[2/7, ArcT
an[x], 1/2])/(2450*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.867941, antiderivative size = 333, normalized size of antiderivative = 1.5, number of steps used = 20, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{3\sqrt{x^4+3x^2+2}\,x}{175\left(5x^2+7\right)}+\frac{1}{75}\sqrt{x^4+3x^2+2}\,x+\frac{9\left(x^2+2\right)x}{175\sqrt{x^4+3x^2+2}}+\frac{44\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{1875\sqrt{x^4+3x^2+2}}+\frac{81\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{8750\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{9\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{175\sqrt{x^4+3x^2+2}}+\frac{3\sqrt{2}\left(x^2+2\right)\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{875\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}-\frac{39\left(x^2+2\right)\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{12250\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (9*x*(2 + x^2))/(175*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 +
 x^4])/75 - (3*x*Sqrt[2 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) - (9*Sq
rt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/
2])/(175*Sqrt[2 + 3*x^2 + x^4]) + (81*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticF[ArcTan[x], 1/2])/(8750*Sqrt[2]*Sqrt[2 + 3*x^2
+ x^4]) + (44*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellipti
cF[ArcTan[x], 1/2])/(1875*Sqrt[2 + 3*x^2 + x^4]) - (39*(2 + x^2)*
EllipticPi[2/7, ArcTan[x], 1/2])/(12250*Sqrt[2]*Sqrt[(2 + x^2)/(1
 + x^2)]*Sqrt[2 + 3*x^2 + x^4]) + (3*Sqrt[2]*(2 + x^2)*EllipticPi
[2/7, ArcTan[x], 1/2])/(875*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*
x^2 + x^4])
```

Rubi in Sympy [A] time = 131.853, size = 258, normalized size = 1.16

$$\frac{x\sqrt{x^4+3x^2+2}}{75}-\frac{18x\sqrt{x^4+3x^2+2}}{25\left(210x^2+294\right)}+\frac{9x\sqrt{x^4+3x^2+2}}{175\left(x^2+1\right)}-\frac{9\sqrt{x^4+3x^2+2}E\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{175\sqrt{\frac{\frac{x^2}{2}+1}{x^2+1}}\left(x^2+1\right)}+\frac{211\sqrt{x^4+3x^2+2}F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{10500\sqrt{\frac{\frac{x^2}{2}+1}{x^2+1}}\left(x^2+1\right)}+\frac{129\sqrt{x^4+3x^2+2}\left(\frac{2}{7};\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{24500\sqrt{\frac{\frac{x^2}{2}+1}{x^2+1}}\left(x^2+1\right)}+\frac{4\sqrt{2}\sqrt{x^4+3x^2+2}\left(-\frac{3}{7};\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|-1\right)}{875\sqrt{\frac{x^2+1}{\frac{x^2}{2}+1}}\left(\frac{x^2}{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7)**2,x)
```

```
[Out] x*sqrt(x**4 + 3*x**2 + 2)/75 - 18*x*sqrt(x**4 + 3*x**2 + 2)/(25*(
210*x**2 + 294)) + 9*x*sqrt(x**4 + 3*x**2 + 2)/(175*(x**2 + 1)) -
 9*sqrt(x**4 + 3*x**2 + 2)*elliptic_e(atan(x), 1/2)/(175*sqrt((x*
*2/2 + 1)/(x**2 + 1))*(x**2 + 1)) + 211*sqrt(x**4 + 3*x**2 + 2)*e
lliptic_f(atan(x), 1/2)/(10500*sqrt((x**2/2 + 1)/(x**2 + 1))*(x**
2 + 1)) + 129*sqrt(x**4 + 3*x**2 + 2)*elliptic_pi(2/7, atan(x), 1
/2)/(24500*sqrt((x**2/2 + 1)/(x**2 + 1))*(x**2 + 1)) + 4*sqrt(2)*
sqrt(x**4 + 3*x**2 + 2)*elliptic_pi(-3/7, atan(sqrt(2)*x/2), -1)/
(875*sqrt((x**2 + 1)/(x**2/2 + 1))*(x**2/2 + 1))
```

Mathematica [C]   time = 0.214174, size = 213, normalized size = 0.96

$$\frac{1225x^7+5075x^5+6650x^3-182i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-945i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)E\left(i\sinh\right.}{18375\left(5x^2+7\right)\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (2800*x + 6650*x^3 + 5075*x^5 + 1225*x^7 - (945*I)*Sqrt[1 + x^2]*
Sqrt[2 + x^2]*(7 + 5*x^2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (1
82*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)*EllipticF[I*ArcSinh
[x/Sqrt[2]], 2] + (189*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[
10/7, I*ArcSinh[x/Sqrt[2]], 2] + (135*I)*x^2*Sqrt[1 + x^2]*Sqrt[2
 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(18375*(7 + 5*
x^2)*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C]   time = 0.028, size = 177, normalized size = 0.8

$$\begin{aligned}
&-\frac{3x}{875x^2+1225}\sqrt{x^4+3x^2+2}+\frac{x}{75}\sqrt{x^4+3x^2+2}\\
&-\frac{13i}{2625}\sqrt{2}\mathit{EllipticF}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}\\
&-\frac{9i}{350}\sqrt{2}\mathit{EllipticE}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}\\
&+\frac{9i}{6125}\sqrt{2}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}\mathit{EllipticPi}\left(\frac{i}{2}\sqrt{2}x,\frac{10}{7},\sqrt{2}\right)\frac{1}{\sqrt{x^4+3x^2+2}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x)
```

```
[Out] -3/175*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)+1/75*x*(x^4+3*x^2+2)^(1/2)
-13/2625*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1
/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-9/350*I*2^(1/2)*(2*x^2+4)^
(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x
,2^(1/2))+9/6125*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3
*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^2,x, algorithm="maxima")

[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{25\,x^4+70\,x^2+49},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^2,x, algorithm="fricas")

[Out] integral((x^4 + 3*x^2 + 2)^(3/2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7)**2,x)

[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^2,x, algorithm="giac")

[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

3.303 $\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3} dx$

Optimal. Leaf size=231

$$\frac{17\sqrt{x^4+3x^2+2}\,x}{9800(5x^2+7)} - \frac{3\sqrt{x^4+3x^2+2}\,x}{350(5x^2+7)^2} + \frac{3(x^2+2)x}{392\sqrt{x^4+3x^2+2}} + \frac{5(x^2+1)\sqrt{\frac{x^2+2}{2x^2+2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{784\sqrt{x^4+3x^2+2}} - \frac{3(x^2+1)\sqrt{\frac{x^2+2}{2x^2+2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{196\sqrt{x^4+3x^2+2}} + \frac{141(x^2+2)\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{27440\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

```
[Out] (3*x*(2 + x^2))/(392*Sqrt[2 + 3*x^2 + x^4]) - (3*x*Sqrt[2 + 3*x^2
 + x^4])/(350*(7 + 5*x^2)^2) + (17*x*Sqrt[2 + 3*x^2 + x^4])/(9800
*(7 + 5*x^2)) - (3*(1 + x^2)*Sqrt[(2 + x^2)/(2 + 2*x^2)]*Elliptic
E[ArcTan[x], 1/2])/(196*Sqrt[2 + 3*x^2 + x^4]) + (5*(1 + x^2)*Sqr
t[(2 + x^2)/(2 + 2*x^2)]*EllipticF[ArcTan[x], 1/2])/(784*Sqrt[2 +
 3*x^2 + x^4]) + (141*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/
(27440*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 1.26706, antiderivative size = 288, normalized size of antiderivative = 1.25, number of steps used = 26, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458

$$\frac{17\sqrt{x^4+3x^2+2}\,x}{9800(5x^2+7)} - \frac{3\sqrt{x^4+3x^2+2}\,x}{350(5x^2+7)^2} + \frac{3(x^2+2)x}{392\sqrt{x^4+3x^2+2}} + \frac{5(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{784\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{6\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{875\sqrt{x^4+3x^2+2}} - \frac{39(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{24500\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{141(x^2+2)\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{27440\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] (3*x*(2 + x^2))/(392*Sqrt[2 + 3*x^2 + x^4]) - (3*x*Sqrt[2 + 3*x^2
 + x^4])/(350*(7 + 5*x^2)^2) + (17*x*Sqrt[2 + 3*x^2 + x^4])/(9800
*(7 + 5*x^2)) - (39*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE
[ArcTan[x], 1/2])/(24500*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (6*Sqrt
[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2]
)/(875*Sqrt[2 + 3*x^2 + x^4]) + (5*(1 + x^2)*Sqrt[(2 + x^2)/(1 +
x^2)]*EllipticF[ArcTan[x], 1/2])/(784*Sqrt[2]*Sqrt[2 + 3*x^2 + x^
4]) + (141*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(27440*Sqrt
[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7)**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.319737, size = 174, normalized size = 0.75

$$\frac{-406i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-525i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+141i\sqrt{x^2+1}\sqrt{x^2+2}\left(\frac{10}{7};i\sinh^{-1}\left(\right.\right.}{68600\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] ((-588*x*(2 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + (119*x*(2 + 3*x^2 + x
^4))/(7 + 5*x^2) - (525*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[
I*ArcSinh[x/Sqrt[2]], 2] - (406*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*El
lipticF[I*ArcSinh[x/Sqrt[2]], 2] + (141*I)*Sqrt[1 + x^2]*Sqrt[2 +
 x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(68600*Sqrt[2 +
3*x^2 + x^4])
```

Maple [C] time = 0.029, size = 186, normalized size = 0.8

$$-\frac{3x}{350\left(5x^2+7\right)^2}\sqrt{x^4+3x^2+2}+\frac{17x}{49000x^2+68600}\sqrt{x^4+3x^2+2}$$
$$-\frac{29i}{9800}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}$$
$$-\frac{3i}{784}\sqrt{2}EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}$$
$$+\frac{141i}{68600}\sqrt{2}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}EllipticPi\left(\frac{i}{2}\sqrt{2}x,\frac{10}{7},\sqrt{2}\right)\frac{1}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x)
```

```
[Out] -3/350*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2+17/9800*x*(x^4+3*x^2+2)^
(1/2)/(5*x^2+7)-29/9800*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(
x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-3/784*I*2^(
1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(
1/2*I*2^(1/2)*x,2^(1/2))+141/68600*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x
^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^3,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{125x^6+525x^4+735x^2+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^3,x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 2)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 3
43), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(x^{2}+1\right)\left(x^{2}+2\right)\right)^{\frac{3}{2}}}{\left(5x^{2}+7\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7)**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^{4}+3\,x^{2}+2\right)^{\frac{3}{2}}}{\left(5\,x^{2}+7\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^3,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

3.304 $\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=157

$$75\sqrt{x^4+3x^2+2}\,x+\frac{135\left(x^2+2\right)x}{\sqrt{x^4+3x^2+2}}+\frac{193\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{135\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+25\sqrt{x^4+3x^2+2}\,x^3$$

```
[Out] (135*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + 75*x*Sqrt[2 + 3*x^2 + x
^4] + 25*x^3*Sqrt[2 + 3*x^2 + x^4] - (135*Sqrt[2]*(1 + x^2)*Sqrt[
(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 +
x^4] + (193*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[
x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.169904, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$75\sqrt{x^4+3x^2+2}\,x+\frac{135\left(x^2+2\right)x}{\sqrt{x^4+3x^2+2}}+\frac{193\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{135\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+25\sqrt{x^4+3x^2+2}\,x^3$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (135*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + 75*x*Sqrt[2 + 3*x^2 + x
^4] + 25*x^3*Sqrt[2 + 3*x^2 + x^4] - (135*Sqrt[2]*(1 + x^2)*Sqrt[
(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 +
x^4] + (193*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[
x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 31.0747, size = 151, normalized size = 0.96

$$25x^3\sqrt{x^4+3x^2+2}+\frac{135x\left(2x^2+4\right)}{2\sqrt{x^4+3x^2+2}}+75x\sqrt{x^4+3x^2+2}-\frac{135\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)E\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{4\sqrt{x^4+3x^2+2}}+\frac{193\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{8\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**3/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] 25*x**3*sqrt(x**4 + 3*x**2 + 2) + 135*x*(2*x**2 + 4)/(2*sqrt(x**4
 + 3*x**2 + 2)) + 75*x*sqrt(x**4 + 3*x**2 + 2) - 135*sqrt((2*x**2
 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_e(atan(x), 1/2)/(4*sqrt(x
**4 + 3*x**2 + 2)) + 193*sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 +
4)*elliptic_f(atan(x), 1/2)/(8*sqrt(x**4 + 3*x**2 + 2))
```

Mathematica [C] time = 0.10496, size = 106, normalized size = 0.68

$$\frac{-58i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-135i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+25x\left(x^6+6x^4+11x^2+6\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (25*x*(6 + 11*x^2 + 6*x^4 + x^6) - (135*I)*Sqrt[1 + x^2]*Sqrt[2 +
 x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (58*I)*Sqrt[1 + x^2]*S
qrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/Sqrt[2 + 3*x^2 +
 x^4]
```

Maple [C] time = 0.025, size = 138, normalized size = 0.9

$$-\frac{193\,i}{2}\sqrt{2}\mathit{EllipticF}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}$$
$$+\frac{135\,i}{2}\sqrt{2}\left(\mathit{EllipticF}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}$$
$$+75\,x\sqrt{x^4+3\,x^2+2}+25\,x^3\sqrt{x^4+3\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -193/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2
)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+135/2*I*2^(1/2)*(2*x^2+4)^(1
/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,
2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))+75*x*(x^4+3*x^2+2)^(
1/2)+25*x^3*(x^4+3*x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{125\,x^6+525\,x^4+735\,x^2+343}{\sqrt{x^4+3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)/sqrt(x^4 + 3*x^2 + 2
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^3}{\sqrt{(x^2+1)(x^2+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**3/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5\,x^2+7)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

3.305 $\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=142

$$\frac{25}{3}\sqrt{x^4+3x^2+2}\,x+\frac{20\left(x^2+2\right)x}{\sqrt{x^4+3x^2+2}}+\frac{97\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{20\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

```
[Out] (20*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (25*x*Sqrt[2 + 3*x^2 + x
^4])/3 - (20*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elliptic
E[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (97*(1 + x^2)*Sqrt[(2
+ x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 +
3*x^2 + x^4])
```

Rubi [A] time = 0.128537, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{25}{3}\sqrt{x^4+3x^2+2}\,x+\frac{20\left(x^2+2\right)x}{\sqrt{x^4+3x^2+2}}+\frac{97\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{20\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (20*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (25*x*Sqrt[2 + 3*x^2 + x
^4])/3 - (20*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elliptic
E[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (97*(1 + x^2)*Sqrt[(2
+ x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 +
3*x^2 + x^4])
```

Rubi in Sympy [A] time = 21.3959, size = 131, normalized size = 0.92

$$\frac{10x\left(2x^2+4\right)}{\sqrt{x^4+3x^2+2}}+\frac{25x\sqrt{x^4+3x^2+2}}{3}-\frac{5\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)E\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{97\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{24\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] 10*x*(2*x**2 + 4)/sqrt(x**4 + 3*x**2 + 2) + 25*x*sqrt(x**4 + 3*x*
*2 + 2)/3 - 5*sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic
_e(atan(x), 1/2)/sqrt(x**4 + 3*x**2 + 2) + 97*sqrt((2*x**2 + 4)/(
x**2 + 1))*(4*x**2 + 4)*elliptic_f(atan(x), 1/2)/(24*sqrt(x**4 +
3*x**2 + 2))
```

Mathematica [C] time = 0.090911, size = 104, normalized size = 0.73

$$\frac{-37i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-60i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+25x\left(x^4+3x^2+2\right)}{3\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (25*x*(2 + 3*x^2 + x^4) - (60*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Elli
pticE[I*ArcSinh[x/Sqrt[2]], 2] - (37*I)*Sqrt[1 + x^2]*Sqrt[2 + x^
2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.01, size = 121, normalized size = 0.9

$$-\frac{97 i}{6}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}} + 10i\sqrt{2}\left(EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)-EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}} + \frac{25x}{3}\sqrt{x^4+3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -97/6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)
*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+10*I*2^(1/2)*(2*x^2+4)^(1/2)*
(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1
/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))+25/3*x*(x^4+3*x^2+2)^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+7\right)^2}{\sqrt{x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{25x^4+70x^2+49}{\sqrt{x^4+3x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)/sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+7\right)^2}{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

3.306 $\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=121

$$\frac{5x(x^2+2)}{\sqrt{x^4+3x^2+2}}+\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

```
[Out] (5*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (5*Sqrt[2]*(1 + x^2)*Sqrt
[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 +
 x^4] + (7*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x
], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0800457, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{5x(x^2+2)}{\sqrt{x^4+3x^2+2}}+\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (5*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (5*Sqrt[2]*(1 + x^2)*Sqrt
[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 +
 x^4] + (7*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x
], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 12.2912, size = 116, normalized size = 0.96

$$\frac{5x(2x^2+4)}{2\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{\frac{2x^2+4}{x^2+1}}(4x^2+4)\,E\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{4\sqrt{x^4+3x^2+2}}+\frac{7\sqrt{\frac{2x^2+4}{x^2+1}}(4x^2+4)\,F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{8\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] 5*x*(2*x**2 + 4)/(2*sqrt(x**4 + 3*x**2 + 2)) - 5*sqrt((2*x**2 + 4
)/(x**2 + 1))*(4*x**2 + 4)*elliptic_e(atan(x), 1/2)/(4*sqrt(x**4
+ 3*x**2 + 2)) + 7*sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*ell
iptic_f(atan(x), 1/2)/(8*sqrt(x**4 + 3*x**2 + 2))
```

Mathematica [C] time = 0.0597962, size = 69, normalized size = 0.57

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\left(2F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+5E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(5*EllipticE[I*ArcSinh[x/Sqrt[2
]], 2] + 2*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 +
```

```
x^4]
```

Maple [C] time = 0.007, size = 106, normalized size = 0.9

$$-\frac{7i}{2}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}+\frac{5i}{2}\sqrt{2}\left(EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)-EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -7/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*
EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+5/2*I*2^(1/2)*(2*x^2+4)^(1/2)*
(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1
/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2+7}{\sqrt{x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{5x^2+7}{\sqrt{x^4+3x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2+7}{\sqrt{(x^2+1)(x^2+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{\sqrt{x^4 + 3x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2), x)
```

3.307 $\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=48

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(
Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0165178, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(
Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 2.69468, size = 42, normalized size = 0.88

$$\frac{\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{8\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_f(atan(x), 1/
2)/(8*sqrt(x**4 + 3*x**2 + 2))
```

Mathematica [C] time = 0.0252985, size = 50, normalized size = 1.04

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]],
 2])/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] time = 0.004, size = 46, normalized size = 1.

$$-\frac{i}{2}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*
EllipticF(1/2*I*2^(1/2)*x,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^4 + 3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{x^4 + 3\,x^2 + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^4 + 3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**4 + 3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^4 + 3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 2), x)
```

3.308 $\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=106

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5(x^2+2)\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(
2*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (5*(2 + x^2)*EllipticPi[2/7, A
rcTan[x], 1/2])/(14*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*
x^2 + x^4])
```

Rubi [A] time = 0.210988, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5(x^2+2)\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(
2*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (5*(2 + x^2)*EllipticPi[2/7, A
rcTan[x], 1/2])/(14*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*
x^2 + x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [C] time = 0.036814, size = 55, normalized size = 0.52

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{7\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((-I/7)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/
Sqrt[2]], 2])/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] time = 0.018, size = 47, normalized size = 0.4

$$-\frac{i}{7}\sqrt{2}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}\mathit{EllipticPi}\left(\frac{i}{2}\sqrt{2}x,\frac{10}{7},\sqrt{2}\right)\frac{1}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -1/7*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2
)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3x^2+2}(5x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{x^4+3x^2+2}(5x^2+7)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2+1)(x^2+2)}(5x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3x^2+2}(5x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)),x, algorithm="giac")

[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

3.309 $\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=209

$$-\frac{25\sqrt{x^4+3x^2+2}\,x}{84\,(5x^2+7)}+\frac{5\left(x^2+2\right)x}{84\sqrt{x^4+3x^2+2}}+\frac{9\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{56\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{42\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{65\left(x^2+2\right)\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{1176\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

```
[Out] (5*x*(2 + x^2))/(84*Sqrt[2 + 3*x^2 + x^4]) - (25*x*Sqrt[2 + 3*x^2
 + x^4])/(84*(7 + 5*x^2)) - (5*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)
]*EllipticE[ArcTan[x], 1/2])/(42*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) +
 (9*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2]
)/(56*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (65*(2 + x^2)*EllipticPi[2
/7, ArcTan[x], 1/2])/(1176*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt
[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.334366, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$-\frac{25\sqrt{x^4+3x^2+2}\,x}{84\,(5x^2+7)}+\frac{5\left(x^2+2\right)x}{84\sqrt{x^4+3x^2+2}}+\frac{9\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{56\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{42\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{65\left(x^2+2\right)\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{1176\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (5*x*(2 + x^2))/(84*Sqrt[2 + 3*x^2 + x^4]) - (25*x*Sqrt[2 + 3*x^2
 + x^4])/(84*(7 + 5*x^2)) - (5*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)
]*EllipticE[ArcTan[x], 1/2])/(42*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) +
 (9*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2]
)/(56*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (65*(2 + x^2)*EllipticPi[2
/7, ArcTan[x], 1/2])/(1176*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt
[2 + 3*x^2 + x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**2+7)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [C] time = 0.218433, size = 208, normalized size = 1.

$$\frac{-175x^5-525x^3-14i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-35i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-}{588\left(5x^2+7\right)\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (-350*x - 525*x^3 - 175*x^5 - (35*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*
(7 + 5*x^2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (14*I)*Sqrt[1 +
x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]
 - (91*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[
x/Sqrt[2]], 2] - (65*I)*x^2*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticP
i[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(588*(7 + 5*x^2)*Sqrt[2 + 3*x^2
 + x^4])
```

Maple [C] time = 0.026, size = 162, normalized size = 0.8

$$-\frac{25\,x}{420\,x^2+588}\sqrt{x^4+3\,x^2+2}-\frac{i}{84}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}$$
$$-\frac{5\,i}{168}\sqrt{2}EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}$$
$$-\frac{13\,i}{588}\sqrt{2}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}EllipticPi\left(\frac{i}{2}\sqrt{2}x,\frac{10}{7},\sqrt{2}\right)\frac{1}{\sqrt{x^4+3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -25/84*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-1/84*I*2^(1/2)*(2*x^2+4)^(
1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,
2^(1/2))-5/168*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2
+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))-13/588*I*2^(1/2)*(1+
1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I
*2^(1/2)*x,10/7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(25\,x^4+70\,x^2+49)\sqrt{x^4+3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2),x, algorithm="fricas")
```

```
[Out] integral(1/((25*x^4 + 70*x^2 + 49)*sqrt(x^4 + 3*x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2+1)(x^2+2)}(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x**2+7)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3x^2+2}(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2), x)
```

3.310 $\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=237

$$-\frac{325\sqrt{x^4+3x^2+2}\,x}{4704\,(5x^2+7)}-\frac{25\sqrt{x^4+3x^2+2}\,x}{168\,(5x^2+7)^2}+\frac{65\,(x^2+2)\,x}{4704\sqrt{x^4+3x^2+2}}+\frac{631\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{9408\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{65\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2352\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{2525\,(x^2+2)\,\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{65856\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

```
[Out] (65*x*(2 + x^2))/(4704*Sqrt[2 + 3*x^2 + x^4]) - (25*x*Sqrt[2 + 3*
x^2 + x^4])/(168*(7 + 5*x^2)^2) - (325*x*Sqrt[2 + 3*x^2 + x^4])/(
4704*(7 + 5*x^2)) - (65*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellip
ticE[ArcTan[x], 1/2])/(2352*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (631
*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(
9408*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (2525*(2 + x^2)*EllipticPi[
2/7, ArcTan[x], 1/2])/(65856*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sq
rt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.527152, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{325\sqrt{x^4+3x^2+2}\,x}{4704\,(5x^2+7)}-\frac{25\sqrt{x^4+3x^2+2}\,x}{168\,(5x^2+7)^2}+\frac{65\,(x^2+2)\,x}{4704\sqrt{x^4+3x^2+2}}+\frac{631\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{9408\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{65\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2352\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{2525\,(x^2+2)\,\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{65856\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (65*x*(2 + x^2))/(4704*Sqrt[2 + 3*x^2 + x^4]) - (25*x*Sqrt[2 + 3*
x^2 + x^4])/(168*(7 + 5*x^2)^2) - (325*x*Sqrt[2 + 3*x^2 + x^4])/(
4704*(7 + 5*x^2)) - (65*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellip
ticE[ArcTan[x], 1/2])/(2352*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (631
*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(
9408*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (2525*(2 + x^2)*EllipticPi[
2/7, ArcTan[x], 1/2])/(65856*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sq
rt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**2+7)**3/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.317175, size = 186, normalized size = 0.78

$$\frac{14i\sqrt{x^2+1}\sqrt{x^2+2}\,(5x^2+7)^2F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-455i\sqrt{x^2+1}\sqrt{x^2+2}\,(5x^2+7)^2E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-505i\sqrt{x^2+1}\cdots}{32928\,(5x^2+7)^2\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (-175*x*(238 + 487*x^2 + 314*x^4 + 65*x^6) - (455*I)*Sqrt[1 + x^2
]*Sqrt[2 + x^2]*(7 + 5*x^2)^2*EllipticE[I*ArcSinh[x/Sqrt[2]], 2]
+ (14*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)^2*EllipticF[I*Ar
cSinh[x/Sqrt[2]], 2] - (505*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5
*x^2)^2*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(32928*(7 + 5*
x^2)^2*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.028, size = 186, normalized size = 0.8

$$-\frac{25\,x}{168\,(5\,x^2+7)^2}\sqrt{x^4+3\,x^2+2}-\frac{325\,x}{23520\,x^2+32928}\sqrt{x^4+3\,x^2+2}$$
$$+\frac{i}{4704}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}$$
$$-\frac{65\,i}{9408}\sqrt{2}EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}$$
$$-\frac{505\,i}{32928}\sqrt{2}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}EllipticPi\left(\frac{i}{2}\sqrt{2}x,\frac{10}{7},\sqrt{2}\right)\frac{1}{\sqrt{x^4+3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -25/168*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2-325/4704*x*(x^4+3*x^2+2
)^(1/2)/(5*x^2+7)+1/4704*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/
(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-65/9408*I*
2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Ellipti
cE(1/2*I*2^(1/2)*x,2^(1/2))-505/32928*I*2^(1/2)*(1+1/2*x^2)^(1/2)
*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/
7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(125\,x^6+525\,x^4+735\,x^2+343)\sqrt{x^4+3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3),x, algorithm="fricas")
```

```
[Out] integral(1/((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(x^4 + 3*x^2
+ 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2+1)(x^2+2)}\,(5x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x**2+7)**3/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3), x)
```

3.311 $\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=189

$$\frac{5000}{3}\sqrt{x^4+3x^2+2}\,x+\frac{7679\left(x^2+2\right)x}{2\sqrt{x^4+3x^2+2}}-\frac{\left(179x^2+115\right)x}{2\sqrt{x^4+3x^2+2}}+\frac{15383\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{7679\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+625\sqrt{x^4+3x^2+2}\,x^3$$

```
[Out] (7679*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) - (x*(115 + 179*x^2)
)/(2*Sqrt[2 + 3*x^2 + x^4]) + (5000*x*Sqrt[2 + 3*x^2 + x^4])/3 +
625*x^3*Sqrt[2 + 3*x^2 + x^4] - (7679*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4
]) + (15383*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[
x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.216293, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{5000}{3}\sqrt{x^4+3x^2+2}\,x+\frac{7679\left(x^2+2\right)x}{2\sqrt{x^4+3x^2+2}}-\frac{\left(179x^2+115\right)x}{2\sqrt{x^4+3x^2+2}}+\frac{15383\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{7679\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+625\sqrt{x^4+3x^2+2}\,x^3$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^5/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (7679*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) - (x*(115 + 179*x^2)
)/(2*Sqrt[2 + 3*x^2 + x^4]) + (5000*x*Sqrt[2 + 3*x^2 + x^4])/3 +
625*x^3*Sqrt[2 + 3*x^2 + x^4] - (7679*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4
]) + (15383*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[
x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 23.5647, size = 177, normalized size = 0.94

$$625x^3\sqrt{x^4+3x^2+2}+\frac{7679x\left(2x^2+4\right)}{4\sqrt{x^4+3x^2+2}}-\frac{x\left(43497x^2+27945\right)}{486\sqrt{x^4+3x^2+2}}+\frac{5000x\sqrt{x^4+3x^2+2}}{3}-\frac{7679\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)E\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{8\sqrt{x^4+3x^2+2}}+\frac{15383\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)F\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{24\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**5/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] 625*x**3*sqrt(x**4 + 3*x**2 + 2) + 7679*x*(2*x**2 + 4)/(4*sqrt(x*
*4 + 3*x**2 + 2)) - x*(43497*x**2 + 27945)/(486*sqrt(x**4 + 3*x**
2 + 2)) + 5000*x*sqrt(x**4 + 3*x**2 + 2)/3 - 7679*sqrt((2*x**2 +
4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_e(atan(x), 1/2)/(8*sqrt(x**4
 + 3*x**2 + 2)) + 15383*sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4
)*elliptic_f(atan(x), 1/2)/(24*sqrt(x**4 + 3*x**2 + 2))
```

Mathematica [C] time = 0.0977977, size = 109, normalized size = 0.58

$$\frac{3750x^7 + 21250x^5 + 36963x^3 - 7729i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 23037i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) + 1}{6\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^5/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (19655*x + 36963*x^3 + 21250*x^5 + 3750*x^7 - (23037*I)*Sqrt[1 +
x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (7729*I)*
Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(
6*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.048, size = 274, normalized size = 1.5

$$-33614\,{\frac{-3/4\,{x}^{3}-5/4\,x}{\sqrt{{x}^{4}+3\,{x}^{2}+2}}}-{\frac{15383\,i}{6}}\sqrt{2}{\it EllipticF}\left({\frac{i}{2}}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,{x}^{2}+4}\sqrt{{x}^{2}+1}{\frac{1}{\sqrt{{x}^{4}+3\,{x}^{2}+2}}}+{\frac{7679\,i}{4}}\sqrt{2}\left({\it EllipticF}\left({\frac{i}{2}}\sqrt{2}x,\sqrt{2}\right)-{\it EllipticE}\left({\frac{i}{2}}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2\,{x}^{2}+4}\sqrt{{x}^{2}+1}{\frac{1}{\sqrt{{x}^{4}+3\,{x}^{2}+2}}}-120050\,{\frac{{x}^{3}+3/2\,x}{\sqrt{{x}^{4}+3\,{x}^{2}+2}}}-171500\,{\frac{-3/2\,{x}^{3}-2\,x}{\sqrt{{x}^{4}+3\,{x}^{2}+2}}}-122500\,{\frac{5/2\,{x}^{3}+3\,x}{\sqrt{{x}^{4}+3\,{x}^{2}+2}}}-43750\,{\frac{-9/2\,{x}^{3}-5\,x}{\sqrt{{x}^{4}+3\,{x}^{2}+2}}}+{\frac{5000\,x}{3}\sqrt{{x}^{4}+3\,{x}^{2}+2}}-6250\,{\frac{17/2\,{x}^{3}+9\,x}{\sqrt{{x}^{4}+3\,{x}^{2}+2}}}+625\,{x}^{3}\sqrt{{x}^{4}+3\,{x}^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -33614*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^(1/2)-15383/6*I*2^(1/2)*(2*
x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^
(1/2)*x,2^(1/2))+7679/4*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(
x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(
1/2*I*2^(1/2)*x,2^(1/2)))-120050*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)-
171500*(-3/2*x^3-2*x)/(x^4+3*x^2+2)^(1/2)-122500*(5/2*x^3+3*x)/(x
^4+3*x^2+2)^(1/2)-43750*(-9/2*x^3-5*x)/(x^4+3*x^2+2)^(1/2)+5000/3
*x*(x^4+3*x^2+2)^(1/2)-6250*(17/2*x^3+9*x)/(x^4+3*x^2+2)^(1/2)+62
5*x^3*(x^4+3*x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^5}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{3125\,x^{10}+21875\,x^{8}+61250\,x^{6}+85750\,x^{4}+60025\,x^{2}+16807}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3125*x^10 + 21875*x^8 + 61250*x^6 + 85750*x^4 + 60025*x
^2 + 16807)/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+7\right)^5}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**5/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**5/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^5}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.312 $\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=170

$$\frac{625}{3}\sqrt{x^4+3x^2+2}\,x+\frac{637\left(x^2+2\right)x}{2\sqrt{x^4+3x^2+2}}+\frac{\left(113x^2+145\right)x}{2\sqrt{x^4+3x^2+2}}+\frac{1067\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{637\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] (637*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(145 + 113*x^2))
/(2*Sqrt[2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/3 - (6
37*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])
/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (1067*Sqrt[2]*(1 + x^2)*Sqrt[(
2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2
+ x^4])
```

Rubi [A] time = 0.173624, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{625}{3}\sqrt{x^4+3x^2+2}\,x+\frac{637\left(x^2+2\right)x}{2\sqrt{x^4+3x^2+2}}+\frac{\left(113x^2+145\right)x}{2\sqrt{x^4+3x^2+2}}+\frac{1067\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{637\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^4/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (637*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(145 + 113*x^2))
/(2*Sqrt[2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/3 - (6
37*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])
/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (1067*Sqrt[2]*(1 + x^2)*Sqrt[(
2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2
+ x^4])
```

Rubi in Sympy [A] time = 16.6691, size = 158, normalized size = 0.93

$$\frac{637x\left(2x^2+4\right)}{4\sqrt{x^4+3x^2+2}}+\frac{x\left(9153x^2+11745\right)}{162\sqrt{x^4+3x^2+2}}+\frac{625x\sqrt{x^4+3x^2+2}}{3}-\frac{637\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)E\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{8\sqrt{x^4+3x^2+2}}+\frac{1067\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)F\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{12\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**4/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] 637*x*(2*x**2 + 4)/(4*sqrt(x**4 + 3*x**2 + 2)) + x*(9153*x**2 + 1
1745)/(162*sqrt(x**4 + 3*x**2 + 2)) + 625*x*sqrt(x**4 + 3*x**2 +
2)/3 - 637*sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_e(
atan(x), 1/2)/(8*sqrt(x**4 + 3*x**2 + 2)) + 1067*sqrt((2*x**2 + 4
)/(x**2 + 1))*(4*x**2 + 4)*elliptic_f(atan(x), 1/2)/(12*sqrt(x**4
 + 3*x**2 + 2))
```

Mathematica [C] time = 0.0884903, size = 104, normalized size = 0.61

$$\frac{1250x^5+4089x^3-2357i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-1911i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+2935x}{6\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^4/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2935*x + 4089*x^3 + 1250*x^5 - (1911*I)*Sqrt[1 + x^2]*Sqrt[2 + x
^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (2357*I)*Sqrt[1 + x^2]*S
qrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(6*Sqrt[2 + 3*x^
2 + x^4])
```

Maple [C] time = 0.014, size = 234, normalized size = 1.4

$$-4802\frac{-3/4x^3-5/4x}{\sqrt{x^4+3x^2+2}}-\frac{1067i}{3}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}+\frac{637i}{4}\sqrt{2}\left(EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)-EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}-13720\frac{x^3+3/2x}{\sqrt{x^4+3x^2+2}}-14700\frac{-3/2x^3-2x}{\sqrt{x^4+3x^2+2}}-7000\frac{5/2x^3+3x}{\sqrt{x^4+3x^2+2}}-1250\frac{-9/2x^3-5x}{\sqrt{x^4+3x^2+2}}+\frac{625x}{3}\sqrt{x^4+3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -4802*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^(1/2)-1067/3*I*2^(1/2)*(2*x^
2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1
/2)*x,2^(1/2))+637/4*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4
+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2
*I*2^(1/2)*x,2^(1/2)))-13720*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)-1470
0*(-3/2*x^3-2*x)/(x^4+3*x^2+2)^(1/2)-7000*(5/2*x^3+3*x)/(x^4+3*x^
2+2)^(1/2)-1250*(-9/2*x^3-5*x)/(x^4+3*x^2+2)^(1/2)+625/3*x*(x^4+3
*x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(5x^2+7)^4}{(x^4+3x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{625x^8+3500x^6+7350x^4+6860x^2+2401}{(x^4+3x^2+2)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)/(x^4 +
 3*x^2 + 2)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+7\right)^4}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**4/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**4/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^4}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.313 $\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=149

$$\frac{x(5-11x^2)}{2\sqrt{x^4+3x^2+2}}+\frac{261x(x^2+2)}{2\sqrt{x^4+3x^2+2}}+\frac{169(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{261(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] (x*(5 - 11*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (261*x*(2 + x^2))/(2
*Sqrt[2 + 3*x^2 + x^4]) - (261*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)
]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (1
69*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])
/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.132147, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{x(5-11x^2)}{2\sqrt{x^4+3x^2+2}}+\frac{261x(x^2+2)}{2\sqrt{x^4+3x^2+2}}+\frac{169(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{261(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^3/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(5 - 11*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (261*x*(2 + x^2))/(2
*Sqrt[2 + 3*x^2 + x^4]) - (261*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)
]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (1
69*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])
/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 11.2422, size = 139, normalized size = 0.93

$$\frac{x(-297x^2+135)}{54\sqrt{x^4+3x^2+2}}+\frac{261x(2x^2+4)}{4\sqrt{x^4+3x^2+2}}-\frac{261\sqrt{\frac{2x^2+4}{x^2+1}}(4x^2+4)E\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{8\sqrt{x^4+3x^2+2}}+\frac{169\sqrt{\frac{2x^2+4}{x^2+1}}(4x^2+4)F\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{8\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**3/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] x*(-297*x**2 + 135)/(54*sqrt(x**4 + 3*x**2 + 2)) + 261*x*(2*x**2
+ 4)/(4*sqrt(x**4 + 3*x**2 + 2)) - 261*sqrt((2*x**2 + 4)/(x**2 +
1))*(4*x**2 + 4)*elliptic_e(atan(x), 1/2)/(8*sqrt(x**4 + 3*x**2 +
 2)) + 169*sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_f(
atan(x), 1/2)/(8*sqrt(x**4 + 3*x**2 + 2))
```

Mathematica [C] time = 0.10541, size = 99, normalized size = 0.66

$$-\frac{11x^3+77i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\Big|2\right)+261i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\Big|2\right)-5x}{2\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^3/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] -(-5*x + 11*x^3 + (261*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I
*ArcSinh[x/Sqrt[2]], 2] + (77*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Elli
pticF[I*ArcSinh[x/Sqrt[2]], 2])/(2*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.011, size = 196, normalized size = 1.3

$$-686\,\frac{-3/4\,x^3-5/4\,x}{\sqrt{x^4+3\,x^2+2}}-\frac{169\,i}{2}\sqrt{2}\mathit{EllipticF}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}+\frac{261\,i}{4}\sqrt{2}\left(\mathit{EllipticF}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}-1470\,\frac{x^3+3/2\,x}{\sqrt{x^4+3\,x^2+2}}-1050\,\frac{-3/2\,x^3-2\,x}{\sqrt{x^4+3\,x^2+2}}-250\,\frac{5/2\,x^3+3\,x}{\sqrt{x^4+3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -686*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^(1/2)-169/2*I*2^(1/2)*(2*x^2+
4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2
)*x,2^(1/2))+261/4*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3
*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I
*2^(1/2)*x,2^(1/2)))-1470*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)-1050*(-
3/2*x^3-2*x)/(x^4+3*x^2+2)^(1/2)-250*(5/2*x^3+3*x)/(x^4+3*x^2+2)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^3}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{125\,x^6+525\,x^4+735\,x^2+343}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)/(x^4 + 3*x^2 + 2)^(3
/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+7\right)^3}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**3/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^3}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.314 $\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{17x(x^2+2)}{2\sqrt{x^4+3x^2+2}}+\frac{x(17x^2+25)}{2\sqrt{x^4+3x^2+2}}+\frac{6\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{17(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] (-17*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(25 + 17*x^2))/(
2*Sqrt[2 + 3*x^2 + x^4]) + (17*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)
]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (6
*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x],
 1/2])/Sqrt[2 + 3*x^2 + x^4]
```

Rubi [A] time = 0.129607, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{17x(x^2+2)}{2\sqrt{x^4+3x^2+2}}+\frac{x(17x^2+25)}{2\sqrt{x^4+3x^2+2}}+\frac{6\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{17(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^2/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (-17*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(25 + 17*x^2))/(
2*Sqrt[2 + 3*x^2 + x^4]) + (17*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)
]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (6
*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x],
 1/2])/Sqrt[2 + 3*x^2 + x^4]
```

Rubi in Sympy [A] time = 11.2299, size = 139, normalized size = 0.93

$$-\frac{17x(2x^2+4)}{4\sqrt{x^4+3x^2+2}}+\frac{x(153x^2+225)}{18\sqrt{x^4+3x^2+2}}+\frac{17\sqrt{\frac{2x^2+4}{x^2+1}}(4x^2+4)E\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{8\sqrt{x^4+3x^2+2}}+\frac{3\sqrt{\frac{2x^2+4}{x^2+1}}(4x^2+4)F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{2\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**2/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] -17*x*(2*x**2 + 4)/(4*sqrt(x**4 + 3*x**2 + 2)) + x*(153*x**2 + 22
5)/(18*sqrt(x**4 + 3*x**2 + 2)) + 17*sqrt((2*x**2 + 4)/(x**2 + 1)
)*(4*x**2 + 4)*elliptic_e(atan(x), 1/2)/(8*sqrt(x**4 + 3*x**2 + 2
)) + 3*sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_f(atan
(x), 1/2)/(2*sqrt(x**4 + 3*x**2 + 2))
```

Mathematica [C] time = 0.081323, size = 99, normalized size = 0.66

$$\frac{17x^3-41i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+17i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+25x}{2\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^2/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (25*x + 17*x^3 + (17*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*A
rcSinh[x/Sqrt[2]], 2] - (41*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ellipt
icF[I*ArcSinh[x/Sqrt[2]], 2])/(2*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.01, size = 173, normalized size = 1.2

$$-98\frac{-3/4\,x^3-5/4\,x}{\sqrt{x^4+3\,x^2+2}}-6\,i\sqrt{2}\mathit{EllipticF}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}-\frac{17\,i}{4}\sqrt{2}\left(\mathit{EllipticF}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}-140\frac{x^3+3/2\,x}{\sqrt{x^4+3\,x^2+2}}-50\frac{-3/2\,x^3-2\,x}{\sqrt{x^4+3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -98*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^(1/2)-6*I*2^(1/2)*(2*x^2+4)^(1
/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2
^(1/2))-17/4*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2
)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/
2)*x,2^(1/2)))-140*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)-50*(-3/2*x^3-2
*x)/(x^4+3*x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5\,x^2+7\right)^2}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{25\,x^4+70\,x^2+49}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5x^2+7\right)^2}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**2/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^2}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.315 $\int \frac{7+5x^2}{(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{x(x^2+2)}{2\sqrt{x^4+3x^2+2}}+\frac{x(x^2+5)}{2\sqrt{x^4+3x^2+2}}+\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] -(x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + x^2))/(2*Sqrt[
2 + 3*x^2 + x^4]) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elliptic
E[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + ((1 + x^2)*S
qrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt
[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.107024, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{x(x^2+2)}{2\sqrt{x^4+3x^2+2}}+\frac{x(x^2+5)}{2\sqrt{x^4+3x^2+2}}+\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] -(x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + x^2))/(2*Sqrt[
2 + 3*x^2 + x^4]) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elliptic
E[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + ((1 + x^2)*S
qrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt
[2 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 10.2562, size = 133, normalized size = 0.92

$$\frac{x(x^2+5)}{2\sqrt{x^4+3x^2+2}}-\frac{x(2x^2+4)}{4\sqrt{x^4+3x^2+2}}+\frac{\sqrt{\frac{2x^2+4}{x^2+1}}(4x^2+4)E\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{8\sqrt{x^4+3x^2+2}}+\frac{\sqrt{\frac{2x^2+4}{x^2+1}}(4x^2+4)F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{8\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] x*(x**2 + 5)/(2*sqrt(x**4 + 3*x**2 + 2)) - x*(2*x**2 + 4)/(4*sqrt
(x**4 + 3*x**2 + 2)) + sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)
*elliptic_e(atan(x), 1/2)/(8*sqrt(x**4 + 3*x**2 + 2)) + sqrt((2*x
**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_f(atan(x), 1/2)/(8*sqr
t(x**4 + 3*x**2 + 2))
```

Mathematica [C] time = 0.0709652, size = 97, normalized size = 0.67

$$\frac{x^3-3i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+5x}{2\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (5*x + x^3 + I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/
Sqrt[2]], 2] - (3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcS
inh[x/Sqrt[2]], 2])/(2*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.008, size = 150, normalized size = 1.

$$-14\,{\frac{-3/4\,{x}^{3}-5/4\,x}{\sqrt{{x}^{4}+3\,{x}^{2}+2}}}-{\frac{i}{2}}\sqrt{2}{\it EllipticF} \left({\frac{i}{2}}\sqrt{2}x,\sqrt{2} \right) \sqrt{2\,{x}^{2}+4}\sqrt{{x}^{2}+1}{\frac{1}{\sqrt{{x}^{4}+3\,{x}^{2}+2}}}-{\frac{i}{4}}\sqrt{2} \left({\it EllipticF} \left({\frac{i}{2}}\sqrt{2}x,\sqrt{2} \right) -{\it EllipticE} \left({\frac{i}{2}}\sqrt{2}x,\sqrt{2} \right) \right) \sqrt{2\,{x}^{2}+4}\sqrt{{x}^{2}+1}{\frac{1}{\sqrt{{x}^{4}+3\,{x}^{2}+2}}}-10\,{\frac{{x}^{3}+3/2\,x}{\sqrt{{x}^{4}+3\,{x}^{2}+2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -14*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^(1/2)-1/2*I*2^(1/2)*(2*x^2+4)^
(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x
,2^(1/2))-1/4*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+
2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1
/2)*x,2^(1/2)))-10*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^{2} + 7}{{\left(x^{4} + 3\,x^{2} + 2\right)}^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{5\,x^{2} + 7}{{\left(x^{4} + 3\,x^{2} + 2\right)}^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((5*x^2 + 7)/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5 x^{2} + 7}{\left(\left(x^{2} + 1\right) \left(x^{2} + 2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)/(x**4+3*x**2+2)**(3/2),x)

[Out] Integral((5*x**2 + 7)/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^2+7}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 2)^(3/2),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.316 $\int \frac{1}{(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{3x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{x\left(3x^2+5\right)}{2\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] (-3*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + 3*x^2))/(2*S
qrt[2 + 3*x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*El
lipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (Sqrt[
2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])
/Sqrt[2 + 3*x^2 + x^4]
```

Rubi [A] time = 0.100618, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{3x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{x\left(3x^2+5\right)}{2\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2 + x^4)^(-3/2),x]
```


```
[Out] (-3*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + 3*x^2))/(2*S
qrt[2 + 3*x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*El
lipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (Sqrt[
2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])
/Sqrt[2 + 3*x^2 + x^4]
```

Rubi in Sympy [A] time = 15.6474, size = 138, normalized size = 0.93

$$-\frac{3x\left(2x^2+4\right)}{4\sqrt{x^4+3x^2+2}}+\frac{x\left(3x^2+5\right)}{2\sqrt{x^4+3x^2+2}}+\frac{3\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)E\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{8\sqrt{x^4+3x^2+2}}-\frac{\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)F\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] -3*x*(2*x**2 + 4)/(4*sqrt(x**4 + 3*x**2 + 2)) + x*(3*x**2 + 5)/(2
*sqrt(x**4 + 3*x**2 + 2)) + 3*sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x*
*2 + 4)*elliptic_e(atan(x), 1/2)/(8*sqrt(x**4 + 3*x**2 + 2)) - sq
rt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_f(atan(x), 1/2)
/(4*sqrt(x**4 + 3*x**2 + 2))
```

Mathematica [C] time = 0.0614227, size = 99, normalized size = 0.66

$$\frac{3x^3+i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+3i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+5x}{2\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] (5*x + 3*x^3 + (3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcS
inh[x/Sqrt[2]], 2] + I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*Ar
cSinh[x/Sqrt[2]], 2])/(2*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.006, size = 129, normalized size = 0.9

$$-2\frac{-3/4\,x^3-5/4\,x}{\sqrt{x^4+3\,x^2+2}}+i\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}-\frac{3\,i}{4}\sqrt{2}\left(EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)-EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -2*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^(1/2)+I*2^(1/2)*(2*x^2+4)^(1/2)
*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1
/2))-3/4*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1
/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x
,2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 2)^(-3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(x^4+3\,x^2+2)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 2)^(-3/2),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 2)^(-3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+3x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((x**4 + 3*x**2 + 2)**(-3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 3x^2 + 2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 2)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(-3/2), x)
```

3.317 $\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{x}{6\sqrt{x^4+3x^2+2}}-\frac{9\left(x^2+1\right)\sqrt{\frac{x^2+2}{2x^2+2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+3x^2+2}}+\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{125\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{84\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] x/(6*Sqrt[2 + 3*x^2 + x^4]) + (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(
1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4]) -
(9*(1 + x^2)*Sqrt[(2 + x^2)/(2 + 2*x^2)]*EllipticF[ArcTan[x], 1/2
])/(4*Sqrt[2 + 3*x^2 + x^4]) + (125*(1 + x^2)*Sqrt[(2 + x^2)/(1 +
 x^2)]*EllipticPi[2/7, ArcTan[x], 1/2])/(84*Sqrt[2]*Sqrt[2 + 3*x^
2 + x^4])
```

Rubi [A] time = 0.367105, antiderivative size = 207, normalized size of antiderivative = 1.2, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{x\left(x^2+2\right)}{3\sqrt{x^4+3x^2+2}}+\frac{x\left(2x^2+5\right)}{6\sqrt{x^4+3x^2+2}}-\frac{9\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{125\left(x^2+2\right)\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{84\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] -(x*(2 + x^2))/(3*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + 2*x^2))/(6*Sqr
t[2 + 3*x^2 + x^4]) + (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)
]*EllipticE[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4]) - (9*(1 +
x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(4*Sqrt
[2]*Sqrt[2 + 3*x^2 + x^4]) + (125*(2 + x^2)*EllipticPi[2/7, ArcTa
n[x], 1/2])/(84*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2
+ x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**2+7)/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [C] time = 0.131034, size = 138, normalized size = 0.8

$$\frac{14x^3-7i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+14i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+25i\sqrt{x^2+1}\sqrt{x^2+2}\left(\frac{10}{7};i\sinh^{-1}\right.}{42\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (35*x + 14*x^3 + (14*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*A
rcSinh[x/Sqrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ellipti
cF[I*ArcSinh[x/Sqrt[2]], 2] + (25*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*
EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(42*Sqrt[2 + 3*x^2 + x
^4])
```

Maple [C] time = 0.022, size = 161, normalized size = 0.9

$$-2\frac{1}{\sqrt{x^4+3x^2+2}}\left(-1/6\,x^3-\frac{5\,x}{12}\right)-\frac{i}{12}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}+\frac{i}{6}\sqrt{2}EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}+\frac{25\,i}{42}\sqrt{2}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}EllipticPi\left(\frac{i}{2}\sqrt{2}x,\frac{10}{7},\sqrt{2}\right)\frac{1}{\sqrt{x^4+3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -2*(-1/6*x^3-5/12*x)/(x^4+3*x^2+2)^(1/2)-1/12*I*2^(1/2)*(2*x^2+4)
^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*
x,2^(1/2))+1/6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2
+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))+25/42*I*2^(1/2)*(1+1
/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*
2^(1/2)*x,10/7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+3\,x^2+2)^{\frac{3}{2}}(5\,x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(5\,x^6+22\,x^4+31\,x^2+14)\sqrt{x^4+3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)),x, algorithm="fricas")
```

```
[Out] integrate(1/((5*x^6 + 22*x^4 + 31*x^2 + 14)*sqrt(x^4 + 3*x^2 + 2))
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x^{2}+1\right)\left(x^{2}+2\right)\right)^{\frac{3}{2}}\left(5x^{2}+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x**2+7)/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^{4}+3\,x^{2}+2\right)^{\frac{3}{2}}\left(5\,x^{2}+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

3.318 $\int \frac{1}{(7+5x^2)^2(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=235

$$\frac{625\sqrt{x^4+3x^2+2}x}{504(5x^2+7)}-\frac{31(x^2+2)x}{56\sqrt{x^4+3x^2+2}}+\frac{(11x^2+20)x}{36\sqrt{x^4+3x^2+2}}-\frac{463(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{336\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{31(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{28\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{375(x^2+2)\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{784\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

```
[Out] (-31*x*(2 + x^2))/(56*Sqrt[2 + 3*x^2 + x^4]) + (x*(20 + 11*x^2))/
(36*Sqrt[2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/(504*(
7 + 5*x^2)) + (31*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[A
rcTan[x], 1/2])/(28*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (463*(1 + x^
2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(336*Sqrt
[2]*Sqrt[2 + 3*x^2 + x^4]) + (375*(2 + x^2)*EllipticPi[2/7, ArcTa
n[x], 1/2])/(784*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2
 + x^4])
```

Rubi [A] time = 0.806312, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$\frac{625\sqrt{x^4+3x^2+2}x}{504(5x^2+7)}-\frac{31(x^2+2)x}{56\sqrt{x^4+3x^2+2}}+\frac{(11x^2+20)x}{36\sqrt{x^4+3x^2+2}}-\frac{463(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{336\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{31(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{28\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{375(x^2+2)\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{784\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-31*x*(2 + x^2))/(56*Sqrt[2 + 3*x^2 + x^4]) + (x*(20 + 11*x^2))/
(36*Sqrt[2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/(504*(
7 + 5*x^2)) + (31*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[A
rcTan[x], 1/2])/(28*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (463*(1 + x^
2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(336*Sqrt
[2]*Sqrt[2 + 3*x^2 + x^4]) + (375*(2 + x^2)*EllipticPi[2/7, ArcTa
n[x], 1/2])/(784*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2
 + x^4])
```

Rubi in Sympy [A] time = 129.892, size = 330, normalized size = 1.4

$$\frac{625x\sqrt{x^4+3x^2+2}}{12(210x^2+294)}-\frac{125x\sqrt{x^4+3x^2+2}}{504(x^2+1)}+\frac{4x\sqrt{x^4+3x^2+2}}{27\left(\frac{x^2}{2}+1\right)(x^2+1)}+\frac{305\sqrt{x^4+3x^2+2}E\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{1512\sqrt{\frac{\frac{x^2}{2}+1}{x^2+1}}(x^2+1)}-\frac{7667\sqrt{x^4+3x^2+2}F\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{6048\sqrt{\frac{\frac{x^2}{2}+1}{x^2+1}}(x^2+1)}+\frac{6875\sqrt{x^4+3x^2+2}\left(\frac{2}{7};\operatorname{atan}(x)|\frac{1}{2}\right)}{14112\sqrt{\frac{\frac{x^2}{2}+1}{x^2+1}}(x^2+1)}+\frac{19\sqrt{2}\sqrt{x^4+3x^2+2}E\left(\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|-1\right)}{108\sqrt{\frac{x^2+1}{\frac{x^2}{2}+1}}\left(\frac{x^2}{2}+1\right)}+\frac{125\sqrt{2}\sqrt{x^4+3x^2+2}\left(-\frac{3}{7};\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|-1\right)}{378\sqrt{\frac{x^2+1}{\frac{x^2}{2}+1}}\left(\frac{x^2}{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**2+7)**2/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] 625*x*sqrt(x**4 + 3*x**2 + 2)/(12*(210*x**2 + 294)) - 125*x*sqrt(
x**4 + 3*x**2 + 2)/(504*(x**2 + 1)) + 4*x*sqrt(x**4 + 3*x**2 + 2)
/(27*(x**2/2 + 1)*(x**2 + 1)) + 305*sqrt(x**4 + 3*x**2 + 2)*ellip
tic_e(atan(x), 1/2)/(1512*sqrt((x**2/2 + 1)/(x**2 + 1))*(x**2 + 1
)) - 7667*sqrt(x**4 + 3*x**2 + 2)*elliptic_f(atan(x), 1/2)/(6048*
sqrt((x**2/2 + 1)/(x**2 + 1))*(x**2 + 1)) + 6875*sqrt(x**4 + 3*x*
*2 + 2)*elliptic_pi(2/7, atan(x), 1/2)/(14112*sqrt((x**2/2 + 1)/(
x**2 + 1))*(x**2 + 1)) + 19*sqrt(2)*sqrt(x**4 + 3*x**2 + 2)*ellip
tic_e(atan(sqrt(2)*x/2), -1)/(108*sqrt((x**2 + 1)/(x**2/2 + 1))*(
x**2/2 + 1)) + 125*sqrt(2)*sqrt(x**4 + 3*x**2 + 2)*elliptic_pi(-3
/7, atan(sqrt(2)*x/2), -1)/(378*sqrt((x**2 + 1)/(x**2/2 + 1))*(x*
*2/2 + 1))
```

Mathematica [C] time = 0.224743, size = 208, normalized size = 0.89

$$\frac{3255x^5+10157x^3+182i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+651i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right.}{1176\left(5x^2+7\right)\sqrt{x^4+3x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (7490*x + 10157*x^3 + 3255*x^5 + (651*I)*Sqrt[1 + x^2]*Sqrt[2 + x
^2]*(7 + 5*x^2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + (182*I)*Sqrt
[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]
], 2] + (1575*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*A
rcSinh[x/Sqrt[2]], 2] + (1125*I)*x^2*Sqrt[1 + x^2]*Sqrt[2 + x^2]*
EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(1176*(7 + 5*x^2)*Sqrt
[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.03, size = 185, normalized size = 0.8

$$\begin{aligned}&\frac{625\,x}{2520\,x^2+3528}\sqrt{x^4+3\,x^2+2}-2\,\frac{1}{\sqrt{x^4+3\,x^2+2}}\left(-\frac{11\,x^3}{72}-\frac{5\,x}{18}\right)\\&+\frac{13\,i}{168}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}\\&+\frac{31\,i}{112}\sqrt{2}EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}\\&+\frac{75\,i}{392}\sqrt{2}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}EllipticPi\left(\frac{i}{2}\sqrt{2}x,\frac{10}{7},\sqrt{2}\right)\frac{1}{\sqrt{x^4+3\,x^2+2}}\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] 625/504*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-2*(-11/72*x^3-5/18*x)/(x^
4+3*x^2+2)^(1/2)+13/168*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(
x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+31/112*I*2^
(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE
(1/2*I*2^(1/2)*x,2^(1/2))+75/392*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2
+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+3x^2+2)^{\frac{3}{2}}(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(25x^8+145x^6+309x^4+287x^2+98)\sqrt{x^4+3x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2),x, algorithm="fricas")
```

```
[Out] integral(1/((25*x^8 + 145*x^6 + 309*x^4 + 287*x^2 + 98)*sqrt(x^4
+ 3*x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2+1)(x^2+2))^{\frac{3}{2}}(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x**2+7)**2/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+3x^2+2)^{\frac{3}{2}}(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

3.319 $\int \frac{1}{(7+5x^2)^3(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=263

$$\frac{41875\sqrt{x^4+3x^2+2}\,x}{84672\,(5x^2+7)}+\frac{625\sqrt{x^4+3x^2+2}\,x}{1008\,(5x^2+7)^2}-\frac{5797\,(x^2+2)\,x}{28224\sqrt{x^4+3x^2+2}}+\frac{(23x^2+50)\,x}{216\sqrt{x^4+3x^2+2}}-\frac{49907\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{56448\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{5797\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{14112\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{192625\,(x^2+2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{395136\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

```
[Out] (-5797*x*(2 + x^2))/(28224*Sqrt[2 + 3*x^2 + x^4]) + (x*(50 + 23*x
^2))/(216*Sqrt[2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/
(1008*(7 + 5*x^2)^2) + (41875*x*Sqrt[2 + 3*x^2 + x^4])/(84672*(7
+ 5*x^2)) + (5797*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[A
rcTan[x], 1/2])/(14112*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (49907*(1
 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(564
48*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (192625*(2 + x^2)*EllipticPi[
2/7, ArcTan[x], 1/2])/(395136*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*S
qrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 1.45916, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{41875\sqrt{x^4+3x^2+2}\,x}{84672\,(5x^2+7)}+\frac{625\sqrt{x^4+3x^2+2}\,x}{1008\,(5x^2+7)^2}-\frac{5797\,(x^2+2)\,x}{28224\sqrt{x^4+3x^2+2}}+\frac{(23x^2+50)\,x}{216\sqrt{x^4+3x^2+2}}-\frac{49907\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{56448\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{5797\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{14112\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{192625\,(x^2+2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{395136\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-5797*x*(2 + x^2))/(28224*Sqrt[2 + 3*x^2 + x^4]) + (x*(50 + 23*x
^2))/(216*Sqrt[2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/
(1008*(7 + 5*x^2)^2) + (41875*x*Sqrt[2 + 3*x^2 + x^4])/(84672*(7
+ 5*x^2)) + (5797*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[A
rcTan[x], 1/2])/(14112*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (49907*(1
 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(564
48*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (192625*(2 + x^2)*EllipticPi[
2/7, ArcTan[x], 1/2])/(395136*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*S
qrt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**2+7)**3/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.552916, size = 159, normalized size = 0.6

$$\frac{-742i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+40579i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+38525i\sqrt{x^2+1}\sqrt{x^2+2}\left(\frac{10}{7};i\sinh\right.}{197568\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] ((7*x*(550550 + 1089803*x^2 + 698290*x^4 + 144925*x^6))/(7 + 5*x^
2)^2 + (40579*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[
x/Sqrt[2]], 2] - (742*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*
ArcSinh[x/Sqrt[2]], 2] + (38525*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*El
lipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(197568*Sqrt[2 + 3*x^2 +
 x^4])
```

Maple [C] time = 0.031, size = 209, normalized size = 0.8

$$\begin{aligned}&\frac{625\,x}{1008\,(5\,x^2+7)^2}\sqrt{x^4+3\,x^2+2}+\frac{41875\,x}{423360\,x^2+592704}\sqrt{x^4+3\,x^2+2}\\&-2\,\frac{1}{\sqrt{x^4+3\,x^2+2}}\left(-\frac{23\,x^3}{432}-\frac{25\,x}{216}\right)\\&-\frac{53\,i}{28224}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}\\&+\frac{5797\,i}{56448}\sqrt{2}EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}\\&+\frac{38525\,i}{197568}\sqrt{2}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}EllipticPi\left(\frac{i}{2}\sqrt{2}x,\frac{10}{7},\sqrt{2}\right)\frac{1}{\sqrt{x^4+3\,x^2+2}}\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] 625/1008*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2+41875/84672*x*(x^4+3*x
^2+2)^(1/2)/(5*x^2+7)-2*(-23/432*x^3-25/216*x)/(x^4+3*x^2+2)^(1/2
)-53/28224*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^
(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+5797/56448*I*2^(1/2)*(2*
x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^
(1/2)*x,2^(1/2))+38525/197568*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)
^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(x^4+3\,x^2+2)^{\frac{3}{2}}(5\,x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(125\,x^{10}+900\,x^{8}+2560\,x^{6}+3598\,x^{4}+2499\,x^{2}+686)\sqrt{x^{4}+3\,x^{2}+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3),x, algorithm="fricas")
```

```
[Out] integral(1/((125*x^10 + 900*x^8 + 2560*x^6 + 3598*x^4 + 2499*x^2
+ 686)*sqrt(x^4 + 3*x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left((x^{2}+1)\,(x^{2}+2)\right)^{\frac{3}{2}}\,(5x^{2}+7)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x**2+7)**3/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(x^{4}+3\,x^{2}+2)^{\frac{3}{2}}(5\,x^{2}+7)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

3.320 $\int (7+5x^2)^4 \sqrt{2+x^2-x^4}\,dx$

Optimal. Leaf size=116

$$-\frac{116100}{77}\left(-x^4+x^2+2\right)^{3/2}x+\frac{1}{231}\left(717372x^2+177953\right)\sqrt{-x^4+x^2+2}x-\frac{625}{11}\left(-x^4+x^2+2\right)^{3/2}x^5-\frac{14500}{33}\left(-x^4+x^2+2\right)^{3/2}x^3-\frac{539419}{77}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{3764813}{231}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(177953 + 717372*x^2)*Sqrt[2 + x^2 - x^4])/231 - (116100*x*(2
+ x^2 - x^4)^(3/2))/77 - (14500*x^3*(2 + x^2 - x^4)^(3/2))/33 - (
625*x^5*(2 + x^2 - x^4)^(3/2))/11 + (3764813*EllipticE[ArcSin[x/S
qrt[2]], -2])/231 - (539419*EllipticF[ArcSin[x/Sqrt[2]], -2])/77
```

Rubi [A] time = 0.262908, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$-\frac{116100}{77}\left(-x^4+x^2+2\right)^{3/2}x+\frac{1}{231}\left(717372x^2+177953\right)\sqrt{-x^4+x^2+2}x-\frac{625}{11}\left(-x^4+x^2+2\right)^{3/2}x^5-\frac{14500}{33}\left(-x^4+x^2+2\right)^{3/2}x^3-\frac{539419}{77}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{3764813}{231}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^4*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (x*(177953 + 717372*x^2)*Sqrt[2 + x^2 - x^4])/231 - (116100*x*(2
+ x^2 - x^4)^(3/2))/77 - (14500*x^3*(2 + x^2 - x^4)^(3/2))/33 - (
625*x^5*(2 + x^2 - x^4)^(3/2))/11 + (3764813*EllipticE[ArcSin[x/S
qrt[2]], -2])/231 - (539419*EllipticF[ArcSin[x/Sqrt[2]], -2])/77
```

Rubi in Sympy [A] time = 54.1714, size = 112, normalized size = 0.97

$$-\frac{625x^5\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{11}-\frac{14500x^3\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{33}+\frac{x\left(\frac{3586860x^2}{77}+\frac{889765}{77}\right)\sqrt{-x^4+x^2+2}}{15}-\frac{116100x\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{77}+\frac{3764813E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{231}-\frac{539419F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{77}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**4*(-x**4+x**2+2)**(1/2),x)
```

```
[Out] -625*x**5*(-x**4 + x**2 + 2)**(3/2)/11 - 14500*x**3*(-x**4 + x**2
 + 2)**(3/2)/33 + x*(3586860*x**2/77 + 889765/77)*sqrt(-x**4 + x*
*2 + 2)/15 - 116100*x*(-x**4 + x**2 + 2)**(3/2)/77 + 3764813*elli
ptic_e(asin(sqrt(2)*x/2), -2)/231 - 539419*elliptic_f(asin(sqrt(2
)*x/2), -2)/77
```

Mathematica [C] time = 0.124879, size = 112, normalized size = 0.97

$$\frac{-13125x^{13}-75250x^{11}-105925x^9+231228x^7+1125819x^5-186503x^3-4838091i\sqrt{-2x^4+2x^2+4}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+}{231\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^4*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-1037294*x - 186503*x^3 + 1125819*x^5 + 231228*x^7 - 105925*x^9
- 75250*x^11 - 13125*x^13 + (3764813*I)*Sqrt[4 + 2*x^2 - 2*x^4]*E
llipticE[I*ArcSinh[x], -1/2] - (4838091*I)*Sqrt[4 + 2*x^2 - 2*x^4
]*EllipticF[I*ArcSinh[x], -1/2])/(231*Sqrt[2 + x^2 - x^4])
```

Maple [A] time = 0.035, size = 193, normalized size = 1.7

$$-\frac{518647 x}{231}\sqrt{-x^4+x^2+2}+\frac{1073278\sqrt{2}}{231}\sqrt{-2x^2+4}\sqrt{x^2+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}} -\frac{3764813\sqrt{2}}{462}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}} +\frac{166072x^3}{231}\sqrt{-x^4+x^2+2}+\frac{20050x^5}{21}\sqrt{-x^4+x^2+2} +\frac{12625x^7}{33}\sqrt{-x^4+x^2+2}+\frac{625x^9}{11}\sqrt{-x^4+x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x)
```

```
[Out] -518647/231*x*(-x^4+x^2+2)^(1/2)+1073278/231*2^(1/2)*(-2*x^2+4)^(
1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2
^(1/2))-3764813/462*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+
x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^
(1/2)*x,I*2^(1/2)))+166072/231*x^3*(-x^4+x^2+2)^(1/2)+20050/21*x^
5*(-x^4+x^2+2)^(1/2)+12625/33*x^7*(-x^4+x^2+2)^(1/2)+625/11*x^9*(
-x^4+x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-x^4+x^2+2}\left(5x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(625x^8+3500x^6+7350x^4+6860x^2+2401\right)\sqrt{-x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^4,x, algorithm="fricas")
```

```
[Out] integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)*sqrt(-
x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**4*(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**4, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^4, x)
```

3.321 $\int \left(7+5x^2\right)^3 \sqrt{2+x^2-x^4}\,dx$

Optimal. Leaf size=95

$$-\frac{1825}{21}\left(-x^4+x^2+2\right)^{3/2}x+\frac{1}{63}\left(14691x^2+5956\right)\sqrt{-x^4+x^2+2}\,x-\frac{125}{9}\left(-x^4+x^2+2\right)^{3/2}x^3-\frac{8735}{21}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{79411}{63}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(5956 + 14691*x^2)*Sqrt[2 + x^2 - x^4])/63 - (1825*x*(2 + x^2
- x^4)^(3/2))/21 - (125*x^3*(2 + x^2 - x^4)^(3/2))/9 + (79411*Ell
ipticE[ArcSin[x/Sqrt[2]], -2])/63 - (8735*EllipticF[ArcSin[x/Sqrt
[2]], -2])/21
```

Rubi [A] time = 0.22413, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$-\frac{1825}{21}\left(-x^4+x^2+2\right)^{3/2}x+\frac{1}{63}\left(14691x^2+5956\right)\sqrt{-x^4+x^2+2}\,x-\frac{125}{9}\left(-x^4+x^2+2\right)^{3/2}x^3-\frac{8735}{21}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{79411}{63}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (x*(5956 + 14691*x^2)*Sqrt[2 + x^2 - x^4])/63 - (1825*x*(2 + x^2
- x^4)^(3/2))/21 - (125*x^3*(2 + x^2 - x^4)^(3/2))/9 + (79411*Ell
ipticE[ArcSin[x/Sqrt[2]], -2])/63 - (8735*EllipticF[ArcSin[x/Sqrt
[2]], -2])/21
```

Rubi in Sympy [A] time = 42.0977, size = 94, normalized size = 0.99

$$-\frac{125x^3\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{9}+\frac{x\left(\frac{24485x^2}{7}+\frac{29780}{21}\right)\sqrt{-x^4+x^2+2}}{15}-\frac{1825x\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{21}+\frac{79411E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{63}-\frac{8735F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**3*(-x**4+x**2+2)**(1/2),x)
```

```
[Out] -125*x**3*(-x**4 + x**2 + 2)**(3/2)/9 + x*(24485*x**2/7 + 29780/2
1)*sqrt(-x**4 + x**2 + 2)/15 - 1825*x*(-x**4 + x**2 + 2)**(3/2)/2
1 + 79411*elliptic_e(asin(sqrt(2)*x/2), -2)/63 - 8735*elliptic_f(
asin(sqrt(2)*x/2), -2)/21
```

Mathematica [C] time = 0.104221, size = 107, normalized size = 1.13

$$\frac{-875x^{11}-3725x^9-1116x^7+21660x^5+9938x^3-106014i\sqrt{-2x^4+2x^2+4}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+79411i\sqrt{-2x^4+2x^2+4}E}{63\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-9988*x + 9938*x^3 + 21660*x^5 - 1116*x^7 - 3725*x^9 - 875*x^11
+ (79411*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2]
 - (106014*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/
2])/(63*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.011, size = 176, normalized size = 1.9

$$-\frac{4994\,x}{63}\sqrt{-x^4+x^2+2}+\frac{26603\,\sqrt{2}}{63}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}-\frac{79411\,\sqrt{2}}{126}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\left(\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}}+\frac{7466\,x^3}{63}\sqrt{-x^4+x^2+2}+\frac{4600\,x^5}{63}\sqrt{-x^4+x^2+2}+\frac{125\,x^7}{9}\sqrt{-x^4+x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x)
```

```
[Out] -4994/63*x*(-x^4+x^2+2)^(1/2)+26603/63*2^(1/2)*(-2*x^2+4)^(1/2)*(
x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2)
)-79411/126*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(
1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,
I*2^(1/2)))+7466/63*x^3*(-x^4+x^2+2)^(1/2)+4600/63*x^5*(-x^4+x^2+
2)^(1/2)+125/9*x^7*(-x^4+x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(125\,x^6+525\,x^4+735\,x^2+343\right)\sqrt{-x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3,x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(-x^4 + x^2 + 2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**3*(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + x^2 + 2}\left(5\,x^2 + 7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3, x)
```

3.322 $\int \left(7+5x^2\right)^2 \sqrt{2+x^2-x^4}\,dx$

Optimal. Leaf size=74

$$-\frac{25}{7}x\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{21}x\left(354x^2+275\right)\sqrt{-x^4+x^2+2}-\frac{79}{7}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{2045}{21}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(275 + 354*x^2)*Sqrt[2 + x^2 - x^4])/21 - (25*x*(2 + x^2 - x^4
)^(3/2))/7 + (2045*EllipticE[ArcSin[x/Sqrt[2]], -2])/21 - (79*Ell
ipticF[ArcSin[x/Sqrt[2]], -2])/7
```

Rubi [A] time = 0.189563, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{25}{7}x\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{21}x\left(354x^2+275\right)\sqrt{-x^4+x^2+2}-\frac{79}{7}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{2045}{21}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (x*(275 + 354*x^2)*Sqrt[2 + x^2 - x^4])/21 - (25*x*(2 + x^2 - x^4
)^(3/2))/7 + (2045*EllipticE[ArcSin[x/Sqrt[2]], -2])/21 - (79*Ell
ipticF[ArcSin[x/Sqrt[2]], -2])/7
```

Rubi in Sympy [A] time = 33.0303, size = 75, normalized size = 1.01

$$\frac{x\left(\frac{1770x^2}{7}+\frac{1375}{7}\right)\sqrt{-x^4+x^2+2}}{15}-\frac{25x\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{7}+\frac{2045E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{21}-\frac{79F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**2*(-x**4+x**2+2)**(1/2),x)
```

```
[Out] x*(1770*x**2/7 + 1375/7)*sqrt(-x**4 + x**2 + 2)/15 - 25*x*(-x**4
+ x**2 + 2)**(3/2)/7 + 2045*elliptic_e(asin(sqrt(2)*x/2), -2)/21
- 79*elliptic_f(asin(sqrt(2)*x/2), -2)/7
```

Mathematica [C] time = 0.100961, size = 102, normalized size = 1.38

$$\frac{-75x^9-204x^7+304x^5+683x^3-2949i\sqrt{-2x^4+2x^2+4}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+2045i\sqrt{-2x^4+2x^2+4}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+2}{21\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (250*x + 683*x^3 + 304*x^5 - 204*x^7 - 75*x^9 + (2045*I)*Sqrt[4 +
 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (2949*I)*Sqrt[4 +
 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(21*Sqrt[2 + x^2 -
```

```
x^4])
```

Maple [B] time = 0.011, size = 159, normalized size = 2.2

$$\frac{125\,x\sqrt{-x^4+x^2+2}}{21}+\frac{904\sqrt{2}\,\sqrt{-2\,x^2+4}\,\sqrt{x^2+1}\,\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)}{21\sqrt{-x^4+x^2+2}}-\frac{2045\sqrt{2}\,\sqrt{-2\,x^2+4}\,\sqrt{x^2+1}\left(\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\right)}{42\sqrt{-x^4+x^2+2}}+\frac{93\,x^3\sqrt{-x^4+x^2+2}}{7}+\frac{25\,x^5\sqrt{-x^4+x^2+2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x)
```

```
[Out] 125/21*x*(-x^4+x^2+2)^(1/2)+904/21*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-20
45/42*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(
EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1
/2)))+93/7*x^3*(-x^4+x^2+2)^(1/2)+25/7*x^5*(-x^4+x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(25\,x^4+70\,x^2+49\right)\sqrt{-x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2,x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(-x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**2*(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2, x)
```

3.323 $\int (7+5x^2)\sqrt{2+x^2-x^4}\,dx$

Optimal. Leaf size=46

$$x\sqrt{-x^4+x^2+2}\left(x^2+2\right)+3F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+7E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] x*(2 + x^2)*Sqrt[2 + x^2 - x^4] + 7*EllipticE[ArcSin[x/Sqrt[2]],
-2] + 3*EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rubi [A] time = 0.153491, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$x\sqrt{-x^4+x^2+2}\left(x^2+2\right)+3F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+7E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] x*(2 + x^2)*Sqrt[2 + x^2 - x^4] + 7*EllipticE[ArcSin[x/Sqrt[2]],
-2] + 3*EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rubi in Sympy [A] time = 24.2568, size = 51, normalized size = 1.11

$$\frac{x\left(15x^2+30\right)\sqrt{-x^4+x^2+2}}{15}+7E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)+3F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)*(-x**4+x**2+2)**(1/2),x)
```

```
[Out] x*(15*x**2 + 30)*sqrt(-x**4 + x**2 + 2)/15 + 7*elliptic_e(asin(sq
rt(2)*x/2), -2) + 3*elliptic_f(asin(sqrt(2)*x/2), -2)
```

Mathematica [C] time = 0.102937, size = 94, normalized size = 2.04

$$\frac{-x^7-x^5+4x^3-12i\sqrt{-2x^4+2x^2+4}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+7i\sqrt{-2x^4+2x^2+4}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+4x}{\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (4*x + 4*x^3 - x^5 - x^7 + (7*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Elliptic
E[I*ArcSinh[x], -1/2] - (12*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[
I*ArcSinh[x], -1/2])/Sqrt[2 + x^2 - x^4]
```

Maple [B] time = 0.008, size = 141, normalized size = 3.1

$$2x\sqrt{-x^4+x^2+2}+5\frac{\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}EllipticF\left(1/2\sqrt{2}x,i\sqrt{2}\right)}{\sqrt{-x^4+x^2+2}} - \frac{7\sqrt{2}}{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(EllipticF\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)-EllipticE\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}} + x^3\sqrt{-x^4+x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)*(-x^4+x^2+2)^(1/2),x)
```

```
[Out] 2*x*(-x^4+x^2+2)^(1/2)+5*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(
-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-7/2*2^(1/2)*
(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*
2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))+x^3*(-x^
4+x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-x^4+x^2+2}\left(5x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^4+x^2+2}\left(5x^2+7\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)*(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + x^2 + 2}\left(5\,x^2 + 7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7),x, algorithm="giac")

[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7), x)
```

3.324 $\int \sqrt{2+x^2-x^4}\,dx$

Optimal. Leaf size=44

$$\frac{1}{3}\sqrt{-x^4+x^2+2}x+F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*Sqrt[2 + x^2 - x^4])/3 + EllipticE[ArcSin[x/Sqrt[2]], -2]/3 +
EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rubi [A] time = 0.130613, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$\frac{1}{3}\sqrt{-x^4+x^2+2}x+F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (x*Sqrt[2 + x^2 - x^4])/3 + EllipticE[ArcSin[x/Sqrt[2]], -2]/3 +
EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rubi in Sympy [A] time = 19.9534, size = 42, normalized size = 0.95

$$\frac{x\sqrt{-x^4+x^2+2}}{3}+\frac{E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{3}+F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+x**2+2)**(1/2),x)
```

```
[Out] x*sqrt(-x**4 + x**2 + 2)/3 + elliptic_e(asin(sqrt(2)*x/2), -2)/3
+ elliptic_f(asin(sqrt(2)*x/2), -2)
```

Mathematica [C] time = 0.0855372, size = 90, normalized size = 2.05

$$\frac{-x^5+x^3-3i\sqrt{-2x^4+2x^2+4}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+i\sqrt{-2x^4+2x^2+4}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+2x}{3\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (2*x + x^3 - x^5 + I*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[
x], -1/2] - (3*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x],
 -1/2])/(3*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.004, size = 125, normalized size = 2.8

$$\frac{x}{3}\sqrt{-x^4+x^2+2}+\frac{2\sqrt{2}}{3}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}}{6}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\left(\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+x^2+2)^(1/2),x)
```

```
[Out] 1/3*x*(-x^4+x^2+2)^(1/2)+2/3*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/
2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-1/6*2^(1
/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(
1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**4 + x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2), x)
```

3.325 $\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx$

Optimal. Leaf size=46

$$\frac{17}{25}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1}{5}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{34}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] -EllipticE[ArcSin[x/Sqrt[2]], -2]/5 + (17*EllipticF[ArcSin[x/Sqrt
[2]], -2])/25 - (34*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/175
```

Rubi [A] time = 0.264677, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{17}{25}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1}{5}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{34}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2),x]
```

```
[Out] -EllipticE[ArcSin[x/Sqrt[2]], -2]/5 + (17*EllipticF[ArcSin[x/Sqrt
[2]], -2])/25 - (34*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/175
```

Rubi in Sympy [A] time = 39.1256, size = 51, normalized size = 1.11

$$-\frac{E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{5}+\frac{17F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{25}-\frac{34\left(-\frac{10}{7};\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7),x)
```

```
[Out] -elliptic_e(asin(sqrt(2)*x/2), -2)/5 + 17*elliptic_f(asin(sqrt(2)
*x/2), -2)/25 - 34*elliptic_pi(-10/7, asin(sqrt(2)*x/2), -2)/175
```

Mathematica [C] time = 0.0853295, size = 51, normalized size = 1.11

$$-\frac{1}{175}i\sqrt{2}\left(7F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+35E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-17\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2),x]
```

```
[Out] (-I/175)*Sqrt[2]*(35*EllipticE[I*ArcSinh[x], -1/2] + 7*EllipticF[
I*ArcSinh[x], -1/2] - 17*EllipticPi[5/7, I*ArcSinh[x], -1/2])
```

Maple [B] time = 0.02, size = 141, normalized size = 3.1

$$\frac{17\sqrt{2}}{50}\sqrt{-2x^2+4}\sqrt{x^2+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}, i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}} - \frac{\sqrt{2}}{10}\sqrt{-2x^2+4}\sqrt{x^2+1}\mathit{EllipticE}\left(\frac{\sqrt{2}x}{2}, i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}} - \frac{34\sqrt{2}}{175}\sqrt{1-\frac{x^2}{2}}\sqrt{x^2+1}\mathit{EllipticPi}\left(\frac{\sqrt{2}x}{2}, -\frac{10}{7}, i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+x^2+2)^(1/2)/(5*x^2+7),x)
```

```
[Out] 17/50*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*E
llipticF(1/2*2^(1/2)*x,I*2^(1/2))-1/10*2^(1/2)*(-2*x^2+4)^(1/2)*(
x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2)
)-34/175*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/
2)*EllipticPi(1/2*2^(1/2)*x,-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+x^2+2}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4+x^2+2}}{5x^2+7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x^2-2)(x^2+1)}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7),x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4 + x^2 + 2}}{5\,x^2 + 7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

3.326 $\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{-x^4+x^2+2}x}{14(5x^2+7)}-\frac{6}{175}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{70}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{99\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2450}$$

```
[Out] (x*Sqrt[2 + x^2 - x^4])/(14*(7 + 5*x^2)) + EllipticE[ArcSin[x/Sqr
t[2]], -2]/70 - (6*EllipticF[ArcSin[x/Sqrt[2]], -2])/175 + (99*El
lipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/2450
```

Rubi [A] time = 0.268251, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{\sqrt{-x^4+x^2+2}x}{14(5x^2+7)}-\frac{6}{175}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{70}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{99\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2450}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (x*Sqrt[2 + x^2 - x^4])/(14*(7 + 5*x^2)) + EllipticE[ArcSin[x/Sqr
t[2]], -2]/70 - (6*EllipticF[ArcSin[x/Sqrt[2]], -2])/175 + (99*El
lipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/2450
```

Rubi in Sympy [A] time = 37.3469, size = 178, normalized size = 2.41

$$\frac{x\sqrt{-x^4+x^2+2}}{70x^2+98}+\frac{\sqrt{2}\sqrt{-x^4+x^2+2}E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{140\sqrt{-\frac{x^2}{2}+1}\sqrt{x^2+1}}-\frac{3\sqrt{2}\sqrt{-x^4+x^2+2}F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{175\sqrt{-\frac{x^2}{2}+1}\sqrt{x^2+1}}+\frac{99\sqrt{2}\sqrt{-x^4+x^2+2}\left(-\frac{10}{7};\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{4900\sqrt{-\frac{x^2}{2}+1}\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7)**2,x)
```

```
[Out] x*sqrt(-x**4 + x**2 + 2)/(70*x**2 + 98) + sqrt(2)*sqrt(-x**4 + x*
*2 + 2)*elliptic_e(asin(sqrt(2)*x/2), -2)/(140*sqrt(-x**2/2 + 1)*
sqrt(x**2 + 1)) - 3*sqrt(2)*sqrt(-x**4 + x**2 + 2)*elliptic_f(asi
n(sqrt(2)*x/2), -2)/(175*sqrt(-x**2/2 + 1)*sqrt(x**2 + 1)) + 99*s
qrt(2)*sqrt(-x**4 + x**2 + 2)*elliptic_pi(-10/7, asin(sqrt(2)*x/2
), -2)/(4900*sqrt(-x**2/2 + 1)*sqrt(x**2 + 1))
```

Mathematica [C] time = 0.267206, size = 196, normalized size = 2.65

$$\frac{-350x^5+350x^3-21i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+70i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-}{4900\left(5x^2+7\right)\sqrt{-x^4+x^2+}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (700*x + 350*x^3 - 350*x^5 + (70*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 +
x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] - (21*I)*Sqrt[2]*(7 + 5*
x^2)*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] - (693*I)*
Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] -
 (495*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSin
h[x], -1/2])/(4900*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.027, size = 165, normalized size = 2.2

$$\frac{x}{70\,x^2+98}\sqrt{-x^4+x^2+2}-\frac{3\sqrt{2}}{175}\sqrt{-2\,x^2+4}\sqrt{x^2+1}EllipticF\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}+\frac{\sqrt{2}}{140}\sqrt{-2\,x^2+4}\sqrt{x^2+1}EllipticE\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}+\frac{99\sqrt{2}}{2450}\sqrt{1-\frac{x^2}{2}}\sqrt{x^2+1}EllipticPi\left(\frac{\sqrt{2}x}{2},-\frac{10}{7},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x)
```

```
[Out] 1/14*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-3/175*2^(1/2)*(-2*x^2+4)^(1/2
)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1
/2))+1/140*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1
/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))+99/2450*2^(1/2)*(1-1/2*x^2
)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*2^(1/2)*x
,-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+x^2+2}}{(5\,x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4+x^2+2}}{25\,x^4+70\,x^2+49},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x^2-2)(x^2+1)}}{(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7)**2,x)

[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+x^2+2}}{(5\,x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^2,x, algorithm="giac")

[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^2, x)
```

3.327 $\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3}\,dx$

Optimal. Leaf size=102

$$-\frac{31\sqrt{-x^4+x^2+2}x}{13328(5x^2+7)}+\frac{\sqrt{-x^4+x^2+2}x}{28(5x^2+7)^2}-\frac{269F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{166600}-\frac{31E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{66640}+\frac{16601\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2332400}$$

```
[Out] (x*Sqrt[2 + x^2 - x^4])/(28*(7 + 5*x^2)^2) - (31*x*Sqrt[2 + x^2 -
 x^4])/(13328*(7 + 5*x^2)) - (31*EllipticE[ArcSin[x/Sqrt[2]], -2]
)/66640 - (269*EllipticF[ArcSin[x/Sqrt[2]], -2])/166600 + (16601*
EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/2332400
```

Rubi [A] time = 0.891212, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458

$$-\frac{31\sqrt{-x^4+x^2+2}x}{13328(5x^2+7)}+\frac{\sqrt{-x^4+x^2+2}x}{28(5x^2+7)^2}-\frac{269F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{166600}-\frac{31E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{66640}+\frac{16601\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2332400}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (x*Sqrt[2 + x^2 - x^4])/(28*(7 + 5*x^2)^2) - (31*x*Sqrt[2 + x^2 -
 x^4])/(13328*(7 + 5*x^2)) - (31*EllipticE[ArcSin[x/Sqrt[2]], -2]
)/66640 - (269*EllipticF[ArcSin[x/Sqrt[2]], -2])/166600 + (16601*
EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/2332400
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7)**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.467603, size = 244, normalized size = 2.39

$$\frac{54250x^7-144900x^5-17850x^3+7021i\sqrt{2}\left(5x^2+7\right)^2\sqrt{-x^4+x^2+2}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-2170i\sqrt{2}\left(5x^2+7\right)^2\sqrt{-x^4+x^2+}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (181300*x - 17850*x^3 - 144900*x^5 + 54250*x^7 - (2170*I)*Sqrt[2]
*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2]
+ (7021*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*
ArcSinh[x], -1/2] - (813449*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*Ellipt
icPi[5/7, I*ArcSinh[x], -1/2] - (1162070*I)*Sqrt[2]*x^2*Sqrt[2 +
x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (415025*I)*Sqrt[
2]*x^4*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(
4664800*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4])
```

Maple [A] time = 0.029, size = 189, normalized size = 1.9

$$\frac{x}{28\,(5\,x^2+7)^2}\sqrt{-x^4+x^2+2}-\frac{31\,x}{66640\,x^2+93296}\sqrt{-x^4+x^2+2}$$
$$-\frac{269\,\sqrt{2}}{333200}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$
$$-\frac{31\,\sqrt{2}}{133280}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathit{EllipticE}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$
$$+\frac{16601\,\sqrt{2}}{2332400}\sqrt{1-\frac{x^2}{2}}\sqrt{x^2+1}\mathit{EllipticPi}\left(\frac{\sqrt{2}x}{2},-\frac{10}{7},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x)
```

```
[Out] 1/28*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-31/13328*x*(-x^4+x^2+2)^(1/
2)/(5*x^2+7)-269/333200*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-
x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-31/133280*2^(
1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(
1/2*2^(1/2)*x,I*2^(1/2))+16601/2332400*2^(1/2)*(1-1/2*x^2)^(1/2)*
(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*2^(1/2)*x,-10/7,I
*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+x^2+2}}{(5\,x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^4+x^2+2}}{125\,x^6+525\,x^4+735\,x^2+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(125*x^6 + 525*x^4 + 735*x^2 + 343)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\left(x^{2}-2\right)\left(x^{2}+1\right)}}{\left(5x^{2}+7\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7)**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^{4}+x^{2}+2}}{\left(5\,x^{2}+7\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^3, x)
```

3.328 $\int \left(7+5x^2\right)^4 \left(2+x^2-x^4\right)^{3/2} \, dx$

Optimal. Leaf size=142

$$-\frac{132300}{143}\left(-x^4+x^2+2\right)^{5/2}x-\frac{\left(69817-1581440x^2\right)\left(-x^4+x^2+2\right)^{3/2}x}{1001}+\frac{3\left(7837383x^2+2193559\right)\sqrt{-x^4+x^2+2}x}{5005}-\frac{125}{3}\left(-x^4+x^2+2\right)^{5/2}x^5-\frac{11750}{39}\left(-x^4+x^2+2\right)^{5/2}x^3-\frac{50794416F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{5005}+\frac{124141422E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{5005}$$

```
[Out] (3*x*(2193559 + 7837383*x^2)*Sqrt[2 + x^2 - x^4])/5005 - (x*(6981
7 - 1581440*x^2)*(2 + x^2 - x^4)^(3/2))/1001 - (132300*x*(2 + x^2
 - x^4)^(5/2))/143 - (11750*x^3*(2 + x^2 - x^4)^(5/2))/39 - (125*
x^5*(2 + x^2 - x^4)^(5/2))/3 + (124141422*EllipticE[ArcSin[x/Sqrt
[2]], -2])/5005 - (50794416*EllipticF[ArcSin[x/Sqrt[2]], -2])/500
5
```

Rubi [A] time = 0.288905, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$-\frac{132300}{143}\left(-x^4+x^2+2\right)^{5/2}x-\frac{\left(69817-1581440x^2\right)\left(-x^4+x^2+2\right)^{3/2}x}{1001}+\frac{3\left(7837383x^2+2193559\right)\sqrt{-x^4+x^2+2}x}{5005}-\frac{125}{3}\left(-x^4+x^2+2\right)^{5/2}x^5-\frac{11750}{39}\left(-x^4+x^2+2\right)^{5/2}x^3-\frac{50794416F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{5005}+\frac{124141422E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{5005}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^4*(2 + x^2 - x^4)^(3/2),x]
```


```
[Out] (3*x*(2193559 + 7837383*x^2)*Sqrt[2 + x^2 - x^4])/5005 - (x*(6981
7 - 1581440*x^2)*(2 + x^2 - x^4)^(3/2))/1001 - (132300*x*(2 + x^2
 - x^4)^(5/2))/143 - (11750*x^3*(2 + x^2 - x^4)^(5/2))/39 - (125*
x^5*(2 + x^2 - x^4)^(5/2))/3 + (124141422*EllipticE[ArcSin[x/Sqrt
[2]], -2])/5005 - (50794416*EllipticF[ArcSin[x/Sqrt[2]], -2])/500
5
```

Rubi in Sympy [A] time = 60.5915, size = 138, normalized size = 0.97

$$-\frac{125x^5\left(-x^4+x^2+2\right)^{\frac{5}{2}}}{3}-\frac{11750x^3\left(-x^4+x^2+2\right)^{\frac{5}{2}}}{39}-\frac{x\left(-\frac{14232960x^2}{143}+\frac{57123}{13}\right)\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{63}+\frac{x\left(\frac{211609341x^2}{143}+\frac{59226093}{143}\right)\sqrt{-x^4+x^2+2}}{315}-\frac{132300x\left(-x^4+x^2+2\right)^{\frac{5}{2}}}{143}+\frac{124141422E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{5005}-\frac{50794416F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{5005}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**4*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] -125*x**5*(-x**4 + x**2 + 2)**(5/2)/3 - 11750*x**3*(-x**4 + x**2
+ 2)**(5/2)/39 - x*(-14232960*x**2/143 + 57123/13)*(-x**4 + x**2
```

```
+ 2)**(3/2)/63 + x*(211609341*x**2/143 + 59226093/143)*sqrt(-x**4
 + x**2 + 2)/315 - 132300*x*(-x**4 + x**2 + 2)**(5/2)/143 + 12414
1422*elliptic_e(asin(sqrt(2)*x/2), -2)/5005 - 50794416*elliptic_f
(asin(sqrt(2)*x/2), -2)/5005
```

Mathematica [C] time = 0.117841, size = 122, normalized size = 0.86

$$\frac{625625x^{17}+2646875x^{15}-1556625x^{13}-24642275x^{11}-36649955x^9+32834763x^7+172881581x^5+48624305x^3-482444}{15015\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^4*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (-75836958*x + 48624305*x^3 + 172881581*x^5 + 32834763*x^7 - 3664
9955*x^9 - 24642275*x^11 - 1556625*x^13 + 2646875*x^15 + 625625*x
^17 + (372424266*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x
], -1/2] - (482444775*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcS
inh[x], -1/2])/(15015*Sqrt[2 + x^2 - x^4])
```

Maple [A] time = 0.032, size = 227, normalized size = 1.6

$$\frac{833561\,x^5}{273}\sqrt{-x^4+x^2+2}+\frac{43271392\,x^3}{15015}\sqrt{-x^4+x^2+2}-\frac{12639493\,x}{5005}\sqrt{-x^4+x^2+2}$$
$$+\frac{36673503\,\sqrt{2}}{5005}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$
$$-\frac{62070711\,\sqrt{2}}{5005}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\left(\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$
$$+\frac{432290\,x^7}{429}\sqrt{-x^4+x^2+2}-\frac{84775\,x^9}{429}\sqrt{-x^4+x^2+2}-\frac{8500\,x^{11}}{39}\sqrt{-x^4+x^2+2}-\frac{125\,x^{13}}{3}\sqrt{-x^4+x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 833561/273*x^5*(-x^4+x^2+2)^(1/2)+43271392/15015*x^3*(-x^4+x^2+2)
^(1/2)-12639493/5005*x*(-x^4+x^2+2)^(1/2)+36673503/5005*2^(1/2)*(
-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^
(1/2)*x,I*2^(1/2))-62070711/5005*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)
^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-Ell
ipticE(1/2*2^(1/2)*x,I*2^(1/2)))+432290/429*x^7*(-x^4+x^2+2)^(1/2
)-84775/429*x^9*(-x^4+x^2+2)^(1/2)-8500/39*x^11*(-x^4+x^2+2)^(1/2
)-125/3*x^13*(-x^4+x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^4,x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(625\,x^{12}+2875\,x^{10}+2600\,x^{8}-7490\,x^{6}-19159\,x^{4}-16121\,x^{2}-4802\right)\sqrt{-x^{4}+x^{2}+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^4,x, algorithm="fricas")
```

```
[Out] integral(-(625*x^12 + 2875*x^10 + 2600*x^8 - 7490*x^6 - 19159*x^4
 - 16121*x^2 - 4802)*sqrt(-x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(x^{2}-2\right)\left(x^{2}+1\right)\right)^{\frac{3}{2}}\left(5x^{2}+7\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**4*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**4, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^{4}+x^{2}+2\right)^{\frac{3}{2}}\left(5\,x^{2}+7\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^4,x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^4, x)
```

3.329 $\int \left(7+5x^2\right)^3 \left(2+x^2-x^4\right)^{3/2} \, dx$

Optimal. Leaf size=121

$$-\frac{7825}{143}\left(-x^4+x^2+2\right)^{5/2}x+\frac{\left(374045x^2+33792\right)\left(-x^4+x^2+2\right)^{3/2}x}{3003}+\frac{\left(5712051x^2+2512273\right)\sqrt{-x^4+x^2+2}\,x}{15015}-\frac{125}{13}\left(-x^4+x^2+2\right)^{5/2}x^3-\frac{3199778F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{5005}+\frac{31072528E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{15015}$$

```
[Out] (x*(2512273 + 5712051*x^2)*Sqrt[2 + x^2 - x^4])/15015 + (x*(33792
 + 374045*x^2)*(2 + x^2 - x^4)^(3/2))/3003 - (7825*x*(2 + x^2 - x
^4)^(5/2))/143 - (125*x^3*(2 + x^2 - x^4)^(5/2))/13 + (31072528*E
llipticE[ArcSin[x/Sqrt[2]], -2])/15015 - (3199778*EllipticF[ArcSi
n[x/Sqrt[2]], -2])/5005
```

Rubi [A] time = 0.253102, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$-\frac{7825}{143}\left(-x^4+x^2+2\right)^{5/2}x+\frac{\left(374045x^2+33792\right)\left(-x^4+x^2+2\right)^{3/2}x}{3003}+\frac{\left(5712051x^2+2512273\right)\sqrt{-x^4+x^2+2}\,x}{15015}-\frac{125}{13}\left(-x^4+x^2+2\right)^{5/2}x^3-\frac{3199778F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{5005}+\frac{31072528E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{15015}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(2512273 + 5712051*x^2)*Sqrt[2 + x^2 - x^4])/15015 + (x*(33792
 + 374045*x^2)*(2 + x^2 - x^4)^(3/2))/3003 - (7825*x*(2 + x^2 - x
^4)^(5/2))/143 - (125*x^3*(2 + x^2 - x^4)^(5/2))/13 + (31072528*E
llipticE[ArcSin[x/Sqrt[2]], -2])/15015 - (3199778*EllipticF[ArcSi
n[x/Sqrt[2]], -2])/5005
```

Rubi in Sympy [A] time = 48.2813, size = 119, normalized size = 0.98

$$-\frac{125x^3\left(-x^4+x^2+2\right)^{\frac{5}{2}}}{13}+\frac{x\left(\frac{1122135x^2}{143}+\frac{9216}{13}\right)\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{63}+\frac{x\left(\frac{17136153x^2}{143}+\frac{7536819}{143}\right)\sqrt{-x^4+x^2+2}}{315}-\frac{7825x\left(-x^4+x^2+2\right)^{\frac{5}{2}}}{143}+\frac{31072528E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{15015}-\frac{3199778F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{5005}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**3*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] -125*x**3*(-x**4 + x**2 + 2)**(5/2)/13 + x*(1122135*x**2/143 + 92
16/13)*(-x**4 + x**2 + 2)**(3/2)/63 + x*(17136153*x**2/143 + 7536
819/143)*sqrt(-x**4 + x**2 + 2)/315 - 7825*x*(-x**4 + x**2 + 2)**
(5/2)/143 + 31072528*elliptic_e(asin(sqrt(2)*x/2), -2)/15015 - 31
```

```
99778*elliptic_f(asin(sqrt(2)*x/2), -2)/5005
```

Mathematica [C] time = 0.11404, size = 117, normalized size = 0.97

$$\frac{144375x^{15} + 388500x^{13} - 1027775x^{11} - 4448240x^9 - 1756521x^7 + 13371048x^5 + 11078615x^3 - 41809125i\sqrt{-2x^4 + 2x^2 + 4}}{15015\sqrt{-x^4 + x^2 + 2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (-872614*x + 11078615*x^3 + 13371048*x^5 - 1756521*x^7 - 4448240*
x^9 - 1027775*x^11 + 388500*x^13 + 144375*x^15 + (31072528*I)*Sqr
t[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (41809125*I)
*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(15015*Sq
rt[2 + x^2 - x^4])
```

Maple [A] time = 0.011, size = 210, normalized size = 1.7

$$\frac{65248\,x^5}{273}\sqrt{-x^4+x^2+2}+\frac{5757461\,x^3}{15015}\sqrt{-x^4+x^2+2}-\frac{436307\,x}{15015}\sqrt{-x^4+x^2+2}$$
$$+\frac{10736597\,\sqrt{2}}{15015}\sqrt{-2\,x^2+4}\sqrt{x^2+1}EllipticF\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$
$$-\frac{15536264\,\sqrt{2}}{15015}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\left(EllipticF\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)-EllipticE\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$
$$+\frac{5890\,x^7}{429}\sqrt{-x^4+x^2+2}-\frac{5075\,x^9}{143}\sqrt{-x^4+x^2+2}-\frac{125\,x^{11}}{13}\sqrt{-x^4+x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 65248/273*x^5*(-x^4+x^2+2)^(1/2)+5757461/15015*x^3*(-x^4+x^2+2)^(
1/2)-436307/15015*x*(-x^4+x^2+2)^(1/2)+10736597/15015*2^(1/2)*(-2
*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1
/2)*x,I*2^(1/2))-15536264/15015*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-Elli
pticE(1/2*2^(1/2)*x,I*2^(1/2)))+5890/429*x^7*(-x^4+x^2+2)^(1/2)-5
075/143*x^9*(-x^4+x^2+2)^(1/2)-125/13*x^11*(-x^4+x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3,x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(125\,x^{10}+400\,x^8-40\,x^6-1442\,x^4-1813\,x^2-686\right)\sqrt{-x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3,x, algorithm="fricas")
```

```
[Out] integral(-(125*x^10 + 400*x^8 - 40*x^6 - 1442*x^4 - 1813*x^2 - 68
6)*sqrt(-x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(x^{2}-2\right)\left(x^{2}+1\right)\right)^{\frac{3}{2}}\left(5x^{2}+7\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**3*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^{4}+x^{2}+2\right)^{\frac{3}{2}}\left(5\,x^{2}+7\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3,x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

3.330 $\int (7+5x^2)^2 (2+x^2-x^4)^{3/2} \, dx$

Optimal. Leaf size=100

$$-\frac{25}{11}x\left(-x^4+x^2+2\right)^{5/2}+\frac{1}{99}x\left(920x^2+363\right)\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{495}x\left(14889x^2+11497\right)\sqrt{-x^4+x^2+2}-\frac{3392}{165}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{85942}{495}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(11497 + 14889*x^2)*Sqrt[2 + x^2 - x^4])/495 + (x*(363 + 920*x
^2)*(2 + x^2 - x^4)^(3/2))/99 - (25*x*(2 + x^2 - x^4)^(5/2))/11 +
 (85942*EllipticE[ArcSin[x/Sqrt[2]], -2])/495 - (3392*EllipticF[A
rcSin[x/Sqrt[2]], -2])/165
```

Rubi [A] time = 0.220222, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{25}{11}x\left(-x^4+x^2+2\right)^{5/2}+\frac{1}{99}x\left(920x^2+363\right)\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{495}x\left(14889x^2+11497\right)\sqrt{-x^4+x^2+2}-\frac{3392}{165}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{85942}{495}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(11497 + 14889*x^2)*Sqrt[2 + x^2 - x^4])/495 + (x*(363 + 920*x
^2)*(2 + x^2 - x^4)^(3/2))/99 - (25*x*(2 + x^2 - x^4)^(5/2))/11 +
 (85942*EllipticE[ArcSin[x/Sqrt[2]], -2])/495 - (3392*EllipticF[A
rcSin[x/Sqrt[2]], -2])/165
```

Rubi in Sympy [A] time = 38.9803, size = 99, normalized size = 0.99

$$\frac{x\left(\frac{6440x^2}{11}+231\right)\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{63}+\frac{x\left(\frac{104223x^2}{11}+\frac{80479}{11}\right)\sqrt{-x^4+x^2+2}}{315}-\frac{25x\left(-x^4+x^2+2\right)^{\frac{5}{2}}}{11}+\frac{85942E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{495}-\frac{3392F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{165}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**2*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] x*(6440*x**2/11 + 231)*(-x**4 + x**2 + 2)**(3/2)/63 + x*(104223*x
**2/11 + 80479/11)*sqrt(-x**4 + x**2 + 2)/315 - 25*x*(-x**4 + x**
2 + 2)**(5/2)/11 + 85942*elliptic_e(asin(sqrt(2)*x/2), -2)/495 -
3392*elliptic_f(asin(sqrt(2)*x/2), -2)/165
```

Mathematica [C] time = 0.11192, size = 112, normalized size = 1.12

$$\frac{1125x^{13}+1225x^{11}-10760x^9-19944x^7+23097x^5+53435x^3-123825i\sqrt{-2x^4+2x^2+4}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+85942i\sqrt{-2x}}{495\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (21254*x + 53435*x^3 + 23097*x^5 - 19944*x^7 - 10760*x^9 + 1225*x
^11 + 1125*x^13 + (85942*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*A
rcSinh[x], -1/2] - (123825*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I
*ArcSinh[x], -1/2])/(495*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.011, size = 193, normalized size = 1.9

$$\frac{112\,x^5}{9}\sqrt{-x^4+x^2+2}+\frac{21404\,x^3}{495}\sqrt{-x^4+x^2+2}+\frac{10627\,x}{495}\sqrt{-x^4+x^2+2} \\ +\frac{37883\,\sqrt{2}}{495}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\,EllipticF\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}} \\ -\frac{42971\,\sqrt{2}}{495}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\left(EllipticF\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)-EllipticE\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}} \\ -\frac{470\,x^7}{99}\sqrt{-x^4+x^2+2}-\frac{25\,x^9}{11}\sqrt{-x^4+x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 112/9*x^5*(-x^4+x^2+2)^(1/2)+21404/495*x^3*(-x^4+x^2+2)^(1/2)+106
27/495*x*(-x^4+x^2+2)^(1/2)+37883/495*2^(1/2)*(-2*x^2+4)^(1/2)*(x
^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))
-42971/495*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1
/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I
*2^(1/2)))-470/99*x^7*(-x^4+x^2+2)^(1/2)-25/11*x^9*(-x^4+x^2+2)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2,x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(25\,x^8+45\,x^6-71\,x^4-189\,x^2-98\right)\sqrt{-x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2,x, algorithm="fricas")
```

```
[Out] integral(-(25*x^8 + 45*x^6 - 71*x^4 - 189*x^2 - 98)*sqrt(-x^4 + x
^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(x^{2}-2\right)\left(x^{2}+1\right)\right)^{\frac{3}{2}}\left(5x^{2}+7\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**2*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^{4}+x^{2}+2\right)^{\frac{3}{2}}\left(5\,x^{2}+7\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2,x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

3.331 $\int (7+5x^2)(2+x^2-x^4)^{3/2}\,dx$

Optimal. Leaf size=81

$$\frac{1}{63}x(35x^2+48)(-x^4+x^2+2)^{3/2}+\frac{1}{315}x(669x^2+1087)\sqrt{-x^4+x^2+2}+\frac{418}{105}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{4432}{315}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(1087 + 669*x^2)*Sqrt[2 + x^2 - x^4])/315 + (x*(48 + 35*x^2)*(
2 + x^2 - x^4)^(3/2))/63 + (4432*EllipticE[ArcSin[x/Sqrt[2]], -2]
)/315 + (418*EllipticF[ArcSin[x/Sqrt[2]], -2])/105
```

Rubi [A] time = 0.178023, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{1}{63}x(35x^2+48)(-x^4+x^2+2)^{3/2}+\frac{1}{315}x(669x^2+1087)\sqrt{-x^4+x^2+2}+\frac{418}{105}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{4432}{315}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(1087 + 669*x^2)*Sqrt[2 + x^2 - x^4])/315 + (x*(48 + 35*x^2)*(
2 + x^2 - x^4)^(3/2))/63 + (4432*EllipticE[ArcSin[x/Sqrt[2]], -2]
)/315 + (418*EllipticF[ArcSin[x/Sqrt[2]], -2])/105
```

Rubi in Sympy [A] time = 29.2614, size = 76, normalized size = 0.94

$$\frac{x(35x^2+48)(-x^4+x^2+2)^{\frac{3}{2}}}{63}+\frac{x(669x^2+1087)\sqrt{-x^4+x^2+2}}{315}+\frac{4432E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{315}+\frac{418F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] x*(35*x**2 + 48)*(-x**4 + x**2 + 2)**(3/2)/63 + x*(669*x**2 + 108
7)*sqrt(-x**4 + x**2 + 2)/315 + 4432*elliptic_e(asin(sqrt(2)*x/2)
, -2)/315 + 418*elliptic_f(asin(sqrt(2)*x/2), -2)/105
```

Mathematica [C] time = 0.101102, size = 107, normalized size = 1.32

$$\frac{175x^{11}-110x^9-1674x^7-438x^5+4085x^3-7275i\sqrt{-2x^4+2x^2+4}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+4432i\sqrt{-2x^4+2x^2+4}E\left(i\sinh^{-1}\right.}{315\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (3134*x + 4085*x^3 - 438*x^5 - 1674*x^7 - 110*x^9 + 175*x^11 + (4
432*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (7
275*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(31
5*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.008, size = 176, normalized size = 2.2

$$-\frac{13\,x^5}{63}\sqrt{-x^4+x^2+2}+\frac{1259\,x^3}{315}\sqrt{-x^4+x^2+2}+\frac{1567\,x}{315}\sqrt{-x^4+x^2+2}+\frac{2843\,\sqrt{2}}{315}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}-\frac{2216\,\sqrt{2}}{315}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\left(\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}}-\frac{5\,x^7}{9}\sqrt{-x^4+x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)*(-x^4+x^2+2)^(3/2),x)
```

```
[Out] -13/63*x^5*(-x^4+x^2+2)^(1/2)+1259/315*x^3*(-x^4+x^2+2)^(1/2)+156
7/315*x*(-x^4+x^2+2)^(1/2)+2843/315*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2
+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-2
216/315*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)
*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^
(1/2)))-5/9*x^7*(-x^4+x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(5\,x^6+2\,x^4-17\,x^2-14\right)\sqrt{-x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7),x, algorithm="fricas")
```

```
[Out] integral(-(5*x^6 + 2*x^4 - 17*x^2 - 14)*sqrt(-x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4 + x^2 + 2\right)^{\frac{3}{2}} \left(5\,x^2 + 7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

3.332 $\int \left(2+x^2-x^4\right)^{3/2} dx$

Optimal. Leaf size=74

$$\frac{1}{7}x\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{35}x\left(3x^2+19\right)\sqrt{-x^4+x^2+2}+\frac{48}{35}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{34}{35}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(19 + 3*x^2)*Sqrt[2 + x^2 - x^4])/35 + (x*(2 + x^2 - x^4)^(3/2
))/7 + (34*EllipticE[ArcSin[x/Sqrt[2]], -2])/35 + (48*EllipticF[A
rcSin[x/Sqrt[2]], -2])/35
```

Rubi [A] time = 0.161289, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$\frac{1}{7}x\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{35}x\left(3x^2+19\right)\sqrt{-x^4+x^2+2}+\frac{48}{35}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{34}{35}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(19 + 3*x^2)*Sqrt[2 + x^2 - x^4])/35 + (x*(2 + x^2 - x^4)^(3/2
))/7 + (34*EllipticE[ArcSin[x/Sqrt[2]], -2])/35 + (48*EllipticF[A
rcSin[x/Sqrt[2]], -2])/35
```

Rubi in Sympy [A] time = 25.7945, size = 70, normalized size = 0.95

$$\frac{x\left(3x^2+19\right)\sqrt{-x^4+x^2+2}}{35}+\frac{x\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{7}+\frac{34E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{35}+\frac{48F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+x**2+2)**(3/2),x)
```

```
[Out] x*(3*x**2 + 19)*sqrt(-x**4 + x**2 + 2)/35 + x*(-x**4 + x**2 + 2)*
*(3/2)/7 + 34*elliptic_e(asin(sqrt(2)*x/2), -2)/35 + 48*elliptic_
f(asin(sqrt(2)*x/2), -2)/35
```

Mathematica [C] time = 0.0888932, size = 102, normalized size = 1.38

$$\frac{5x^9-13x^7-31x^5+45x^3-75i\sqrt{-2x^4+2x^2+4}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+34i\sqrt{-2x^4+2x^2+4}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+58x}{35\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (58*x + 45*x^3 - 31*x^5 - 13*x^7 + 5*x^9 + (34*I)*Sqrt[4 + 2*x^2
- 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (75*I)*Sqrt[4 + 2*x^2 -
2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(35*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.005, size = 159, normalized size = 2.2

$$-\frac{x^5}{7}\sqrt{-x^4+x^2+2}+\frac{8x^3}{35}\sqrt{-x^4+x^2+2}+\frac{29x}{35}\sqrt{-x^4+x^2+2}+\frac{41\sqrt{2}}{35}\sqrt{-2x^2+4}\sqrt{x^2+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}-\frac{17\sqrt{2}}{35}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+x^2+2)^(3/2),x)
```

```
[Out] -1/7*x^5*(-x^4+x^2+2)^(1/2)+8/35*x^3*(-x^4+x^2+2)^(1/2)+29/35*x*(
-x^4+x^2+2)^(1/2)+41/35*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-
x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-17/35*2^(1/2)
*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2
*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-x^4+x^2+2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-x^4+x^2+2\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-x^4 + x^2 + 2)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-x^4+x^2+2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-x**4 + x**2 + 2)**(3/2), x)
```

<u>GIAC/XCAS</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4 + x^2 + 2\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2), x)
```

3.333 $\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx$

Optimal. Leaf size=72

$$\frac{1}{75}x\sqrt{-x^4+x^2+2}\,(13-3x^2)-\frac{178}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{92}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1156\,\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{4375}$$

```
[Out] (x*(13 - 3*x^2)*Sqrt[2 + x^2 - x^4])/75 + (92*EllipticE[ArcSin[x/
Sqrt[2]], -2])/375 - (178*EllipticF[ArcSin[x/Sqrt[2]], -2])/625 +
 (1156*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/4375
```

Rubi [A] time = 0.440763, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{75}x\sqrt{-x^4+x^2+2}\,(13-3x^2)-\frac{178}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{92}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1156\,\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{4375}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (x*(13 - 3*x^2)*Sqrt[2 + x^2 - x^4])/75 + (92*EllipticE[ArcSin[x/
Sqrt[2]], -2])/375 - (178*EllipticF[ArcSin[x/Sqrt[2]], -2])/625 +
 (1156*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/4375
```

Rubi in Sympy [A] time = 82.1792, size = 172, normalized size = 2.39

$$-\frac{x^3\sqrt{-x^4+x^2+2}}{25}+\frac{13x\sqrt{-x^4+x^2+2}}{75}+\frac{92E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{375}-\frac{22F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{125}-\frac{68\sqrt{-x^4+x^2+2}F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{625\sqrt{-2x^2+4}\sqrt{\frac{x^2}{2}+\frac{1}{2}}}+\frac{1156\sqrt{-x^4+x^2+2}\,\Pi\left(-\frac{10}{7};\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{4375\sqrt{-2x^2+4}\sqrt{\frac{x^2}{2}+\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7),x)
```

```
[Out] -x**3*sqrt(-x**4 + x**2 + 2)/25 + 13*x*sqrt(-x**4 + x**2 + 2)/75
+ 92*elliptic_e(asin(sqrt(2)*x/2), -2)/375 - 22*elliptic_f(asin(s
qrt(2)*x/2), -2)/125 - 68*sqrt(-x**4 + x**2 + 2)*elliptic_f(asin(
sqrt(2)*x/2), -2)/(625*sqrt(-2*x**2 + 4)*sqrt(x**2/2 + 1/2)) + 11
56*sqrt(-x**4 + x**2 + 2)*elliptic_pi(-10/7, asin(sqrt(2)*x/2), -
2)/(4375*sqrt(-2*x**2 + 4)*sqrt(x**2/2 + 1/2))
```

Mathematica [C] time = 0.177663, size = 130, normalized size = 1.81

$$\frac{525x^7-2800x^5+1225x^3-2961i\sqrt{-2x^4+2x^2+4}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+3220i\sqrt{-2x^4+2x^2+4}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-1734i\sqrt{}}{13125\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (4550*x + 1225*x^3 - 2800*x^5 + 525*x^7 + (3220*I)*Sqrt[4 + 2*x^2
 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (2961*I)*Sqrt[4 + 2*x^2
 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2] - (1734*I)*Sqrt[4 + 2*x^2
 - 2*x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(13125*Sqrt[2 + x^
2 - x^4])
```

Maple [B] time = 0.021, size = 173, normalized size = 2.4

$$-\frac{x^3}{25}\sqrt{-x^4+x^2+2}+\frac{13\,x}{75}\sqrt{-x^4+x^2+2} -\frac{89\sqrt{2}}{625}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}} +\frac{46\sqrt{2}}{375}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathit{EllipticE}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}} +\frac{1156\sqrt{2}}{4375}\sqrt{1-\frac{x^2}{2}}\sqrt{x^2+1}\mathit{EllipticPi}\left(\frac{\sqrt{2}x}{2},-\frac{10}{7},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+x^2+2)^(3/2)/(5*x^2+7),x)
```

```
[Out] -1/25*x^3*(-x^4+x^2+2)^(1/2)+13/75*x*(-x^4+x^2+2)^(1/2)-89/625*2^
(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF
(1/2*2^(1/2)*x,I*2^(1/2))+46/375*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)
^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))+1156
/4375*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*
EllipticPi(1/2*2^(1/2)*x,-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{5\,x^2+7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7),x, algorithm="fricas")
```

```
[Out] integral((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)/(5*x**2 + 7), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

3.334 $\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2}\,dx$

Optimal. Leaf size=93

$$-\frac{17\sqrt{-x^4+x^2+2}x}{175\left(5x^2+7\right)}-\frac{1}{75}\sqrt{-x^4+x^2+2}x+\frac{458}{875}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{97}{525}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1241\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{6125}$$

```
[Out] -(x*Sqrt[2 + x^2 - x^4])/75 - (17*x*Sqrt[2 + x^2 - x^4])/(175*(7
+ 5*x^2)) - (97*EllipticE[ArcSin[x/Sqrt[2]], -2])/525 + (458*Elli
pticF[ArcSin[x/Sqrt[2]], -2])/875 - (1241*EllipticPi[-10/7, ArcSi
n[x/Sqrt[2]], -2])/6125
```

Rubi [A] time = 0.722154, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{17\sqrt{-x^4+x^2+2}x}{175\left(5x^2+7\right)}-\frac{1}{75}\sqrt{-x^4+x^2+2}x+\frac{458}{875}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{97}{525}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1241\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{6125}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] -(x*Sqrt[2 + x^2 - x^4])/75 - (17*x*Sqrt[2 + x^2 - x^4])/(175*(7
+ 5*x^2)) - (97*EllipticE[ArcSin[x/Sqrt[2]], -2])/525 + (458*Elli
pticF[ArcSin[x/Sqrt[2]], -2])/875 - (1241*EllipticPi[-10/7, ArcSi
n[x/Sqrt[2]], -2])/6125
```

Rubi in Sympy [A] time = 114.061, size = 241, normalized size = 2.59

$$-\frac{x\sqrt{-x^4+x^2+2}}{75}-\frac{578x\sqrt{-x^4+x^2+2}}{25\left(1190x^2+1666\right)}-\frac{97\sqrt{2}\sqrt{-x^4+x^2+2}E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{1050\sqrt{-\frac{x^2}{2}+1}\sqrt{x^2+1}}-\frac{209\sqrt{2}\sqrt{-x^4+x^2+2}F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{8750\sqrt{-\frac{x^2}{2}+1}\sqrt{x^2+1}}-\frac{1207\sqrt{2}\sqrt{-x^4+x^2+2}\left(-\frac{10}{7};\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{61250\sqrt{-\frac{x^2}{2}+1}\sqrt{x^2+1}}+\frac{51\sqrt{-x^4+x^2+2}\left(\frac{2}{7};\operatorname{atan}(x)\middle|\frac{3}{2}\right)}{250\sqrt{\frac{-\frac{x^2}{2}+1}{x^2+1}}\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7)**2,x)
```

```
[Out] -x*sqrt(-x**4 + x**2 + 2)/75 - 578*x*sqrt(-x**4 + x**2 + 2)/(25*(
1190*x**2 + 1666)) - 97*sqrt(2)*sqrt(-x**4 + x**2 + 2)*elliptic_e
(asin(sqrt(2)*x/2), -2)/(1050*sqrt(-x**2/2 + 1)*sqrt(x**2 + 1)) -
 209*sqrt(2)*sqrt(-x**4 + x**2 + 2)*elliptic_f(asin(sqrt(2)*x/2),
 -2)/(8750*sqrt(-x**2/2 + 1)*sqrt(x**2 + 1)) - 1207*sqrt(2)*sqrt(
-x**4 + x**2 + 2)*elliptic_pi(-10/7, asin(sqrt(2)*x/2), -2)/(6125
```

```
0*sqrt(-x**2/2 + 1)*sqrt(x**2 + 1)) + 51*sqrt(-x**4 + x**2 + 2)*e
lliptic_pi(2/7, atan(x), 3/2)/(250*sqrt((-x**2/2 + 1)/(x**2 + 1))
*(x**2 + 1))
```

Mathematica [C] time = 0.292483, size = 201, normalized size = 2.16

$$\frac{2450x^7 + 4550x^5 - 11900x^3 + 567i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right) - 6790i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}E\left(i\,\mathrm{si}\right.}{36750\left(5x^2+7\right)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (-14000*x - 11900*x^3 + 4550*x^5 + 2450*x^7 - (6790*I)*Sqrt[2]*(7
 + 5*x^2)*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] + (56
7*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[
x], -1/2] + (26061*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7,
 I*ArcSinh[x], -1/2] + (18615*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*
EllipticPi[5/7, I*ArcSinh[x], -1/2])/(36750*(7 + 5*x^2)*Sqrt[2 +
x^2 - x^4])
```

Maple [B] time = 0.028, size = 180, normalized size = 1.9

$$\begin{aligned}&-\frac{17\,x}{875\,x^2+1225}\sqrt{-x^4+x^2+2}-\frac{x}{75}\sqrt{-x^4+x^2+2}\\&+\frac{229\sqrt{2}}{875}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}\\&-\frac{97\sqrt{2}}{1050}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathit{EllipticE}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}\\&-\frac{1241\sqrt{2}}{6125}\sqrt{1-\frac{x^2}{2}}\sqrt{x^2+1}\mathit{EllipticPi}\left(\frac{\sqrt{2}x}{2},-\frac{10}{7},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x)
```

```
[Out] -17/175*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-1/75*x*(-x^4+x^2+2)^(1/2)+
229/875*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)
*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-97/1050*2^(1/2)*(-2*x^2+4)^(1
/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^
(1/2))-1241/6125*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^
2+2)^(1/2)*EllipticPi(1/2*2^(1/2)*x,-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^2,x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{25\,x^4+70\,x^2+49},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^2,x, algorithm="fricas")
```

```
[Out] integral((-x^4 + x^2 + 2)^(3/2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)/(5*x**2 + 7)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^2,x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

3.335 $\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3}\,dx$

Optimal. Leaf size=102

$$\frac{563\sqrt{-x^4+x^2+2}x}{9800(5x^2+7)}-\frac{17\sqrt{-x^4+x^2+2}x}{350(5x^2+7)^2}-\frac{1251F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{24500}+\frac{191E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{9800}+\frac{9879\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{343000}$$

```
[Out] (-17*x*Sqrt[2 + x^2 - x^4])/(350*(7 + 5*x^2)^2) + (563*x*Sqrt[2 +
 x^2 - x^4])/(9800*(7 + 5*x^2)) + (191*EllipticE[ArcSin[x/Sqrt[2]
], -2])/9800 - (1251*EllipticF[ArcSin[x/Sqrt[2]], -2])/24500 + (9
879*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/343000
```

Rubi [A] time = 1.10595, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583

$$\frac{563\sqrt{-x^4+x^2+2}x}{9800(5x^2+7)}-\frac{17\sqrt{-x^4+x^2+2}x}{350(5x^2+7)^2}-\frac{1251F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{24500}+\frac{191E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{9800}+\frac{9879\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{343000}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] (-17*x*Sqrt[2 + x^2 - x^4])/(350*(7 + 5*x^2)^2) + (563*x*Sqrt[2 +
 x^2 - x^4])/(9800*(7 + 5*x^2)) + (191*EllipticE[ArcSin[x/Sqrt[2]
], -2])/9800 - (1251*EllipticF[ArcSin[x/Sqrt[2]], -2])/24500 + (9
879*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/343000
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7)**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.354785, size = 244, normalized size = 2.39

$$\frac{-197050x^7-45500x^5+636650x^3-2541i\sqrt{2}\left(5x^2+7\right)^2\sqrt{-x^4+x^2+2}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+13370i\sqrt{2}\left(5x^2+7\right)^2\sqrt{-x^4+x}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] (485100*x + 636650*x^3 - 45500*x^5 - 197050*x^7 + (13370*I)*Sqrt[
2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2
] - (2541*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[
I*ArcSinh[x], -1/2] - (484071*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*Elli
pticPi[5/7, I*ArcSinh[x], -1/2] - (691530*I)*Sqrt[2]*x^2*Sqrt[2 +
 x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (246975*I)*Sqrt
[2]*x^4*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/
(686000*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4])
```

Maple [A] time = 0.03, size = 189, normalized size = 1.9

$$-\frac{17\,x}{350\,\left(5\,x^{2}+7\right)^{2}}\sqrt{-x^{4}+x^{2}+2}+\frac{563\,x}{49000\,x^{2}+68600}\sqrt{-x^{4}+x^{2}+2}-\frac{1251\,\sqrt{2}}{49000}\sqrt{-2\,x^{2}+4}\sqrt{x^{2}+1}EllipticF\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^{4}+x^{2}+2}}+\frac{191\,\sqrt{2}}{19600}\sqrt{-2\,x^{2}+4}\sqrt{x^{2}+1}EllipticE\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^{4}+x^{2}+2}}+\frac{9879\,\sqrt{2}}{343000}\sqrt{1-\frac{x^{2}}{2}}\sqrt{x^{2}+1}EllipticPi\left(\frac{\sqrt{2}x}{2},-\frac{10}{7},i\sqrt{2}\right)\frac{1}{\sqrt{-x^{4}+x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x)
```

```
[Out] -17/350*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2+563/9800*x*(-x^4+x^2+2)^
(1/2)/(5*x^2+7)-1251/49000*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)
/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+191/19600*
2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Ellipti
cE(1/2*2^(1/2)*x,I*2^(1/2))+9879/343000*2^(1/2)*(1-1/2*x^2)^(1/2)
*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*2^(1/2)*x,-10/7,
I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^{4}+x^{2}+2\right)^{\frac{3}{2}}}{\left(5\,x^{2}+7\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^3,x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-x^{4}+x^{2}+2\right)^{\frac{3}{2}}}{125\,x^{6}+525\,x^{4}+735\,x^{2}+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^3,x, algorithm="fricas")
```

```
[Out] integral((-x^4 + x^2 + 2)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 34
3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(x^{2}-2\right)\left(x^{2}+1\right)\right)^{\frac{3}{2}}}{\left(5x^{2}+7\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)/(5*x**2 + 7)**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^{4}+x^{2}+2\right)^{\frac{3}{2}}}{\left(5\,x^{2}+7\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^3,x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

3.336 $\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=65

$$-\frac{625}{3}\sqrt{-x^4+x^2+2}\,x-25\sqrt{-x^4+x^2+2}\,x^3-542F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{3905}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (-625*x*Sqrt[2 + x^2 - x^4])/3 - 25*x^3*Sqrt[2 + x^2 - x^4] + (39
05*EllipticE[ArcSin[x/Sqrt[2]], -2])/3 - 542*EllipticF[ArcSin[x/S
qrt[2]], -2]
```

Rubi [A] time = 0.198078, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{625}{3}\sqrt{-x^4+x^2+2}\,x-25\sqrt{-x^4+x^2+2}\,x^3-542F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{3905}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^3/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-625*x*Sqrt[2 + x^2 - x^4])/3 - 25*x^3*Sqrt[2 + x^2 - x^4] + (39
05*EllipticE[ArcSin[x/Sqrt[2]], -2])/3 - 542*EllipticF[ArcSin[x/S
qrt[2]], -2]
```

Rubi in Sympy [A] time = 39.0364, size = 65, normalized size = 1.

$$-25x^3\sqrt{-x^4+x^2+2}-\frac{625x\sqrt{-x^4+x^2+2}}{3}+\frac{3905E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{3}-542F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**3/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] -25*x**3*sqrt(-x**4 + x**2 + 2) - 625*x*sqrt(-x**4 + x**2 + 2)/3
+ 3905*elliptic_e(asin(sqrt(2)*x/2), -2)/3 - 542*elliptic_f(asin(
sqrt(2)*x/2), -2)
```

Mathematica [C] time = 0.123721, size = 97, normalized size = 1.49

$$\frac{150x^7+1100x^5-1550x^3-10089i\sqrt{-2x^4+2x^2+4}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+7810i\sqrt{-2x^4+2x^2+4}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-2500x}{6\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^3/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-2500*x - 1550*x^3 + 1100*x^5 + 150*x^7 + (7810*I)*Sqrt[4 + 2*x^
2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (10089*I)*Sqrt[4 + 2*x
^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(6*Sqrt[2 + x^2 - x^4]
)
```

Maple [B] time = 0.024, size = 142, normalized size = 2.2

$$\frac{2279\sqrt{2}}{6}\sqrt{-2x^2+4}\sqrt{x^2+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}, i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}} - \frac{3905\sqrt{2}}{6}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}, i\sqrt{2}\right) - \mathit{EllipticE}\left(\frac{\sqrt{2}x}{2}, i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}} - \frac{625x}{3}\sqrt{-x^4+x^2+2} - 25x^3\sqrt{-x^4+x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] 2279/6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*
EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-3905/6*2^(1/2)*(-2*x^2+4)^(1/2
)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(
1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))-625/3*x*(-x^4+x^2+2)^(1
/2)-25*x^3*(-x^4+x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^3}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^3/sqrt(-x^4 + x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(-x^4 + x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{125x^6+525x^4+735x^2+343}{\sqrt{-x^4+x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^3/sqrt(-x^4 + x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)/sqrt(-x^4 + x^2 + 2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^3}{\sqrt{-(x^2-2)(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**3/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^3}{\sqrt{-x^4 + x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^3/sqrt(-x^4 + x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(-x^4 + x^2 + 2), x)
```

3.337 $\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=46

$$-\frac{25}{3}\sqrt{-x^4+x^2+2}x-21F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{260}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/3 + (260*EllipticE[ArcSin[x/Sqrt[2]],
 -2])/3 - 21*EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rubi [A] time = 0.164553, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{25}{3}\sqrt{-x^4+x^2+2}x-21F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{260}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^2/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/3 + (260*EllipticE[ArcSin[x/Sqrt[2]],
 -2])/3 - 21*EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rubi in Sympy [A] time = 29.0104, size = 48, normalized size = 1.04

$$-\frac{25x\sqrt{-x^4+x^2+2}}{3}+\frac{260E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{3}-21F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**2/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] -25*x*sqrt(-x**4 + x**2 + 2)/3 + 260*elliptic_e(asin(sqrt(2)*x/2)
, -2)/3 - 21*elliptic_f(asin(sqrt(2)*x/2), -2)
```

Mathematica [C] time = 0.108627, size = 92, normalized size = 2.

$$\frac{50x^5-50x^3-717i\sqrt{-2x^4+2x^2+4}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+520i\sqrt{-2x^4+2x^2+4}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-100x}{6\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^2/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-100*x - 50*x^3 + 50*x^5 + (520*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Ellip
ticE[I*ArcSinh[x], -1/2] - (717*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Ellipt
icF[I*ArcSinh[x], -1/2])/(6*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.01, size = 125, normalized size = 2.7

$$\frac{197\sqrt{2}}{6}\sqrt{-2x^2+4}\sqrt{x^2+1}\,\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}, i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}} - \frac{130\sqrt{2}}{3}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}, i\sqrt{2}\right) - \mathit{EllipticE}\left(\frac{\sqrt{2}x}{2}, i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}} - \frac{25x}{3}\sqrt{-x^4+x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] 197/6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*E
llipticF(1/2*2^(1/2)*x,I*2^(1/2))-130/3*2^(1/2)*(-2*x^2+4)^(1/2)*
(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/
2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))-25/3*x*(-x^4+x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^2}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^2/sqrt(-x^4 + x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(-x^4 + x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{25x^4+70x^2+49}{\sqrt{-x^4+x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^2/sqrt(-x^4 + x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)/sqrt(-x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^2}{\sqrt{-(x^2-2)(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**2/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^2}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^2/sqrt(-x^4 + x^2 + 2),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)^2/sqrt(-x^4 + x^2 + 2), x)
```

3.338 $\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=25

$$2F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+5E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] 5*EllipticE[ArcSin[x/Sqrt[2]], -2] + 2*EllipticF[ArcSin[x/Sqrt[2]
], -2]
```

Rubi [A] time = 0.127942, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$2F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+5E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] 5*EllipticE[ArcSin[x/Sqrt[2]], -2] + 2*EllipticF[ArcSin[x/Sqrt[2]
], -2]
```

Rubi in Sympy [A] time = 19.73, size = 29, normalized size = 1.16

$$5E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)+2F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] 5*elliptic_e(asin(sqrt(2)*x/2), -2) + 2*elliptic_f(asin(sqrt(2)*x
/2), -2)
```

Mathematica [C] time = 0.0584724, size = 34, normalized size = 1.36

$$\frac{i\left(10E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-17F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (I*(10*EllipticE[I*ArcSinh[x], -1/2] - 17*EllipticF[I*ArcSinh[x],
 -1/2]))/Sqrt[2]
```

Maple [B] time = 0.007, size = 110, normalized size = 4.4

$$\frac{7\sqrt{2}}{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,EllipticF\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}-\frac{5\sqrt{2}}{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(EllipticF\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)-EllipticE\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((5*x^2+7)/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] 7/2*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Ell
ipticF(1/2*2^(1/2)*x,I*2^(1/2))-5/2*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2
+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-
EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^2+7}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((5*x^2 + 7)/sqrt(-x^4 + x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(-x^4 + x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{5\,x^2+7}{\sqrt{-x^4+x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((5*x^2 + 7)/sqrt(-x^4 + x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral((5*x^2 + 7)/sqrt(-x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2+7}{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((5*x**2+7)/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^2+7}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((5*x^2 + 7)/sqrt(-x^4 + x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(-x^4 + x^2 + 2), x)
```

3.339 $\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=10

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rubi [A] time = 0.0313686, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + x^2 - x^4],x]

[Out] EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rubi in Sympy [A] time = 6.49142, size = 12, normalized size = 1.2

$$F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**4+x**2+2)**(1/2),x)

[Out] elliptic_f(asin(sqrt(2)*x/2), -2)
```

Mathematica [C] time = 0.0275006, size = 19, normalized size = 1.9

$$-\frac{iF\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + x^2 - x^4],x]

[Out] ((-I)*EllipticF[I*ArcSinh[x], -1/2])/Sqrt[2]
```

Maple [B] time = 0.005, size = 47, normalized size = 4.7

$$\frac{\sqrt{2}}{2}\sqrt{-2\,x^2+4}\sqrt{x^2+1}EllipticF\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Ell
ipticF(1/2*2^(1/2)*x,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^4 + x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 + x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-x^4+x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^4 + x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**4 + x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^4 + x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + x^2 + 2), x)
```

3.340 $\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=17

$$\frac{1}{7}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2]/7
```

Rubi [A] time = 0.109498, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{1}{7}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((7 + 5*x^2)*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2]/7
```

Rubi in Sympy [A] time = 17.7466, size = 17, normalized size = 1.

$$\frac{\Pi\left(-\frac{10}{7};\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**2+7)/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] elliptic_pi(-10/7, asin(sqrt(2)*x/2), -2)/7
```

Mathematica [C] time = 0.0443375, size = 24, normalized size = 1.41

$$-\frac{i\,\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{7\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((7 + 5*x^2)*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] ((-I/7)*EllipticPi[5/7, I*ArcSinh[x], -1/2])/Sqrt[2]
```

Maple [B] time = 0.018, size = 48, normalized size = 2.8

$$\frac{\sqrt{2}}{7}\sqrt{1-\frac{x^2}{2}}\sqrt{x^2+1}\,EllipticPi\left(\frac{\sqrt{2}x}{2},-\frac{10}{7},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] 1/7*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*El
lipticPi(1/2*2^(1/2)*x,-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x^2-2)(x^2+1)}\,(5x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x**2+7)/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)), x)
```

3.341 $\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=74

$$-\frac{25\sqrt{-x^4+x^2+2}\,x}{476\,(5x^2+7)}-\frac{1}{238}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{5}{476}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{167\,\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3332}$$

```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/(476*(7 + 5*x^2)) - (5*EllipticE[ArcS
in[x/Sqrt[2]], -2])/476 - EllipticF[ArcSin[x/Sqrt[2]], -2]/238 +
(167*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/3332
```

Rubi [A] time = 0.262785, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$-\frac{25\sqrt{-x^4+x^2+2}\,x}{476\,(5x^2+7)}-\frac{1}{238}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{5}{476}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{167\,\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3332}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/(476*(7 + 5*x^2)) - (5*EllipticE[ArcS
in[x/Sqrt[2]], -2])/476 - EllipticF[ArcSin[x/Sqrt[2]], -2]/238 +
(167*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/3332
```

Rubi in Sympy [A] time = 38.5835, size = 73, normalized size = 0.99

$$-\frac{25x\sqrt{-x^4+x^2+2}}{2380x^2+3332}-\frac{5E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{476}-\frac{F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{238}+\frac{167\,\Pi\left(-\frac{10}{7};\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{3332}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**2+7)**2/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] -25*x*sqrt(-x**4 + x**2 + 2)/(2380*x**2 + 3332) - 5*elliptic_e(as
in(sqrt(2)*x/2), -2)/476 - elliptic_f(asin(sqrt(2)*x/2), -2)/238
+ 167*elliptic_pi(-10/7, asin(sqrt(2)*x/2), -2)/3332
```

Mathematica [C] time = 0.316699, size = 196, normalized size = 2.65

$$\frac{350x^5-350x^3+119i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-70i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-}{6664\left(5x^2+7\right)\sqrt{-x^4+x^2+}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] (-700*x - 350*x^3 + 350*x^5 - (70*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 +
 x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] + (119*I)*Sqrt[2]*(7 +
5*x^2)*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] - (1169*
I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2
] - (835*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*Arc
```

```
Sinh[x], -1/2])/(6664*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.026, size = 165, normalized size = 2.2

$$-\frac{25\,x}{2380\,x^2+3332}\sqrt{-x^4+x^2+2}-\frac{\sqrt{2}}{476}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}-\frac{5\,\sqrt{2}}{952}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathit{EllipticE}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}+\frac{167\,\sqrt{2}}{3332}\sqrt{1-\frac{x^2}{2}}\sqrt{x^2+1}\mathit{EllipticPi}\left(\frac{\sqrt{2}x}{2},-\frac{10}{7},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] -25/476*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-1/476*2^(1/2)*(-2*x^2+4)^(
1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2
^(1/2))-5/952*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)
^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))+167/3332*2^(1/2)*(1-1/2
*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*2^(1/
2)*x,-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{(25\,x^4+70\,x^2+49)\sqrt{-x^4+x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2),x, algorithm="fricas")
```

```
[Out] integral(1/((25*x^4 + 70*x^2 + 49)*sqrt(-x^4 + x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x^2-2)(x^2+1)}\,(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x**2+7)**2/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4 + x^2 + 2}(5\,x^2 + 7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2), x)
```

3.342 $\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=102

$$-\frac{12525\sqrt{-x^4+x^2+2}\,x}{453152\,(5x^2+7)}-\frac{25\sqrt{-x^4+x^2+2}\,x}{952\,(5x^2+7)^2}-\frac{263F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{226576}-\frac{2505E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{453152}+\frac{58915\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3172064}$$

```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/(952*(7 + 5*x^2)^2) - (12525*x*Sqrt[2
 + x^2 - x^4])/(453152*(7 + 5*x^2)) - (2505*EllipticE[ArcSin[x/Sq
rt[2]], -2])/453152 - (263*EllipticF[ArcSin[x/Sqrt[2]], -2])/2265
76 + (58915*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/3172064
```

Rubi [A] time = 0.436637, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{12525\sqrt{-x^4+x^2+2}\,x}{453152\,(5x^2+7)}-\frac{25\sqrt{-x^4+x^2+2}\,x}{952\,(5x^2+7)^2}-\frac{263F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{226576}-\frac{2505E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{453152}+\frac{58915\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3172064}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/(952*(7 + 5*x^2)^2) - (12525*x*Sqrt[2
 + x^2 - x^4])/(453152*(7 + 5*x^2)) - (2505*EllipticE[ArcSin[x/Sq
rt[2]], -2])/453152 - (263*EllipticF[ArcSin[x/Sqrt[2]], -2])/2265
76 + (58915*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/3172064
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**2+7)**3/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.425966, size = 108, normalized size = 1.06

$$\frac{\frac{350x(2505x^6+1478x^4-8993x^2-7966)}{(5x^2+7)^2\sqrt{-x^4+x^2+2}}+56287i\sqrt{2}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-35070i\sqrt{2}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-58915i\sqrt{2}\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{6344128}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] ((350*x*(-7966 - 8993*x^2 + 1478*x^4 + 2505*x^6))/((7 + 5*x^2)^2*
Sqrt[2 + x^2 - x^4]) - (35070*I)*Sqrt[2]*EllipticE[I*ArcSinh[x],
```

```
-1/2] + (56287*I)*Sqrt[2]*EllipticF[I*ArcSinh[x], -1/2] - (58915*
I)*Sqrt[2]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/6344128
```

Maple [A] time = 0.028, size = 189, normalized size = 1.9

$$-\frac{25\,x}{952\,\left(5\,x^{2}+7\right)^{2}}\sqrt{-x^{4}+x^{2}+2}-\frac{12525\,x}{2265760\,x^{2}+3172064}\sqrt{-x^{4}+x^{2}+2}-\frac{263\,\sqrt{2}}{453152}\sqrt{-2\,x^{2}+4}\sqrt{x^{2}+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^{4}+x^{2}+2}}-\frac{2505\,\sqrt{2}}{906304}\sqrt{-2\,x^{2}+4}\sqrt{x^{2}+1}\mathit{EllipticE}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^{4}+x^{2}+2}}+\frac{58915\,\sqrt{2}}{3172064}\sqrt{1-\frac{x^{2}}{2}}\sqrt{x^{2}+1}\mathit{EllipticPi}\left(\frac{\sqrt{2}x}{2},-\frac{10}{7},i\sqrt{2}\right)\frac{1}{\sqrt{-x^{4}+x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] -25/952*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-12525/453152*x*(-x^4+x^2
+2)^(1/2)/(5*x^2+7)-263/453152*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(
1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-2505/9
06304*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*E
llipticE(1/2*2^(1/2)*x,I*2^(1/2))+58915/3172064*2^(1/2)*(1-1/2*x^
2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*2^(1/2)*
x,-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^{4}+x^{2}+2}(5\,x^{2}+7)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{(125\,x^{6}+525\,x^{4}+735\,x^{2}+343)\sqrt{-x^{4}+x^{2}+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3),x, algorithm="fricas")
```

```
[Out] integral(1/((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(-x^4 + x^2 +
 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(x^{2}-2\right)\left(x^{2}+1\right)}\left(5x^{2}+7\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x**2+7)**3/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+x^2+2}\,(5\,x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3), x)
```

3.343 $\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{27500}{3}\sqrt{-x^4+x^2+2}x+\frac{(1419793x^2+1419985)\,x}{18\sqrt{-x^4+x^2+2}}+625\sqrt{-x^4+x^2+2}x^3+\frac{627857}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{3482293}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(1419985 + 1419793*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (27500*x*S
qrt[2 + x^2 - x^4])/3 + 625*x^3*Sqrt[2 + x^2 - x^4] - (3482293*El
lipticE[ArcSin[x/Sqrt[2]], -2])/18 + (627857*EllipticF[ArcSin[x/S
qrt[2]], -2])/6
```

Rubi [A] time = 0.232379, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{27500}{3}\sqrt{-x^4+x^2+2}x+\frac{(1419793x^2+1419985)\,x}{18\sqrt{-x^4+x^2+2}}+625\sqrt{-x^4+x^2+2}x^3+\frac{627857}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{3482293}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^5/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(1419985 + 1419793*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (27500*x*S
qrt[2 + x^2 - x^4])/3 + 625*x^3*Sqrt[2 + x^2 - x^4] - (3482293*El
lipticE[ArcSin[x/Sqrt[2]], -2])/18 + (627857*EllipticF[ArcSin[x/S
qrt[2]], -2])/6
```

Rubi in Sympy [A] time = 53.0862, size = 88, normalized size = 0.95

$$625x^3\sqrt{-x^4+x^2+2}+\frac{x\,(1419793x^2+1419985)}{18\sqrt{-x^4+x^2+2}}+\frac{27500x\sqrt{-x^4+x^2+2}}{3}-\frac{3482293E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{18}+\frac{627857F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**5/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] 625*x**3*sqrt(-x**4 + x**2 + 2) + x*(1419793*x**2 + 1419985)/(18*
sqrt(-x**4 + x**2 + 2)) + 27500*x*sqrt(-x**4 + x**2 + 2)/3 - 3482
293*elliptic_e(asin(sqrt(2)*x/2), -2)/18 + 627857*elliptic_f(asin
(sqrt(2)*x/2), -2)/6
```

Mathematica [C] time = 0.126462, size = 97, normalized size = 1.04

$$\frac{-11250x^7-153750x^5+1607293x^3+4281654i\sqrt{-2x^4+2x^2+4}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-3482293i\sqrt{-2x^4+2x^2+4}E\left(i\sinh^{-1}(\right.}{18\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^5/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (1749985*x + 1607293*x^3 - 153750*x^5 - 11250*x^7 - (3482293*I)*S
qrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] + (4281654*I
)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(18*Sqrt
[2 + x^2 - x^4])
```

Maple [B] time = 0.048, size = 280, normalized size = 3.

$$
\begin{aligned}
&33614\,\frac{1}{\sqrt{-x^4+x^2+2}}\left(\frac{5\,x}{36}-1/36\,x^3\right)\\
&-\frac{799361\,\sqrt{2}}{18}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}\\
&+\frac{3482293\,\sqrt{2}}{36}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\left(\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}}\\
&+120050\,\frac{1/9\,x^3-x/18}{\sqrt{-x^4+x^2+2}}+171500\,\frac{1/18\,x^3+2/9\,x}{\sqrt{-x^4+x^2+2}}+122500\,\frac{1}{\sqrt{-x^4+x^2+2}}\left(\frac{5\,x^3}{18}+x/9\right)\\
&+43750\,\frac{1}{\sqrt{-x^4+x^2+2}}\left(\frac{7\,x^3}{18}+5/9\,x\right)+\frac{27500\,x}{3}\sqrt{-x^4+x^2+2}\\
&+6250\,\frac{1}{\sqrt{-x^4+x^2+2}}\left(\frac{17\,x^3}{18}+\frac{7\,x}{9}\right)+625\,x^3\sqrt{-x^4+x^2+2}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 33614*(5/36*x-1/36*x^3)/(-x^4+x^2+2)^(1/2)-799361/18*2^(1/2)*(-2*
x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/
2)*x,I*2^(1/2))+3482293/36*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)
/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE
(1/2*2^(1/2)*x,I*2^(1/2)))+120050*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(
1/2)+171500*(1/18*x^3+2/9*x)/(-x^4+x^2+2)^(1/2)+122500*(5/18*x^3+
1/9*x)/(-x^4+x^2+2)^(1/2)+43750*(7/18*x^3+5/9*x)/(-x^4+x^2+2)^(1/
2)+27500/3*x*(-x^4+x^2+2)^(1/2)+6250*(17/18*x^3+7/9*x)/(-x^4+x^2+
2)^(1/2)+625*x^3*(-x^4+x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^5}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^5/(-x^4 + x^2 + 2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^5/(-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{3125\,x^{10}+21875\,x^8+61250\,x^6+85750\,x^4+60025\,x^2+16807}{\left(x^4-x^2-2\right)\sqrt{-x^4+x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^5/(-x^4 + x^2 + 2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(3125*x^10 + 21875*x^8 + 61250*x^6 + 85750*x^4 + 60025*
x^2 + 16807)/((x^4 - x^2 - 2)*sqrt(-x^4 + x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+7\right)^5}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**5/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**5/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^5}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^5/(-x^4 + x^2 + 2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^5/(-x^4 + x^2 + 2)^(3/2), x)
```

3.344 $\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{625}{3}\sqrt{-x^4+x^2+2}x+\frac{(83489x^2+83585)\,x}{18\sqrt{-x^4+x^2+2}}+\frac{31921}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{165239}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(83585 + 83489*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2
+ x^2 - x^4])/3 - (165239*EllipticE[ArcSin[x/Sqrt[2]], -2])/18 +
(31921*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rubi [A] time = 0.20344, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{625}{3}\sqrt{-x^4+x^2+2}x+\frac{(83489x^2+83585)\,x}{18\sqrt{-x^4+x^2+2}}+\frac{31921}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{165239}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^4/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(83585 + 83489*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2
+ x^2 - x^4])/3 - (165239*EllipticE[ArcSin[x/Sqrt[2]], -2])/18 +
(31921*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rubi in Sympy [A] time = 39.6363, size = 71, normalized size = 0.96

$$\frac{x\,(83489x^2+83585)}{18\sqrt{-x^4+x^2+2}}+\frac{625x\sqrt{-x^4+x^2+2}}{3}-\frac{165239E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{18}+\frac{31921F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**4/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] x*(83489*x**2 + 83585)/(18*sqrt(-x**4 + x**2 + 2)) + 625*x*sqrt(-
x**4 + x**2 + 2)/3 - 165239*elliptic_e(asin(sqrt(2)*x/2), -2)/18
+ 31921*elliptic_f(asin(sqrt(2)*x/2), -2)/6
```

Mathematica [C] time = 0.116434, size = 92, normalized size = 1.24

$$\frac{-3750x^5+87239x^3+199977i\sqrt{-2x^4+2x^2+4}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-165239i\sqrt{-2x^4+2x^2+4}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+91085x}{18\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^4/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (91085*x + 87239*x^3 - 3750*x^5 - (165239*I)*Sqrt[4 + 2*x^2 - 2*x
^4]*EllipticE[I*ArcSinh[x], -1/2] + (199977*I)*Sqrt[4 + 2*x^2 - 2
*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(18*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.012, size = 240, normalized size = 3.2

$$4802\,\frac{1}{\sqrt{-x^4+x^2+2}}\left(\frac{5\,x}{36}-1/36\,x^3\right)-\frac{17369\,\sqrt{2}}{9}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\,EllipticF\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}+\frac{165239\,\sqrt{2}}{36}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\left(EllipticF\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)-EllipticE\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}}+13720\,\frac{1/9\,x^3-x/18}{\sqrt{-x^4+x^2+2}}+14700\,\frac{1/18\,x^3+2/9\,x}{\sqrt{-x^4+x^2+2}}+7000\,\frac{1}{\sqrt{-x^4+x^2+2}}\left(\frac{5\,x^3}{18}+x/9\right)+1250\,\frac{1}{\sqrt{-x^4+x^2+2}}\left(\frac{7\,x^3}{18}+5/9\,x\right)+\frac{625\,x}{3}\sqrt{-x^4+x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 4802*(5/36*x-1/36*x^3)/(-x^4+x^2+2)^(1/2)-17369/9*2^(1/2)*(-2*x^2
+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*
x,I*2^(1/2))+165239/36*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x
^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2
*2^(1/2)*x,I*2^(1/2)))+13720*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+
14700*(1/18*x^3+2/9*x)/(-x^4+x^2+2)^(1/2)+7000*(5/18*x^3+1/9*x)/(
-x^4+x^2+2)^(1/2)+1250*(7/18*x^3+5/9*x)/(-x^4+x^2+2)^(1/2)+625/3*
x*(-x^4+x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^4}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^4/(-x^4 + x^2 + 2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^4/(-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{625\,x^8+3500\,x^6+7350\,x^4+6860\,x^2+2401}{(x^4-x^2-2)\sqrt{-x^4+x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^4/(-x^4 + x^2 + 2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)/((x^4
 - x^2 - 2)*sqrt(-x^4 + x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+7\right)^4}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**4/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**4/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^4}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^4/(-x^4 + x^2 + 2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^4/(-x^4 + x^2 + 2)^(3/2), x)
```

3.345 $\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\, dx$

Optimal. Leaf size=55

$$\frac{x\left(4897x^2+4945\right)}{18\sqrt{-x^4+x^2+2}}+\frac{1763}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{7147}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(4945 + 4897*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (7147*EllipticE[
ArcSin[x/Sqrt[2]], -2])/18 + (1763*EllipticF[ArcSin[x/Sqrt[2]], -
2])/6
```

Rubi [A] time = 0.168468, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{x\left(4897x^2+4945\right)}{18\sqrt{-x^4+x^2+2}}+\frac{1763}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{7147}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^3/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(4945 + 4897*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (7147*EllipticE[
ArcSin[x/Sqrt[2]], -2])/18 + (1763*EllipticF[ArcSin[x/Sqrt[2]], -
2])/6
```

Rubi in Sympy [A] time = 29.5979, size = 54, normalized size = 0.98

$$\frac{x\left(4897x^2+4945\right)}{18\sqrt{-x^4+x^2+2}}-\frac{7147E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{18}+\frac{1763F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**3/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] x*(4897*x**2 + 4945)/(18*sqrt(-x**4 + x**2 + 2)) - 7147*elliptic_
e(asin(sqrt(2)*x/2), -2)/18 + 1763*elliptic_f(asin(sqrt(2)*x/2),
-2)/6
```

Mathematica [C] time = 0.125959, size = 79, normalized size = 1.44

$$\frac{1}{18}\left(\frac{4945x}{\sqrt{-x^4+x^2+2}}+\frac{4897x^3}{\sqrt{-x^4+x^2+2}}+8076i\sqrt{2}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-7147i\sqrt{2}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^3/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] ((4945*x)/Sqrt[2 + x^2 - x^4] + (4897*x^3)/Sqrt[2 + x^2 - x^4] -
(7147*I)*Sqrt[2]*EllipticE[I*ArcSinh[x], -1/2] + (8076*I)*Sqrt[2]
*EllipticF[I*ArcSinh[x], -1/2])/18
```

Maple [B] time = 0.01, size = 202, normalized size = 3.7

$$
\begin{aligned}
&686\frac{1}{\sqrt{-x^4+x^2+2}}\left(\frac{5x}{36}-1/36\,x^3\right)\\
&-\frac{929\sqrt{2}}{18}\sqrt{-2x^2+4}\sqrt{x^2+1}\,\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}\\
&+\frac{7147\sqrt{2}}{36}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}}\\
&+1470\frac{1/9\,x^3-x/18}{\sqrt{-x^4+x^2+2}}+1050\frac{1/18\,x^3+2/9\,x}{\sqrt{-x^4+x^2+2}}+250\frac{1}{\sqrt{-x^4+x^2+2}}\left(\frac{5x^3}{18}+x/9\right)
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 686*(5/36*x-1/36*x^3)/(-x^4+x^2+2)^(1/2)-929/18*2^(1/2)*(-2*x^2+4
)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,
I*2^(1/2))+7147/36*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x
^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(
1/2)*x,I*2^(1/2)))+1470*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+1050*
(1/18*x^3+2/9*x)/(-x^4+x^2+2)^(1/2)+250*(5/18*x^3+1/9*x)/(-x^4+x^
2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^3}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^3/(-x^4 + x^2 + 2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/(-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{125x^6+525x^4+735x^2+343}{(x^4-x^2-2)\sqrt{-x^4+x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^3/(-x^4 + x^2 + 2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(125*x^6 + 525*x^4 + 735*x^2 + 343)/((x^4 - x^2 - 2)*sq
rt(-x^4 + x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^3}{(-(x^2-2)(x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**3/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^3}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^3/(-x^4 + x^2 + 2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/(-x^4 + x^2 + 2)^(3/2), x)
```

3.346 $\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}+\frac{139}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{281}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(305 + 281*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (281*EllipticE[Arc
Sin[x/Sqrt[2]], -2])/18 + (139*EllipticF[ArcSin[x/Sqrt[2]], -2])/
6
```

Rubi [A] time = 0.169027, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}+\frac{139}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{281}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(305 + 281*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (281*EllipticE[Arc
Sin[x/Sqrt[2]], -2])/18 + (139*EllipticF[ArcSin[x/Sqrt[2]], -2])/
6
```

Rubi in Sympy [A] time = 29.5493, size = 54, normalized size = 0.98

$$\frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}-\frac{281E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{18}+\frac{139F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((5*x**2+7)**2/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] x*(281*x**2 + 305)/(18*sqrt(-x**4 + x**2 + 2)) - 281*elliptic_e(a
sin(sqrt(2)*x/2), -2)/18 + 139*elliptic_f(asin(sqrt(2)*x/2), -2)/
6
```

Mathematica [C] time = 0.124175, size = 79, normalized size = 1.44

$$\frac{1}{18}\left(\frac{305x}{\sqrt{-x^4+x^2+2}}+\frac{281x^3}{\sqrt{-x^4+x^2+2}}+213i\sqrt{2}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-281i\sqrt{2}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] ((305*x)/Sqrt[2 + x^2 - x^4] + (281*x^3)/Sqrt[2 + x^2 - x^4] - (2
81*I)*Sqrt[2]*EllipticE[I*ArcSinh[x], -1/2] + (213*I)*Sqrt[2]*Ell
ipticF[I*ArcSinh[x], -1/2])/18
```

Maple [B] time = 0.01, size = 179, normalized size = 3.3

$$98\frac{1}{\sqrt{-x^4+x^2+2}}\left(\frac{5x}{36}-1/36x^3\right)+\frac{34\sqrt{2}}{9}\sqrt{-2x^2+4}\sqrt{x^2+1}EllipticF\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}+\frac{281\sqrt{2}}{36}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(EllipticF\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)-EllipticE\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}}+140\frac{1/9x^3-x/18}{\sqrt{-x^4+x^2+2}}+50\frac{1/18x^3+2/9x}{\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 98*(5/36*x-1/36*x^3)/(-x^4+x^2+2)^(1/2)+34/9*2^(1/2)*(-2*x^2+4)^(
1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2
^(1/2))+281/36*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2
)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)
*x,I*2^(1/2)))+140*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+50*(1/18*x
^3+2/9*x)/(-x^4+x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^2}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^2/(-x^4 + x^2 + 2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/(-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{25x^4+70x^2+49}{(x^4-x^2-2)\sqrt{-x^4+x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^2/(-x^4 + x^2 + 2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(25*x^4 + 70*x^2 + 49)/((x^4 - x^2 - 2)*sqrt(-x^4 + x^2
 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^2}{(-(x^2-2)(x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**2/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^2}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^2/(-x^4 + x^2 + 2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/(-x^4 + x^2 + 2)^(3/2), x)
```

3.347 $\int \frac{7+5x^2}{(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{x\left(13x^2+25\right)}{18\sqrt{-x^4+x^2+2}}+\frac{17}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{13}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(25 + 13*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (13*EllipticE[ArcSin
[x/Sqrt[2]], -2])/18 + (17*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rubi [A] time = 0.154854, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{x\left(13x^2+25\right)}{18\sqrt{-x^4+x^2+2}}+\frac{17}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{13}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(25 + 13*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (13*EllipticE[ArcSin
[x/Sqrt[2]], -2])/18 + (17*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rubi in Sympy [A] time = 24.0928, size = 54, normalized size = 0.98

$$\frac{x\left(13x^2+25\right)}{18\sqrt{-x^4+x^2+2}}-\frac{13E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{18}+\frac{17F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] x*(13*x**2 + 25)/(18*sqrt(-x**4 + x**2 + 2)) - 13*elliptic_e(asin
(sqrt(2)*x/2), -2)/18 + 17*elliptic_f(asin(sqrt(2)*x/2), -2)/6
```

Mathematica [C] time = 0.115975, size = 79, normalized size = 1.44

$$\frac{1}{18}\left(\frac{25x}{\sqrt{-x^4+x^2+2}}+\frac{13x^3}{\sqrt{-x^4+x^2+2}}-6i\sqrt{2}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-13i\sqrt{2}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] ((25*x)/Sqrt[2 + x^2 - x^4] + (13*x^3)/Sqrt[2 + x^2 - x^4] - (13*
I)*Sqrt[2]*EllipticE[I*ArcSinh[x], -1/2] - (6*I)*Sqrt[2]*Elliptic
F[I*ArcSinh[x], -1/2])/18
```

Maple [B] time = 0.008, size = 156, normalized size = 2.8

$$14\frac{1}{\sqrt{-x^4+x^2+2}}\left(\frac{5x}{36}-1/36\,x^3\right)+\frac{19\sqrt{2}}{18}\sqrt{-2\,x^2+4}\sqrt{x^2+1}EllipticF\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}+\frac{13\sqrt{2}}{36}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\left(EllipticF\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)-EllipticE\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}}+10\,\frac{1/9\,x^3-x/18}{\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 14*(5/36*x-1/36*x^3)/(-x^4+x^2+2)^(1/2)+19/18*2^(1/2)*(-2*x^2+4)^
(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*
2^(1/2))+13/36*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2
)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)
*x,I*2^(1/2)))+10*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^2+7}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)/(-x^4 + x^2 + 2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/(-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{5\,x^2+7}{(x^4-x^2-2)\sqrt{-x^4+x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)/(-x^4 + x^2 + 2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(5*x^2 + 7)/((x^4 - x^2 - 2)*sqrt(-x^4 + x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2+7}{(-(x^2-2)(x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2+7}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)/(-x^4 + x^2 + 2)^(3/2),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)/(-x^4 + x^2 + 2)^(3/2), x)
```

3.348 $\int \frac{1}{(2+x^2-x^4)^{3/2}} \, dx$

Optimal. Leaf size=55

$$\frac{x(5-x^2)}{18\sqrt{-x^4+x^2+2}}+\frac{1}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(5 - x^2))/(18*Sqrt[2 + x^2 - x^4]) + EllipticE[ArcSin[x/Sqrt[
2]], -2]/18 + EllipticF[ArcSin[x/Sqrt[2]], -2]/6
```

Rubi [A] time = 0.137175, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\frac{x(5-x^2)}{18\sqrt{-x^4+x^2+2}}+\frac{1}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x^2 - x^4)^(-3/2),x]
```

```
[Out] (x*(5 - x^2))/(18*Sqrt[2 + x^2 - x^4]) + EllipticE[ArcSin[x/Sqrt[
2]], -2]/18 + EllipticF[ArcSin[x/Sqrt[2]], -2]/6
```

Rubi in Sympy [A] time = 21.0142, size = 49, normalized size = 0.89

$$\frac{x(-x^2+5)}{18\sqrt{-x^4+x^2+2}}+\frac{E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{18}+\frac{F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] x*(-x**2 + 5)/(18*sqrt(-x**4 + x**2 + 2)) + elliptic_e(asin(sqrt(
2)*x/2), -2)/18 + elliptic_f(asin(sqrt(2)*x/2), -2)/6
```

Mathematica [C] time = 0.10784, size = 79, normalized size = 1.44

$$\frac{1}{18}\left(\frac{5x}{\sqrt{-x^4+x^2+2}}-\frac{x^3}{\sqrt{-x^4+x^2+2}}-3i\sqrt{2}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+i\sqrt{2}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x^2 - x^4)^(-3/2),x]
```

```
[Out] ((5*x)/Sqrt[2 + x^2 - x^4] - x^3/Sqrt[2 + x^2 - x^4] + I*Sqrt[2]*
EllipticE[I*ArcSinh[x], -1/2] - (3*I)*Sqrt[2]*EllipticF[I*ArcSinh
[x], -1/2])/18
```

Maple [B] time = 0.006, size = 133, normalized size = 2.4

$$2\frac{1}{\sqrt{-x^4+x^2+2}}\left(\frac{5x}{36}-1/36x^3\right)+\frac{\sqrt{2}}{9}\sqrt{-2x^2+4}\sqrt{x^2+1}EllipticF\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}}{36}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(EllipticF\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)-EllipticE\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 2*(5/36*x-1/36*x^3)/(-x^4+x^2+2)^(1/2)+1/9*2^(1/2)*(-2*x^2+4)^(1/
2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(
1/2))-1/36*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1
/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I
*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(-3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{(x^4-x^2-2)\sqrt{-x^4+x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(-3/2),x, algorithm="fricas")
```

```
[Out] integral(-1/((x^4 - x^2 - 2)*sqrt(-x^4 + x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-x**4 + x**2 + 2)**(-3/2), x)
```

<u>GIAC/XCAS</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + x^2 + 2)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(-3/2), x)
```

3.349 $\int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{x(35-16x^2)}{306\sqrt{-x^4+x^2+2}}+\frac{1}{102}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{8}{153}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{25}{238}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(35 - 16*x^2))/(306*Sqrt[2 + x^2 - x^4]) + (8*EllipticE[ArcSin
[x/Sqrt[2]], -2])/153 + EllipticF[ArcSin[x/Sqrt[2]], -2]/102 - (2
5*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/238
```

Rubi [A] time = 0.296828, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{x(35-16x^2)}{306\sqrt{-x^4+x^2+2}}+\frac{1}{102}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{8}{153}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{25}{238}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (x*(35 - 16*x^2))/(306*Sqrt[2 + x^2 - x^4]) + (8*EllipticE[ArcSin
[x/Sqrt[2]], -2])/153 + EllipticF[ArcSin[x/Sqrt[2]], -2]/102 - (2
5*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/238
```

Rubi in Sympy [A] time = 81.7535, size = 73, normalized size = 1.01

$$\frac{x(-32x^2+70)}{612\sqrt{-x^4+x^2+2}}+\frac{8E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{153}+\frac{F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{102}-\frac{25\Pi\left(-\frac{10}{7};\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{238}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(5*x**2+7)/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] x*(-32*x**2 + 70)/(612*sqrt(-x**4 + x**2 + 2)) + 8*elliptic_e(asi
n(sqrt(2)*x/2), -2)/153 + elliptic_f(asin(sqrt(2)*x/2), -2)/102 -
 25*elliptic_pi(-10/7, asin(sqrt(2)*x/2), -2)/238
```

Mathematica [C] time = 0.212783, size = 101, normalized size = 1.4

$$\frac{\frac{490x}{\sqrt{-x^4+x^2+2}}-\frac{224x^3}{\sqrt{-x^4+x^2+2}}-357i\sqrt{2}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+224i\sqrt{2}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+225i\sqrt{2}\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{4284}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] ((490*x)/Sqrt[2 + x^2 - x^4] - (224*x^3)/Sqrt[2 + x^2 - x^4] + (2
24*I)*Sqrt[2]*EllipticE[I*ArcSinh[x], -1/2] - (357*I)*Sqrt[2]*Ell
ipticF[I*ArcSinh[x], -1/2] + (225*I)*Sqrt[2]*EllipticPi[5/7, I*Ar
cSinh[x], -1/2])/4284
```

Maple [B] time = 0.023, size = 164, normalized size = 2.3

$$2\frac{1}{\sqrt{-x^4+x^2+2}}\left(-\frac{4x^3}{153}+\frac{35x}{612}\right)+\frac{\sqrt{2}}{204}\sqrt{-2x^2+4}\sqrt{x^2+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}+\frac{4\sqrt{2}}{153}\sqrt{-2x^2+4}\sqrt{x^2+1}\mathit{EllipticE}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}-\frac{25\sqrt{2}}{238}\sqrt{1-\frac{x^2}{2}}\sqrt{x^2+1}\mathit{EllipticPi}\left(\frac{\sqrt{2}x}{2},-\frac{10}{7},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 2*(-4/153*x^3+35/612*x)/(-x^4+x^2+2)^(1/2)+1/204*2^(1/2)*(-2*x^2+
4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x
,I*2^(1/2))+4/153*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^
2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))-25/238*2^(1/2)*(1-1
/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*2^(
1/2)*x,-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}(5x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{1}{(5x^6+2x^4-17x^2-14)\sqrt{-x^4+x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)),x, algorithm="fricas")
```

```
[Out] integral(-1/((5*x^6 + 2*x^4 - 17*x^2 - 14)*sqrt(-x^4 + x^2 + 2)),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(-(x^2-2)(x^2+1))^{\frac{3}{2}}(5x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x**2+7)/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral(1/((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^4 + x^2 + 2)^{\frac{3}{2}}(5\,x^2 + 7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

3.350 $\int \frac{1}{(7+5x^2)^2(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{625\sqrt{-x^4+x^2+2}\,x}{16184\,(5x^2+7)}+\frac{(580-287x^2)\,x}{10404\sqrt{-x^4+x^2+2}}+\frac{89F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{24276}+\frac{5143E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{145656}-\frac{10825\,\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{113288}$$

```
[Out] (x*(580 - 287*x^2))/(10404*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 +
 x^2 - x^4])/(16184*(7 + 5*x^2)) + (5143*EllipticE[ArcSin[x/Sqrt[
2]], -2])/145656 + (89*EllipticF[ArcSin[x/Sqrt[2]], -2])/24276 -
(10825*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/113288
```

Rubi [A] time = 0.662267, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$\frac{625\sqrt{-x^4+x^2+2}\,x}{16184\,(5x^2+7)}+\frac{(580-287x^2)\,x}{10404\sqrt{-x^4+x^2+2}}+\frac{89F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{24276}+\frac{5143E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{145656}-\frac{10825\,\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{113288}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (x*(580 - 287*x^2))/(10404*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 +
 x^2 - x^4])/(16184*(7 + 5*x^2)) + (5143*EllipticE[ArcSin[x/Sqrt[
2]], -2])/145656 + (89*EllipticF[ArcSin[x/Sqrt[2]], -2])/24276 -
(10825*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/113288
```

Rubi in Sympy [A] time = 144.434, size = 299, normalized size = 2.99

$$\frac{625x\sqrt{-x^4+x^2+2}}{68\,(1190x^2+1666)}+\frac{1595x\sqrt{-x^4+x^2+2}}{58956\,(x^2+1)}-\frac{47x\sqrt{-x^4+x^2+2}}{176868\left(-\frac{x^2}{2}+1\right)}+\frac{16x\sqrt{-x^4+x^2+2}}{14739\left(-\frac{x^2}{2}+1\right)(x^2+1)}+\frac{5143\sqrt{2}\sqrt{-x^4+x^2+2}\,E\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{291312\sqrt{-\frac{x^2}{2}+1}\sqrt{x^2+1}}-\frac{9797\sqrt{2}\sqrt{-x^4+x^2+2}\,F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{97104\sqrt{-\frac{x^2}{2}+1}\sqrt{x^2+1}}-\frac{4175\sqrt{2}\sqrt{-x^4+x^2+2}\,\Pi\left(-\frac{10}{7};\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{226576\sqrt{-\frac{x^2}{2}+1}\sqrt{x^2+1}}+\frac{2375\sqrt{-x^4+x^2+2}\,\Pi\left(\frac{2}{7};\operatorname{atan}(x)\middle|\frac{3}{2}\right)}{32368\sqrt{\frac{-\frac{x^2}{2}+1}{x^2+1}}\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**2+7)**2/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] 625*x*sqrt(-x**4 + x**2 + 2)/(68*(1190*x**2 + 1666)) + 1595*x*sqr
t(-x**4 + x**2 + 2)/(58956*(x**2 + 1)) - 47*x*sqrt(-x**4 + x**2 +
 2)/(176868*(-x**2/2 + 1)) + 16*x*sqrt(-x**4 + x**2 + 2)/(14739*(
-x**2/2 + 1)*(x**2 + 1)) + 5143*sqrt(2)*sqrt(-x**4 + x**2 + 2)*el
liptic_e(asin(sqrt(2)*x/2), -2)/(291312*sqrt(-x**2/2 + 1)*sqrt(x*
*2 + 1)) - 9797*sqrt(2)*sqrt(-x**4 + x**2 + 2)*elliptic_f(asin(sq
rt(2)*x/2), -2)/(97104*sqrt(-x**2/2 + 1)*sqrt(x**2 + 1)) - 4175*s
```

```
qrt(2)*sqrt(-x**4 + x**2 + 2)*elliptic_pi(-10/7, asin(sqrt(2)*x/2
), -2)/(226576*sqrt(-x**2/2 + 1)*sqrt(x**2 + 1)) + 2375*sqrt(-x**
4 + x**2 + 2)*elliptic_pi(2/7, atan(x), 3/2)/(32368*sqrt((-x**2/2
 + 1)/(x**2 + 1))*(x**2 + 1))
```

Mathematica [C] time = 0.292675, size = 196, normalized size = 1.96

$$\frac{-360010x^5+253386x^3-111741i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}F\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)+72002i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}E\left(i\,\text{si}\right.}{2039184\left(5x^2+7\right.}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (953260*x + 253386*x^3 - 360010*x^5 + (72002*I)*Sqrt[2]*(7 + 5*x^
2)*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] - (111741*I)
*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x],
-1/2] + (681975*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*
ArcSinh[x], -1/2] + (487125*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*El
lipticPi[5/7, I*ArcSinh[x], -1/2])/(2039184*(7 + 5*x^2)*Sqrt[2 +
x^2 - x^4])
```

Maple [B] time = 0.03, size = 188, normalized size = 1.9

$$\frac{625\,x}{80920\,x^2+113288}\sqrt{-x^4+x^2+2}+2\,\frac{1}{\sqrt{-x^4+x^2+2}}\left(-\frac{287\,x^3}{20808}+\frac{145\,x}{5202}\right)$$
$$+\frac{89\,\sqrt{2}}{48552}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$
$$+\frac{5143\,\sqrt{2}}{291312}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathit{EllipticE}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$
$$-\frac{10825\,\sqrt{2}}{113288}\sqrt{1-\frac{x^2}{2}}\sqrt{x^2+1}\mathit{EllipticPi}\left(\frac{\sqrt{2}x}{2},-\frac{10}{7},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 625/16184*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)+2*(-287/20808*x^3+145/52
02*x)/(-x^4+x^2+2)^(1/2)+89/48552*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1
)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+514
3/291312*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2
)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))-10825/113288*2^(1/2)*(1-1/2*
x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*2^(1/2
)*x,-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{(25\,x^8+45\,x^6-71\,x^4-189\,x^2-98)\sqrt{-x^4+x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2),x, algorithm="fricas")
```

```
[Out] integral(-1/((25*x^8 + 45*x^6 - 71*x^4 - 189*x^2 - 98)*sqrt(-x^4
+ x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x**2+7)**2/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral(1/((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}(5\,x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

3.351 $\int \frac{1}{(7+5x^2)^3(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=128

$$\frac{645625\sqrt{-x^4+x^2+2}\,x}{15407168\,(5x^2+7)}+\frac{625\sqrt{-x^4+x^2+2}\,x}{32368\,(5x^2+7)^2}+\frac{(9830-4909x^2)\,x}{353736\sqrt{-x^4+x^2+2}}+\frac{60409F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{23110752}+\frac{3086453E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{138664512}-\frac{6898575\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{107850176}$$

```
[Out] (x*(9830 - 4909*x^2))/(353736*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[
2 + x^2 - x^4])/(32368*(7 + 5*x^2)^2) + (645625*x*Sqrt[2 + x^2 -
x^4])/(15407168*(7 + 5*x^2)) + (3086453*EllipticE[ArcSin[x/Sqrt[2
]], -2])/138664512 + (60409*EllipticF[ArcSin[x/Sqrt[2]], -2])/231
10752 - (6898575*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/107850
176
```

Rubi [A] time = 1.2707, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{645625\sqrt{-x^4+x^2+2}\,x}{15407168\,(5x^2+7)}+\frac{625\sqrt{-x^4+x^2+2}\,x}{32368\,(5x^2+7)^2}+\frac{(9830-4909x^2)\,x}{353736\sqrt{-x^4+x^2+2}}+\frac{60409F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{23110752}+\frac{3086453E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{138664512}-\frac{6898575\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{107850176}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (x*(9830 - 4909*x^2))/(353736*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[
2 + x^2 - x^4])/(32368*(7 + 5*x^2)^2) + (645625*x*Sqrt[2 + x^2 -
x^4])/(15407168*(7 + 5*x^2)) + (3086453*EllipticE[ArcSin[x/Sqrt[2
]], -2])/138664512 + (60409*EllipticF[ArcSin[x/Sqrt[2]], -2])/231
10752 - (6898575*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/107850
176
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**2+7)**3/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.379555, size = 244, normalized size = 1.91

$$-1080258550x^7-737347940x^5+3876617542x^3-67352691i\sqrt{2}\,(5x^2+7)^2\sqrt{-x^4+x^2+2}F\left(i\sinh^{-1}(x)\middle|-\tfrac{1}{2}\right)+43210342i\sqrt{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (3857257460*x + 3876617542*x^3 - 737347940*x^5 - 1080258550*x^7 +
 (43210342*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticE
[I*ArcSinh[x], -1/2] - (67352691*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2
+ x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] + (3042271575*I)*Sqrt[
2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] + (434
6102250*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcS
inh[x], -1/2] + (1552179375*I)*Sqrt[2]*x^4*Sqrt[2 + x^2 - x^4]*El
lipticPi[5/7, I*ArcSinh[x], -1/2])/(1941303168*(7 + 5*x^2)^2*Sqrt
[2 + x^2 - x^4])
```

Maple [A] time = 0.033, size = 212, normalized size = 1.7

$$\frac{625\,x}{32368\,\left(5\,x^2+7\right)^2}\sqrt{-x^4+x^2+2}+\frac{645625\,x}{77035840\,x^2+107850176}\sqrt{-x^4+x^2+2}$$
$$+2\,\frac{1}{\sqrt{-x^4+x^2+2}}\left(-\frac{4909\,x^3}{707472}+\frac{4915\,x}{353736}\right)$$
$$+\frac{60409\,\sqrt{2}}{46221504}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$
$$+\frac{3086453\,\sqrt{2}}{277329024}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathit{EllipticE}\left(\frac{\sqrt{2}x}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$
$$-\frac{6898575\,\sqrt{2}}{107850176}\sqrt{1-\frac{x^2}{2}}\sqrt{x^2+1}\mathit{EllipticPi}\left(\frac{\sqrt{2}x}{2},-\frac{10}{7},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 625/32368*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2+645625/15407168*x*(-x^
4+x^2+2)^(1/2)/(5*x^2+7)+2*(-4909/707472*x^3+4915/353736*x)/(-x^4
+x^2+2)^(1/2)+60409/46221504*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/
2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+3086453/
277329024*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/
2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))-6898575/107850176*2^(1/2)*(
1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*
2^(1/2)*x,-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{1}{\left(125\,x^{10}+400\,x^8-40\,x^6-1442\,x^4-1813\,x^2-686\right)\sqrt{-x^4+x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3),x, algorithm="fricas")
```

```
[Out] integral(-1/((125*x^10 + 400*x^8 - 40*x^6 - 1442*x^4 - 1813*x^2 -
 686)*sqrt(-x^4 + x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x**2+7)**3/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral(1/((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

3.352 $\int \left(7+5x^2\right)^4 \sqrt{4+3x^2+x^4}\,dx$

Optimal. Leaf size=242

$$\frac{3050}{11}\left(x^4+3x^2+4\right)^{3/2}x+\frac{1}{33}\left(4516x^2+18727\right)\sqrt{x^4+3x^2+4}\,x+\frac{51665\sqrt{x^4+3x^2+4}\,x}{33\left(x^2+2\right)}+\frac{33159\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{11\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{51665\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{33\sqrt{x^4+3x^2+4}}+\frac{625}{11}\left(x^4+3x^2+4\right)^{3/2}x^5+\frac{23500}{99}\left(x^4+3x^2+4\right)^{3/2}x^3$$

```
[Out] (51665*x*Sqrt[4 + 3*x^2 + x^4])/(33*(2 + x^2)) + (x*(18727 + 4516
*x^2)*Sqrt[4 + 3*x^2 + x^4])/33 + (3050*x*(4 + 3*x^2 + x^4)^(3/2)
)/11 + (23500*x^3*(4 + 3*x^2 + x^4)^(3/2))/99 + (625*x^5*(4 + 3*x
^2 + x^4)^(3/2))/11 - (51665*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 +
x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(33*Sqrt[4
 + 3*x^2 + x^4]) + (33159*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x
^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(11*Sqrt[2]*Sqrt[4 +
3*x^2 + x^4])
```

Rubi [A] time = 0.28012, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{3050}{11}\left(x^4+3x^2+4\right)^{3/2}x+\frac{1}{33}\left(4516x^2+18727\right)\sqrt{x^4+3x^2+4}\,x+\frac{51665\sqrt{x^4+3x^2+4}\,x}{33\left(x^2+2\right)}+\frac{33159\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{11\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{51665\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{33\sqrt{x^4+3x^2+4}}+\frac{625}{11}\left(x^4+3x^2+4\right)^{3/2}x^5+\frac{23500}{99}\left(x^4+3x^2+4\right)^{3/2}x^3$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^4*Sqrt[4 + 3*x^2 + x^4],x]
```


```
[Out] (51665*x*Sqrt[4 + 3*x^2 + x^4])/(33*(2 + x^2)) + (x*(18727 + 4516
*x^2)*Sqrt[4 + 3*x^2 + x^4])/33 + (3050*x*(4 + 3*x^2 + x^4)^(3/2)
)/11 + (23500*x^3*(4 + 3*x^2 + x^4)^(3/2))/99 + (625*x^5*(4 + 3*x
^2 + x^4)^(3/2))/11 - (51665*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 +
x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(33*Sqrt[4
 + 3*x^2 + x^4]) + (33159*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x
^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(11*Sqrt[2]*Sqrt[4 +
3*x^2 + x^4])
```

Rubi in Sympy [A] time = 56.6838, size = 241, normalized size = 1.

$$\frac{625x^5\left(x^4+3x^2+4\right)^{\frac{3}{2}}}{11}+\frac{23500x^3\left(x^4+3x^2+4\right)^{\frac{3}{2}}}{99}+\frac{x\left(\frac{22580x^2}{11}+\frac{93635}{11}\right)\sqrt{x^4+3x^2+4}}{15}+\frac{3050x\left(x^4+3x^2+4\right)^{\frac{3}{2}}}{11}+\frac{103330x\sqrt{x^4+3x^2+4}}{33\left(2x^2+4\right)}-\frac{51665\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{33\sqrt{x^4+3x^2+4}}+\frac{33159\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{22\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**4*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] 625*x**5*(x**4 + 3*x**2 + 4)**(3/2)/11 + 23500*x**3*(x**4 + 3*x**
2 + 4)**(3/2)/99 + x*(22580*x**2/11 + 93635/11)*sqrt(x**4 + 3*x**
2 + 4)/15 + 3050*x*(x**4 + 3*x**2 + 4)**(3/2)/11 + 103330*x*sqrt(
x**4 + 3*x**2 + 4)/(33*(2*x**2 + 4)) - 51665*sqrt(2)*sqrt((x**4 +
 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_e(2*atan(sqrt
(2)*x/2), 1/8)/(33*sqrt(x**4 + 3*x**2 + 4)) + 33159*sqrt(2)*sqrt(
(x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_f(2*at
an(sqrt(2)*x/2), 1/8)/(22*sqrt(x**4 + 3*x**2 + 4))
```

Mathematica [C] time = 0.988836, size = 354, normalized size = 1.46

$$\frac{3\sqrt{2}\left(51665\sqrt{7}-36253i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-154995\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}}{396}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^4*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(663924 + 1257535*x^2 + 1217475*
x^4 + 712748*x^6 + 264075*x^8 + 57250*x^10 + 5625*x^12) - 154995*
Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I +
 Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*Elli
pticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])
/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-36253*I + 51665*Sqrt[7])*Sqrt[(-3
*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] +
 (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*
I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(396*Sqrt[(-I
)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.208, size = 292, normalized size = 1.2

$$\frac{55327x}{33}\sqrt{x^4+3x^2+4}$$
$$+\frac{382496}{33\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\mathit{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$-\frac{1653280}{33\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\left(\mathit{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)-\mathit{Elliptic}\right.$$
$$+\frac{189898x^3}{99}\sqrt{x^4+3x^2+4}+\frac{3650x^5}{3}\sqrt{x^4+3x^2+4}+\frac{40375x^7}{99}\sqrt{x^4+3x^2+4}+\frac{625x^9}{11}\sqrt{x^4+3x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 55327/33*x*(x^4+3*x^2+4)^(1/2)+382496/33/(-6+2*I*7^(1/2))^(1/2)*(
1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1
/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/
4*(2+6*I*7^(1/2))^(1/2))-1653280/33/(-6+2*I*7^(1/2))^(1/2)*(1-(-3
/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(
x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2)
)^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2
))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+189898/99*x^3*(x^4+3*x^2+4)^
```

```
(1/2)+3650/3*x^5*(x^4+3*x^2+4)^(1/2)+40375/99*x^7*(x^4+3*x^2+4)^(
1/2)+625/11*x^9*(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+3x^2+4}(5x^2+7)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((625x^8+3500x^6+7350x^4+6860x^2+2401)\sqrt{x^4+3x^2+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^4,x, algorithm="fricas")
```

```
[Out] integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)*sqrt(x
^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^2-x+2)(x^2+x+2)}\,(5x^2+7)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**4*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**4, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+3x^2+4}(5x^2+7)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^4, x)
```

3.353 $\int \left(7+5x^2\right)^3 \sqrt{4+3x^2+x^4}\,dx$

Optimal. Leaf size=221

$$\frac{275}{7}\left(x^4+3x^2+4\right)^{3/2}x+\frac{1}{21}\left(407x^2+1708\right)\sqrt{x^4+3x^2+4}x+\frac{4717\sqrt{x^4+3x^2+4}x}{21\left(x^2+2\right)}+\frac{1301\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{4717\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{21\sqrt{x^4+3x^2+4}}+\frac{125}{9}\left(x^4+3x^2+4\right)^{3/2}x^3$$

```
[Out] (4717*x*Sqrt[4 + 3*x^2 + x^4])/(21*(2 + x^2)) + (x*(1708 + 407*x^
2)*Sqrt[4 + 3*x^2 + x^4])/21 + (275*x*(4 + 3*x^2 + x^4)^(3/2))/7
+ (125*x^3*(4 + 3*x^2 + x^4)^(3/2))/9 - (4717*Sqrt[2]*(2 + x^2)*S
qrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]],
 1/8])/(21*Sqrt[4 + 3*x^2 + x^4]) + (1301*(2 + x^2)*Sqrt[(4 + 3*x
^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sq
rt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.232639, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{275}{7}\left(x^4+3x^2+4\right)^{3/2}x+\frac{1}{21}\left(407x^2+1708\right)\sqrt{x^4+3x^2+4}x+\frac{4717\sqrt{x^4+3x^2+4}x}{21\left(x^2+2\right)}+\frac{1301\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{4717\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{21\sqrt{x^4+3x^2+4}}+\frac{125}{9}\left(x^4+3x^2+4\right)^{3/2}x^3$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4717*x*Sqrt[4 + 3*x^2 + x^4])/(21*(2 + x^2)) + (x*(1708 + 407*x^
2)*Sqrt[4 + 3*x^2 + x^4])/21 + (275*x*(4 + 3*x^2 + x^4)^(3/2))/7
+ (125*x^3*(4 + 3*x^2 + x^4)^(3/2))/9 - (4717*Sqrt[2]*(2 + x^2)*S
qrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]],
 1/8])/(21*Sqrt[4 + 3*x^2 + x^4]) + (1301*(2 + x^2)*Sqrt[(4 + 3*x
^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sq
rt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 44.3811, size = 219, normalized size = 0.99

$$\frac{125x^3\left(x^4+3x^2+4\right)^{\frac{3}{2}}}{9}+\frac{x\left(\frac{2035x^2}{7}+1220\right)\sqrt{x^4+3x^2+4}}{15}+\frac{275x\left(x^4+3x^2+4\right)^{\frac{3}{2}}}{7}+\frac{9434x\sqrt{x^4+3x^2+4}}{21\left(2x^2+4\right)}-\frac{4717\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{21\sqrt{x^4+3x^2+4}}+\frac{1301\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{6\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**3*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] 125*x**3*(x**4 + 3*x**2 + 4)**(3/2)/9 + x*(2035*x**2/7 + 1220)*sq
rt(x**4 + 3*x**2 + 4)/15 + 275*x*(x**4 + 3*x**2 + 4)**(3/2)/7 + 9
434*x*sqrt(x**4 + 3*x**2 + 4)/(21*(2*x**2 + 4)) - 4717*sqrt(2)*sq
rt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_e(2
*atan(sqrt(2)*x/2), 1/8)/(21*sqrt(x**4 + 3*x**2 + 4)) + 1301*sqrt
(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*ellipt
ic_f(2*atan(sqrt(2)*x/2), 1/8)/(6*sqrt(x**4 + 3*x**2 + 4))
```

Mathematica [C] time = 0.910092, size = 349, normalized size = 1.58

$$\frac{3\sqrt{2}\left(4717\sqrt{7}-3409i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-14151\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\cdots}}{252\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^{\cdots}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(60096 + 93656*x^2 + 71862*x^4 +
 30946*x^6 + 7725*x^8 + 875*x^10) - 14151*Sqrt[2]*(3*I + Sqrt[7])
*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I +
Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-
2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 3*S
qrt[2]*(-3409*I + 4717*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)
/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7
])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I -
Sqrt[7])/(3*I + Sqrt[7])])/(252*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[
4 + 3*x^2 + x^4])
```

Maple [C] time = 0.013, size = 275, normalized size = 1.2

$$\frac{5008\,x}{21}\sqrt{x^4+3\,x^2+4}$$
$$+\frac{35120}{21\sqrt{-6+2\,i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4},\frac{\sqrt{2+6\,i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3\,x^2+4}}$$
$$-\frac{150944}{21\sqrt{-6+2\,i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4},\frac{\sqrt{2+6\,i\sqrt{7}}}{4}\right)-Elliptic\cdots\right.$$
$$+\frac{12146\,x^3}{63}\sqrt{x^4+3\,x^2+4}+\frac{1700\,x^5}{21}\sqrt{x^4+3\,x^2+4}+\frac{125\,x^7}{9}\sqrt{x^4+3\,x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 5008/21*x*(x^4+3*x^2+4)^(1/2)+35120/21/(-6+2*I*7^(1/2))^(1/2)*(1-
(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2
)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*
(2+6*I*7^(1/2))^(1/2))-150944/21/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+
1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4
+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(
1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^
(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+12146/63*x^3*(x^4+3*x^2+4)^(1/2
)+1700/21*x^5*(x^4+3*x^2+4)^(1/2)+125/9*x^7*(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 4}\left(5x^2 + 7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(125x^6 + 525x^4 + 735x^2 + 343\right)\sqrt{x^4 + 3x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3,x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(x^4 + 3*x^2 + 4
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^2 - x + 2)(x^2 + x + 2)}\left(5x^2 + 7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**3*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 4}\left(5x^2 + 7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3, x)
```

3.354 $\int (7+5x^2)^2 \sqrt{4+3x^2+x^4}\,dx$

Optimal. Leaf size=198

$$\frac{25}{7}x\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{7}x\left(38x^2+119\right)\sqrt{x^4+3x^2+4}+\frac{319x\sqrt{x^4+3x^2+4}}{7\left(x^2+2\right)}+\frac{81\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{319\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{7\sqrt{x^4+3x^2+4}}$$

```
[Out] (319*x*Sqrt[4 + 3*x^2 + x^4])/(7*(2 + x^2)) + (x*(119 + 38*x^2)*S
qrt[4 + 3*x^2 + x^4])/7 + (25*x*(4 + 3*x^2 + x^4)^(3/2))/7 - (319
*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[
2*ArcTan[x/Sqrt[2]], 1/8])/(7*Sqrt[4 + 3*x^2 + x^4]) + (81*(2 + x
^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt
[2]], 1/8])/(Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.186034, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{25}{7}x\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{7}x\left(38x^2+119\right)\sqrt{x^4+3x^2+4}+\frac{319x\sqrt{x^4+3x^2+4}}{7\left(x^2+2\right)}+\frac{81\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{319\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{7\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (319*x*Sqrt[4 + 3*x^2 + x^4])/(7*(2 + x^2)) + (x*(119 + 38*x^2)*S
qrt[4 + 3*x^2 + x^4])/7 + (25*x*(4 + 3*x^2 + x^4)^(3/2))/7 - (319
*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[
2*ArcTan[x/Sqrt[2]], 1/8])/(7*Sqrt[4 + 3*x^2 + x^4]) + (81*(2 + x
^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt
[2]], 1/8])/(Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 35.3093, size = 199, normalized size = 1.01

$$\frac{x\left(\frac{570x^2}{7}+255\right)\sqrt{x^4+3x^2+4}}{15}+\frac{25x\left(x^4+3x^2+4\right)^{\frac{3}{2}}}{7}+\frac{638x\sqrt{x^4+3x^2+4}}{7\left(2x^2+4\right)}-\frac{319\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{7\sqrt{x^4+3x^2+4}}+\frac{81\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{2\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**2*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] x*(570*x**2/7 + 255)*sqrt(x**4 + 3*x**2 + 4)/15 + 25*x*(x**4 + 3*
x**2 + 4)**(3/2)/7 + 638*x*sqrt(x**4 + 3*x**2 + 4)/(7*(2*x**2 + 4
)) - 319*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/
2 + 1)*elliptic_e(2*atan(sqrt(2)*x/2), 1/8)/(7*sqrt(x**4 + 3*x**2
 + 4)) + 81*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x*
*2/2 + 1)*elliptic_f(2*atan(sqrt(2)*x/2), 1/8)/(2*sqrt(x**4 + 3*x
**2 + 4))
```

Mathematica [C] time = 1.037, size = 343, normalized size = 1.73

$$\frac{\sqrt{2}\left(319\sqrt{7}-35i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-319\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+}{\sqrt{7}+3i}}}{28\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(876 + 1109*x^2 + 658*x^4 + 188*
x^6 + 25*x^8) - 319*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7]
- (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(
3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]
*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-35*I + 319*Sqrt
[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*
I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqr
t[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/
(28*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.011, size = 258, normalized size = 1.3

$$\frac{219x}{7}\sqrt{x^4+3x^2+4}$$
$$+\frac{1984}{7\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\mathit{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$-\frac{10208}{7\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\left(\mathit{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)-\mathit{Elliptic}\right.$$
$$+\frac{113x^3}{7}\sqrt{x^4+3x^2+4}+\frac{25x^5}{7}\sqrt{x^4+3x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 219/7*x*(x^4+3*x^2+4)^(1/2)+1984/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/
8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x
^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6
*I*7^(1/2))^(1/2))-10208/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*
7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2
+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1
/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),
1/4*(2+6*I*7^(1/2))^(1/2)))+113/7*x^3*(x^4+3*x^2+4)^(1/2)+25/7*x^
5*(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^4+3x^2+4}\left(5x^2+7\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(25\,x^4+70\,x^2+49\right)\sqrt{x^4+3\,x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2,x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{(x^2-x+2)\,(x^2+x+2)}\,\left(5x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**2*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2, x)
```

3.355 $\int \left(7+5x^2\right) \sqrt{4+3x^2+x^4}\,dx$

Optimal. Leaf size=177

$$\frac{1}{3}\left(3x^2+10\right)\sqrt{x^4+3x^2+4}\,x+\frac{9\sqrt{x^4+3x^2+4}\,x}{x^2+2}+\frac{49\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{9\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}$$

```
[Out] (9*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) + (x*(10 + 3*x^2)*Sqrt[4 +
3*x^2 + x^4])/3 - (9*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x
^4] + (49*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF
[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.13205, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{1}{3}\left(3x^2+10\right)\sqrt{x^4+3x^2+4}\,x+\frac{9\sqrt{x^4+3x^2+4}\,x}{x^2+2}+\frac{49\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{9\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (9*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) + (x*(10 + 3*x^2)*Sqrt[4 +
3*x^2 + x^4])/3 - (9*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x
^4] + (49*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF
[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 26.6132, size = 177, normalized size = 1.

$$\frac{x\left(15x^2+50\right)\sqrt{x^4+3x^2+4}}{15}+\frac{18x\sqrt{x^4+3x^2+4}}{2x^2+4}-\frac{9\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}+\frac{49\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{6\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] x*(15*x**2 + 50)*sqrt(x**4 + 3*x**2 + 4)/15 + 18*x*sqrt(x**4 + 3*
x**2 + 4)/(2*x**2 + 4) - 9*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2
/2 + 1)**2)*(x**2/2 + 1)*elliptic_e(2*atan(sqrt(2)*x/2), 1/8)/sqr
t(x**4 + 3*x**2 + 4) + 49*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/
2 + 1)**2)*(x**2/2 + 1)*elliptic_f(2*atan(sqrt(2)*x/2), 1/8)/(6*s
qrt(x**4 + 3*x**2 + 4))
```

Mathematica [C] time = 0.776209, size = 338, normalized size = 1.91

$$\frac{\sqrt{2}\left(27\sqrt{7}-7i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-27\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}E}{12\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(40 + 42*x^2 + 19*x^4 + 3*x^6) -
 27*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3
*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*
EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt
[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-7*I + 27*Sqrt[7])*Sqrt[(-3*I +
Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I
)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + S
qrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(12*Sqrt[(-I)/(-3*
I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.008, size = 240, normalized size = 1.4

$$\frac{10x}{3}\sqrt{x^4+3x^2+4}+\frac{176}{3\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}-288\frac{\sqrt{1-\left(-3/8+i/8\sqrt{7}\right)x^2}\sqrt{1-\left(-3/8-i/8\sqrt{7}\right)x^2}\left(EllipticF\left(1/4\,x\sqrt{-6+2i\sqrt{7}},1/4\sqrt{2+6i\sqrt{7}}\right)-EllipticE\left(1/4\,x\right.\right.}{\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}\left(i\sqrt{7}+3\right)}+x^3\sqrt{x^4+3x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 10/3*x*(x^4+3*x^2+4)^(1/2)+176/3/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+
1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4
+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I
*7^(1/2))^(1/2))-288/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2
))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1
/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+
6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2
+6*I*7^(1/2))^(1/2)))+x^3*(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^4+3x^2+4}\left(5x^2+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{(x^2-x+2)\,(x^2+x+2)}\,\left(5x^2+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7), x)
```

3.356 $\int \sqrt{4+3x^2+x^4}\,dx$

Optimal. Leaf size=169

$$\frac{\sqrt{x^4+3x^2+4}\,x}{x^2+2}+\frac{1}{3}\sqrt{x^4+3x^2+4}\,x+\frac{7\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}$$

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/3 + (x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2)
 - (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellipti
cE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (7*(2 + x^2
)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2
]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.117189, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{\sqrt{x^4+3x^2+4}\,x}{x^2+2}+\frac{1}{3}\sqrt{x^4+3x^2+4}\,x+\frac{7\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/3 + (x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2)
 - (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellipti
cE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (7*(2 + x^2
)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2
]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 30.9604, size = 168, normalized size = 0.99

$$\frac{x\sqrt{x^4+3x^2+4}}{3}+\frac{2x\sqrt{x^4+3x^2+4}}{2x^2+4}-\frac{\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}+\frac{7\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{6\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+3*x**2+4)**(1/2),x)
```

```
[Out] x*sqrt(x**4 + 3*x**2 + 4)/3 + 2*x*sqrt(x**4 + 3*x**2 + 4)/(2*x**2
 + 4) - sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2
 + 1)*elliptic_e(2*atan(sqrt(2)*x/2), 1/8)/sqrt(x**4 + 3*x**2 + 4
) + 7*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 +
 1)*elliptic_f(2*atan(sqrt(2)*x/2), 1/8)/(6*sqrt(x**4 + 3*x**2 +
4))
```

Mathematica [C] time = 0.716904, size = 331, normalized size = 1.96

$$\frac{\sqrt{2}\left(3\sqrt{7}-7i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-3\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}E\left(i\right.}{12\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) - 3*Sqrt[2]*(3
*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]
*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*Ar
cSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sq
rt[7])] + Sqrt[2]*(-7*I + 3*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)
*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + S
qrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3
*I - Sqrt[7])/(3*I + Sqrt[7])])/(12*Sqrt[(-I)/(-3*I + Sqrt[7])]*S
qrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.005, size = 224, normalized size = 1.3

$$\frac{x}{3}\sqrt{x^4+3x^2+4}+\frac{32}{3\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}-32\frac{\sqrt{1-\left(-3/8+i/8\sqrt{7}\right)x^2}\sqrt{1-\left(-3/8-i/8\sqrt{7}\right)x^2}\left(EllipticF\left(1/4\,x\sqrt{-6+2i\sqrt{7}},1/4\sqrt{2+6i\sqrt{7}}\right)-EllipticE\left(1/4\,x\sqrt{\right.\right.}{\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}\left(i\sqrt{7}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 1/3*x*(x^4+3*x^2+4)^(1/2)+32/3/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/
8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3
*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7
^(1/2))^(1/2))-32/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*
x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I
*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*
I*7^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^4+3x^2+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x^4 + 3\,x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+3*x**2+4)**(1/2),x)

[Out] Integral(sqrt(x**4 + 3*x**2 + 4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3\,x^2 + 4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4),x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 3*x^2 + 4), x)
```

3.357 $\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx$

Optimal. Leaf size=322

$$\frac{\sqrt{x^4+3x^2+4}\,x}{5(x^2+2)}+\frac{1}{5}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)-\frac{11\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{75\sqrt{x^4+3x^2+4}}+\frac{9(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{25\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{5\sqrt{x^4+3x^2+4}}+\frac{187(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{525\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/(5*(2 + x^2)) + (Sqrt[11/35]*ArcTan[(2*
Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/5 - (Sqrt[2]*(2 + x^2)*Sqr
t[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1
/8])/(5*Sqrt[4 + 3*x^2 + x^4]) + (9*(2 + x^2)*Sqrt[(4 + 3*x^2 + x
^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(25*Sqrt[2]
*Sqrt[4 + 3*x^2 + x^4]) - (11*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 +
 x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(75*Sqrt[
4 + 3*x^2 + x^4]) + (187*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^
2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(525*Sqrt[2]*
Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.233921, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{\sqrt{x^4+3x^2+4}\,x}{5(x^2+2)}+\frac{1}{5}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)-\frac{11\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{75\sqrt{x^4+3x^2+4}}+\frac{9(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{25\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{5\sqrt{x^4+3x^2+4}}+\frac{187(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{525\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```



```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/(5*(2 + x^2)) + (Sqrt[11/35]*ArcTan[(2*
Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/5 - (Sqrt[2]*(2 + x^2)*Sqr
t[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1
/8])/(5*Sqrt[4 + 3*x^2 + x^4]) + (9*(2 + x^2)*Sqrt[(4 + 3*x^2 + x
^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(25*Sqrt[2]
*Sqrt[4 + 3*x^2 + x^4]) - (11*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 +
 x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(75*Sqrt[
4 + 3*x^2 + x^4]) + (187*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^
2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(525*Sqrt[2]*
Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 33.4957, size = 253, normalized size = 0.79

$$\frac{2x\sqrt{x^4+3x^2+4}}{5(2x^2+4)} - \frac{\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{5\sqrt{x^4+3x^2+4}} + \frac{\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{30\sqrt{x^4+3x^2+4}} + \frac{187\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)\left(-\frac{9}{280};2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{1050\sqrt{x^4+3x^2+4}} + \frac{\sqrt{385}\,\mathrm{atan}\left(\frac{2\sqrt{385}x}{35\sqrt{x^4+3x^2+4}}\right)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7),x)
```

```
[Out] 2*x*sqrt(x**4 + 3*x**2 + 4)/(5*(2*x**2 + 4)) - sqrt(2)*sqrt((x**4
 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_e(2*atan(sq
rt(2)*x/2), 1/8)/(5*sqrt(x**4 + 3*x**2 + 4)) + sqrt(2)*sqrt((x**4
 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_f(2*atan(sq
rt(2)*x/2), 1/8)/(30*sqrt(x**4 + 3*x**2 + 4)) + 187*sqrt(2)*sqrt(
(x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_pi(-9/
280, 2*atan(sqrt(2)*x/2), 1/8)/(1050*sqrt(x**4 + 3*x**2 + 4)) + s
qrt(385)*atan(2*sqrt(385)*x/(35*sqrt(x**4 + 3*x**2 + 4)))/175
```

Mathematica [C] time = 0.327502, size = 283, normalized size = 0.88

$$-\frac{\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(\left(-35\sqrt{7}+7i\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+35\left(\sqrt{7}+3i\right)E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)\right.}{350\sqrt{2}\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] -(Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*
I + Sqrt[7])]*(35*(3*I + Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)
/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (7*I -
35*Sqrt[7])*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x],
 (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (88*I)*EllipticPi[(5*(3 + I*S
qrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - S
qrt[7])/(3*I + Sqrt[7])]))/(350*Sqrt[2]*Sqrt[(-I)/(-3*I + Sqrt[7]
)]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.08, size = 386, normalized size = 1.2

$$\frac{32}{25\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\mathit{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$-\frac{32}{5\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\mathit{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$+\frac{32}{5\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\mathit{EllipticE}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$+\frac{44}{175\sqrt{-3/8+i/8\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\mathit{EllipticPi}\left(\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}x,-\frac{5}{-\frac{21}{8}+\frac{7i}{8}\sqrt{7}},\frac{\sqrt{-\frac{3}{8}-\frac{i}{8}\sqrt{7}}}{\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}}\right)\frac{1}{\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x)
```

```
[Out] 32/25/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*
(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF
(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-32/5/(-6
+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^
2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*Elli
pticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+32/
5/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3
/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)
*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)
)+44/175/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))
^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*El
lipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(
-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 3\,x^2 + 4}}{5\,x^2 + 7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3\,x^2 + 4}}{5\,x^2 + 7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2 - x + 2)\,(x^2 + x + 2)}}{5x^2 + 7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 3\,x^2 + 4}}{5\,x^2 + 7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7),x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)
```

3.358 $\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx$

Optimal. Leaf size=284

$$-\frac{\sqrt{x^4+3x^2+4}\,x}{70\,(x^2+2)}+\frac{\sqrt{x^4+3x^2+4}\,x}{14\,(5x^2+7)}+\frac{51\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{280\sqrt{385}}-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{289\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{9800\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] -(x*Sqrt[4 + 3*x^2 + x^4])/(70*(2 + x^2)) + (x*Sqrt[4 + 3*x^2 + x
^4])/(14*(7 + 5*x^2)) + (51*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x
^2 + x^4]])/(280*Sqrt[385]) + ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(
2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sqrt[2]*Sqrt
[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^
2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sqrt[2]*Sqrt[4 + 3*x^
2 + x^4]) + (289*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*El
lipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(9800*Sqrt[2]*Sqrt[4
+ 3*x^2 + x^4])
```

Rubi [A] time = 0.230263, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$-\frac{\sqrt{x^4+3x^2+4}\,x}{70\,(x^2+2)}+\frac{\sqrt{x^4+3x^2+4}\,x}{14\,(5x^2+7)}+\frac{51\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{280\sqrt{385}}-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{289\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{9800\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```


```
[Out] -(x*Sqrt[4 + 3*x^2 + x^4])/(70*(2 + x^2)) + (x*Sqrt[4 + 3*x^2 + x
^4])/(14*(7 + 5*x^2)) + (51*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x
^2 + x^4]])/(280*Sqrt[385]) + ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(
2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sqrt[2]*Sqrt
[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^
2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sqrt[2]*Sqrt[4 + 3*x^
2 + x^4]) + (289*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*El
lipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(9800*Sqrt[2]*Sqrt[4
+ 3*x^2 + x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7)**2,x)
```

```
[Out] Exception raised: TypeError
```

Mathematica [C] time = 1.24195, size = 481, normalized size = 1.69

$$\frac{-98i\left(5x^2+7\right)\sqrt{2-\frac{4ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-102i\left(5x^2+7\right)\sqrt{2-\frac{4ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(\frac{5}{14}\left(3+i\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (700*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) + 35*(3*I +
Sqrt[7])*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[
1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*(EllipticE[I*ArcSinh[Sqrt[(-2*I)
/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - Ellipti
cF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3
*I + Sqrt[7])]) - (98*I)*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I +
 Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcS
inh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt
[7])] - (102*I)*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])
]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt
[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt
[7])/(3*I + Sqrt[7])])/(9800*Sqrt[(-I)/(-3*I + Sqrt[7])]*(7 + 5*x
^2)*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.032, size = 410, normalized size = 1.4

$$\frac{x}{70x^2+98}\sqrt{x^4+3x^2+4}$$
$$+\frac{2}{25\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$+\frac{16}{35\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$-\frac{16}{35\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticE\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$+\frac{51}{2450\sqrt{-3/8+i/8\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticPi\left(\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}x,-\frac{5}{-\frac{21}{8}+\frac{7i}{8}\sqrt{7}},\frac{\sqrt{-\frac{3}{8}-\frac{i}{8}\sqrt{7}}}{\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}}\right)\frac{}{\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x)
```

```
[Out] 1/14*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+2/25/(-6+2*I*7^(1/2))^(1/2)*
(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))
^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2)
,1/4*(2+6*I*7^(1/2))^(1/2))+16/35/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x
^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(
x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2))
^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-16/35/(-6+2*I*7^(1/2))^(1/2)*(1
+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(
1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^
(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+51/2450/(-3/8+1/8*I*7^(1/
2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^
2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/
2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/
(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+3x^2+4}}{(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^2,x, algorithm="maxima")

[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+3x^2+4}}{25x^4+70x^2+49}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^2,x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2-x+2)(x^2+x+2)}}{(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7)**2,x)

[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+3x^2+4}}{(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^2,x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^2, x)
```

3.359 $\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx$

Optimal. Leaf size=312

$$-\frac{139\sqrt{x^4+3x^2+4}\,x}{86240\,(x^2+2)}+\frac{139\sqrt{x^4+3x^2+4}\,x}{17248\,(5x^2+7)}+\frac{\sqrt{x^4+3x^2+4}\,x}{28\,(5x^2+7)^2}+\frac{14999\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{344960\sqrt{385}}-\frac{23\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2940\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{139\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{43120\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{254983\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{36220800\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] (-139*x*Sqrt[4 + 3*x^2 + x^4])/(86240*(2 + x^2)) + (x*Sqrt[4 + 3*
x^2 + x^4])/(28*(7 + 5*x^2)^2) + (139*x*Sqrt[4 + 3*x^2 + x^4])/(1
7248*(7 + 5*x^2)) + (14999*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^
2 + x^4]])/(344960*Sqrt[385]) + (139*(2 + x^2)*Sqrt[(4 + 3*x^2 +
x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(43120*Sqr
t[2]*Sqrt[4 + 3*x^2 + x^4]) - (23*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4
)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2940*Sqrt[2]
*Sqrt[4 + 3*x^2 + x^4]) + (254983*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4
)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(362
20800*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.849995, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{139\sqrt{x^4+3x^2+4}\,x}{86240\,(x^2+2)}+\frac{139\sqrt{x^4+3x^2+4}\,x}{17248\,(5x^2+7)}+\frac{\sqrt{x^4+3x^2+4}\,x}{28\,(5x^2+7)^2}+\frac{14999\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{344960\sqrt{385}}-\frac{23\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2940\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{139\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{43120\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{254983\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{36220800\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (-139*x*Sqrt[4 + 3*x^2 + x^4])/(86240*(2 + x^2)) + (x*Sqrt[4 + 3*
x^2 + x^4])/(28*(7 + 5*x^2)^2) + (139*x*Sqrt[4 + 3*x^2 + x^4])/(1
7248*(7 + 5*x^2)) + (14999*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^
2 + x^4]])/(344960*Sqrt[385]) + (139*(2 + x^2)*Sqrt[(4 + 3*x^2 +
x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(43120*Sqr
t[2]*Sqrt[4 + 3*x^2 + x^4]) - (23*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4
)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2940*Sqrt[2]
*Sqrt[4 + 3*x^2 + x^4]) + (254983*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4
)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(362
20800*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7)**3,x)
```

```
[Out] Exception raised: TypeError
```

Mathematica [C] time = 0.88365, size = 308, normalized size = 0.99

$$\frac{\frac{700x\left(695x^2+1589\right)\left(x^4+3x^2+4\right)}{\left(5x^2+7\right)^2}+i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(\left(-9597+4865i\sqrt{7}\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+486\right.}{12073600\sqrt{x^4+3x^2+}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] ((700*x*(1589 + 695*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + I*Sqr
t[6 + (2*I)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[
1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*(4865*(3 - I*Sqrt[7])*EllipticE[
I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I
+ Sqrt[7])] + (-9597 + (4865*I)*Sqrt[7])*EllipticF[I*ArcSinh[Sqrt
[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] -
29998*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-
3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(12073600*
Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.034, size = 434, normalized size = 1.4

$$\frac{x}{28\left(5x^2+7\right)^2}\sqrt{x^4+3x^2+4}+\frac{139x}{86240x^2+120736}\sqrt{x^4+3x^2+4}$$
$$-\frac{51}{15400\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\,EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$+\frac{139}{2695\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\,EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+}}$$
$$-\frac{139}{2695\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\,EllipticE\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+}}$$
$$+\frac{14999}{3018400\sqrt{-3/8+i/8\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\,EllipticPi\left(\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}x,-\frac{5}{-\frac{21}{8}+\frac{7i}{8}\sqrt{7}},\frac{\sqrt{-\frac{3}{8}-\frac{i}{8}\sqrt{7}}}{\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x)
```

```
[Out] 1/28*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+139/17248*x*(x^4+3*x^2+4)^
(1/2)/(5*x^2+7)-51/15400/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*
x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2
+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/
2))^(1/2))+139/2695/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7
^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(
1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+
6*I*7^(1/2))^(1/2))-139/2695/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/
8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3
*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2
),1/4*(2+6*I*7^(1/2))^(1/2))+14999/3018400/(-3/8+1/8*I*7^(1/2))^(
1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(
1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(
1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8
```

```
+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 3\,x^2 + 4}}{\left(5\,x^2 + 7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3\,x^2 + 4}}{125\,x^6 + 525\,x^4 + 735\,x^2 + 343}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^6 + 525*x^4 + 735*x^2 + 343
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2 - x + 2)\,(x^2 + x + 2)}}{\left(5x^2 + 7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7)**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 3\,x^2 + 4}}{\left(5\,x^2 + 7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^3, x)
```

3.360 $\int \left(7+5x^2\right)^4 \left(4+3x^2+x^4\right)^{3/2} dx$

Optimal. Leaf size=268

$$\frac{92150}{429}\left(x^4+3x^2+4\right)^{5/2}x+\frac{\left(131080x^2+452001\right)\left(x^4+3x^2+4\right)^{3/2}x}{1287}+\frac{7\left(174989x^2+661429\right)\sqrt{x^4+3x^2+4}\,x}{2145}+\frac{12665086\sqrt{x^4+3x^2+4}\,x}{2145\left(x^2+2\right)}+\frac{2383556\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{429\sqrt{x^4+3x^2+4}}-\frac{12665086\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2145\sqrt{x^4+3x^2+4}}+\frac{125}{3}\left(x^4+3x^2+4\right)^{5/2}x^5+\frac{2250}{13}\left(x^4+3x^2+4\right)^{5/2}x^3$$

```
[Out] (12665086*x*Sqrt[4 + 3*x^2 + x^4])/(2145*(2 + x^2)) + (7*x*(66142
9 + 174989*x^2)*Sqrt[4 + 3*x^2 + x^4])/2145 + (x*(452001 + 131080
*x^2)*(4 + 3*x^2 + x^4)^(3/2))/1287 + (92150*x*(4 + 3*x^2 + x^4)^
(5/2))/429 + (2250*x^3*(4 + 3*x^2 + x^4)^(5/2))/13 + (125*x^5*(4
+ 3*x^2 + x^4)^(5/2))/3 - (12665086*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3
*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(21
45*Sqrt[4 + 3*x^2 + x^4]) + (2383556*Sqrt[2]*(2 + x^2)*Sqrt[(4 +
3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(4
29*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.320155, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{92150}{429}\left(x^4+3x^2+4\right)^{5/2}x+\frac{\left(131080x^2+452001\right)\left(x^4+3x^2+4\right)^{3/2}x}{1287}+\frac{7\left(174989x^2+661429\right)\sqrt{x^4+3x^2+4}\,x}{2145}+\frac{12665086\sqrt{x^4+3x^2+4}\,x}{2145\left(x^2+2\right)}+\frac{2383556\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{429\sqrt{x^4+3x^2+4}}-\frac{12665086\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2145\sqrt{x^4+3x^2+4}}+\frac{125}{3}\left(x^4+3x^2+4\right)^{5/2}x^5+\frac{2250}{13}\left(x^4+3x^2+4\right)^{5/2}x^3$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^4*(4 + 3*x^2 + x^4)^(3/2),x]
```


```
[Out] (12665086*x*Sqrt[4 + 3*x^2 + x^4])/(2145*(2 + x^2)) + (7*x*(66142
9 + 174989*x^2)*Sqrt[4 + 3*x^2 + x^4])/2145 + (x*(452001 + 131080
*x^2)*(4 + 3*x^2 + x^4)^(3/2))/1287 + (92150*x*(4 + 3*x^2 + x^4)^
(5/2))/429 + (2250*x^3*(4 + 3*x^2 + x^4)^(5/2))/13 + (125*x^5*(4
+ 3*x^2 + x^4)^(5/2))/3 - (12665086*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3
*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(21
45*Sqrt[4 + 3*x^2 + x^4]) + (2383556*Sqrt[2]*(2 + x^2)*Sqrt[(4 +
3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(4
29*Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 63.0751, size = 269, normalized size = 1.

$$\frac{125x^5(x^4+3x^2+4)^{\frac{5}{2}}}{3}+\frac{2250x^3(x^4+3x^2+4)^{\frac{5}{2}}}{13}+\frac{x\left(\frac{917560x^2}{143}+\frac{287637}{13}\right)(x^4+3x^2+4)^{\frac{3}{2}}}{63}+\frac{x\left(\frac{25723383x^2}{143}+\frac{97230063}{143}\right)\sqrt{x^4+3x^2+4}}{315}+\frac{92150x(x^4+3x^2+4)^{\frac{5}{2}}}{429}+\frac{25330172x\sqrt{x^4+3x^2+4}}{2145(2x^2+4)}-\frac{12665086\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{2145\sqrt{x^4+3x^2+4}}+\frac{2383556\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{429\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**4*(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] 125*x**5*(x**4 + 3*x**2 + 4)**(5/2)/3 + 2250*x**3*(x**4 + 3*x**2
+ 4)**(5/2)/13 + x*(917560*x**2/143 + 287637/13)*(x**4 + 3*x**2 +
 4)**(3/2)/63 + x*(25723383*x**2/143 + 97230063/143)*sqrt(x**4 +
3*x**2 + 4)/315 + 92150*x*(x**4 + 3*x**2 + 4)**(5/2)/429 + 253301
72*x*sqrt(x**4 + 3*x**2 + 4)/(2145*(2*x**2 + 4)) - 12665086*sqrt(
2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*ellipti
c_e(2*atan(sqrt(2)*x/2), 1/8)/(2145*sqrt(x**4 + 3*x**2 + 4)) + 23
83556*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 +
 1)*elliptic_f(2*atan(sqrt(2)*x/2), 1/8)/(429*sqrt(x**4 + 3*x**2
+ 4))
```

Mathematica [C] time = 1.19101, size = 364, normalized size = 1.36

$$21\sqrt{2}\left(904649\sqrt{7}-477617i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-18997629\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2}{\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^4*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(180184116 + 391419623*x^2 + 472
235001*x^4 + 377574349*x^6 + 212188905*x^8 + 83076275*x^10 + 2186
2875*x^12 + 3526875*x^14 + 268125*x^16) - 18997629*Sqrt[2]*(3*I +
 Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqr
t[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSin
h[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7
])] + 21*Sqrt[2]*(-477617*I + 904649*Sqrt[7])*Sqrt[(-3*I + Sqrt[7
] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)
/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7]
)]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(12870*Sqrt[(-I)/(-3*I +
 Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.049, size = 326, normalized size = 1.2

$$\frac{356027 x^5}{39}\sqrt{x^4+3x^2+4}+\frac{64070384 x^3}{6435}\sqrt{x^4+3x^2+4}+\frac{15015343 x}{2145}\sqrt{x^4+3x^2+4}$$
$$+\frac{89363792}{2145\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$-\frac{405282752}{2145\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)-Ellip\right.$$
$$+\frac{6863530 x^7}{1287}\sqrt{x^4+3x^2+4}+\frac{841525 x^9}{429}\sqrt{x^4+3x^2+4}+\frac{5500 x^{11}}{13}\sqrt{x^4+3x^2+4}+\frac{125 x^{13}}{3}\sqrt{x^4+3x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] 356027/39*x^5*(x^4+3*x^2+4)^(1/2)+64070384/6435*x^3*(x^4+3*x^2+4)
^(1/2)+15015343/2145*x*(x^4+3*x^2+4)^(1/2)+89363792/2145/(-6+2*I*
7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*
7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7
^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-405282752/2145/(-6+2*I*7
^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7
^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1
/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(
1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+6863530/
1287*x^7*(x^4+3*x^2+4)^(1/2)+841525/429*x^9*(x^4+3*x^2+4)^(1/2)+5
500/13*x^11*(x^4+3*x^2+4)^(1/2)+125/3*x^13*(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^4+3x^2+4\right)^{\frac{3}{2}}\left(5x^2+7\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^4,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(625 x^{12}+5375 x^{10}+20350 x^8+42910 x^6+52381 x^4+34643 x^2+9604\right)\sqrt{x^4+3x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^4,x, algorithm="fricas")
```

```
[Out] integral((625*x^12 + 5375*x^10 + 20350*x^8 + 42910*x^6 + 52381*x^
4 + 34643*x^2 + 9604)*sqrt(x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**4*(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**4,
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+3x^2+4\right)^{\frac{3}{2}}\left(5x^2+7\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^4,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^4, x)
```

3.361 $\int \left(7+5x^2\right)^3 \left(4+3x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=247

$$\frac{3825}{143}\left(x^4+3x^2+4\right)^{5/2}x+\frac{\left(15365x^2+53504\right)\left(x^4+3x^2+4\right)^{3/2}x}{1001}+\frac{\left(435441x^2+1653701\right)\sqrt{x^4+3x^2+4}\,x}{5005}+\frac{4525662\sqrt{x^4+3x^2+4}\,x}{5005\left(x^2+2\right)}+\frac{121826\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{143\sqrt{x^4+3x^2+4}}-\frac{4525662\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{5005\sqrt{x^4+3x^2+4}}+\frac{125}{13}\left(x^4+3x^2+4\right)^{5/2}x^3$$

```
[Out] (4525662*x*Sqrt[4 + 3*x^2 + x^4])/(5005*(2 + x^2)) + (x*(1653701
+ 435441*x^2)*Sqrt[4 + 3*x^2 + x^4])/5005 + (x*(53504 + 15365*x^2
)*(4 + 3*x^2 + x^4)^(3/2))/1001 + (3825*x*(4 + 3*x^2 + x^4)^(5/2)
)/143 + (125*x^3*(4 + 3*x^2 + x^4)^(5/2))/13 - (4525662*Sqrt[2]*(
2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x
/Sqrt[2]], 1/8])/(5005*Sqrt[4 + 3*x^2 + x^4]) + (121826*Sqrt[2]*(
2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x
/Sqrt[2]], 1/8])/(143*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.272914, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{3825}{143}\left(x^4+3x^2+4\right)^{5/2}x+\frac{\left(15365x^2+53504\right)\left(x^4+3x^2+4\right)^{3/2}x}{1001}+\frac{\left(435441x^2+1653701\right)\sqrt{x^4+3x^2+4}\,x}{5005}+\frac{4525662\sqrt{x^4+3x^2+4}\,x}{5005\left(x^2+2\right)}+\frac{121826\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{143\sqrt{x^4+3x^2+4}}-\frac{4525662\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{5005\sqrt{x^4+3x^2+4}}+\frac{125}{13}\left(x^4+3x^2+4\right)^{5/2}x^3$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2),x]
```


```
[Out] (4525662*x*Sqrt[4 + 3*x^2 + x^4])/(5005*(2 + x^2)) + (x*(1653701
+ 435441*x^2)*Sqrt[4 + 3*x^2 + x^4])/5005 + (x*(53504 + 15365*x^2
)*(4 + 3*x^2 + x^4)^(3/2))/1001 + (3825*x*(4 + 3*x^2 + x^4)^(5/2)
)/143 + (125*x^3*(4 + 3*x^2 + x^4)^(5/2))/13 - (4525662*Sqrt[2]*(
2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x
/Sqrt[2]], 1/8])/(5005*Sqrt[4 + 3*x^2 + x^4]) + (121826*Sqrt[2]*(
2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x
/Sqrt[2]], 1/8])/(143*Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 50.9606, size = 248, normalized size = 1.

$$\frac{125x^3\left(x^4+3x^2+4\right)^{\frac{5}{2}}}{13}+\frac{x\left(\frac{138285x^2}{143}+\frac{43776}{13}\right)\left(x^4+3x^2+4\right)^{\frac{3}{2}}}{63}+\frac{x\left(\frac{3918969x^2}{143}+\frac{14883309}{143}\right)\sqrt{x^4+3x^2+4}}{315}+\frac{3825x\left(x^4+3x^2+4\right)^{\frac{5}{2}}}{143}+\frac{9051324x\sqrt{x^4+3x^2+4}}{5005\left(2x^2+4\right)}-\frac{4525662\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{5005\sqrt{x^4+3x^2+4}}+\frac{121826\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{143\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**3*(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] 125*x**3*(x**4 + 3*x**2 + 4)**(5/2)/13 + x*(138285*x**2/143 + 437
76/13)*(x**4 + 3*x**2 + 4)**(3/2)/63 + x*(3918969*x**2/143 + 1488
3309/143)*sqrt(x**4 + 3*x**2 + 4)/315 + 3825*x*(x**4 + 3*x**2 + 4
)**(5/2)/143 + 9051324*x*sqrt(x**4 + 3*x**2 + 4)/(5005*(2*x**2 +
4)) - 4525662*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(
x**2/2 + 1)*elliptic_e(2*atan(sqrt(2)*x/2), 1/8)/(5005*sqrt(x**4
+ 3*x**2 + 4)) + 121826*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2
+ 1)**2)*(x**2/2 + 1)*elliptic_f(2*atan(sqrt(2)*x/2), 1/8)/(143*s
qrt(x**4 + 3*x**2 + 4))
```

Mathematica [C] time = 0.992859, size = 358, normalized size = 1.45

$$\sqrt{2}\left(2262831\sqrt{7}-1215823i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-2262831\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+}{\sqrt{7}-}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(19463124 + 36710547*x^2 + 37166
164*x^4 + 24107711*x^6 + 10713970*x^8 + 3158575*x^10 + 567000*x^1
2 + 48125*x^14) - 2262831*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sq
rt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*
x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqr
t[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-1215823*I
 + 2262831*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqr
t[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*Elliptic
F[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*
I + Sqrt[7])])/(10010*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2
+ x^4])
```

Maple [C] time = 0.01, size = 309, normalized size = 1.3

$$\frac{71434 x^5}{91}\sqrt{x^4+3x^2+4}+\frac{5528301 x^3}{5005}\sqrt{x^4+3x^2+4}+\frac{4865781 x}{5005}\sqrt{x^4+3x^2+4}$$
$$+\frac{32017264}{5005\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$-\frac{144821184}{5005\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)-Ellip\right.$$
$$+\frac{48520 x^7}{143}\sqrt{x^4+3x^2+4}+\frac{12075 x^9}{143}\sqrt{x^4+3x^2+4}+\frac{125 x^{11}}{13}\sqrt{x^4+3x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] 71434/91*x^5*(x^4+3*x^2+4)^(1/2)+5528301/5005*x^3*(x^4+3*x^2+4)^(
1/2)+4865781/5005*x*(x^4+3*x^2+4)^(1/2)+32017264/5005/(-6+2*I*7^(
1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(
1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1
/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-144821184/5005/(-6+2*I*7^(1
/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1
/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*
x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4
*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+48520/143*x
^7*(x^4+3*x^2+4)^(1/2)+12075/143*x^9*(x^4+3*x^2+4)^(1/2)+125/13*x
^11*(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+3x^2+4\right)^{\frac{3}{2}}\left(5x^2+7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(125x^{10}+900x^8+2810x^6+4648x^4+3969x^2+1372\right)\sqrt{x^4+3x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3,x, algorithm="fricas")
```

```
[Out] integral((125*x^10 + 900*x^8 + 2810*x^6 + 4648*x^4 + 3969*x^2 + 1
372)*sqrt(x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**3*(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**3,
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4 + 3\,x^2 + 4\right)^{\frac{3}{2}} \left(5\,x^2 + 7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3, x)
```

3.362 $\int \left(7+5x^2\right)^2 \left(4+3x^2+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=226

$$\frac{25}{11}x\left(x^4+3x^2+4\right)^{5/2}+\frac{1}{693}x\left(2240x^2+6831\right)\left(x^4+3x^2+4\right)^{3/2}+\frac{x\left(18253x^2+64533\right)\sqrt{x^4+3x^2+4}}{1155}+\frac{175346x\sqrt{x^4+3x^2+4}}{1155\left(x^2+2\right)}+\frac{4628\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{33\sqrt{x^4+3x^2+4}}-\frac{175346\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{1155\sqrt{x^4+3x^2+4}}$$

```
[Out] (175346*x*Sqrt[4 + 3*x^2 + x^4])/(1155*(2 + x^2)) + (x*(64533 + 1
8253*x^2)*Sqrt[4 + 3*x^2 + x^4])/1155 + (x*(6831 + 2240*x^2)*(4 +
 3*x^2 + x^4)^(3/2))/693 + (25*x*(4 + 3*x^2 + x^4)^(5/2))/11 - (1
75346*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellip
ticE[2*ArcTan[x/Sqrt[2]], 1/8])/(1155*Sqrt[4 + 3*x^2 + x^4]) + (4
628*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellipti
cF[2*ArcTan[x/Sqrt[2]], 1/8])/(33*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.222728, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{25}{11}x\left(x^4+3x^2+4\right)^{5/2}+\frac{1}{693}x\left(2240x^2+6831\right)\left(x^4+3x^2+4\right)^{3/2}+\frac{x\left(18253x^2+64533\right)\sqrt{x^4+3x^2+4}}{1155}+\frac{175346x\sqrt{x^4+3x^2+4}}{1155\left(x^2+2\right)}+\frac{4628\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{33\sqrt{x^4+3x^2+4}}-\frac{175346\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{1155\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (175346*x*Sqrt[4 + 3*x^2 + x^4])/(1155*(2 + x^2)) + (x*(64533 + 1
8253*x^2)*Sqrt[4 + 3*x^2 + x^4])/1155 + (x*(6831 + 2240*x^2)*(4 +
 3*x^2 + x^4)^(3/2))/693 + (25*x*(4 + 3*x^2 + x^4)^(5/2))/11 - (1
75346*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellip
ticE[2*ArcTan[x/Sqrt[2]], 1/8])/(1155*Sqrt[4 + 3*x^2 + x^4]) + (4
628*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellipti
cF[2*ArcTan[x/Sqrt[2]], 1/8])/(33*Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 41.6745, size = 226, normalized size = 1.

$$\frac{x\left(\frac{2240x^2}{11}+621\right)\left(x^4+3x^2+4\right)^{\frac{3}{2}}}{63}+\frac{x\left(\frac{54759x^2}{11}+\frac{193599}{11}\right)\sqrt{x^4+3x^2+4}}{315}+\frac{25x\left(x^4+3x^2+4\right)^{\frac{5}{2}}}{11}+\frac{350692x\sqrt{x^4+3x^2+4}}{1155\left(2x^2+4\right)}-\frac{175346\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{1155\sqrt{x^4+3x^2+4}}+\frac{4628\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{33\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**2*(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] x*(2240*x**2/11 + 621)*(x**4 + 3*x**2 + 4)**(3/2)/63 + x*(54759*x
**2/11 + 193599/11)*sqrt(x**4 + 3*x**2 + 4)/315 + 25*x*(x**4 + 3*
x**2 + 4)**(5/2)/11 + 350692*x*sqrt(x**4 + 3*x**2 + 4)/(1155*(2*x
**2 + 4)) - 175346*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)*
*2)*(x**2/2 + 1)*elliptic_e(2*atan(sqrt(2)*x/2), 1/8)/(1155*sqrt(
x**4 + 3*x**2 + 4)) + 4628*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2
/2 + 1)**2)*(x**2/2 + 1)*elliptic_f(2*atan(sqrt(2)*x/2), 1/8)/(33
*sqrt(x**4 + 3*x**2 + 4))
```

Mathematica [C]   time = 0.911577, size = 354, normalized size = 1.57

$$\frac{3\sqrt{2}\left(87673\sqrt{7}-34209i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-263019\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}}{69}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(1824876 + 2932753*x^2 + 2435811
*x^4 + 1229714*x^6 + 408480*x^8 + 82075*x^10 + 7875*x^12) - 26301
9*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I
 + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*El
lipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7
])/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-34209*I + 87673*Sqrt[7])*Sqrt[(
-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7]
 + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-
3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(6930*Sqrt[
(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C]   time = 0.013, size = 292, normalized size = 1.3

$$\frac{1222x^5}{21}\sqrt{x^4+3x^2+4}+\frac{391024x^3}{3465}\sqrt{x^4+3x^2+4}+\frac{50691x}{385}\sqrt{x^4+3x^2+4}$$
$$+\frac{396304}{385\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$-\frac{5611072}{1155\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)-Ellip\right.$$
$$+\frac{1670x^7}{99}\sqrt{x^4+3x^2+4}+\frac{25x^9}{11}\sqrt{x^4+3x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] 1222/21*x^5*(x^4+3*x^2+4)^(1/2)+391024/3465*x^3*(x^4+3*x^2+4)^(1/
2)+50691/385*x*(x^4+3*x^2+4)^(1/2)+396304/385/(-6+2*I*7^(1/2))^(1
/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^
2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/
2),1/4*(2+6*I*7^(1/2))^(1/2))-5611072/1155/(-6+2*I*7^(1/2))^(1/2)
*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^
(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*
7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I
*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+1670/99*x^7*(x^4+3*x^
2+4)^(1/2)+25/11*x^9*(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 4)^{\frac{3}{2}} (5x^2 + 7)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((25x^8 + 145x^6 + 359x^4 + 427x^2 + 196)\sqrt{x^4 + 3x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2,x, algorithm="fricas")
```

```
[Out] integral((25*x^8 + 145*x^6 + 359*x^4 + 427*x^2 + 196)*sqrt(x^4 +
3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int ((x^2 - x + 2)(x^2 + x + 2))^{\frac{3}{2}} (5x^2 + 7)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**2*(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**2,
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 4)^{\frac{3}{2}} (5x^2 + 7)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2, x)
```

3.363 $\int (7+5x^2)\,(4+3x^2+x^4)^{3/2}\,dx$

Optimal. Leaf size=207

$$\frac{1}{63}x\left(35x^2+108\right)\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{105}x\left(289x^2+1029\right)\sqrt{x^4+3x^2+4}+\frac{2798x\sqrt{x^4+3x^2+4}}{105\left(x^2+2\right)}+\frac{74\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{x^4+3x^2+4}}-\frac{2798\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{105\sqrt{x^4+3x^2+4}}$$

```
[Out] (2798*x*Sqrt[4 + 3*x^2 + x^4])/(105*(2 + x^2)) + (x*(1029 + 289*x
^2)*Sqrt[4 + 3*x^2 + x^4])/105 + (x*(108 + 35*x^2)*(4 + 3*x^2 + x
^4)^(3/2))/63 - (2798*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(105*Sqrt[4 + 3*x
^2 + x^4]) + (74*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^
2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[4 + 3*x^2 + x^
4])
```

Rubi [A] time = 0.169919, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{1}{63}x\left(35x^2+108\right)\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{105}x\left(289x^2+1029\right)\sqrt{x^4+3x^2+4}+\frac{2798x\sqrt{x^4+3x^2+4}}{105\left(x^2+2\right)}+\frac{74\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{x^4+3x^2+4}}-\frac{2798\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{105\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2798*x*Sqrt[4 + 3*x^2 + x^4])/(105*(2 + x^2)) + (x*(1029 + 289*x
^2)*Sqrt[4 + 3*x^2 + x^4])/105 + (x*(108 + 35*x^2)*(4 + 3*x^2 + x
^4)^(3/2))/63 - (2798*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(105*Sqrt[4 + 3*x
^2 + x^4]) + (74*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^
2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[4 + 3*x^2 + x^
4])
```

Rubi in Sympy [A] time = 31.7825, size = 202, normalized size = 0.98

$$\frac{x\left(35x^2+108\right)\left(x^4+3x^2+4\right)^{\frac{3}{2}}}{63}+\frac{x\left(867x^2+3087\right)\sqrt{x^4+3x^2+4}}{315}+\frac{5596x\sqrt{x^4+3x^2+4}}{105\left(2x^2+4\right)}-\frac{2798\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{105\sqrt{x^4+3x^2+4}}+\frac{74\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{3\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)*(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] x*(35*x**2 + 108)*(x**4 + 3*x**2 + 4)**(3/2)/63 + x*(867*x**2 + 3
087)*sqrt(x**4 + 3*x**2 + 4)/315 + 5596*x*sqrt(x**4 + 3*x**2 + 4)
/(105*(2*x**2 + 4)) - 2798*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2
/2 + 1)**2)*(x**2/2 + 1)*elliptic_e(2*atan(sqrt(2)*x/2), 1/8)/(10
5*sqrt(x**4 + 3*x**2 + 4)) + 74*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/
(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_f(2*atan(sqrt(2)*x/2), 1/8
```

```
)/(3*sqrt(x**4 + 3*x**2 + 4))
```

Mathematica [C] time = 1.04044, size = 349, normalized size = 1.69

$$\frac{3\sqrt{2}\left(1399\sqrt{7}-567i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-4197\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2}{}}}{630\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(20988 + 28489*x^2 + 19068*x^4 +
 7082*x^6 + 1590*x^8 + 175*x^10) - 4197*Sqrt[2]*(3*I + Sqrt[7])*S
qrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sq
rt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*
I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 3*Sqr
t[2]*(-567*I + 1399*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-
3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]
*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqr
t[7])/(3*I + Sqrt[7])])/(630*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 +
 3*x^2 + x^4])
```

Maple [C] time = 0.01, size = 275, normalized size = 1.3

$$\frac{71x^5}{21}\sqrt{x^4+3x^2+4}+\frac{3187x^3}{315}\sqrt{x^4+3x^2+4}+\frac{583x}{35}\sqrt{x^4+3x^2+4}$$
$$+\frac{6352}{35\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$-\frac{89536}{105\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)-Ellipt\right.$$
$$+\frac{5x^7}{9}\sqrt{x^4+3x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] 71/21*x^5*(x^4+3*x^2+4)^(1/2)+3187/315*x^3*(x^4+3*x^2+4)^(1/2)+58
3/35*x*(x^4+3*x^2+4)^(1/2)+6352/35/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/
8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x
^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6
*I*7^(1/2))^(1/2))-89536/105/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*
I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x
^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2)
,1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2
),1/4*(2+6*I*7^(1/2))^(1/2)))+5/9*x^7*(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^4+3x^2+4\right)^{\frac{3}{2}}\left(5x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7),x, algorithm="maxima")

[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(5\,x^{6}+22\,x^{4}+41\,x^{2}+28\right)\sqrt{x^{4}+3\,x^{2}+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7),x, algorithm="fricas")

[Out] integral((5*x^6 + 22*x^4 + 41*x^2 + 28)*sqrt(x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(x^{2}-x+2\right)\left(x^{2}+x+2\right)\right)^{\frac{3}{2}}\left(5x^{2}+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)*(x**4+3*x**2+4)**(3/2),x)

[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^{4}+3\,x^{2}+4\right)^{\frac{3}{2}}\left(5\,x^{2}+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7),x, algorithm="giac")

[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7), x)
```

3.364 $\int \left(4+3x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=198

$$\frac{1}{7}x\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{35}x\left(9x^2+49\right)\sqrt{x^4+3x^2+4}+\frac{138x\sqrt{x^4+3x^2+4}}{35\left(x^2+2\right)}+\frac{4\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{138\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{x^4+3x^2+4}}$$

```
[Out] (138*x*Sqrt[4 + 3*x^2 + x^4])/(35*(2 + x^2)) + (x*(49 + 9*x^2)*Sq
rt[4 + 3*x^2 + x^4])/35 + (x*(4 + 3*x^2 + x^4)^(3/2))/7 - (138*Sq
rt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*A
rcTan[x/Sqrt[2]], 1/8])/(35*Sqrt[4 + 3*x^2 + x^4]) + (4*Sqrt[2]*(
2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x
/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]
```

Rubi [A] time = 0.157542, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\frac{1}{7}x\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{35}x\left(9x^2+49\right)\sqrt{x^4+3x^2+4}+\frac{138x\sqrt{x^4+3x^2+4}}{35\left(x^2+2\right)}+\frac{4\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{138\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (138*x*Sqrt[4 + 3*x^2 + x^4])/(35*(2 + x^2)) + (x*(49 + 9*x^2)*Sq
rt[4 + 3*x^2 + x^4])/35 + (x*(4 + 3*x^2 + x^4)^(3/2))/7 - (138*Sq
rt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*A
rcTan[x/Sqrt[2]], 1/8])/(35*Sqrt[4 + 3*x^2 + x^4]) + (4*Sqrt[2]*(
2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x
/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]
```

Rubi in Sympy [A] time = 36.5299, size = 194, normalized size = 0.98

$$\frac{x\left(9x^2+49\right)\sqrt{x^4+3x^2+4}}{35}+\frac{x\left(x^4+3x^2+4\right)^{\frac{3}{2}}}{7}+\frac{276x\sqrt{x^4+3x^2+4}}{35\left(2x^2+4\right)}-\frac{138\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{35\sqrt{x^4+3x^2+4}}+\frac{4\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+3*x**2+4)**(3/2),x)
```

```
[Out] x*(9*x**2 + 49)*sqrt(x**4 + 3*x**2 + 4)/35 + x*(x**4 + 3*x**2 + 4
)**(3/2)/7 + 276*x*sqrt(x**4 + 3*x**2 + 4)/(35*(2*x**2 + 4)) - 13
8*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*
elliptic_e(2*atan(sqrt(2)*x/2), 1/8)/(35*sqrt(x**4 + 3*x**2 + 4))
 + 4*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 +
1)*elliptic_f(2*atan(sqrt(2)*x/2), 1/8)/sqrt(x**4 + 3*x**2 + 4)
```

Mathematica [C] time = 0.921861, size = 343, normalized size = 1.73

$$\frac{\sqrt{2}\left(69\sqrt{7}-77i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-69\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}}{70\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(276 + 303*x^2 + 161*x^4 + 39*x^
6 + 5*x^8) - 69*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2
*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I
+ Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x],
 (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-77*I + 69*Sqrt[7])*
Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + S
qrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2
*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(70*S
qrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.005, size = 258, normalized size = 1.3

$$\frac{x^5}{7}\sqrt{x^4+3x^2+4}+\frac{24x^3}{35}\sqrt{x^4+3x^2+4}+\frac{69x}{35}\sqrt{x^4+3x^2+4}$$
$$+\frac{1136}{35\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$-\frac{4416}{35\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)-Elliptic\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+3*x^2+4)^(3/2),x)
```

```
[Out] 1/7*x^5*(x^4+3*x^2+4)^(1/2)+24/35*x^3*(x^4+3*x^2+4)^(1/2)+69/35*x
*(x^4+3*x^2+4)^(1/2)+1136/35/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*
I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x
^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(
1/2))^(1/2))-4416/35/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2
))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1
/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+
6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2
+6*I*7^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^4+3x^2+4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((x^4+3x^2+4)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(3/2),x, algorithm="fricas")

[Out] integral((x^4 + 3*x^2 + 4)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4+3x^2+4)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+3*x**2+4)**(3/2),x)

[Out] Integral((x**4 + 3*x**2 + 4)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4+3x^2+4)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(3/2),x, algorithm="giac")

[Out] integrate((x^4 + 3*x^2 + 4)^(3/2), x)
```

3.365 $\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2} \, dx$

Optimal. Leaf size=284

$$\frac{1}{75}\left(3x^2+11\right)\sqrt{x^4+3x^2+4}\,x+\frac{94\sqrt{x^4+3x^2+4}\,x}{125\left(x^2+2\right)}+\frac{44}{125}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)+\frac{54\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{125\sqrt{x^4+3x^2+4}}-\frac{94\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{125\sqrt{x^4+3x^2+4}}+\frac{4114\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13125\sqrt{x^4+3x^2+4}}$$

```
[Out] (94*x*Sqrt[4 + 3*x^2 + x^4])/(125*(2 + x^2)) + (x*(11 + 3*x^2)*Sq
rt[4 + 3*x^2 + x^4])/75 + (44*Sqrt[11/35]*ArcTan[(2*Sqrt[11/35]*x
)/Sqrt[4 + 3*x^2 + x^4]])/125 - (94*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3
*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(12
5*Sqrt[4 + 3*x^2 + x^4]) + (54*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2
+ x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(125*Sqr
t[4 + 3*x^2 + x^4]) + (4114*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x
^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(1
3125*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.403498, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{1}{75}\left(3x^2+11\right)\sqrt{x^4+3x^2+4}\,x+\frac{94\sqrt{x^4+3x^2+4}\,x}{125\left(x^2+2\right)}+\frac{44}{125}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)+\frac{54\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{125\sqrt{x^4+3x^2+4}}-\frac{94\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{125\sqrt{x^4+3x^2+4}}+\frac{4114\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13125\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```


```
[Out] (94*x*Sqrt[4 + 3*x^2 + x^4])/(125*(2 + x^2)) + (x*(11 + 3*x^2)*Sq
rt[4 + 3*x^2 + x^4])/75 + (44*Sqrt[11/35]*ArcTan[(2*Sqrt[11/35]*x
)/Sqrt[4 + 3*x^2 + x^4]])/125 - (94*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3
*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(12
5*Sqrt[4 + 3*x^2 + x^4]) + (54*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2
+ x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(125*Sqr
t[4 + 3*x^2 + x^4]) + (4114*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x
^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(1
3125*Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 88.1831, size = 301, normalized size = 1.06

$$\frac{x\left(3x^2+3\right)\sqrt{x^4+3x^2+4}}{75}+\frac{8x\sqrt{x^4+3x^2+4}}{75}+\frac{188x\sqrt{x^4+3x^2+4}}{125\left(2x^2+4\right)}-\frac{94\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{125\sqrt{x^4+3x^2+4}}+\frac{54\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{125\sqrt{x^4+3x^2+4}}+\frac{4114\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)\Pi\left(-\frac{9}{280};2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{13125\sqrt{x^4+3x^2+4}}+\frac{44\sqrt{385}\operatorname{atan}\left(\frac{2\sqrt{385}x}{35\sqrt{x^4+3x^2+4}}\right)}{4375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7),x)
```

```
[Out] x*(3*x**2 + 3)*sqrt(x**4 + 3*x**2 + 4)/75 + 8*x*sqrt(x**4 + 3*x**
2 + 4)/75 + 188*x*sqrt(x**4 + 3*x**2 + 4)/(125*(2*x**2 + 4)) - 94
*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*e
lliptic_e(2*atan(sqrt(2)*x/2), 1/8)/(125*sqrt(x**4 + 3*x**2 + 4))
 + 54*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 +
 1)*elliptic_f(2*atan(sqrt(2)*x/2), 1/8)/(125*sqrt(x**4 + 3*x**2
+ 4)) + 4114*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x
**2/2 + 1)*elliptic_pi(-9/280, 2*atan(sqrt(2)*x/2), 1/8)/(13125*s
qrt(x**4 + 3*x**2 + 4)) + 44*sqrt(385)*atan(2*sqrt(385)*x/(35*sqr
t(x**4 + 3*x**2 + 4)))/4375
```

Mathematica [C] time = 1.77037, size = 477, normalized size = 1.68

$$7\sqrt{2}\left(705\sqrt{7}-241i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-4935\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+}{\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (350*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(44 + 45*x^2 + 20*x^4 + 3*x^6)
 - 4935*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)
/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7
])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I -
Sqrt[7])/(3*I + Sqrt[7])] + 7*Sqrt[2]*(-241*I + 705*Sqrt[7])*Sqrt
[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[
7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/
(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - (5808*I)
*Sqrt[2]*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt
[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 +
I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I
- Sqrt[7])/(3*I + Sqrt[7])])/(26250*Sqrt[(-I)/(-3*I + Sqrt[7])]*S
qrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.026, size = 418, normalized size = 1.5

$$\frac{x^3}{25}\sqrt{x^4+3x^2+4}+\frac{11x}{75}\sqrt{x^4+3x^2+4}$$
$$+\frac{9424}{1875\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\,EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$-\frac{3008}{125\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\,EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$+\frac{3008}{125\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\,EllipticE\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$+\frac{1936}{4375\sqrt{-3/8+i/8\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\,EllipticPi\left(\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}x,-\frac{5}{-\frac{21}{8}+\frac{7i}{8}\sqrt{7}},\frac{\sqrt{-\frac{3}{8}-\frac{i}{8}\sqrt{7}}}{\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x)
```

```
[Out] 1/25*x^3*(x^4+3*x^2+4)^(1/2)+11/75*x*(x^4+3*x^2+4)^(1/2)+9424/187
5/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3
/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4
*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-3008/125/(-6
+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^
2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*Elli
pticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+300
8/125/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*
(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2
)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(
1/2))+1936/4375/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7
^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(
1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(
1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^4+3x^2+4\right)^{\frac{3}{2}}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x^4+3x^2+4\right)^{\frac{3}{2}}}{5x^2+7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(x^2 - x + 2\right)\left(x^2 + x + 2\right)\right)^{\frac{3}{2}}}{5x^2 + 7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7),x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4 + 3\,x^2 + 4\right)^{\frac{3}{2}}}{5\,x^2 + 7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

3.366 $\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2} \, dx$

Optimal. Leaf size=305

$$\frac{4\sqrt{x^4+3x^2+4}\,x}{175\left(x^2+2\right)}+\frac{22\sqrt{x^4+3x^2+4}\,x}{175\left(5x^2+7\right)}+\frac{1}{75}\sqrt{x^4+3x^2+4}\,x+\frac{13}{350}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)+\frac{4\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{175\sqrt{x^4+3x^2+4}}-\frac{4\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{175\sqrt{x^4+3x^2+4}}+\frac{2431\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{36750\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/75 + (4*x*Sqrt[4 + 3*x^2 + x^4])/(175*(
2 + x^2)) + (22*x*Sqrt[4 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) + (13*
Sqrt[11/35]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/350
- (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellipt
icE[2*ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt[4 + 3*x^2 + x^4]) + (4*S
qrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*
ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt[4 + 3*x^2 + x^4]) + (2431*(2 +
 x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*Ar
cTan[x/Sqrt[2]], 1/8])/(36750*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.711411, antiderivative size = 372, normalized size of antiderivative = 1.22, number of steps used = 18, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{4\sqrt{x^4+3x^2+4}\,x}{175\left(x^2+2\right)}+\frac{22\sqrt{x^4+3x^2+4}\,x}{175\left(5x^2+7\right)}+\frac{1}{75}\sqrt{x^4+3x^2+4}\,x+\frac{13}{350}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)+\frac{4\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{175\sqrt{x^4+3x^2+4}}-\frac{4\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{175\sqrt{x^4+3x^2+4}}+\frac{187\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{13125\sqrt{x^4+3x^2+4}}+\frac{6919\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{183750\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/75 + (4*x*Sqrt[4 + 3*x^2 + x^4])/(175*(
2 + x^2)) + (22*x*Sqrt[4 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) + (13*
Sqrt[11/35]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/350
- (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellipt
icE[2*ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt[4 + 3*x^2 + x^4]) + (4*S
qrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*
ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt[4 + 3*x^2 + x^4]) + (6919*(2 +
 x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*Ar
cTan[x/Sqrt[2]], 1/8])/(183750*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (
187*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellipti
cPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(13125*Sqrt[4 + 3*x^2 + x^
4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.726759, size = 309, normalized size = 1.01

$$\frac{\frac{175x(7x^2+23)(x^4+3x^2+4)}{5x^2+7} - i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(7\left(158+15i\sqrt{7}\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+105\left(3-i\right.\right.}{18375\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] ((175*x*(23 + 7*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2) - I*Sqrt[6 +
(2*I)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((
2*I)*x^2)/(3*I + Sqrt[7])]*(105*(3 - I*Sqrt[7])*EllipticE[I*ArcSi
nh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[
7])] + 7*(158 + (15*I)*Sqrt[7])*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(
-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 429*Ellip
ticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[
7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(18375*Sqrt[4 + 3*x^2
 + x^4])
```

Maple [C] time = 0.034, size = 425, normalized size = 1.4

$$\frac{22x}{875x^2+1225}\sqrt{x^4+3x^2+4}+\frac{x}{75}\sqrt{x^4+3x^2+4}$$
$$+\frac{232}{375\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$-\frac{128}{175\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+\cdots}}$$
$$+\frac{128}{175\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticE\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4\cdots}}$$
$$+\frac{286}{6125\sqrt{-3/8+i/8\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticPi\left(\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}x,-\frac{5}{-\frac{21}{8}+\frac{7i}{8}\sqrt{7}},\frac{\sqrt{-\frac{3}{8}-\frac{i}{8}\sqrt{7}}}{\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}}\right)\frac{\cdots}{\sqrt{x^4\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x)
```

```
[Out] 22/175*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+1/75*x*(x^4+3*x^2+4)^(1/2)
+232/375/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/
2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*Ellipt
icF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-128/1
75/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+
3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3
)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2
))+128/175/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(
1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7
^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/
2))^(1/2))+286/6125/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x
^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+
4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I
```

```
*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4+3x^2+4)^{\frac{3}{2}}}{(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(x^4+3x^2+4)^{\frac{3}{2}}}{25x^4+70x^2+49},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^2,x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 4)^(3/2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}}}{(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7)**2,
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4+3x^2+4)^{\frac{3}{2}}}{(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^2, x)
```

3.367 $\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx$

Optimal. Leaf size=440

$$\frac{9\sqrt{x^4+3x^2+4}\,x}{1960\left(x^2+2\right)}+\frac{167\sqrt{x^4+3x^2+4}\,x}{9800\left(5x^2+7\right)}+\frac{11\sqrt{x^4+3x^2+4}\,x}{175\left(5x^2+7\right)^2}+\frac{1347\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{7840\sqrt{385}}-\frac{22\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13125\sqrt{x^4+3x^2+4}}-\frac{817\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{91875\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{6\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{875\sqrt{x^4+3x^2+4}}+\frac{111\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{24500\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{7633\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{274400\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] (9*x*Sqrt[4 + 3*x^2 + x^4])/(1960*(2 + x^2)) + (11*x*Sqrt[4 + 3*x
^2 + x^4])/(175*(7 + 5*x^2)^2) + (167*x*Sqrt[4 + 3*x^2 + x^4])/(9
800*(7 + 5*x^2)) + (1347*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2
+ x^4]])/(7840*Sqrt[385]) + (111*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)
/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(24500*Sqrt[2]
*Sqrt[4 + 3*x^2 + x^4]) - (6*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 +
x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(875*Sqrt[
4 + 3*x^2 + x^4]) - (817*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^
2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(91875*Sqrt[2]*Sqrt[4
+ 3*x^2 + x^4]) - (22*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(13125*Sqrt[4 + 3
*x^2 + x^4]) + (7633*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2
]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(274400*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 1.0593, antiderivative size = 440, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$\frac{9\sqrt{x^4+3x^2+4}\,x}{1960\left(x^2+2\right)}+\frac{167\sqrt{x^4+3x^2+4}\,x}{9800\left(5x^2+7\right)}+\frac{11\sqrt{x^4+3x^2+4}\,x}{175\left(5x^2+7\right)^2}+\frac{1347\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{7840\sqrt{385}}-\frac{22\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13125\sqrt{x^4+3x^2+4}}-\frac{817\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{91875\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{6\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{875\sqrt{x^4+3x^2+4}}+\frac{111\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{24500\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{7633\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{274400\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```


```
[Out] (9*x*Sqrt[4 + 3*x^2 + x^4])/(1960*(2 + x^2)) + (11*x*Sqrt[4 + 3*x
^2 + x^4])/(175*(7 + 5*x^2)^2) + (167*x*Sqrt[4 + 3*x^2 + x^4])/(9
800*(7 + 5*x^2)) + (1347*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2
+ x^4]])/(7840*Sqrt[385]) + (111*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)
/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(24500*Sqrt[2]
*Sqrt[4 + 3*x^2 + x^4]) - (6*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 +
x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(875*Sqrt[
4 + 3*x^2 + x^4]) - (817*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^
2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(91875*Sqrt[2]*Sqrt[4
```

```
+ 3*x^2 + x^4]) - (22*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(13125*Sqrt[4 + 3
*x^2 + x^4]) + (7633*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2
]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(274400*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7)**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.884546, size = 309, normalized size = 0.7

$$\frac{\frac{140x(167x^2+357)(x^4+3x^2+4)}{(5x^2+7)^2} - i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(7\left(103+45i\sqrt{7}\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+315\left(3-\right.\right.}{274400\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] ((140*x*(357 + 167*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 - I*Sqrt
[6 + (2*I)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1
 + ((2*I)*x^2)/(3*I + Sqrt[7])]*(315*(3 - I*Sqrt[7])*EllipticE[I*
ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I +
Sqrt[7])] + 7*(103 + (45*I)*Sqrt[7])*EllipticF[I*ArcSinh[Sqrt[(-2
*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 2694
*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I +
 Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(274400*Sqrt[4
+ 3*x^2 + x^4])
```

Maple [C] time = 0.034, size = 434, normalized size = 1.

$$\frac{11\,x}{175\,(5\,x^2+7)^2}\sqrt{x^4+3\,x^2+4}+\frac{167\,x}{49000\,x^2+68600}\sqrt{x^4+3\,x^2+4}$$
$$+\frac{17}{350\sqrt{-6+2\,i\sqrt{7}}}\sqrt{1+\frac{3\,x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3\,x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticF\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4},\frac{\sqrt{2+6\,i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3\,x^2+4}}$$
$$-\frac{36}{245\sqrt{-6+2\,i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3\,x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3\,x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticF\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4},\frac{\sqrt{2+6\,i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3\,x^2+}}$$
$$+\frac{36}{245\sqrt{-6+2\,i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3\,x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3\,x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticE\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4},\frac{\sqrt{2+6\,i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3\,x^2+4}}$$
$$+\frac{1347}{68600\sqrt{-3/8+i/8\sqrt{7}}}\sqrt{1+\frac{3\,x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3\,x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticPi\left(\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}x,-\frac{5}{-\frac{21}{8}+\frac{7\,i}{8}\sqrt{7}},\frac{\sqrt{-\frac{3}{8}-\frac{i}{8}\sqrt{7}}}{\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}}\right)\frac{}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x)
```

```
[Out] 11/175*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+167/9800*x*(x^4+3*x^2+4)
^(1/2)/(5*x^2+7)+17/350/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x
^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+
4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2
))^(1/2))-36/245/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1
/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2
)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I
*7^(1/2))^(1/2))+36/245/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x
^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+
4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4
*(2+6*I*7^(1/2))^(1/2))+1347/68600/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+
3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1
/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-
5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7
^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^3,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{125\,x^6+525\,x^4+735\,x^2+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^3,x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 4)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 3
43), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7)**3,
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^3,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^3, x)
```

3.368 $\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=187

$$-\frac{15\sqrt{x^4+3x^2+4}\,x}{x^2+2}+75\sqrt{x^4+3x^2+4}\,x+\frac{13\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{15\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}+25\sqrt{x^4+3x^2+4}\,x^3$$

```
[Out] 75*x*Sqrt[4 + 3*x^2 + x^4] + 25*x^3*Sqrt[4 + 3*x^2 + x^4] - (15*x
*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) + (15*Sqrt[2]*(2 + x^2)*Sqrt[(4
 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])
/Sqrt[4 + 3*x^2 + x^4] + (13*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*Sqrt[4
+ 3*x^2 + x^4])
```

Rubi [A] time = 0.188988, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{15\sqrt{x^4+3x^2+4}\,x}{x^2+2}+75\sqrt{x^4+3x^2+4}\,x+\frac{13\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{15\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}+25\sqrt{x^4+3x^2+4}\,x^3$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^3/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] 75*x*Sqrt[4 + 3*x^2 + x^4] + 25*x^3*Sqrt[4 + 3*x^2 + x^4] - (15*x
*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) + (15*Sqrt[2]*(2 + x^2)*Sqrt[(4
 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])
/Sqrt[4 + 3*x^2 + x^4] + (13*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*Sqrt[4
+ 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 39.207, size = 189, normalized size = 1.01

$$25x^3\sqrt{x^4+3x^2+4}+75x\sqrt{x^4+3x^2+4}-\frac{30x\sqrt{x^4+3x^2+4}}{2x^2+4}+\frac{15\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}+\frac{13\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{4\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**3/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] 25*x**3*sqrt(x**4 + 3*x**2 + 4) + 75*x*sqrt(x**4 + 3*x**2 + 4) -
30*x*sqrt(x**4 + 3*x**2 + 4)/(2*x**2 + 4) + 15*sqrt(2)*sqrt((x**4
 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_e(2*atan(sq
rt(2)*x/2), 1/8)/sqrt(x**4 + 3*x**2 + 4) + 13*sqrt(2)*sqrt((x**4
+ 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_f(2*atan(sqr
t(2)*x/2), 1/8)/(4*sqrt(x**4 + 3*x**2 + 4))
```

Mathematica [C] time = 0.843927, size = 337, normalized size = 1.8

$$\frac{-\sqrt{2}\left(15\sqrt{7}+131i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+15\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+\cdots}{\sqrt{7}+3i}}}{4\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^3/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (100*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(12 + 13*x^2 + 6*x^4 + x^6) +
15*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*
I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*E
llipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[
7])/(3*I + Sqrt[7])] - Sqrt[2]*(131*I + 15*Sqrt[7])*Sqrt[(-3*I +
Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I
)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + S
qrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(4*Sqrt[(-I)/(-3*I
 + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.038, size = 241, normalized size = 1.3

$$172\frac{\sqrt{1-\left(-3/8+i/8\sqrt{7}\right)x^2}\sqrt{1-\left(-3/8-i/8\sqrt{7}\right)x^2}\mathit{EllipticF}\left(1/4\,x\sqrt{-6+2\,i\sqrt{7}},1/4\,\sqrt{2+6\,i\sqrt{7}}\right)}{\sqrt{-6+2\,i\sqrt{7}}\sqrt{x^4+3\,x^2+4}}$$
$$+480\frac{\sqrt{1-\left(-3/8+i/8\sqrt{7}\right)x^2}\sqrt{1-\left(-3/8-i/8\sqrt{7}\right)x^2}\left(\mathit{EllipticF}\left(1/4\,x\sqrt{-6+2\,i\sqrt{7}},1/4\,\sqrt{2+6\,i\sqrt{7}}\right)-\mathit{EllipticE}\left(1/4\,x\sqrt{\cdots}\right.\right.}{\sqrt{-6+2\,i\sqrt{7}}\sqrt{x^4+3\,x^2+4}\left(i\sqrt{7}+3\right)}$$
$$+75\,x\sqrt{x^4+3\,x^2+4}+25\,x^3\sqrt{x^4+3\,x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 172/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-
(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4
*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+480/(-6+2*I*
7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*
7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(
1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE
(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+75*x*(x
^4+3*x^2+4)^(1/2)+25*x^3*(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5\,x^2+7\right)^3}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 4),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{125\,x^6+525\,x^4+735\,x^2+343}{\sqrt{x^4+3\,x^2+4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 4),x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)/sqrt(x^4 + 3*x^2 + 4
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5x^2+7\right)^3}{\sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**3/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5\,x^2+7\right)^3}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 4),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 4), x)
```

3.369 $\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=170

$$\frac{20\sqrt{x^4+3x^2+4}\,x}{x^2+2}+\frac{25}{3}\sqrt{x^4+3x^2+4}\,x+\frac{167\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{20\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}$$

```
[Out] (25*x*Sqrt[4 + 3*x^2 + x^4])/3 + (20*x*Sqrt[4 + 3*x^2 + x^4])/(2
+ x^2) - (20*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2
]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (1
67*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcT
an[x/Sqrt[2]], 1/8])/(6*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.146686, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{20\sqrt{x^4+3x^2+4}\,x}{x^2+2}+\frac{25}{3}\sqrt{x^4+3x^2+4}\,x+\frac{167\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{20\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^2/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (25*x*Sqrt[4 + 3*x^2 + x^4])/3 + (20*x*Sqrt[4 + 3*x^2 + x^4])/(2
+ x^2) - (20*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2
]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (1
67*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcT
an[x/Sqrt[2]], 1/8])/(6*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 29.766, size = 172, normalized size = 1.01

$$\frac{25x\sqrt{x^4+3x^2+4}}{3}+\frac{40x\sqrt{x^4+3x^2+4}}{2x^2+4}-\frac{20\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}+\frac{167\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{12\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**2/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] 25*x*sqrt(x**4 + 3*x**2 + 4)/3 + 40*x*sqrt(x**4 + 3*x**2 + 4)/(2*
x**2 + 4) - 20*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*
(x**2/2 + 1)*elliptic_e(2*atan(sqrt(2)*x/2), 1/8)/sqrt(x**4 + 3*x
**2 + 4) + 167*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*
(x**2/2 + 1)*elliptic_f(2*atan(sqrt(2)*x/2), 1/8)/(12*sqrt(x**4 +
 3*x**2 + 4))
```

Mathematica [C] time = 0.757333, size = 331, normalized size = 1.95

$$\frac{\sqrt{2}\left(30\sqrt{7}+43i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-30\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}E}{6\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^2/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (50*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) - 30*Sqrt[2]*
(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7]
)]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*
ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I +
Sqrt[7])] + Sqrt[2]*(43*I + 30*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2
*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I
+ Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x],
 (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(6*Sqrt[(-I)/(-3*I + Sqrt[7])]
*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.01, size = 224, normalized size = 1.3

$$\frac{188}{3\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\mathit{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$-640\frac{\sqrt{1-\left(-3/8+i/8\sqrt{7}\right)x^2}\sqrt{1-\left(-3/8-i/8\sqrt{7}\right)x^2}\left(\mathit{EllipticF}\left(1/4x\sqrt{-6+2i\sqrt{7}},1/4\sqrt{2+6i\sqrt{7}}\right)-\mathit{EllipticE}\left(1/4x\sqrt{\right.\right.}{\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}\left(i\sqrt{7}+3\right)}$$
$$+\frac{25x}{3}\sqrt{x^4+3x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 188/3/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(
1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1
/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-640/(-6+2*
I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*
I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(Elliptic
F(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-Ellipti
cE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+25/3*
x*(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5x^2+7\right)^2}{\sqrt{x^4+3x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 4),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{25\,x^{4}+70\,x^{2}+49}{\sqrt{x^{4}+3\,x^{2}+4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 4),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)/sqrt(x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5x^{2}+7\right)^{2}}{\sqrt{\left(x^{2}-x+2\right)\left(x^{2}+x+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**2/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5\,x^{2}+7\right)^{2}}{\sqrt{x^{4}+3\,x^{2}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 4),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 4), x)
```

3.370 $\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=151

$$\frac{5\sqrt{x^4+3x^2+4}\,x}{x^2+2}+\frac{17\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{5\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}$$

```
[Out] (5*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (5*Sqrt[2]*(2 + x^2)*Sqrt
[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/
8])/Sqrt[4 + 3*x^2 + x^4] + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*Sqrt
[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0958973, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{5\sqrt{x^4+3x^2+4}\,x}{x^2+2}+\frac{17\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{5\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (5*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (5*Sqrt[2]*(2 + x^2)*Sqrt
[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/
8])/Sqrt[4 + 3*x^2 + x^4] + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*Sqrt
[4 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 20.6707, size = 153, normalized size = 1.01

$$\frac{10x\sqrt{x^4+3x^2+4}}{2x^2+4}-\frac{5\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}+\frac{17\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{4\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] 10*x*sqrt(x**4 + 3*x**2 + 4)/(2*x**2 + 4) - 5*sqrt(2)*sqrt((x**4
+ 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_e(2*atan(sqr
t(2)*x/2), 1/8)/sqrt(x**4 + 3*x**2 + 4) + 17*sqrt(2)*sqrt((x**4 +
 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_f(2*atan(sqrt
(2)*x/2), 1/8)/(4*sqrt(x**4 + 3*x**2 + 4))
```

Mathematica [C] time = 0.264273, size = 214, normalized size = 1.42

$$\frac{\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(\left(5\sqrt{7}+i\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-5\left(\sqrt{7}+3i\right)E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)\right)}{2\sqrt{2}\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
 + Sqrt[7])]*(-5*(3*I + Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/
(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (I + 5*S
qrt[7])*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*
I - Sqrt[7])/(3*I + Sqrt[7])]))/(2*Sqrt[2]*Sqrt[(-I)/(-3*I + Sqrt
[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.007, size = 209, normalized size = 1.4

$$28\,\frac{\sqrt{1-\left(-3/8+i/8\sqrt{7}\right)x^{2}}\sqrt{1-\left(-3/8-i/8\sqrt{7}\right)x^{2}}\,EllipticF\left(1/4\,x\sqrt{-6+2\,i\sqrt{7}},1/4\,\sqrt{2+6\,i\sqrt{7}}\right)}{\sqrt{-6+2\,i\sqrt{7}}\sqrt{x^{4}+3\,x^{2}+4}}$$

$$-160\,\frac{\sqrt{1-\left(-3/8+i/8\sqrt{7}\right)x^{2}}\sqrt{1-\left(-3/8-i/8\sqrt{7}\right)x^{2}}\left(EllipticF\left(1/4\,x\sqrt{-6+2\,i\sqrt{7}},1/4\,\sqrt{2+6\,i\sqrt{7}}\right)-EllipticE\left(1/4\,x\right.\right.}{\sqrt{-6+2\,i\sqrt{7}}\sqrt{x^{4}+3\,x^{2}+4}\left(i\sqrt{7}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 28/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(
-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*
x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-160/(-6+2*I*7
^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7
^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1
/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(
1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^{2}+7}{\sqrt{x^{4}+3\,x^{2}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{5\,x^{2}+7}{\sqrt{x^{4}+3\,x^{2}+4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4),x, algorithm="fricas")
```

```
[Out] integral((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{\sqrt{(x^2 - x + 2)(x^2 + x + 2)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)/(x**4+3*x**2+4)**(1/2),x)

[Out] Integral((5*x**2 + 7)/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^2 + 7}{\sqrt{x^4 + 3\,x^2 + 4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4), x)
```

3.371 $\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=64

$$\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan
[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0212322, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan
[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 9.96456, size = 63, normalized size = 0.98

$$\frac{\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{4\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*el
liptic_f(2*atan(sqrt(2)*x/2), 1/8)/(4*sqrt(x**4 + 3*x**2 + 4))
```

Mathematica [C] time = 0.0955351, size = 142, normalized size = 2.22

$$-\frac{i\sqrt{1-\frac{2x^2}{-3-i\sqrt{7}}}\sqrt{1-\frac{2x^2}{-3+i\sqrt{7}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-3-i\sqrt{7}}}x\right)|\frac{-3-i\sqrt{7}}{-3+i\sqrt{7}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-3-i\sqrt{7}}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-3 - I*Sqrt[7])]*Sqrt[1 - (2*x^2)/(-3 + I
*Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[-2/(-3 - I*Sqrt[7])]*x], (-3
- I*Sqrt[7])/(-3 + I*Sqrt[7])])/(Sqrt[2]*Sqrt[-(-3 - I*Sqrt[7])^(
-1)]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.004, size = 85, normalized size = 1.3

$$4\,\frac{\sqrt{1-\left(-3/8+i/8\sqrt{7}\right)x^{2}}\sqrt{1-\left(-3/8-i/8\sqrt{7}\right)x^{2}}\,\mathit{EllipticF}\left(1/4\,x\sqrt{-6+2\,i\sqrt{7}},1/4\,\sqrt{2+6\,i\sqrt{7}}\right)}{\sqrt{-6+2\,i\sqrt{7}}\sqrt{x^{4}+3\,x^{2}+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 4/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-
3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x
*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^{4}+3\,x^{2}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^4 + 3*x^2 + 4),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{x^{4}+3\,x^{2}+4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^4 + 3*x^2 + 4),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^{4}+3x^{2}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**4 + 3*x**2 + 4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^{4}+3\,x^{2}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^4 + 3*x^2 + 4),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 4), x)
```

3.372 $\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=168

$$\frac{1}{4}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{17(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{84\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] (Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/4 -
((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan
[x/Sqrt[2]], 1/8])/(6*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (17*(2 + x
^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcT
an[x/Sqrt[2]], 1/8])/(84*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0851049, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{1}{4}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{17(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{84\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/4 -
((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan
[x/Sqrt[2]], 1/8])/(6*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (17*(2 + x
^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcT
an[x/Sqrt[2]], 1/8])/(84*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 17.0271, size = 165, normalized size = 0.98

$$-\frac{\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{12\sqrt{x^4+3x^2+4}}+\frac{17\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)\left(-\frac{9}{280};2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{168\sqrt{x^4+3x^2+4}}+\frac{\sqrt{385}\,\mathrm{atan}\left(\frac{2\sqrt{385}x}{35\sqrt{x^4+3x^2+4}}\right)}{308}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**2+7)/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] -sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*e
lliptic_f(2*atan(sqrt(2)*x/2), 1/8)/(12*sqrt(x**4 + 3*x**2 + 4))
+ 17*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 +
1)*elliptic_pi(-9/280, 2*atan(sqrt(2)*x/2), 1/8)/(168*sqrt(x**4 +
 3*x**2 + 4)) + sqrt(385)*atan(2*sqrt(385)*x/(35*sqrt(x**4 + 3*x*
*2 + 4)))/308
```

Mathematica [C] time = 0.112374, size = 159, normalized size = 0.95

$$-\frac{i\sqrt{1-\frac{2x^2}{-3-i\sqrt{7}}}\sqrt{1-\frac{2x^2}{-3+i\sqrt{7}}}\,\Pi\left(-\frac{5}{14}\left(-3-i\sqrt{7}\right);i\sinh^{-1}\left(\sqrt{-\frac{2}{-3-i\sqrt{7}}}x\right)|\frac{-3-i\sqrt{7}}{-3+i\sqrt{7}}\right)}{7\sqrt{2}\sqrt{-\frac{1}{-3-i\sqrt{7}}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] ((-I/7)*Sqrt[1 - (2*x^2)/(-3 - I*Sqrt[7])]*Sqrt[1 - (2*x^2)/(-3 +
 I*Sqrt[7])]*EllipticPi[(-5*(-3 - I*Sqrt[7]))/14, I*ArcSinh[Sqrt[
-2/(-3 - I*Sqrt[7])]*x], (-3 - I*Sqrt[7])/(-3 + I*Sqrt[7])])/(Sqr
t[2]*Sqrt[-(-3 - I*Sqrt[7])^(-1)]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.023, size = 107, normalized size = 0.6

$$\frac{1}{7\sqrt{-3/8+i/8\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\mathit{EllipticPi}\left(\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}x,-\frac{5}{-\frac{21}{8}+\frac{7i}{8}\sqrt{7}},\frac{\sqrt{-\frac{3}{8}-\frac{i}{8}\sqrt{7}}}{\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 1/7/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2
)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*Ellipti
cPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-
1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3x^2+4}(5x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{x^4+3x^2+4}(5x^2+7)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 - x + 2)(x^2 + x + 2)}(5x^2 + 7)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x**2+7)/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3\,x^2 + 4}(5\,x^2 + 7)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)), x)
```

3.373 $\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=286

$$-\frac{5\sqrt{x^4+3x^2+4}\,x}{616\,(x^2+2)}+\frac{25\sqrt{x^4+3x^2+4}\,x}{616\,(5x^2+7)}+\frac{37\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{2464}-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{42\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{5\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{308\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{629\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{51744\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] (-5*x*Sqrt[4 + 3*x^2 + x^4])/(616*(2 + x^2)) + (25*x*Sqrt[4 + 3*x
^2 + x^4])/(616*(7 + 5*x^2)) + (37*Sqrt[5/77]*ArcTan[(2*Sqrt[11/3
5]*x)/Sqrt[4 + 3*x^2 + x^4]])/2464 + (5*(2 + x^2)*Sqrt[(4 + 3*x^2
 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(308*Sq
rt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(42*Sqrt[2]*Sqr
t[4 + 3*x^2 + x^4]) + (629*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(51744*Sqrt
[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.232554, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$-\frac{5\sqrt{x^4+3x^2+4}\,x}{616\,(x^2+2)}+\frac{25\sqrt{x^4+3x^2+4}\,x}{616\,(5x^2+7)}+\frac{37\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{2464}-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{42\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{5\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{308\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{629\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{51744\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4]),x]
```


```
[Out] (-5*x*Sqrt[4 + 3*x^2 + x^4])/(616*(2 + x^2)) + (25*x*Sqrt[4 + 3*x
^2 + x^4])/(616*(7 + 5*x^2)) + (37*Sqrt[5/77]*ArcTan[(2*Sqrt[11/3
5]*x)/Sqrt[4 + 3*x^2 + x^4]])/2464 + (5*(2 + x^2)*Sqrt[(4 + 3*x^2
 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(308*Sq
rt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(42*Sqrt[2]*Sqr
t[4 + 3*x^2 + x^4]) + (629*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(51744*Sqrt
[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 32.9841, size = 280, normalized size = 0.98

$$\frac{25x\sqrt{x^4+3x^2+4}}{3080x^2+4312}-\frac{5x\sqrt{x^4+3x^2+4}}{308\left(2x^2+4\right)}+\frac{5\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{616\sqrt{x^4+3x^2+4}}-\frac{\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{84\sqrt{x^4+3x^2+4}}+\frac{629\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)\left(-\frac{9}{280};2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{103488\sqrt{x^4+3x^2+4}}+\frac{37\sqrt{385}\operatorname{atan}\left(\frac{2\sqrt{385}x}{35\sqrt{x^4+3x^2+4}}\right)}{189728}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**2+7)**2/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] 25*x*sqrt(x**4 + 3*x**2 + 4)/(3080*x**2 + 4312) - 5*x*sqrt(x**4 +
 3*x**2 + 4)/(308*(2*x**2 + 4)) + 5*sqrt(2)*sqrt((x**4 + 3*x**2 +
 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_e(2*atan(sqrt(2)*x/2),
 1/8)/(616*sqrt(x**4 + 3*x**2 + 4)) - sqrt(2)*sqrt((x**4 + 3*x**2
 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_f(2*atan(sqrt(2)*x/2
), 1/8)/(84*sqrt(x**4 + 3*x**2 + 4)) + 629*sqrt(2)*sqrt((x**4 + 3
*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_pi(-9/280, 2*at
an(sqrt(2)*x/2), 1/8)/(103488*sqrt(x**4 + 3*x**2 + 4)) + 37*sqrt(
385)*atan(2*sqrt(385)*x/(35*sqrt(x**4 + 3*x**2 + 4)))/189728
```

Mathematica [C] time = 1.2245, size = 481, normalized size = 1.68

$$98i\left(5x^2+7\right)\sqrt{2-\frac{4ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-74i\left(5x^2+7\right)\sqrt{2-\frac{4ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(\frac{5}{14}\left(3+i\sqrt{7}\right)\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (700*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) + 35*(3*I +
Sqrt[7])*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[
1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*(EllipticE[I*ArcSinh[Sqrt[(-2*I)
/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - Ellipti
cF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3
*I + Sqrt[7])]) + (98*I)*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I +
 Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcS
inh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt
[7])] - (74*I)*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]
*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[
7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[
7])/(3*I + Sqrt[7])])/(17248*Sqrt[(-I)/(-3*I + Sqrt[7])]*(7 + 5*x
^2)*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.031, size = 410, normalized size = 1.4

$$\frac{25x}{3080x^2+4312}\sqrt{x^4+3x^2+4} - \frac{1}{22\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}} + \frac{20}{77\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}} - \frac{20}{77\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticE\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}} + \frac{37}{4312\sqrt{-3/8+i/8\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticPi\left(\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}x,-\frac{5}{-\frac{21}{8}+\frac{7i}{8}\sqrt{7}},\frac{\sqrt{-\frac{3}{8}-\frac{i}{8}\sqrt{7}}}{\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}}\right)\frac{1}{\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 25/616*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)-1/22/(-6+2*I*7^(1/2))^(1/2
)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2
))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/
2),1/4*(2+6*I*7^(1/2))^(1/2))+20/77/(-6+2*I*7^(1/2))^(1/2)*(1+3/8
*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)
/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2
))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-20/77/(-6+2*I*7^(1/2))^(1/2)*
(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))
^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*
7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+37/4312/(-3/8+1/8*I*7^(
1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*
x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(
1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2
)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3x^2+4}(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(25x^4+70x^2+49)\sqrt{x^4+3x^2+4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2),x, algorithm="fricas")
```

```
[Out] integral(1/((25*x^4 + 70*x^2 + 49)*sqrt(x^4 + 3*x^2 + 4)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 - x + 2)(x^2 + x + 2)}(5x^2 + 7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x**2+7)**2/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**2),
 x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3\,x^2 + 4}(5\,x^2 + 7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2), x)
```

3.374 $\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=314

$$-\frac{555\sqrt{x^4+3x^2+4}\,x}{758912\,(x^2+2)}+\frac{2775\sqrt{x^4+3x^2+4}\,x}{758912\,(5x^2+7)}+\frac{25\sqrt{x^4+3x^2+4}\,x}{1232\,(5x^2+7)^2}-\frac{3285\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{3035648}-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{8624\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{555\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{379456\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{18615\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{21249536\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] (-555*x*Sqrt[4 + 3*x^2 + x^4])/(758912*(2 + x^2)) + (25*x*Sqrt[4
+ 3*x^2 + x^4])/(1232*(7 + 5*x^2)^2) + (2775*x*Sqrt[4 + 3*x^2 + x
^4])/(758912*(7 + 5*x^2)) - (3285*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35
]*x)/Sqrt[4 + 3*x^2 + x^4]])/3035648 + (555*(2 + x^2)*Sqrt[(4 + 3
*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(37
9456*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4 + 3*x^2
+ x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(8624*Sq
rt[2]*Sqrt[4 + 3*x^2 + x^4]) - (18615*(2 + x^2)*Sqrt[(4 + 3*x^2 +
 x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/
(21249536*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.438426, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$-\frac{555\sqrt{x^4+3x^2+4}\,x}{758912\,(x^2+2)}+\frac{2775\sqrt{x^4+3x^2+4}\,x}{758912\,(5x^2+7)}+\frac{25\sqrt{x^4+3x^2+4}\,x}{1232\,(5x^2+7)^2}-\frac{3285\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{3035648}-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{8624\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{555\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{379456\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{18615\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{21249536\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4]),x]
```


```
[Out] (-555*x*Sqrt[4 + 3*x^2 + x^4])/(758912*(2 + x^2)) + (25*x*Sqrt[4
+ 3*x^2 + x^4])/(1232*(7 + 5*x^2)^2) + (2775*x*Sqrt[4 + 3*x^2 + x
^4])/(758912*(7 + 5*x^2)) - (3285*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35
]*x)/Sqrt[4 + 3*x^2 + x^4]])/3035648 + (555*(2 + x^2)*Sqrt[(4 + 3
*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(37
9456*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4 + 3*x^2
+ x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(8624*Sq
rt[2]*Sqrt[4 + 3*x^2 + x^4]) - (18615*(2 + x^2)*Sqrt[(4 + 3*x^2 +
 x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/
(21249536*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**2+7)**3/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.3073, size = 308, normalized size = 0.98

$$\frac{\frac{700x(555x^2+1393)(x^4+3x^2+4)}{(5x^2+7)^2} + i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(\left(-9401+3885i\sqrt{7}\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+388\right.}{21249536\sqrt{x^4+3x^2+}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] ((700*x*(1393 + 555*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + I*Sqr
t[6 + (2*I)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[
1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*(3885*(3 - I*Sqrt[7])*EllipticE[
I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I
+ Sqrt[7])] + (-9401 + (3885*I)*Sqrt[7])*EllipticF[I*ArcSinh[Sqrt
[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] +
6570*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3
*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(21249536*S
qrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.031, size = 434, normalized size = 1.4

$$\frac{25x}{1232(5x^2+7)^2}\sqrt{x^4+3x^2+4}+\frac{2775x}{3794560x^2+5312384}\sqrt{x^4+3x^2+4}$$
$$-\frac{23}{27104\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\mathit{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$+\frac{555}{23716\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\mathit{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2}}$$
$$-\frac{555}{23716\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\mathit{EllipticE}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2}}$$
$$-\frac{3285}{5312384\sqrt{-3/8+i/8\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\mathit{EllipticPi}\left(\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}x,-\frac{5}{-\frac{21}{8}+\frac{7i}{8}\sqrt{7}},\frac{\sqrt{-\frac{3}{8}-\frac{i}{8}\sqrt{7}}}{\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 25/1232*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+2775/758912*x*(x^4+3*x^
2+4)^(1/2)/(5*x^2+7)-23/27104/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1
/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+
3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*
7^(1/2))^(1/2))+555/23716/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I
*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^
2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1
/4*(2+6*I*7^(1/2))^(1/2))-555/23716/(-6+2*I*7^(1/2))^(1/2)*(1+3/8
*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)
/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2
))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-3285/5312384/(-3/8+1/8*I*7^(1
/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x
^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1
/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)
```

```
/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 4}(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(125x^6 + 525x^4 + 735x^2 + 343)\sqrt{x^4 + 3x^2 + 4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3),x, algorithm="fricas")
```

```
[Out] integral(1/((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(x^4 + 3*x^2
+ 4)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 - x + 2)(x^2 + x + 2)}(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x**2+7)**3/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**3),
 x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 4}(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3), x)
```

3.375 $\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=219

$$-\frac{220779\sqrt{x^4+3x^2+4}\,x}{28\,(x^2+2)}+\frac{5000}{3}\sqrt{x^4+3x^2+4}\,x+\frac{(45779x^2+99493)\,x}{28\sqrt{x^4+3x^2+4}}-\frac{130729\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{220779\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}+625\sqrt{x^4+3x^2+4}\,x^3$$

```
[Out] (x*(99493 + 45779*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (5000*x*Sqrt
[4 + 3*x^2 + x^4])/3 + 625*x^3*Sqrt[4 + 3*x^2 + x^4] - (220779*x*
Sqrt[4 + 3*x^2 + x^4])/(28*(2 + x^2)) + (220779*(2 + x^2)*Sqrt[(4
 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])
/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (130729*(2 + x^2)*Sqrt[(4 +
 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(
12*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.238617, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{220779\sqrt{x^4+3x^2+4}\,x}{28\,(x^2+2)}+\frac{5000}{3}\sqrt{x^4+3x^2+4}\,x+\frac{(45779x^2+99493)\,x}{28\sqrt{x^4+3x^2+4}}-\frac{130729\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{220779\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}+625\sqrt{x^4+3x^2+4}\,x^3$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^5/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(99493 + 45779*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (5000*x*Sqrt
[4 + 3*x^2 + x^4])/3 + 625*x^3*Sqrt[4 + 3*x^2 + x^4] - (220779*x*
Sqrt[4 + 3*x^2 + x^4])/(28*(2 + x^2)) + (220779*(2 + x^2)*Sqrt[(4
 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])
/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (130729*(2 + x^2)*Sqrt[(4 +
 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(
12*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 55.2237, size = 216, normalized size = 0.99

$$625x^3\sqrt{x^4+3x^2+4}+\frac{x\,(11124297x^2+24176799)}{6804\sqrt{x^4+3x^2+4}}+\frac{5000x\sqrt{x^4+3x^2+4}}{3}-\frac{220779x\sqrt{x^4+3x^2+4}}{14\,(2x^2+4)}+\frac{220779\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{28\sqrt{x^4+3x^2+4}}-\frac{130729\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{24\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((5*x**2+7)**5/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] 625*x**3*sqrt(x**4 + 3*x**2 + 4) + x*(11124297*x**2 + 24176799)/(
6804*sqrt(x**4 + 3*x**2 + 4)) + 5000*x*sqrt(x**4 + 3*x**2 + 4)/3
- 220779*x*sqrt(x**4 + 3*x**2 + 4)/(14*(2*x**2 + 4)) + 220779*sqr
t(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*ellip
tic_e(2*atan(sqrt(2)*x/2), 1/8)/(28*sqrt(x**4 + 3*x**2 + 4)) - 13
0729*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 +
1)*elliptic_f(2*atan(sqrt(2)*x/2), 1/8)/(24*sqrt(x**4 + 3*x**2 +
4))
```

Mathematica [C]   time = 0.903403, size = 339, normalized size = 1.55

$$\frac{-\sqrt{2}\left(662337\sqrt{7}+975947i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+662337\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-}{\sqrt{7}-3i}}}{336\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(7 + 5*x^2)^5/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(858479 + 767337*x^2 + 297500*x^
4 + 52500*x^6) + 662337*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt
[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^
2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[
7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - Sqrt[2]*(975947*I + 6
62337*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])
]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*A
rcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + S
qrt[7])])/(336*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C]   time = 0.067, size = 379, normalized size = 1.7

$$-33614\frac{1}{\sqrt{x^4+3x^2+4}}\left(\frac{x}{56}+\frac{3x^3}{56}\right)$$
$$-\frac{505532}{21\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\mathit{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$+\frac{1766232}{7\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\left(\mathit{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)-\mathit{EllipticE}\right.$$
$$-120050\frac{-1/7x^3-3/14x}{\sqrt{x^4+3x^2+4}}-171500\frac{3/14x^3+4/7x}{\sqrt{x^4+3x^2+4}}-122500\frac{-1/14x^3-6/7x}{\sqrt{x^4+3x^2+4}}$$
$$-43750\frac{1}{\sqrt{x^4+3x^2+4}}\left(-\frac{9x^3}{14}+2/7x\right)+\frac{5000x}{3}\sqrt{x^4+3x^2+4}$$
$$-6250\frac{1}{\sqrt{x^4+3x^2+4}}\left(\frac{31x^3}{14}+\frac{18x}{7}\right)+625x^3\sqrt{x^4+3x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -33614*(1/56*x+3/56*x^3)/(x^4+3*x^2+4)^(1/2)-505532/21/(-6+2*I*7^
(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^
(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(
1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+1766232/7/(-6+2*I*7^(1/2))
^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))
*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-
```

```
6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(
-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))-120050*(-1/7*x^
3-3/14*x)/(x^4+3*x^2+4)^(1/2)-171500*(3/14*x^3+4/7*x)/(x^4+3*x^2+
4)^(1/2)-122500*(-1/14*x^3-6/7*x)/(x^4+3*x^2+4)^(1/2)-43750*(-9/1
4*x^3+2/7*x)/(x^4+3*x^2+4)^(1/2)+5000/3*x*(x^4+3*x^2+4)^(1/2)-625
0*(31/14*x^3+18/7*x)/(x^4+3*x^2+4)^(1/2)+625*x^3*(x^4+3*x^2+4)^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^5}{(x^4+3x^2+4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3125\,x^{10}+21875\,x^8+61250\,x^6+85750\,x^4+60025\,x^2+16807}{(x^4+3x^2+4)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3125*x^10 + 21875*x^8 + 61250*x^6 + 85750*x^4 + 60025*x
^2 + 16807)/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^5}{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**5/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**5/((x**2 - x + 2)*(x**2 + x + 2))**(3/2),
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^5}{(x^4+3x^2+4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.376 $\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=200

$$\frac{14523\sqrt{x^4+3x^2+4}\,x}{28\,(x^2+2)}+\frac{625}{3}\sqrt{x^4+3x^2+4}\,x+\frac{(2719-4023x^2)\,x}{28\sqrt{x^4+3x^2+4}}+\frac{4243\,(x^2+2)\,\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{14523\,(x^2+2)\,\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] (x*(2719 - 4023*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (625*x*Sqrt[4
+ 3*x^2 + x^4])/3 + (14523*x*Sqrt[4 + 3*x^2 + x^4])/(28*(2 + x^2)
) - (14523*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Elliptic
E[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) +
 (4243*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*
ArcTan[x/Sqrt[2]], 1/8])/(12*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.193057, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{14523\sqrt{x^4+3x^2+4}\,x}{28\,(x^2+2)}+\frac{625}{3}\sqrt{x^4+3x^2+4}\,x+\frac{(2719-4023x^2)\,x}{28\sqrt{x^4+3x^2+4}}+\frac{4243\,(x^2+2)\,\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{14523\,(x^2+2)\,\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^4/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(2719 - 4023*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (625*x*Sqrt[4
+ 3*x^2 + x^4])/3 + (14523*x*Sqrt[4 + 3*x^2 + x^4])/(28*(2 + x^2)
) - (14523*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Elliptic
E[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) +
 (4243*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*
ArcTan[x/Sqrt[2]], 1/8])/(12*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 41.7213, size = 197, normalized size = 0.98

$$\frac{x\left(-325863x^2+220239\right)}{2268\sqrt{x^4+3x^2+4}}+\frac{625x\sqrt{x^4+3x^2+4}}{3}+\frac{14523x\sqrt{x^4+3x^2+4}}{14\,(2x^2+4)}-\frac{14523\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{28\sqrt{x^4+3x^2+4}}+\frac{4243\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{24\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**4/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] x*(-325863*x**2 + 220239)/(2268*sqrt(x**4 + 3*x**2 + 4)) + 625*x*
sqrt(x**4 + 3*x**2 + 4)/3 + 14523*x*sqrt(x**4 + 3*x**2 + 4)/(14*(
2*x**2 + 4)) - 14523*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2/2 + 1
)**2)*(x**2/2 + 1)*elliptic_e(2*atan(sqrt(2)*x/2), 1/8)/(28*sqrt(
```

```
x**4 + 3*x**2 + 4)) + 4243*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/(x**2
/2 + 1)**2)*(x**2/2 + 1)*elliptic_f(2*atan(sqrt(2)*x/2), 1/8)/(24
*sqrt(x**4 + 3*x**2 + 4))
```

Mathematica [C] time = 0.797547, size = 333, normalized size = 1.66

$$\frac{\sqrt{2}\left(43569\sqrt{7}+186179i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-43569\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{}}{336\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^4/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(78157 + 40431*x^2 + 17500*x^4)
- 43569*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)
/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7
])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I -
Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(186179*I + 43569*Sqrt[7])*Sq
rt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqr
t[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I
)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(336*Sq
rt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.014, size = 339, normalized size = 1.7

$$-4802\,\frac{1}{\sqrt{x^4+3\,x^2+4}}\left(\frac{x}{56}+\frac{3\,x^3}{56}\right)$$
$$-\frac{27736}{21\sqrt{-6+2\,i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4},\frac{\sqrt{2+6\,i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3\,x^2+4}}$$
$$-\frac{116184}{7\sqrt{-6+2\,i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4},\frac{\sqrt{2+6\,i\sqrt{7}}}{4}\right)-Elliptic\right.$$
$$-13720\,\frac{-1/7\,x^3-3/14\,x}{\sqrt{x^4+3\,x^2+4}}-14700\,\frac{3/14\,x^3+4/7\,x}{\sqrt{x^4+3\,x^2+4}}-7000\,\frac{-1/14\,x^3-6/7\,x}{\sqrt{x^4+3\,x^2+4}}$$
$$-1250\,\frac{1}{\sqrt{x^4+3\,x^2+4}}\left(-\frac{9\,x^3}{14}+2/7\,x\right)+\frac{625\,x}{3}\sqrt{x^4+3\,x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -4802*(1/56*x+3/56*x^3)/(x^4+3*x^2+4)^(1/2)-27736/21/(-6+2*I*7^(1
/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1
/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/
2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-116184/7/(-6+2*I*7^(1/2))^(1
/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^
2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2
*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+
2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))-13720*(-1/7*x^3-3/
14*x)/(x^4+3*x^2+4)^(1/2)-14700*(3/14*x^3+4/7*x)/(x^4+3*x^2+4)^(1
/2)-7000*(-1/14*x^3-6/7*x)/(x^4+3*x^2+4)^(1/2)-1250*(-9/14*x^3+2/
7*x)/(x^4+3*x^2+4)^(1/2)+625/3*x*(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^4}{(x^4+3x^2+4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{625\,x^8+3500\,x^6+7350\,x^4+6860\,x^2+2401}{(x^4+3\,x^2+4)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)/(x^4 +
 3*x^2 + 4)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^4}{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**4/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**4/((x**2 - x + 2)*(x**2 + x + 2))**(3/2),
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^4}{(x^4+3x^2+4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.377 $\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=181

$$\frac{4449\sqrt{x^4+3x^2+4}\,x}{28\,(x^2+2)}-\frac{(949x^2+2323)\,x}{28\sqrt{x^4+3x^2+4}}+\frac{973\,(x^2+2)\,\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{4449\,(x^2+2)\,\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] -(x*(2323 + 949*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (4449*x*Sqrt[4
 + 3*x^2 + x^4])/(28*(2 + x^2)) - (4449*(2 + x^2)*Sqrt[(4 + 3*x^2
 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqr
t[2]*Sqrt[4 + 3*x^2 + x^4]) + (973*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^
4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(4*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.148654, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{4449\sqrt{x^4+3x^2+4}\,x}{28\,(x^2+2)}-\frac{(949x^2+2323)\,x}{28\sqrt{x^4+3x^2+4}}+\frac{973\,(x^2+2)\,\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{4449\,(x^2+2)\,\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^3/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] -(x*(2323 + 949*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (4449*x*Sqrt[4
 + 3*x^2 + x^4])/(28*(2 + x^2)) - (4449*(2 + x^2)*Sqrt[(4 + 3*x^2
 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqr
t[2]*Sqrt[4 + 3*x^2 + x^4]) + (973*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^
4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(4*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 31.3756, size = 178, normalized size = 0.98

$$-\frac{x\,(25623x^2+62721)}{756\sqrt{x^4+3x^2+4}}+\frac{4449x\sqrt{x^4+3x^2+4}}{14\,(2x^2+4)}-\frac{4449\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{28\sqrt{x^4+3x^2+4}}+\frac{973\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{8\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**3/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] -x*(25623*x**2 + 62721)/(756*sqrt(x**4 + 3*x**2 + 4)) + 4449*x*sq
rt(x**4 + 3*x**2 + 4)/(14*(2*x**2 + 4)) - 4449*sqrt(2)*sqrt((x**4
 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_e(2*atan(sq
rt(2)*x/2), 1/8)/(28*sqrt(x**4 + 3*x**2 + 4)) + 973*sqrt(2)*sqrt(
(x**4 + 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_f(2*at
```

```
an(sqrt(2)*x/2), 1/8)/(8*sqrt(x**4 + 3*x**2 + 4))
```

Mathematica [C] time = 0.690509, size = 328, normalized size = 1.81

$$\frac{-4\sqrt{-\frac{i}{\sqrt{7}-3i}}x\left(949x^2+2323\right)+\sqrt{2}\left(4449\sqrt{7}+3899i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-4449\sqrt{}}{112\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^3/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (-4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(2323 + 949*x^2) - 4449*Sqrt[2]
*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7
])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I
*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I +
 Sqrt[7])] + Sqrt[2]*(3899*I + 4449*Sqrt[7])*Sqrt[(-3*I + Sqrt[7]
 - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/
(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])
]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I + Sq
rt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.01, size = 301, normalized size = 1.7

$$-686\frac{1}{\sqrt{x^4+3x^2+4}}\left(\frac{x}{56}+\frac{3x^3}{56}\right)$$
$$+\frac{4724}{7\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$-\frac{35592}{7\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)-EllipticE\right.$$
$$-1470\frac{-1/7x^3-3/14x}{\sqrt{x^4+3x^2+4}}-1050\frac{3/14x^3+4/7x}{\sqrt{x^4+3x^2+4}}-250\frac{-1/14x^3-6/7x}{\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -686*(1/56*x+3/56*x^3)/(x^4+3*x^2+4)^(1/2)+4724/7/(-6+2*I*7^(1/2)
)^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2)
)*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))
^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-35592/7/(-6+2*I*7^(1/2))^(1/2)*
(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(
1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7
^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*
7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))-1470*(-1/7*x^3-3/14*x)
/(x^4+3*x^2+4)^(1/2)-1050*(3/14*x^3+4/7*x)/(x^4+3*x^2+4)^(1/2)-25
0*(-1/14*x^3-6/7*x)/(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5x^2+7\right)^3}{\left(x^4+3x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{125\,x^6 + 525\,x^4 + 735\,x^2 + 343}{\left(x^4 + 3\,x^2 + 4\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)/(x^4 + 3*x^2 + 4)^(3
/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2 + 7\right)^3}{\left(\left(x^2 - x + 2\right)\left(x^2 + x + 2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/((x**2 - x + 2)*(x**2 + x + 2))**(3/2),
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2 + 7\right)^3}{\left(x^4 + 3\,x^2 + 4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.378 $\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{113\sqrt{x^4+3x^2+4}\,x}{28\,(x^2+2)}-\frac{(9-113x^2)\,x}{28\sqrt{x^4+3x^2+4}}+\frac{9\,(x^2+2)\,\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{113\,(x^2+2)\,\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] -(x*(9 - 113*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) - (113*x*Sqrt[4 + 3
*x^2 + x^4])/(28*(2 + x^2)) + (113*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^
4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*
Sqrt[4 + 3*x^2 + x^4]) + (9*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
 x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(4*Sqrt[2]*Sqrt[4 +
 3*x^2 + x^4])
```

Rubi [A] time = 0.146023, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{113\sqrt{x^4+3x^2+4}\,x}{28\,(x^2+2)}-\frac{(9-113x^2)\,x}{28\sqrt{x^4+3x^2+4}}+\frac{9\,(x^2+2)\,\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{113\,(x^2+2)\,\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)^2/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] -(x*(9 - 113*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) - (113*x*Sqrt[4 + 3
*x^2 + x^4])/(28*(2 + x^2)) + (113*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^
4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*
Sqrt[4 + 3*x^2 + x^4]) + (9*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
 x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(4*Sqrt[2]*Sqrt[4 +
 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 31.6843, size = 178, normalized size = 0.98

$$-\frac{x\left(-1017x^2+81\right)}{252\sqrt{x^4+3x^2+4}}-\frac{113x\sqrt{x^4+3x^2+4}}{14\,(2x^2+4)}+\frac{113\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{28\sqrt{x^4+3x^2+4}}+\frac{9\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{8\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)**2/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] -x*(-1017*x**2 + 81)/(252*sqrt(x**4 + 3*x**2 + 4)) - 113*x*sqrt(x
**4 + 3*x**2 + 4)/(14*(2*x**2 + 4)) + 113*sqrt(2)*sqrt((x**4 + 3*
x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_e(2*atan(sqrt(2)
*x/2), 1/8)/(28*sqrt(x**4 + 3*x**2 + 4)) + 9*sqrt(2)*sqrt((x**4 +
 3*x**2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_f(2*atan(sqrt
```

```
(2)*x/2), 1/8)/(8*sqrt(x**4 + 3*x**2 + 4))
```

Mathematica [C] time = 0.703893, size = 329, normalized size = 1.82

$$\frac{4\sqrt{-\frac{i}{\sqrt{7}-3i}}x\left(113x^2-9\right)-\sqrt{2}\left(113\sqrt{7}+1043i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+113\sqrt{2}\left(\sqrt{7}+\right.}{112\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)^2/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(-9 + 113*x^2) + 113*Sqrt[2]*(3*
I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*
Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*Arc
Sinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqr
t[7])] - Sqrt[2]*(1043*I + 113*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2
*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I
+ Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x],
 (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I + Sqrt[7]
)]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.01, size = 278, normalized size = 1.5

$$-98\frac{1}{\sqrt{x^4+3x^2+4}}\left(\frac{x}{56}+\frac{3x^3}{56}\right)$$
$$+\frac{352}{7\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$+\frac{904}{7\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)-EllipticE\right.$$
$$-140\frac{-1/7x^3-3/14x}{\sqrt{x^4+3x^2+4}}-50\frac{3/14x^3+4/7x}{\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -98*(1/56*x+3/56*x^3)/(x^4+3*x^2+4)^(1/2)+352/7/(-6+2*I*7^(1/2))^
(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*
x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(
1/2),1/4*(2+6*I*7^(1/2))^(1/2))+904/7/(-6+2*I*7^(1/2))^(1/2)*(1-(
-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)
/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/
2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1
/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))-140*(-1/7*x^3-3/14*x)/(x^4
+3*x^2+4)^(1/2)-50*(3/14*x^3+4/7*x)/(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+7\right)^2}{\left(x^4+3x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{25\,x^{4}+70\,x^{2}+49}{\left(x^{4}+3\,x^{2}+4\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5x^{2}+7\right)^{2}}{\left(\left(x^{2}-x+2\right)\left(x^{2}+x+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)**2/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/((x**2 - x + 2)*(x**2 + x + 2))**(3/2),
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5\,x^{2}+7\right)^{2}}{\left(x^{4}+3\,x^{2}+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.379 $\int \frac{7+5x^2}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{19\sqrt{x^4+3x^2+4}\,x}{28(x^2+2)}+\frac{(19x^2+53)\,x}{28\sqrt{x^4+3x^2+4}}-\frac{3(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{19(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] (x*(53 + 19*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) - (19*x*Sqrt[4 + 3*x
^2 + x^4])/(28*(2 + x^2)) + (19*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqr
t[4 + 3*x^2 + x^4]) - (3*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^
2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(4*Sqrt[2]*Sqrt[4 + 3*
x^2 + x^4])
```

Rubi [A] time = 0.134979, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{19\sqrt{x^4+3x^2+4}\,x}{28(x^2+2)}+\frac{(19x^2+53)\,x}{28\sqrt{x^4+3x^2+4}}-\frac{3(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{19(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x^2)/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(53 + 19*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) - (19*x*Sqrt[4 + 3*x
^2 + x^4])/(28*(2 + x^2)) + (19*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqr
t[4 + 3*x^2 + x^4]) - (3*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^
2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(4*Sqrt[2]*Sqrt[4 + 3*
x^2 + x^4])
```

Rubi in Sympy [A] time = 26.0778, size = 178, normalized size = 0.98

$$\frac{x(19x^2+53)}{28\sqrt{x^4+3x^2+4}}-\frac{19x\sqrt{x^4+3x^2+4}}{14(2x^2+4)}+\frac{19\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{28\sqrt{x^4+3x^2+4}}-\frac{3\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{8\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+7)/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] x*(19*x**2 + 53)/(28*sqrt(x**4 + 3*x**2 + 4)) - 19*x*sqrt(x**4 +
3*x**2 + 4)/(14*(2*x**2 + 4)) + 19*sqrt(2)*sqrt((x**4 + 3*x**2 +
4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_e(2*atan(sqrt(2)*x/2),
1/8)/(28*sqrt(x**4 + 3*x**2 + 4)) - 3*sqrt(2)*sqrt((x**4 + 3*x**2
 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_f(2*atan(sqrt(2)*x/2
```

```
), 1/8)/(8*sqrt(x**4 + 3*x**2 + 4))
```

Mathematica [C] time = 0.800061, size = 329, normalized size = 1.82

$$\frac{4\sqrt{-\frac{i}{\sqrt{7}-3i}}x\left(19x^2+53\right)-\sqrt{2}\left(19\sqrt{7}+49i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+19\sqrt{2}\left(\sqrt{7}+3i\right)}{112\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x^2)/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(53 + 19*x^2) + 19*Sqrt[2]*(3*I
+ Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sq
rt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSi
nh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[
7])] - Sqrt[2]*(49*I + 19*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x
^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqr
t[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I
 - Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sq
rt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.008, size = 255, normalized size = 1.4

$$-14\frac{1}{\sqrt{x^4+3x^2+4}}\left(\frac{x}{56}+\frac{3x^3}{56}\right)$$
$$-\frac{4}{7\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\,\mathit{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$+\frac{152}{7\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\left(\mathit{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)-\mathit{EllipticE}\right.$$
$$-10\frac{-1/7x^3-3/14x}{\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -14*(1/56*x+3/56*x^3)/(x^4+3*x^2+4)^(1/2)-4/7/(-6+2*I*7^(1/2))^(1
/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^
2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/
2),1/4*(2+6*I*7^(1/2))^(1/2))+152/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3
/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(
x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2)
)^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2
))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))-10*(-1/7*x^3-3/14*x)/(x^4+3*
x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{5x^2+7}{\left(x^4+3x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 4)^(3/2),x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{5\,x^2+7}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 4)^(3/2),x, algorithm="fricas")

[Out] integral((5*x^2 + 7)/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2+7}{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+7)/(x**4+3*x**2+4)**(3/2),x)

[Out] Integral((5*x**2 + 7)/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^2+7}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 4)^(3/2),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.380 $\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=181

$$\frac{3\sqrt{x^4+3x^2+4}\,x}{28\,(x^2+2)}-\frac{(3x^2+1)\,x}{28\sqrt{x^4+3x^2+4}}+\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{3\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] -(x*(1 + 3*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (3*x*Sqrt[4 + 3*x^2
 + x^4])/(28*(2 + x^2)) - (3*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4
 + 3*x^2 + x^4]) + ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 +
 x^4])
```

Rubi [A] time = 0.11957, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{3\sqrt{x^4+3x^2+4}\,x}{28\,(x^2+2)}-\frac{(3x^2+1)\,x}{28\sqrt{x^4+3x^2+4}}+\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{3\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] -(x*(1 + 3*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (3*x*Sqrt[4 + 3*x^2
 + x^4])/(28*(2 + x^2)) - (3*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4
 + 3*x^2 + x^4]) + ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 +
 x^4])
```

Rubi in Sympy [A] time = 30.6522, size = 177, normalized size = 0.98

$$-\frac{x\,(3x^2+1)}{28\sqrt{x^4+3x^2+4}}+\frac{3x\sqrt{x^4+3x^2+4}}{14\,(2x^2+4)}-\frac{3\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{28\sqrt{x^4+3x^2+4}}+\frac{\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{8\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] -x*(3*x**2 + 1)/(28*sqrt(x**4 + 3*x**2 + 4)) + 3*x*sqrt(x**4 + 3*
x**2 + 4)/(14*(2*x**2 + 4)) - 3*sqrt(2)*sqrt((x**4 + 3*x**2 + 4)/
(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_e(2*atan(sqrt(2)*x/2), 1/8
)/(28*sqrt(x**4 + 3*x**2 + 4)) + sqrt(2)*sqrt((x**4 + 3*x**2 + 4)
/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_f(2*atan(sqrt(2)*x/2), 1/
```

```
8)/(8*sqrt(x**4 + 3*x**2 + 4))
```

Mathematica [C] time = 0.669947, size = 328, normalized size = 1.81

$$\frac{-4\sqrt{-\frac{i}{\sqrt{7}-3i}}x\left(3x^2+1\right)+\sqrt{2}\left(3\sqrt{7}-7i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-3\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{-\cdots}}{112\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] (-4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(1 + 3*x^2) - 3*Sqrt[2]*(3*I +
Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt
[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh
[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7]
)] + Sqrt[2]*(-7*I + 3*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)
/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7
])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I -
Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[
4 + 3*x^2 + x^4])
```

Maple [C] time = 0.006, size = 232, normalized size = 1.3

$$-2\frac{1}{\sqrt{x^4+3x^2+4}}\left(\frac{x}{56}+\frac{3x^3}{56}\right)$$
$$+\frac{8}{7\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}$$
$$-\frac{24}{7\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)-EllipticE\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -2*(1/56*x+3/56*x^3)/(x^4+3*x^2+4)^(1/2)+8/7/(-6+2*I*7^(1/2))^(1/
2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2
)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2
),1/4*(2+6*I*7^(1/2))^(1/2))-24/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8
+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^
4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^
(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))
^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(x^4+3x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(-3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(-3/2),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 4)^(-3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(x^4+3x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((x**4 + 3*x**2 + 4)**(-3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 3*x^2 + 4)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(-3/2), x)
```

3.381 $\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=284

$$\frac{\sqrt{x^4+3x^2+4}\,x}{77(x^2+2)}-\frac{(4x^2+13)\,x}{308\sqrt{x^4+3x^2+4}}+\frac{25}{176}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{77\sqrt{x^4+3x^2+4}}+\frac{425(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3696\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] -(x*(13 + 4*x^2))/(308*Sqrt[4 + 3*x^2 + x^4]) + (x*Sqrt[4 + 3*x^2
 + x^4])/(77*(2 + x^2)) + (25*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)
/Sqrt[4 + 3*x^2 + x^4]])/176 - (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2
 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(77*Sqr
t[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)
^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(12*Sqrt[2]*Sqrt[4 + 3*x
^2 + x^4]) + (425*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*E
llipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(3696*Sqrt[2]*Sqrt[4
 + 3*x^2 + x^4])
```

Rubi [A] time = 0.271502, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\sqrt{x^4+3x^2+4}\,x}{77(x^2+2)}-\frac{(4x^2+13)\,x}{308\sqrt{x^4+3x^2+4}}+\frac{25}{176}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{77\sqrt{x^4+3x^2+4}}+\frac{425(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3696\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] -(x*(13 + 4*x^2))/(308*Sqrt[4 + 3*x^2 + x^4]) + (x*Sqrt[4 + 3*x^2
 + x^4])/(77*(2 + x^2)) + (25*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)
/Sqrt[4 + 3*x^2 + x^4]])/176 - (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2
 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(77*Sqr
t[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)
^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(12*Sqrt[2]*Sqrt[4 + 3*x
^2 + x^4]) + (425*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*E
llipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(3696*Sqrt[2]*Sqrt[4
 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 78.0203, size = 279, normalized size = 0.98

$$-\frac{x\left(16x^2+52\right)}{1232\sqrt{x^4+3x^2+4}}+\frac{2x\sqrt{x^4+3x^2+4}}{77\left(2x^2+4\right)}-\frac{\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{77\sqrt{x^4+3x^2+4}}-\frac{\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{24\sqrt{x^4+3x^2+4}}+\frac{425\sqrt{2}\sqrt{\frac{x^4+3x^2+4}{\left(\frac{x^2}{2}+1\right)^2}}\left(\frac{x^2}{2}+1\right)\Pi\left(-\frac{9}{280};2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|\frac{1}{8}\right)}{7392\sqrt{x^4+3x^2+4}}+\frac{25\sqrt{385}\,\mathrm{atan}\left(\frac{2\sqrt{385}x}{35\sqrt{x^4+3x^2+4}}\right)}{13552}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**2+7)/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] -x*(16*x**2 + 52)/(1232*sqrt(x**4 + 3*x**2 + 4)) + 2*x*sqrt(x**4
+ 3*x**2 + 4)/(77*(2*x**2 + 4)) - sqrt(2)*sqrt((x**4 + 3*x**2 + 4
)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_e(2*atan(sqrt(2)*x/2), 1
/8)/(77*sqrt(x**4 + 3*x**2 + 4)) - sqrt(2)*sqrt((x**4 + 3*x**2 +
4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_f(2*atan(sqrt(2)*x/2),
1/8)/(24*sqrt(x**4 + 3*x**2 + 4)) + 425*sqrt(2)*sqrt((x**4 + 3*x*
*2 + 4)/(x**2/2 + 1)**2)*(x**2/2 + 1)*elliptic_pi(-9/280, 2*atan(
sqrt(2)*x/2), 1/8)/(7392*sqrt(x**4 + 3*x**2 + 4)) + 25*sqrt(385)*
atan(2*sqrt(385)*x/(35*sqrt(x**4 + 3*x**2 + 4)))/13552
```

Mathematica [C] time = 0.955656, size = 483, normalized size = 1.7

$$\frac{-8\sqrt{-\frac{i}{\sqrt{7}-3i}}x^3+\sqrt{2}\left(2\sqrt{7}+7i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-2\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}}{616}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-26*Sqrt[(-I)/(-3*I + Sqrt[7])]*x - 8*Sqrt[(-I)/(-3*I + Sqrt[7])
]*x^3 - 2*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^
2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt
[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I
- Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(7*I + 2*Sqrt[7])*Sqrt[(-3*
I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] +
(2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I
 + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - (25*I)*Sqrt[2
]*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I +
 Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[
7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[
7])/(3*I + Sqrt[7])])/(616*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3
*x^2 + x^4])
```

Maple [C] time = 0.028, size = 409, normalized size = 1.4

$$-2\frac{1}{\sqrt{x^4+3x^2+4}}\left(\frac{x^3}{154}+\frac{13x}{616}\right)-\frac{1}{77\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}-\frac{32}{77\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticF\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}+\frac{32}{77\sqrt{-6+2i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticE\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}+\frac{25}{308\sqrt{-3/8+i/8\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticPi\left(\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}x,-\frac{5}{-\frac{21}{8}+\frac{7i}{8}\sqrt{7}},\frac{\sqrt{-\frac{3}{8}-\frac{i}{8}\sqrt{7}}}{\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}}\right)\frac{1}{\sqrt{x^4 \cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -2*(1/154*x^3+13/616*x)/(x^4+3*x^2+4)^(1/2)-1/77/(-6+2*I*7^(1/2))
^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7
^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2)
)^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-32/77/(-6+2*I*7^(1/2))^(1/2)*(
1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^
(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7
^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+32/77/(-6+2*I*7^(1/2))^(
1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(
1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6
+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+25/308/(-3/8+1/8*I
*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/
8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I
*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^
(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+3x^2+4)^{\frac{3}{2}}(5x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(5x^6+22x^4+41x^2+28)\sqrt{x^4+3x^2+4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)),x, algorithm="fricas")
```

```
[Out] integral(1/((5*x^6 + 22*x^4 + 41*x^2 + 28)*sqrt(x^4 + 3*x^2 + 4))
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x^{2}-x+2\right)\left(x^{2}+x+2\right)\right)^{\frac{3}{2}}\left(5x^{2}+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x**2+7)/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)),
 x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^{4}+3\,x^{2}+4\right)^{\frac{3}{2}}\left(5\,x^{2}+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)), x)
```

3.382 $\int \frac{1}{(7+5x^2)^2(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=312

$$-\frac{199\sqrt{x^4+3x^2+4}\,x}{27104\,(x^2+2)}+\frac{625\sqrt{x^4+3x^2+4}\,x}{27104\,(5x^2+7)}+\frac{(37x^2+24)\,x}{13552\sqrt{x^4+3x^2+4}}+\frac{575\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{108416}-\frac{2\sqrt{2}\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{231\sqrt{x^4+3x^2+4}}+\frac{199\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13552\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{9775\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2276736\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] (x*(24 + 37*x^2))/(13552*Sqrt[4 + 3*x^2 + x^4]) - (199*x*Sqrt[4 +
 3*x^2 + x^4])/(27104*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2 + x^4])/
(27104*(7 + 5*x^2)) + (575*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sq
rt[4 + 3*x^2 + x^4]])/108416 + (199*(2 + x^2)*Sqrt[(4 + 3*x^2 + x
^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(13552*Sqrt
[2]*Sqrt[4 + 3*x^2 + x^4]) - (2*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2
 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(231*Sq
rt[4 + 3*x^2 + x^4]) + (9775*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(2276736*
Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.625473, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$-\frac{199\sqrt{x^4+3x^2+4}\,x}{27104\,(x^2+2)}+\frac{625\sqrt{x^4+3x^2+4}\,x}{27104\,(5x^2+7)}+\frac{(37x^2+24)\,x}{13552\sqrt{x^4+3x^2+4}}+\frac{575\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{108416}-\frac{2\sqrt{2}\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{231\sqrt{x^4+3x^2+4}}+\frac{199\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13552\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{9775\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2276736\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2)),x]
```


```
[Out] (x*(24 + 37*x^2))/(13552*Sqrt[4 + 3*x^2 + x^4]) - (199*x*Sqrt[4 +
 3*x^2 + x^4])/(27104*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2 + x^4])/
(27104*(7 + 5*x^2)) + (575*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sq
rt[4 + 3*x^2 + x^4]])/108416 + (199*(2 + x^2)*Sqrt[(4 + 3*x^2 + x
^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(13552*Sqrt
[2]*Sqrt[4 + 3*x^2 + x^4]) - (2*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2
 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(231*Sq
rt[4 + 3*x^2 + x^4]) + (9775*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(2276736*
Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**2+7)**2/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

Mathematica [C] time = 0.78243, size = 311, normalized size = 1.

$$\frac{28x\left(995x^4+2633x^2+2836\right)+i\sqrt{6+2i\sqrt{7}}\left(5x^2+7\right)\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(7\left(101+199i\sqrt{7}\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)\right|\right.}{758912\left(5x^2+7\right)\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (28*x*(2836 + 2633*x^2 + 995*x^4) + I*Sqrt[6 + (2*I)*Sqrt[7]]*(7
+ 5*x^2)*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x
^2)/(3*I + Sqrt[7])]*(1393*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sq
rt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]
+ 7*(101 + (199*I)*Sqrt[7])*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I
 + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - 1150*Elliptic
Pi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])
]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(758912*(7 + 5*x^2)*Sqrt
[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.035, size = 433, normalized size = 1.4

$$\frac{625\,x}{135520\,x^2+189728}\sqrt{x^4+3\,x^2+4}-2\,\frac{1}{\sqrt{x^4+3\,x^2+4}}\left(-\frac{37\,x^3}{27104}-\frac{3\,x}{3388}\right)$$
$$-\frac{349}{6776\,\sqrt{-6+2\,i\sqrt{7}}}\sqrt{1+\frac{3\,x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3\,x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticF\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4},\frac{\sqrt{2+6\,i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3\,x^2+4}}$$
$$+\frac{199}{847\,\sqrt{-6+2\,i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3\,x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3\,x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticF\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4},\frac{\sqrt{2+6\,i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3\,x^2+4}}$$
$$-\frac{199}{847\,\sqrt{-6+2\,i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3\,x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3\,x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticE\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4},\frac{\sqrt{2+6\,i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3\,x^2+}}$$
$$+\frac{575}{189728\,\sqrt{-3/8+i/8\sqrt{7}}}\sqrt{1+\frac{3\,x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3\,x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticPi\left(\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}x,-\frac{5}{-\frac{21}{8}+\frac{7\,i}{8}\sqrt{7}},\frac{\sqrt{-\frac{3}{8}-\frac{i}{8}\sqrt{7}}}{\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] 625/27104*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)-2*(-37/27104*x^3-3/3388
*x)/(x^4+3*x^2+4)^(1/2)-349/6776/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^
2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x
^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6
*I*7^(1/2))^(1/2))+199/847/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*
I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x
^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),
1/4*(2+6*I*7^(1/2))^(1/2))-199/847/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*
x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/
(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2)
)^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+575/189728/(-3/8+1/8*I*7^(1/2)
)^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*
7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2)
)^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-
```

```
3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+3x^2+4)^{\frac{3}{2}}(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(25x^8+145x^6+359x^4+427x^2+196)\sqrt{x^4+3x^2+4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2),x, algorithm="fricas")
```

```
[Out] integral(1/((25*x^8 + 145*x^6 + 359*x^4 + 427*x^2 + 196)*sqrt(x^4
 + 3*x^2 + 4)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}}(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x**2+7)**2/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**
2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+3x^2+4)^{\frac{3}{2}}(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2), x)
```

3.383 $\int \frac{1}{(7+5x^2)^3(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=340

$$-\frac{18159\sqrt{x^4+3x^2+4}\,x}{33392128\,(x^2+2)}+\frac{51875\sqrt{x^4+3x^2+4}\,x}{33392128\,(5x^2+7)}+\frac{625\sqrt{x^4+3x^2+4}\,x}{54208\,(5x^2+7)^2}+\frac{(139x^2+548)\,x}{596288\sqrt{x^4+3x^2+4}}-\frac{529425\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{133568512}+\frac{843\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{379456\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{18159\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{16696064\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{3000075\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{934979584\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] (x*(548 + 139*x^2))/(596288*Sqrt[4 + 3*x^2 + x^4]) - (18159*x*Sqr
t[4 + 3*x^2 + x^4])/(33392128*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2
+ x^4])/(54208*(7 + 5*x^2)^2) + (51875*x*Sqrt[4 + 3*x^2 + x^4])/(
33392128*(7 + 5*x^2)) - (529425*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*
x)/Sqrt[4 + 3*x^2 + x^4]])/133568512 + (18159*(2 + x^2)*Sqrt[(4 +
 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(
16696064*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (843*(2 + x^2)*Sqrt[(4
+ 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/
(379456*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (3000075*(2 + x^2)*Sqrt[
(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt
[2]], 1/8])/(934979584*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 1.21423, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$-\frac{18159\sqrt{x^4+3x^2+4}\,x}{33392128\,(x^2+2)}+\frac{51875\sqrt{x^4+3x^2+4}\,x}{33392128\,(5x^2+7)}+\frac{625\sqrt{x^4+3x^2+4}\,x}{54208\,(5x^2+7)^2}+\frac{(139x^2+548)\,x}{596288\sqrt{x^4+3x^2+4}}-\frac{529425\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{133568512}+\frac{843\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{379456\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{18159\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{16696064\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{3000075\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{934979584\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2)),x]
```


```
[Out] (x*(548 + 139*x^2))/(596288*Sqrt[4 + 3*x^2 + x^4]) - (18159*x*Sqr
t[4 + 3*x^2 + x^4])/(33392128*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2
+ x^4])/(54208*(7 + 5*x^2)^2) + (51875*x*Sqrt[4 + 3*x^2 + x^4])/(
33392128*(7 + 5*x^2)) - (529425*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*
x)/Sqrt[4 + 3*x^2 + x^4]])/133568512 + (18159*(2 + x^2)*Sqrt[(4 +
 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(
16696064*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (843*(2 + x^2)*Sqrt[(4
+ 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/
(379456*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (3000075*(2 + x^2)*Sqrt[
(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt
[2]], 1/8])/(934979584*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**2+7)**3/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.01467, size = 320, normalized size = 0.94

$$\frac{28x\left(453975x^6+2838330x^4+5811451x^2+4496212\right)+3i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(5x^2+7\right)^2\left(7i\left(6053\sqrt{7}+23633\right.\right.}{93497}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (28*x*(4496212 + 5811451*x^2 + 2838330*x^4 + 453975*x^6) + (3*I)*
Sqrt[6 + (2*I)*Sqrt[7]]*(7 + 5*x^2)^2*Sqrt[1 - ((2*I)*x^2)/(-3*I
+ Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*(42371*(3 - I*S
qrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*
I - Sqrt[7])/(3*I + Sqrt[7])] + (7*I)*(23633*I + 6053*Sqrt[7])*El
lipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7
])/(3*I + Sqrt[7])] + 352950*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I
*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I +
 Sqrt[7])]))/(934979584*(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.037, size = 457, normalized size = 1.3

$$\frac{625\,x}{54208\,\left(5\,x^2+7\right)^2}\sqrt{x^4+3\,x^2+4}+\frac{51875\,x}{166960640\,x^2+233744896}\sqrt{x^4+3\,x^2+4}$$
$$-2\,\frac{1}{\sqrt{x^4+3\,x^2+4}}\left(-\frac{139\,x^3}{1192576}-\frac{137\,x}{298144}\right)$$
$$+\frac{1173}{1192576\,\sqrt{-6+2\,i\sqrt{7}}}\sqrt{1+\frac{3\,x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3\,x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticF\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4},\frac{\sqrt{2+6\,i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3\,x^2+4}}$$
$$+\frac{18159}{1043504\,\sqrt{-6+2\,i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3\,x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3\,x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticF\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4},\frac{\sqrt{2+6\,i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3\,x}}$$
$$-\frac{18159}{1043504\,\sqrt{-6+2\,i\sqrt{7}}\left(i\sqrt{7}+3\right)}\sqrt{1+\frac{3\,x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3\,x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticE\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4},\frac{\sqrt{2+6\,i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3}}$$
$$-\frac{529425}{233744896\,\sqrt{-3/8+i/8\sqrt{7}}}\sqrt{1+\frac{3\,x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3\,x^2}{8}+\frac{i}{8}x^2\sqrt{7}}EllipticPi\left(\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}x,-\frac{5}{-\frac{21}{8}+\frac{7\,i}{8}\sqrt{7}},\frac{\sqrt{-\frac{3}{8}-\frac{i}{8}\sqrt{7}}}{\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] 625/54208*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+51875/33392128*x*(x^4
+3*x^2+4)^(1/2)/(5*x^2+7)-2*(-139/1192576*x^3-137/298144*x)/(x^4+
3*x^2+4)^(1/2)+1173/1192576/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8
*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*
```

```
x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^
(1/2))^(1/2))+18159/1043504/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8
*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*
x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2)
,1/4*(2+6*I*7^(1/2))^(1/2))-18159/1043504/(-6+2*I*7^(1/2))^(1/2)*
(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))
^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*
7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-529425/233744896/(-3/8+
1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x
^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+
1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1
/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+3x^2+4)^{\frac{3}{2}}(5x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(125x^{10}+900x^8+2810x^6+4648x^4+3969x^2+1372)\sqrt{x^4+3x^2+4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3),x, algorithm="fricas")
```

```
[Out] integral(1/((125*x^10 + 900*x^8 + 2810*x^6 + 4648*x^4 + 3969*x^2
+ 1372)*sqrt(x^4 + 3*x^2 + 4)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}}(5x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**
3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+3x^2+4)^{\frac{3}{2}}(5x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3),x, algorithm="giac")

[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3), x)
```

3.384 $\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=467

$$\frac{ex\sqrt{a+bx^2+cx^4}\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{11/4}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(e\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)+\frac{\sqrt{c}(4abe^3-15acde^2+15c^2d^3)}{\sqrt{a}}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2\right.\right.}{30c^{11/4}\sqrt{a+bx^2+cx^4}} + \frac{e^2x\sqrt{a+bx^2+cx^4}(15cd-4be)}{15c^2} + \frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c}$$

```
[Out] (e^2*(15*c*d - 4*b*e)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^2) + (e^3*
x^3*Sqrt[a + b*x^2 + c*x^4])/(5*c) + (e*(45*c^2*d^2 + 8*b^2*e^2 -
 3*c*e*(10*b*d + 3*a*e))*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(5/2)*(
Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*
c*e*(10*b*d + 3*a*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a
^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(11/4)*Sqrt[a + b*x^
2 + c*x^4]) + (a^(1/4)*((Sqrt[c]*(15*c^2*d^3 - 15*a*c*d*e^2 + 4*a
*b*e^3))/Sqrt[a] + e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*
a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b
/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(11/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.850589, antiderivative size = 467, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{ex\sqrt{a+bx^2+cx^4}\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{11/4}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(e\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)+\frac{\sqrt{c}(4abe^3-15acde^2+15c^2d^3)}{\sqrt{a}}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2\right.\right.}{30c^{11/4}\sqrt{a+bx^2+cx^4}} + \frac{e^2x\sqrt{a+bx^2+cx^4}(15cd-4be)}{15c^2} + \frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(d + e*x^2)^3/Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] (e^2*(15*c*d - 4*b*e)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^2) + (e^3*
x^3*Sqrt[a + b*x^2 + c*x^4])/(5*c) + (e*(45*c^2*d^2 + 8*b^2*e^2 -
 3*c*e*(10*b*d + 3*a*e))*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(5/2)*(
Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*
c*e*(10*b*d + 3*a*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a
^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(11/4)*Sqrt[a + b*x^
2 + c*x^4]) + (a^(1/4)*((Sqrt[c]*(15*c^2*d^3 - 15*a*c*d*e^2 + 4*a
*b*e^3))/Sqrt[a] + e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*
a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b
/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(11/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 105.783, size = 444, normalized size = 0.95

$$-\frac{\sqrt[4]{a}e\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(8b^2e^2+45c^2d^2-3ce\left(3ae+10bd\right)\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{15c^{\frac{11}{4}}\sqrt{a+bx^2+cx^4}}+\frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c}-\frac{e^2x\left(4be-15cd\right)\sqrt{a+bx^2+cx^4}}{15c^2}+\frac{ex\sqrt{a+bx^2+cx^4}\left(8b^2e^2+45c^2d^2-3ce\left(3ae+10bd\right)\right)}{15c^{\frac{5}{2}}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}e\left(8b^2e^2+45c^2d^2-3ce\left(3ae+10bd\right)\right)+\sqrt{c}\left(4abe^3-15acde^2+15c^2d^3\right)\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{30\sqrt[4]{a}c^{\frac{11}{4}}\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -a**(1/4)*e*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**
2)*(sqrt(a) + sqrt(c)*x**2)*(8*b**2*e**2 + 45*c**2*d**2 - 3*c*e*(
3*a*e + 10*b*d))*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/
(4*sqrt(a)*sqrt(c)))/(15*c**(11/4)*sqrt(a + b*x**2 + c*x**4)) + e
**3*x**3*sqrt(a + b*x**2 + c*x**4)/(5*c) - e**2*x*(4*b*e - 15*c*d
)*sqrt(a + b*x**2 + c*x**4)/(15*c**2) + e*x*sqrt(a + b*x**2 + c*x
**4)*(8*b**2*e**2 + 45*c**2*d**2 - 3*c*e*(3*a*e + 10*b*d))/(15*c*
*(5/2)*(sqrt(a) + sqrt(c)*x**2)) + sqrt((a + b*x**2 + c*x**4)/(sq
rt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(sqrt(a)*e*(8*
b**2*e**2 + 45*c**2*d**2 - 3*c*e*(3*a*e + 10*b*d)) + sqrt(c)*(4*a
*b*e**3 - 15*a*c*d*e**2 + 15*c**2*d**3))*elliptic_f(2*atan(c**(1/
4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(30*a**(1/4)*c**(11/
4)*sqrt(a + b*x**2 + c*x**4))
```

Mathematica [C] time = 5.23951, size = 584, normalized size = 1.25

$$ie\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^3/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e^2*x*(a + b*x^2 + c*x^4)*(-
4*b*e + 3*c*(5*d + e*x^2)) + I*(-b + Sqrt[b^2 - 4*a*c])*e*(45*c^2
*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e))*Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2
- 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[
Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c
])/(b - Sqrt[b^2 - 4*a*c])] - I*(30*c^3*d^3 + 8*b^2*(-b + Sqrt[b^
2 - 4*a*c])*e^3 + 15*c^2*d*e*(-3*b*d + 3*Sqrt[b^2 - 4*a*c]*d - 2*
a*e) + c*e^2*(30*b^2*d - 30*b*Sqrt[b^2 - 4*a*c]*d + 17*a*b*e - 9*
a*Sqrt[b^2 - 4*a*c]*e))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)
/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b +
 Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
4*a*c])])/(60*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2
+ c*x^4])
```

Maple [B] time = 0.023, size = 1186, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*d^3*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+
e^3*(1/5/c*x^3*(c*x^4+b*x^2+a)^(1/2)-4/15*b/c^2*x*(c*x^4+b*x^2+a)
^(1/2)+1/15*b/c^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-
2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/
a/c)^(1/2))-1/2*(-3/5/c*a+8/15*b^2/c^2)*a*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2
*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4
*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Ellipti
cE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))-3/2*d^2*e*a*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(
b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-El
lipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))+3*e^2*d*(1/3/c*x*(c*x^4+b*
x^2+a)^(1/2)-1/12/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(
4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2)
)/a/c)^(1/2))+1/3*b/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(
EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4
+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^3}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)/sqrt(c*x^4 +
 b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**3/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^3}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

3.385 $\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=356

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}(3cd^2-ae^2)}{\sqrt{a}}+2e(3cd-be)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2ex\sqrt{a+bx^2+cx^4}(3cd-be)}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{2\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(3cd-be)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{e^2x\sqrt{a+bx^2+cx^4}}{3c}$$

```
[Out] (e^2*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) + (2*e*(3*c*d - b*e)*x*Sqrt
[a + b*x^2 + c*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(
1/4)*e*(3*c*d - b*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(7/4)*Sqrt[a + b*x^2 +
 c*x^4]) + (a^(1/4)*(2*e*(3*c*d - b*e) + (Sqrt[c]*(3*c*d^2 - a*e^
2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sq
rt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
(2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.46093, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}(3cd^2-ae^2)}{\sqrt{a}}+2e(3cd-be)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2ex\sqrt{a+bx^2+cx^4}(3cd-be)}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{2\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(3cd-be)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{e^2x\sqrt{a+bx^2+cx^4}}{3c}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(d + e*x^2)^2/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (e^2*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) + (2*e*(3*c*d - b*e)*x*Sqrt
[a + b*x^2 + c*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(
1/4)*e*(3*c*d - b*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(7/4)*Sqrt[a + b*x^2 +
 c*x^4]) + (a^(1/4)*(2*e*(3*c*d - b*e) + (Sqrt[c]*(3*c*d^2 - a*e^
2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sq
rt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
(2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 54.2477, size = 328, normalized size = 0.92

$$\frac{2\sqrt[4]{a}e\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)(be-3cd)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{3c^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}}+\frac{e^2x\sqrt{a+bx^2+cx^4}}{3c}-\frac{2ex(be-3cd)\sqrt{a+bx^2+cx^4}}{3c^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-\sqrt{a}e(2be-6cd)-a\sqrt{c}e^2+3c^{\frac{3}{2}}d^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{6\sqrt[4]{a}c^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] 2*a**(1/4)*e*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)*
*2)*(sqrt(a) + sqrt(c)*x**2)*(b*e - 3*c*d)*elliptic_e(2*atan(c**(
1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(3*c**(7/4)*sqrt(a
 + b*x**2 + c*x**4)) + e**2*x*sqrt(a + b*x**2 + c*x**4)/(3*c) - 2
*e*x*(b*e - 3*c*d)*sqrt(a + b*x**2 + c*x**4)/(3*c**(3/2)*(sqrt(a)
 + sqrt(c)*x**2)) + sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)
*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(-sqrt(a)*e*(2*b*e - 6*c*d) -
 a*sqrt(c)*e**2 + 3*c**(3/2)*d**2)*elliptic_f(2*atan(c**(1/4)*x/a
**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(6*a**(1/4)*c**(7/4)*sqrt(
a + b*x**2 + c*x**4))
```

Mathematica [C] time = 2.98893, size = 488, normalized size = 1.37

$$i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\left(ce\left(-3d\sqrt{b^2-4ac}+ae+3bd\right)+be^2\left(\sqrt{b^2-4ac}-b\right)-3c^2d^2\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{}{b}}\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^2/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e^2*x*(a + b*x^2 + c*x^4) -
I*(-b + Sqrt[b^2 - 4*a*c])*e*(-3*c*d + b*e)*Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2
 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh
[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-3*c^2*d^2 + b*(-b + Sqrt[b^2 -
 4*a*c])*e^2 + c*e*(3*b*d - 3*Sqrt[b^2 - 4*a*c]*d + a*e))*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*
b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ellip
ticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + S
qrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(6*c^2*Sqrt[c/(b + Sq
rt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [B] time = 0.012, size = 756, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*d^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+
e^2*(1/3/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12/c*a*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4
+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Elli
pticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/3*b/c*a*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*
(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b
+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(
1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Ell
ipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*
b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))-d*e*a*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(
4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+
(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Elli
pticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^4+2\,dex^2+d^2}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**2/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2}{\sqrt{cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

3.386 $\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=283

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{ex\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[Out] (e*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) -
 (a^(1/4)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (
2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) +
(a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sq
rt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.229919, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{ex\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (e*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) -
 (a^(1/4)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (
2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) +
(a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sq
rt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 30.1592, size = 257, normalized size = 0.91

$$-\frac{\sqrt[4]{a}e\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{c^{\frac{3}{4}}\sqrt{a+bx^2+cx^4}} + \frac{ex\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}e+\sqrt{c}d\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2\sqrt[4]{a}c^{\frac{3}{4}}\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -a**(1/4)*e*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**
2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)
), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(c**(3/4)*sqrt(a + b*x**2 + c*x**
4)) + e*x*sqrt(a + b*x**2 + c*x**4)/(sqrt(c)*(sqrt(a) + sqrt(c)*x
**2)) + sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(
sqrt(a) + sqrt(c)*x**2)*(sqrt(a)*e + sqrt(c)*d)*elliptic_f(2*atan
(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(2*a**(1/4)*c
**(3/4)*sqrt(a + b*x**2 + c*x**4))
```

Mathematica [C] time = 0.430882, size = 302, normalized size = 1.07

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(\left(e\left(b-\sqrt{b^2-4ac}\right)-2cd\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+e\left(\sqrt{b^2-4ac}\right.\right.}{2\sqrt{2}c\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*((-b + Sqrt[b^2
 - 4*a*c])*e*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (-
2*c*d + (b - Sqrt[b^2 - 4*a*c])*e)*EllipticF[I*ArcSinh[Sqrt[2]*Sq
rt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sq
rt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.006, size = 362, normalized size = 1.3

$$\frac{d\sqrt{2}}{4}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\,\frac{b\left(b\right.}{}}\right.$$

$$-\frac{ae\sqrt{2}}{2}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+
b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/
2*e*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+
b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^
(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^
2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(
1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + d}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")

[Out] integral((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral((d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")

[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.387 $\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=386

$$\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{ae}{d}-b+\frac{cd}{e}}}{\sqrt{a+bx^2+cx^4}}\right)}{2d\sqrt{\frac{ae}{d}-b+\frac{cd}{e}}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$
$$-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)}$$

```
[Out] ArcTan[(Sqrt[-b + (c*d)/e + (a*e)/d]*x)/Sqrt[a + b*x^2 + c*x^4]]/
(2*d*Sqrt[-b + (c*d)/e + (a*e)/d]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2
*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^2 + c*x^4]) - (((Sq
rt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[a]*((Sqrt[c]*d
)/Sqrt[a] - e)^2)/(4*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*((Sqrt[c]*d)/
Sqrt[a] - e)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.500089, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{ae}{d}-b+\frac{cd}{e}}}{\sqrt{a+bx^2+cx^4}}\right)}{2d\sqrt{\frac{ae}{d}-b+\frac{cd}{e}}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$
$$-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{c}d\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[-b + (c*d)/e + (a*e)/d]*x)/Sqrt[a + b*x^2 + c*x^4]]/
(2*d*Sqrt[-b + (c*d)/e + (a*e)/d]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2
*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1
/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b
*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d -
 Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*c^(1/4)*d*(Sqrt[c]*d - Sqrt
[a]*e)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 29.3117, size = 332, normalized size = 0.86

$$\frac{\operatorname{atan}\left(\frac{x\sqrt{\frac{ae}{d}-b+\frac{cd}{e}}}{\sqrt{a+bx^2+cx^4}}\right)}{2d\sqrt{\frac{ae}{d}-b+\frac{cd}{e}}}-\frac{\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}e-\sqrt{c}d\right)\sqrt{a+bx^2+cx^4}}+\frac{\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}e+\sqrt{c}d\right)\Pi\left(-\frac{\sqrt{a}\left(e-\frac{\sqrt{c}d}{\sqrt{a}}\right)^2}{4\sqrt{c}de};2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{a}e-\sqrt{c}d\right)\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] atan(x*sqrt(a*e/d - b + c*d/e)/sqrt(a + b*x**2 + c*x**4))/(2*d*sq
rt(a*e/d - b + c*d/e)) - c**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqr
t(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_f(2*at
an(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(2*a**(1/4)
*(sqrt(a)*e - sqrt(c)*d)*sqrt(a + b*x**2 + c*x**4)) + sqrt((a + b
*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x
**2)*(sqrt(a)*e + sqrt(c)*d)*elliptic_pi(-sqrt(a)*(e - sqrt(c)*d/
sqrt(a))**2/(4*sqrt(c)*d*e), 2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b
/(4*sqrt(a)*sqrt(c)))/(4*a**(1/4)*c**(1/4)*d*(sqrt(a)*e - sqrt(c)
*d)*sqrt(a + b*x**2 + c*x**4))
```

Mathematica [C] time = 0.22829, size = 214, normalized size = 0.55

$$-\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}d\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticPi[((b +
 Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*Sqrt[a + b*x^2 +
 c*x^4])
```

Maple [A] time = 0.038, size = 200, normalized size = 0.5

$$\frac{\sqrt{2}}{d}\sqrt{1+\frac{bx^2}{2a}-\frac{x^2}{2a}\sqrt{-4ac+b^2}}\sqrt{1+\frac{bx^2}{2a}+\frac{x^2}{2a}\sqrt{-4ac+b^2}}\,EllipticPi\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},-2\frac{ae}{\left(-b+\sqrt{-4ac+b\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/
2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2/a*x^2*(-4*a*
```

```
c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a*
e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}(ex^2 + d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^2)\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.388 $\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=703

$$\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d(d+ex^2)(ae^2-bde+cd^2)}-\frac{\sqrt{c}ex\sqrt{a+bx^2+cx^4}}{2d\left(\sqrt{a}+\sqrt{c}x^2\right)(ae^2-bde+cd^2)}$$
$$+\frac{\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2d\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}$$
$$-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(3cd^2-e(2bd-ae)\right)\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{8\sqrt[4]{c}d^2\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)(ae^2-bde+cd^2)}$$
$$+\frac{\left(3cd^2-e(2bd-ae)\right)\tan^{-1}\left(\frac{x\sqrt{\frac{ae}{d}-b+\frac{cd}{e}}}{\sqrt{a+bx^2+cx^4}}\right)}{4d^3e\left(\frac{ae}{d}-b+\frac{cd}{e}\right)^{3/2}}$$
$$-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^2+cx^4}\left(\sqrt{a}e-\sqrt{c}d\right)}$$

```
[Out] -(Sqrt[c]*e*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a*e^
2)*(Sqrt[a] + Sqrt[c]*x^2)) + (e^2*x*Sqrt[a + b*x^2 + c*x^4])/(2*
d*(c*d^2 - b*d*e + a*e^2)*(d + e*x^2)) + ((3*c*d^2 - e*(2*b*d - a
*e))*ArcTan[(Sqrt[-b + (c*d)/e + (a*e)/d]*x)/Sqrt[a + b*x^2 + c*x
^4]])/(4*d^3*e*(-b + (c*d)/e + (a*e)/d)^(3/2)) + (a^(1/4)*c^(1/4)
*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(2*d*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 +
 c*x^4]) - (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], 1/2 - b/(4*Sqrt[a]*Sqrt[c])])/(2*a^(1/4)*d*(-(Sqrt[c]*d) +
 Sqrt[a]*e)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*((Sqrt[c]*d)/Sqrt
[a] + e)*(3*c*d^2 - e*(2*b*d - a*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt
[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x
)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(8*c^(1/4)*d^2*(Sqrt[c]
*d - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 1.45479, antiderivative size = 889, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$\frac{x\sqrt{cx^4+bx^2+a}e^2}{2d(cd^2-bed+ae^2)(ex^2+d)} + \frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)e}{2d(cd^2-bed+ae^2)\sqrt{cx^4+bx^2+a}}$$
$$-\frac{\sqrt{c}x\sqrt{cx^4+bx^2+a}e}{2d(cd^2-bed+ae^2)\left(\sqrt{c}x^2+\sqrt{a}\right)}$$
$$+\frac{\sqrt[4]{c}\left(3cd^2-e(2bd-ae)\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2-bed+ae^2)\sqrt{cx^4+bx^2+a}}$$
$$-\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4d(cd^2-bed+ae^2)\sqrt{cx^4+bx^2+a}}$$
$$-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(3cd^2-e(2bd-ae)\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{8\sqrt[4]{a}\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2-bed+ae^2)\sqrt{cx^4+bx^2+a}}$$
$$+\frac{\left(3cd^2-e(2bd-ae)\right)\tan^{-1}\left(\frac{\sqrt{-b+\frac{ae}{d}+\frac{cd}{e}}x}{\sqrt{cx^4+bx^2+a}}\right)}{4d^3\left(-b+\frac{ae}{d}+\frac{cd}{e}\right)^{3/2}e}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[c]*e*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a*e^
2)*(Sqrt[a] + Sqrt[c]*x^2)) + (e^2*x*Sqrt[a + b*x^2 + c*x^4])/(2*
d*(c*d^2 - b*d*e + a*e^2)*(d + e*x^2)) + ((3*c*d^2 - e*(2*b*d - a
*e))*ArcTan[(Sqrt[-b + (c*d)/e + (a*e)/d]*x)/Sqrt[a + b*x^2 + c*x
^4]])/(4*d^3*e*(-b + (c*d)/e + (a*e)/d)^(3/2)) + (a^(1/4)*c^(1/4)
*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4]])/(2*d*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 +
 c*x^4]) - (a^(1/4)*c^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*
EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(4*d*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + (c^
(1/4)*(3*c*d^2 - e*(2*b*d - a*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*d*(
Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c
*x^4]) - ((Sqrt[c]*d + Sqrt[a]*e)*(3*c*d^2 - e*(2*b*d - a*e))*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*
x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*
d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]
)/(8*a^(1/4)*c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e +
 a*e^2)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 143.104, size = 784, normalized size = 1.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] a**(1/4)*c**(1/4)*e*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)
*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/
a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(2*d*sqrt(a + b*x**2 + c*
```

```
x**4)*(a*e**2 - b*d*e + c*d**2)) - sqrt(c)*e*x*sqrt(a + b*x**2 +
c*x**4)/(2*d*(sqrt(a) + sqrt(c)*x**2)*(a*e**2 - b*d*e + c*d**2))
+ e**2*x*sqrt(a + b*x**2 + c*x**4)/(2*d*(d + e*x**2)*(a*e**2 - b*
d*e + c*d**2)) + (a*e**2 - 2*b*d*e + 3*c*d**2)*atan(x*sqrt(a*e/d
- b + c*d/e)/sqrt(a + b*x**2 + c*x**4))/(4*d**3*e*(a*e/d - b + c*
d/e)**(3/2)) - c**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqr
t(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(sqrt(a)*e + sqrt(c)*d)*e
lliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)
))/(4*a**(1/4)*d*sqrt(a + b*x**2 + c*x**4)*(a*e**2 - b*d*e + c*d*
*2)) - c**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**
2)**2)*(sqrt(a) + sqrt(c)*x**2)*(a*e**2 - 2*b*d*e + 3*c*d**2)*ell
iptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))
/(4*a**(1/4)*d*(sqrt(a)*e - sqrt(c)*d)*sqrt(a + b*x**2 + c*x**4)*
(a*e**2 - b*d*e + c*d**2)) + sqrt((a + b*x**2 + c*x**4)/(sqrt(a)
+ sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(sqrt(a)*e + sqrt(c)
*d)*(a*e**2 - 2*b*d*e + 3*c*d**2)*elliptic_pi(-sqrt(a)*(e - sqrt(
c)*d/sqrt(a))**2/(4*sqrt(c)*d*e), 2*atan(c**(1/4)*x/a**(1/4)), 1/
2 - b/(4*sqrt(a)*sqrt(c)))/(8*a**(1/4)*c**(1/4)*d**2*(sqrt(a)*e -
 sqrt(c)*d)*sqrt(a + b*x**2 + c*x**4)*(a*e**2 - b*d*e + c*d**2))
```

Mathematica [C] time = 3.09192, size = 1069, normalized size = 1.52

$$2i\sqrt{2}c\sqrt{\frac{2cx^2+b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,(ex^2+d)\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)d^2-6i\sqrt{2}c\sqrt{\frac{2cx^2+b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}}\ \sqrt{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e^2*x*(a + b*x^2 + c*x^4) +
I*Sqrt[2]*(b - Sqrt[b^2 - 4*a*c])*d*e*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c])]*(d + e*x^2)*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) +
(2*I)*Sqrt[2]*c*d^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d
 + e*x^2)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*
c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - (6*I)
*Sqrt[2]*c*d^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*
x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sq
rt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])
/(b - Sqrt[b^2 - 4*a*c])] + (4*I)*Sqrt[2]*b*d*e*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)
/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 -
 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a
*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - (2*I
)*Sqrt[2]*a*e^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e
*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[S
qrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c]
)/(b - Sqrt[b^2 - 4*a*c])])/(8*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*
(c*d^3 + d*e*(-(b*d) + a*e))*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.039, size = 1279, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*e^2*x*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)-1
/8*c/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1
/2)*(4+2*b*x^2/a-2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2
/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(
1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-
4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/4*e*c/(a*e^2-b*d*e+c*d^2)/d*a*2^(
1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2/a*x^2*(-4
*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2/a*x^2*(-4*a*c+b^2)^(1/2))^(
1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c
+b^2)^(1/2))/a/c)^(1/2))-1/4*e*c/(a*e^2-b*d*e+c*d^2)/d*a*2^(1/2)/
(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2/a*x^2*(-4*a*c+
b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/
(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)
^(1/2))/a/c)^(1/2))+1/2/(a*e^2-b*d*e+c*d^2)/d^2*e^2*2^(1/2)/(-b/a
+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2/a*x^2*(-4*a*c+b
^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/
2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*
a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
))*a-1/(a*e^2-b*d*e+c*d^2)/d*e*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/
2))^(1/2)*(1+1/2*b*x^2/a-1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1
/2*b*x^2/a+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1
/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2
/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))*b+3/2/(a*e^2-b*d*e
+c*d^2)*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/
a-1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2/a*x^2*(-
4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^
(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2)
)*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}(ex^2 + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)**2*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}(ex^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

3.389 $\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=553

$$-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4}}}\right.\right.}{30\sqrt{2}c^{7/2}\sqrt{a+bx^2-cx^4}}$$
$$+\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{2c(4abe^3+15acde^2+15c^2d^3)}{b-\sqrt{4ac+b^2}}+e\left(3ce(3ae+10bd)+8b^2e^2+4\right.\right.}{30\sqrt{2}c^{7/2}\sqrt{a+bx^2-cx^4}}$$
$$-\frac{e^2x\sqrt{a+bx^2-cx^4}(4be+15cd)}{15c^2}-\frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c}$$

```
[Out] -(e^2*(15*c*d + 4*b*e)*x*Sqrt[a + b*x^2 - c*x^4])/(15*c^2) - (e^3
*x^3*Sqrt[a + b*x^2 - c*x^4])/(5*c) - ((b - Sqrt[b^2 + 4*a*c])*Sq
rt[b + Sqrt[b^2 + 4*a*c]]*e*(45*c^2*d^2 + 8*b^2*e^2 + 3*c*e*(10*b
*d + 3*a*e))*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 -
 (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b
- Sqrt[b^2 + 4*a*c])])/(30*Sqrt[2]*c^(7/2)*Sqrt[a + b*x^2 - c*x^4
]) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*((2*c*(
15*c^2*d^3 + 15*a*c*d*e^2 + 4*a*b*e^3))/(b - Sqrt[b^2 + 4*a*c]) +
 e*(45*c^2*d^2 + 8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e)))*Sqrt[1 - (2
*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2
 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(
30*Sqrt[2]*c^(7/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] time = 2.61891, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4}}}\right.\right.}{30\sqrt{2}c^{7/2}\sqrt{a+bx^2-cx^4}}$$
$$+\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{2c(4abe^3+15acde^2+15c^2d^3)}{b-\sqrt{4ac+b^2}}+e\left(3ce(3ae+10bd)+8b^2e^2+4\right.\right.}{30\sqrt{2}c^{7/2}\sqrt{a+bx^2-cx^4}}$$
$$-\frac{e^2x\sqrt{a+bx^2-cx^4}(4be+15cd)}{15c^2}-\frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^3/Sqrt[a + b*x^2 - c*x^4],x]
```


```
[Out] -(e^2*(15*c*d + 4*b*e)*x*Sqrt[a + b*x^2 - c*x^4])/(15*c^2) - (e^3
*x^3*Sqrt[a + b*x^2 - c*x^4])/(5*c) - ((b - Sqrt[b^2 + 4*a*c])*Sq
rt[b + Sqrt[b^2 + 4*a*c]]*e*(45*c^2*d^2 + 8*b^2*e^2 + 3*c*e*(10*b
*d + 3*a*e))*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 -
 (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b
- Sqrt[b^2 + 4*a*c])])/(30*Sqrt[2]*c^(7/2)*Sqrt[a + b*x^2 - c*x^4
]) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*((2*c*(
15*c^2*d^3 + 15*a*c*d*e^2 + 4*a*b*e^3))/(b - Sqrt[b^2 + 4*a*c]) +
 e*(45*c^2*d^2 + 8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e)))*Sqrt[1 - (2
*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2
 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(
30*Sqrt[2]*c^(7/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**3/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 4.44418, size = 596, normalized size = 1.08

$$-i\sqrt{2}e\left(\sqrt{4ac+b^2}-b\right)\sqrt{\frac{\sqrt{4ac+b^2}+b-2cx^2}{\sqrt{4ac+b^2}+b}}\sqrt{\frac{\sqrt{4ac+b^2}-b+2cx^2}{\sqrt{4ac+b^2}-b}}\left(3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (-4*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*e^2*x*(a + b*x^2 - c*x^4
)*(4*b*e + 3*c*(5*d + e*x^2)) - I*Sqrt[2]*(-b + Sqrt[b^2 + 4*a*c]
)*e*(45*c^2*d^2 + 8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e))*Sqrt[(b + S
qrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + S
qrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*EllipticE[I
*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt
[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[2]*(-30*c^3*d^3
+ 8*b^2*(-b + Sqrt[b^2 + 4*a*c])*e^3 + 15*c^2*d*e*(-3*b*d + 3*Sqr
t[b^2 + 4*a*c]*d - 2*a*e) + c*e^2*(-30*b^2*d + 30*b*Sqrt[b^2 + 4*
a*c]*d - 17*a*b*e + 9*a*Sqrt[b^2 + 4*a*c]*e))*Sqrt[(b + Sqrt[b^2
+ 4*a*c] - 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2
+ 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh
[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4
*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(60*c^3*Sqrt[-(c/(b + Sqrt[b^2 +
 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Maple [B] time = 0.022, size = 1195, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*d^3*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+
b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/
(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(
1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))+e^3*
(-1/5/c*x^3*(-c*x^4+b*x^2+a)^(1/2)-4/15*b/c^2*x*(-c*x^4+b*x^2+a)^
(1/2)+1/15*b/c^2*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*
(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4
*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^
(1/2))-1/2*(3/5/c*a+8/15*b^2/c^2)*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2
))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)
^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)
,1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^
(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)
```

```
^(1/2))/a/c)^(1/2))))-3/2*d^2*e*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))
/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c
+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(
1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1
/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(
1/2))/a/c)^(1/2)))+3*e^2*d*(-1/3*x*(-c*x^4+b*x^2+a)^(1/2)/c+1/12/
c*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*
x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/3*b/c*
a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^
4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(
(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))
/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(
1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^3}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}{\sqrt{-cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)/sqrt(-c*x^4
+ b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**3/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**3/sqrt(a + b*x**2 - c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^3}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + b*x^2 + a), x)
```

3.390 $\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=454

$$-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}(be+3cd)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}+\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(ce\left(-3d\sqrt{4ac+b^2}+ae+3bd\right)+be^2\left(b-\sqrt{4ac+b^2}\right)+3c^2d^2\right)F\left(\sin^{-1}\right.}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}-\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c}$$

```
[Out] -(e^2*x*Sqrt[a + b*x^2 - c*x^4])/(3*c) - ((b - Sqrt[b^2 + 4*a*c])
*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*(3*c*d + b*e)*Sqrt[1 - (2*c*x^2)/(
b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]
)]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c
]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(3*Sqrt[2]
*c^(5/2)*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*
(3*c^2*d^2 + b*(b - Sqrt[b^2 + 4*a*c])*e^2 + c*e*(3*b*d - 3*Sqrt[
b^2 + 4*a*c]*d + a*e))*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])
]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4
*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^
2 - c*x^4])
```

Rubi [A] time = 1.81206, antiderivative size = 454, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}(be+3cd)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}+\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(ce\left(-3d\sqrt{4ac+b^2}+ae+3bd\right)+be^2\left(b-\sqrt{4ac+b^2}\right)+3c^2d^2\right)F\left(\sin^{-1}\right.}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}-\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^2/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -(e^2*x*Sqrt[a + b*x^2 - c*x^4])/(3*c) - ((b - Sqrt[b^2 + 4*a*c])
*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*(3*c*d + b*e)*Sqrt[1 - (2*c*x^2)/(
b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]
)]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c
]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(3*Sqrt[2]
*c^(5/2)*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*
(3*c^2*d^2 + b*(b - Sqrt[b^2 + 4*a*c])*e^2 + c*e*(3*b*d - 3*Sqrt[
b^2 + 4*a*c]*d + a*e))*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])
]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4
*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^
2 - c*x^4])
```

Rubi in Sympy [A] time = 142.436, size = 405, normalized size = 0.89

$$-\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c}-\frac{\sqrt{2}e\left(b-\sqrt{4ac+b^2}\right)\sqrt{b+\sqrt{4ac+b^2}}\,(be+3cd)\sqrt{-\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{-\frac{2cx^2}{b+\sqrt{4ac+b^2}}+1}E\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{4ac+b^2}}}\right)\middle|\frac{b+\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{6c^{\frac{5}{2}}\sqrt{a+bx^2-cx^4}}+\frac{\sqrt{2}\sqrt{b+\sqrt{4ac+b^2}}\sqrt{-\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{-\frac{2cx^2}{b+\sqrt{4ac+b^2}}+1}\left(be^2\left(b-\sqrt{4ac+b^2}\right)+3cde\left(b-\sqrt{4ac+b^2}\right)+c\left(ae^2+3cd^2\right)\right)F\left(\right.}{6c^{\frac{5}{2}}\sqrt{a+bx^2-cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -e**2*x*sqrt(a + b*x**2 - c*x**4)/(3*c) - sqrt(2)*e*(b - sqrt(4*a
*c + b**2))*sqrt(b + sqrt(4*a*c + b**2))*(b*e + 3*c*d)*sqrt(-2*c*
x**2/(b - sqrt(4*a*c + b**2)) + 1)*sqrt(-2*c*x**2/(b + sqrt(4*a*c
 + b**2)) + 1)*elliptic_e(asin(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(4*
a*c + b**2))), (b + sqrt(4*a*c + b**2))/(b - sqrt(4*a*c + b**2)))
/(6*c**(5/2)*sqrt(a + b*x**2 - c*x**4)) + sqrt(2)*sqrt(b + sqrt(4
*a*c + b**2))*sqrt(-2*c*x**2/(b - sqrt(4*a*c + b**2)) + 1)*sqrt(-
2*c*x**2/(b + sqrt(4*a*c + b**2)) + 1)*(b*e**2*(b - sqrt(4*a*c +
b**2)) + 3*c*d*e*(b - sqrt(4*a*c + b**2)) + c*(a*e**2 + 3*c*d**2)
)*elliptic_f(asin(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(4*a*c + b**2)))
, (b + sqrt(4*a*c + b**2))/(b - sqrt(4*a*c + b**2)))/(6*c**(5/2)*
sqrt(a + b*x**2 - c*x**4))
```

Mathematica [C] time = 2.44266, size = 503, normalized size = 1.11

$$i\sqrt{2}\sqrt{\frac{\sqrt{4ac+b^2}+b-2cx^2}{\sqrt{4ac+b^2}+b}}\sqrt{\frac{\sqrt{4ac+b^2}-b+2cx^2}{\sqrt{4ac+b^2}-b}}\left(-ce\left(-3d\sqrt{4ac+b^2}+ae+3bd\right)+be^2\left(\sqrt{4ac+b^2}-b\right)-3c^2d^2\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^2/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (2*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*e^2*x*(-a - b*x^2 + c*x^4
) - I*Sqrt[2]*(-b + Sqrt[b^2 + 4*a*c])*e*(3*c*d + b*e)*Sqrt[(b +
Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b +
Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*EllipticE[
I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqr
t[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[2]*(-3*c^2*d^2
+ b*(-b + Sqrt[b^2 + 4*a*c])*e^2 - c*e*(3*b*d - 3*Sqrt[b^2 + 4*a*
c]*d + a*e))*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt[b^2
 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^
2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 +
 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/
(6*c^2*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4]
)
```

Maple [A] time = 0.012, size = 761, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*d^2*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+
b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/
(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(
1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))+e^2*
(-1/3*x*(-c*x^4+b*x^2+a)^(1/2)/c+1/12/c*a*2^(1/2)/((-b+(4*a*c+b^2
)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(
b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*Elliptic
F(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(
4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/3*b/c*a*2^(1/2)/((-b+(4*a*c+b^2)^
(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+
(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+
b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(
1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*
x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+
b^2)^(1/2))/a/c)^(1/2))))-d*e*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a
)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/
2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2
*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2
)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/
2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^4+2\,dex^2+d^2}{\sqrt{-cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/sqrt(-c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**2/sqrt(a + b*x**2 - c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

3.391 $\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=385

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}$$

```
[Out] -((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*Sqrt[1 -
(2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b
^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])
/(2*Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt[b^2
 + 4*a*c]]*(2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*Sqrt[1 - (2*c*x^2)
/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*
c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a
*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[
2]*c^(3/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] time = 0.960136, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*Sqrt[1 -
(2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b
^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])
/(2*Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt[b^2
 + 4*a*c]]*(2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*Sqrt[1 - (2*c*x^2)
/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*
c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a
*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[
2]*c^(3/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rubi in Sympy [A] time = 103.691, size = 340, normalized size = 0.88

$$-\frac{\sqrt{2}e\left(b-\sqrt{4ac+b^2}\right)\sqrt{b+\sqrt{4ac+b^2}}\sqrt{-\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{-\frac{2cx^2}{b+\sqrt{4ac+b^2}}+1}E\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{4ac+b^2}}}\right)\middle|\frac{b+\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{4c^{\frac{3}{2}}\sqrt{a+bx^2-cx^4}}+\frac{\sqrt{2}\sqrt{b+\sqrt{4ac+b^2}}\left(2cd+e\left(b-\sqrt{4ac+b^2}\right)\right)\sqrt{-\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{-\frac{2cx^2}{b+\sqrt{4ac+b^2}}+1}F\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{4ac+b^2}}}\right)\middle|\frac{b+\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{4c^{\frac{3}{2}}\sqrt{a+bx^2-cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(2)*e*(b - sqrt(4*a*c + b**2))*sqrt(b + sqrt(4*a*c + b**2))*
sqrt(-2*c*x**2/(b - sqrt(4*a*c + b**2)) + 1)*sqrt(-2*c*x**2/(b +
sqrt(4*a*c + b**2)) + 1)*elliptic_e(asin(sqrt(2)*sqrt(c)*x/sqrt(b
 + sqrt(4*a*c + b**2))), (b + sqrt(4*a*c + b**2))/(b - sqrt(4*a*c
 + b**2)))/(4*c**(3/2)*sqrt(a + b*x**2 - c*x**4)) + sqrt(2)*sqrt(
b + sqrt(4*a*c + b**2))*(2*c*d + e*(b - sqrt(4*a*c + b**2)))*sqrt
(-2*c*x**2/(b - sqrt(4*a*c + b**2)) + 1)*sqrt(-2*c*x**2/(b + sqrt
(4*a*c + b**2)) + 1)*elliptic_f(asin(sqrt(2)*sqrt(c)*x/sqrt(b + s
qrt(4*a*c + b**2))), (b + sqrt(4*a*c + b**2))/(b - sqrt(4*a*c + b
**2)))/(4*c**(3/2)*sqrt(a + b*x**2 - c*x**4))
```

Mathematica [C] time = 0.446691, size = 293, normalized size = 0.76

$$-\frac{i\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}-b}+1}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)+e\left(\sqrt{4ac+b^2}\right.\right.}{2\sqrt{2}c\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*
c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*((-b + Sqrt[b^2 + 4*a*c])*e*Ellip
ticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b
+ Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + (2*c*d + (b - Sqr
t[b^2 + 4*a*c])*e)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt
[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*
c])]))/(Sqrt[2]*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x
^2 - c*x^4])
```

Maple [A] time = 0.007, size = 364, normalized size = 1.

$$\frac{d\sqrt{2}}{4}\sqrt{4-2\,\frac{\left(-b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{4\,ac+b^2}\right)x^2}{a}}EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4-2\,\frac{b\left(b+\sqrt{4\ldots}\right.}{ac}}\right.$$

$$-\frac{ae\sqrt{2}}{2}\sqrt{4-2\,\frac{\left(-b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4-2\,\frac{b\left(b+\ldots\right.}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*d*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^
2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-
c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/
2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*e*
a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^
4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(
(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))
/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(
1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{-cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + d}{\sqrt{-cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{\sqrt{a + bx^2 - cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(a + b*x**2 - c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{-cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 + a), x)
```

3.392 $\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=197

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}}$$

```
[Out] (Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4
*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[-(
(b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2
 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] time = 0.523005, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)*Sqrt[a + b*x^2 - c*x^4]),x]
```


```
[Out] (Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4
*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[-(
(b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2
 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*Sqrt[a + b*x^2 - c*x^4])
```

Rubi in Sympy [A] time = 61.4541, size = 173, normalized size = 0.88

$$\frac{\sqrt{2}\sqrt{b+\sqrt{4ac+b^2}}\sqrt{-\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{-\frac{2cx^2}{b+\sqrt{4ac+b^2}}+1}\,\Pi\left(-\frac{e\left(b+\sqrt{4ac+b^2}\right)}{2cd};\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{4ac+b^2}}}\right)\middle|\frac{b+\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{2\sqrt{c}d\sqrt{a+bx^2-cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(b + sqrt(4*a*c + b**2))*sqrt(-2*c*x**2/(b - sqrt(4*a
*c + b**2)) + 1)*sqrt(-2*c*x**2/(b + sqrt(4*a*c + b**2)) + 1)*ell
iptic_pi(-e*(b + sqrt(4*a*c + b**2))/(2*c*d), asin(sqrt(2)*sqrt(c
)*x/sqrt(b + sqrt(4*a*c + b**2))), (b + sqrt(4*a*c + b**2))/(b -
sqrt(4*a*c + b**2)))/(2*sqrt(c)*d*sqrt(a + b*x**2 - c*x**4))
```

Mathematica [C] time = 0.222401, size = 205, normalized size = 1.04

$$-\frac{i\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}-b}+1}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|-\frac{b+\sqrt{b^2+4ac}}{\sqrt{b^2+4ac}-b}\right)}{\sqrt{2}d\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*
x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[-((b + Sqrt[b^2 + 4*a*c]
)*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))
]*x], -((b + Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c]))])/(Sqrt
[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*d*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A] time = 0.038, size = 201, normalized size = 1.

$$\frac{\sqrt{2}}{d}\sqrt{1+\frac{bx^2}{2a}-\frac{x^2}{2a}\sqrt{4ac+b^2}}\sqrt{1+\frac{bx^2}{2a}+\frac{x^2}{2a}\sqrt{4ac+b^2}}\,EllipticPi\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{4ac+b^2}\right)},-2\frac{ae}{\left(-b+\sqrt{4ac+b^2}\right)d},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2
/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2/a*x^2*(4*a*c+b
^2)^(1/2))^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*
((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(4*a*c+b^2)^(1/2))*a*e/d,
(-1/2*(b+(4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(4*a*c+b^2)^(1/
2))/a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+bx^2+a}(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(a + b*x**2 - c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 + a}(ex^2 + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

3.393 $\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=718

$$-\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{4\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}\left(e(bd-ae)+cd^2\right)}+\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{4\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}\left(e(bd-ae)+cd^2\right)}+\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e(2bd-ae)+3cd^2\right)\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}\sqrt{c}d^2\sqrt{a+bx^2-cx^4}\left(e(bd-ae)+cd^2\right)}-\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d(d+ex^2)(-ae^2+bde+cd^2)}$$

```
[Out] -(e^2*x*Sqrt[a + b*x^2 - c*x^4])/(2*d*(c*d^2 + b*d*e - a*e^2)*(d
+ e*x^2)) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*
e*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/
(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2
+ 4*a*c])])/(4*Sqrt[2]*Sqrt[c]*d*(c*d^2 + e*(b*d - a*e))*Sqrt[a +
 b*x^2 - c*x^4]) - (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(2*c*d + (b - Sqr
t[b^2 + 4*a*c])*e)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sq
rt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c
])/(b - Sqrt[b^2 + 4*a*c])])/(4*Sqrt[2]*Sqrt[c]*d*(c*d^2 + e*(b*d
 - a*e))*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*
(3*c*d^2 + e*(2*b*d - a*e))*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*
a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[-((
b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2
+ 4*a*c])])/(2*Sqrt[2]*Sqrt[c]*d^2*(c*d^2 + e*(b*d - a*e))*Sqrt[a
 + b*x^2 - c*x^4])
```

Rubi [A] time = 1.81231, antiderivative size = 718, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$-\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{4\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}\left(e(bd-ae)+cd^2\right)}+\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{4\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}\left(e(bd-ae)+cd^2\right)}+\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e(2bd-ae)+3cd^2\right)\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}\sqrt{c}d^2\sqrt{a+bx^2-cx^4}\left(e(bd-ae)+cd^2\right)}-\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d(d+ex^2)\left(e(bd-ae)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -(e^2*x*Sqrt[a + b*x^2 - c*x^4])/(2*d*(c*d^2 + e*(b*d - a*e))*(d
+ e*x^2)) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*
e*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/
(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqr
```

```
t[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2
+ 4*a*c])])/(4*Sqrt[2]*Sqrt[c]*d*(c*d^2 + e*(b*d - a*e))*Sqrt[a +
 b*x^2 - c*x^4]) - (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(2*c*d + (b - Sqr
t[b^2 + 4*a*c])*e)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sq
rt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c
])/(b - Sqrt[b^2 + 4*a*c])])/(4*Sqrt[2]*Sqrt[c]*d*(c*d^2 + e*(b*d
 - a*e))*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*
(3*c*d^2 + e*(2*b*d - a*e))*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*
a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[-((
b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2
+ 4*a*c])])/(2*Sqrt[2]*Sqrt[c]*d^2*(c*d^2 + e*(b*d - a*e))*Sqrt[a
 + b*x^2 - c*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 6.49224, size = 1341, normalized size = 1.87

$$\frac{\left(ex^2+d\right)\sqrt{-cx^4+bx^2+a}\left(-\frac{cex^2}{2d(cd^2+bed-ae^2)\sqrt{-cx^4+bx^2+a}}-\frac{c}{2(cd^2+bed-ae^2)\sqrt{-cx^4+bx^2+a}}+\frac{3cd^2+2bed-ae^2}{2d(cd^2+bed-ae^2)(ex^2+d)\sqrt{-cx^4+bx^2+a}}\right)\Big(}{}$$

$$-\frac{e^2x\sqrt{-cx^4+bx^2+a}}{2d\left(cd^2+bed-ae^2\right)\left(ex^2+d\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -(e^2*x*Sqrt[a + b*x^2 - c*x^4])/(2*d*(c*d^2 + b*d*e - a*e^2)*(d
+ e*x^2)) + ((d + e*x^2)*Sqrt[a + b*x^2 - c*x^4]*(-c/(2*(c*d^2 +
b*d*e - a*e^2)*Sqrt[a + b*x^2 - c*x^4]) - (c*e*x^2)/(2*d*(c*d^2 +
 b*d*e - a*e^2)*Sqrt[a + b*x^2 - c*x^4]) + (3*c*d^2 + 2*b*d*e - a
*e^2)/(2*d*(c*d^2 + b*d*e - a*e^2)*(d + e*x^2)*Sqrt[a + b*x^2 - c
*x^4]))*(((I/2)*(-b + Sqrt[b^2 + 4*a*c])*e*Sqrt[1 + (2*c*x^2)/(-b
 + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])
]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]
*x], -((b + Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c]))] - Ellip
ticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], -((
b + Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c]))]))/(Sqrt[2]*Sqrt
[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4]) + (I*c*d*
Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(
b + Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b +
 Sqrt[b^2 + 4*a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^
2 + 4*a*c]))])/(Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a
 + b*x^2 - c*x^4]) - ((3*I)*c*d*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2
 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticP
i[-((b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[-(
c/(b + Sqrt[b^2 + 4*a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c])/(-b + S
qrt[b^2 + 4*a*c]))])/(Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*
Sqrt[a + b*x^2 - c*x^4]) - (I*Sqrt[2]*b*e*Sqrt[1 + (2*c*x^2)/(-b
+ Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]
```

```
*EllipticPi[-((b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[
2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c
])/(-b + Sqrt[b^2 + 4*a*c]))])/(Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))
]*Sqrt[a + b*x^2 - c*x^4]) + (I*a*e^2*Sqrt[1 + (2*c*x^2)/(-b + Sq
rt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Ell
ipticPi[-((b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*S
qrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c])/(
-b + Sqrt[b^2 + 4*a*c]))])/(Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*
c]))]*d*Sqrt[a + b*x^2 - c*x^4])))/(2*c*d^2 + 2*b*d*e - a*e^2 - 2
*c*d*e*x^2 - c*e^2*x^4)
```

Maple [B] time = 0.039, size = 1293, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*e^2/(a*e^2-b*d*e-c*d^2)/d*x*(-c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)+
1/8*c/(a*e^2-b*d*e-c*d^2)*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1
/2)*(4+2*b*x^2/a-2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2/
a*x^2*(4*a*c+b^2)^(1/2))^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1
/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a
*c+b^2)^(1/2))/a/c)^(1/2))-1/4*e*c/(a*e^2-b*d*e-c*d^2)/d*a*2^(1/2
)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2/a*x^2*(4*a*c+
b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)/(
-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/
2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1
/2))/a/c)^(1/2))+1/4*e*c/(a*e^2-b*d*e-c*d^2)/d*a*2^(1/2)/(-b/a+1/
a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2/a*x^2*(4*a*c+b^2)^(1/2)
)^(1/2)*(4+2*b*x^2/a+2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)/(-c*x^4+b*x
^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1/2)*((-b+(4
*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^
(1/2))+1/2/(a*e^2-b*d*e-c*d^2)/d^2*e^2*2^(1/2)/(-b/a+1/a*(4*a*c+b
^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2
)*(1+1/2*b*x^2/a+1/2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)/(-c*x^4+b*x^2
+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/
2),-2/(-b+(4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(4*a*c+b^2)^(1/2))/a)
^(1/2)*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))*a-1/(a*e^2-b*d*e
-c*d^2)/d*e*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x
^2/a-1/2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2/a*x^2*
(4*a*c+b^2)^(1/2))^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*
2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(4*a*c+b^2)^(1/2)
)*a*e/d,(-1/2*(b+(4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(4*a*c+
b^2)^(1/2))/a)^(1/2))*b-3/2/(a*e^2-b*d*e-c*d^2)*2^(1/2)/(-b/a+1/a
*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2/a*x^2*(4*a*c+b^2)^(1
/2))^(1/2)*(1+1/2*b*x^2/a+1/2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)/(-c*
x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2
))/a)^(1/2),-2/(-b+(4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(4*a*c+b^2)^
(1/2))/a)^(1/2)*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 + a}(ex^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)**2*sqrt(a + b*x**2 - c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+bx^2+a}(ex^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

3.394 $\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=479

$$-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)E\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}+\frac{d\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}+\frac{ex\left(b-\sqrt{4ac+b^2}\right)\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)}{2c\sqrt{-a+bx^2+cx^4}}$$

```
[Out] ((b - Sqrt[b^2 + 4*a*c])*e*x*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c
])))/(2*c*Sqrt[-a + b*x^2 + c*x^4]) - ((b - Sqrt[b^2 + 4*a*c])*Sq
rt[b + Sqrt[b^2 + 4*a*c]]*e*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]
))*EllipticE[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c
]]], (-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*
c^(3/2)*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^
2)/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[-a + b*x^2 + c*x^4]) + (Sqrt[b
+ Sqrt[b^2 + 4*a*c]]*d*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))*El
lipticF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]],
(-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]
*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b +
 Sqrt[b^2 + 4*a*c]))]*Sqrt[-a + b*x^2 + c*x^4])
```

Rubi [A] time = 1.37875, antiderivative size = 479, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)E\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}+\frac{d\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}+\frac{ex\left(b-\sqrt{4ac+b^2}\right)\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)}{2c\sqrt{-a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/Sqrt[-a + b*x^2 + c*x^4],x]
```

```
[Out] ((b - Sqrt[b^2 + 4*a*c])*e*x*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c
])))/(2*c*Sqrt[-a + b*x^2 + c*x^4]) - ((b - Sqrt[b^2 + 4*a*c])*Sq
rt[b + Sqrt[b^2 + 4*a*c]]*e*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]
))*EllipticE[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c
]]], (-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*
c^(3/2)*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^
2)/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[-a + b*x^2 + c*x^4]) + (Sqrt[b
+ Sqrt[b^2 + 4*a*c]]*d*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))*El
lipticF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]],
(-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]
```

```
*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b +
 Sqrt[b^2 + 4*a*c]))]*Sqrt[-a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 142.476, size = 420, normalized size = 0.88

$$\frac{ex\left(b-\sqrt{4ac+b^2}\right)\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)}{2c\sqrt{-a+bx^2+cx^4}} + \frac{\sqrt{2}d\sqrt{b+\sqrt{4ac+b^2}}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)F\left(\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{4ac+b^2}}}\right)\middle|-\frac{2\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{2\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{b+\sqrt{4ac+b^2}}+1}}\sqrt{-a+bx^2+cx^4}} - \frac{\sqrt{2}e\left(b-\sqrt{4ac+b^2}\right)\sqrt{b+\sqrt{4ac+b^2}}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)E\left(\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{4ac+b^2}}}\right)\middle|-\frac{2\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{4c^{\frac{3}{2}}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{b+\sqrt{4ac+b^2}}+1}}\sqrt{-a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] e*x*(b - sqrt(4*a*c + b**2))*(2*c*x**2/(b - sqrt(4*a*c + b**2)) +
 1)/(2*c*sqrt(-a + b*x**2 + c*x**4)) + sqrt(2)*d*sqrt(b + sqrt(4*
a*c + b**2))*(2*c*x**2/(b - sqrt(4*a*c + b**2)) + 1)*elliptic_f(a
tan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(4*a*c + b**2))), -2*sqrt(4*a*
c + b**2)/(b - sqrt(4*a*c + b**2)))/(2*sqrt(c)*sqrt((2*c*x**2/(b
- sqrt(4*a*c + b**2)) + 1)/(2*c*x**2/(b + sqrt(4*a*c + b**2)) + 1
))*sqrt(-a + b*x**2 + c*x**4)) - sqrt(2)*e*(b - sqrt(4*a*c + b**2
))*sqrt(b + sqrt(4*a*c + b**2))*(2*c*x**2/(b - sqrt(4*a*c + b**2)
) + 1)*elliptic_e(atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(4*a*c + b*
*2))), -2*sqrt(4*a*c + b**2)/(b - sqrt(4*a*c + b**2)))/(4*c**(3/2
)*sqrt((2*c*x**2/(b - sqrt(4*a*c + b**2)) + 1)/(2*c*x**2/(b + sqr
t(4*a*c + b**2)) + 1))*sqrt(-a + b*x**2 + c*x**4))
```

Mathematica [C] time = 0.476807, size = 304, normalized size = 0.63

$$\frac{i\sqrt{\frac{\sqrt{4ac+b^2}+b+2cx^2}{\sqrt{4ac+b^2}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\left(\left(e\left(b-\sqrt{4ac+b^2}\right)-2cd\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)+e\left(\sqrt{4ac+b^2}-\right.\right.}{2\sqrt{2}c\sqrt{\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{-a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/Sqrt[-a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 + 4*a
*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*((-b + Sqrt[b^2
 + 4*a*c])*e*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4
*a*c])]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + (-
2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*EllipticF[I*ArcSinh[Sqrt[2]*Sq
rt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sq
rt[b^2 + 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*Sq
rt[-a + b*x^2 + c*x^4])
```

Maple [A] time = 0.013, size = 355, normalized size = 0.7

$$\frac{d}{2}\sqrt{4+2\,\frac{\left(-b+\sqrt{4\,ac+b^{2}}\right)x^{2}}{a}}\sqrt{4-2\,\frac{\left(b+\sqrt{4\,ac+b^{2}}\right)x^{2}}{a}}\,\mathit{EllipticF}\left(\frac{x}{2}\sqrt{-2\,\frac{-b+\sqrt{4\,ac+b^{2}}}{a}},\frac{1}{2}\sqrt{-4-2\,\frac{b\left(b+\sqrt{4\,ac+b^{2}}\right.}{ac}}\right.$$

$$+ae\sqrt{4+2\,\frac{\left(-b+\sqrt{4\,ac+b^{2}}\right)x^{2}}{a}}\sqrt{4-2\,\frac{\left(b+\sqrt{4\,ac+b^{2}}\right)x^{2}}{a}}\left(\mathit{EllipticF}\left(\frac{x}{2}\sqrt{-2\,\frac{-b+\sqrt{4\,ac+b^{2}}}{a}},\frac{1}{2}\sqrt{-4-2\,\frac{b\left(b+\sqrt{4\,ac+\right.}}{ac}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/2*d/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+
b*x^2-a)^(1/2)*EllipticF(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))+e*a/(-2*(-b+(4*a*
c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(
4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2-a)^(1/2)/(b+(
4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2
*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)
^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{cx^4+bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ex^2+d}{\sqrt{cx^4+bx^2-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(c*x**4+b*x**2-a)**(1/2),x)

[Out] Integral((d + e*x**2)/sqrt(-a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{cx^4 + bx^2 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a),x, algorithm="giac")

[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a), x)
```

3.395 $\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=204

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\,\Pi\left(\frac{\left(b-\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2+4ac}-b}}\right)|\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}d\sqrt{-a+bx^2+cx^4}}$$

```
[Out] (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 +
4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[(
(b - Sqrt[b^2 + 4*a*c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[-b + Sqrt[b^2 + 4*a*c]]], (b - Sqrt[b^2 + 4*a*c])/(b + Sqrt[b^
2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*Sqrt[-a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.578772, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\,\Pi\left(\frac{\left(b-\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2+4ac}-b}}\right)|\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}d\sqrt{-a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 +
4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[(
(b - Sqrt[b^2 + 4*a*c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[-b + Sqrt[b^2 + 4*a*c]]], (b - Sqrt[b^2 + 4*a*c])/(b + Sqrt[b^
2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*Sqrt[-a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 153.612, size = 411, normalized size = 2.01

$$-\frac{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{4ac+b^2}}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)F\left(\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{4ac+b^2}}}\right)\middle|-\frac{2\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{b+\sqrt{4ac+b^2}}+1}}\sqrt{-a+bx^2+cx^4}\left(be-2cd+e\sqrt{4ac+b^2}\right)}+\frac{\sqrt{2}e\left(b+\sqrt{4ac+b^2}\right)^{\frac{3}{2}}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)\Pi\left(1-\frac{e\left(b+\sqrt{4ac+b^2}\right)}{2cd};\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{4ac+b^2}}}\right)\middle|-\frac{2\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{2\sqrt{c}d\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{b+\sqrt{4ac+b^2}}+1}}\sqrt{-a+bx^2+cx^4}\left(be-2cd+e\sqrt{4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] -sqrt(2)*sqrt(c)*sqrt(b + sqrt(4*a*c + b**2))*(2*c*x**2/(b - sqrt
(4*a*c + b**2)) + 1)*elliptic_f(atan(sqrt(2)*sqrt(c)*x/sqrt(b + s
qrt(4*a*c + b**2))), -2*sqrt(4*a*c + b**2)/(b - sqrt(4*a*c + b**2
)))/(sqrt((2*c*x**2/(b - sqrt(4*a*c + b**2)) + 1)/(2*c*x**2/(b +
sqrt(4*a*c + b**2)) + 1))*sqrt(-a + b*x**2 + c*x**4)*(b*e - 2*c*d
 + e*sqrt(4*a*c + b**2))) + sqrt(2)*e*(b + sqrt(4*a*c + b**2))**(
3/2)*(2*c*x**2/(b - sqrt(4*a*c + b**2)) + 1)*elliptic_pi(1 - e*(b
 + sqrt(4*a*c + b**2))/(2*c*d), atan(sqrt(2)*sqrt(c)*x/sqrt(b + s
```

```
qrt(4*a*c + b**2))), -2*sqrt(4*a*c + b**2)/(b - sqrt(4*a*c + b**2
)))/(2*sqrt(c)*d*sqrt((2*c*x**2/(b - sqrt(4*a*c + b**2)) + 1)/(2*
c*x**2/(b + sqrt(4*a*c + b**2)) + 1))*sqrt(-a + b*x**2 + c*x**4)*
(b*e - 2*c*d + e*sqrt(4*a*c + b**2)))
```

Mathematica [C] time = 0.226377, size = 216, normalized size = 1.06

$$-\frac{i\sqrt{\frac{\sqrt{4ac+b^2}+b+2cx^2}{\sqrt{4ac+b^2}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\left(\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}d\sqrt{\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{-a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 + 4*a*
c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*EllipticPi[((b +
 Sqrt[b^2 + 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqr
t[b^2 + 4*a*c])]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*
c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*d*Sqrt[-a + b*x^2
+ c*x^4])
```

Maple [A] time = 0.036, size = 198, normalized size = 1.

$$\frac{1}{d}\sqrt{1-\frac{bx^2}{2a}+\frac{x^2}{2a}\sqrt{4ac+b^2}}\sqrt{1-\frac{bx^2}{2a}-\frac{x^2}{2a}\sqrt{4ac+b^2}}EllipticPi\left(\sqrt{-\frac{1}{2a}\left(-b+\sqrt{4ac+b^2}\right)}x,2\frac{ae}{\left(-b+\sqrt{4ac+b^2}\right)d},\frac{\sqrt{2}}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/d/(1/2*b/a-1/2/a*(4*a*c+b^2)^(1/2))^(1/2)*(1-1/2*b*x^2/a+1/2/a*
x^2*(4*a*c+b^2)^(1/2))^(1/2)*(1-1/2*b*x^2/a-1/2/a*x^2*(4*a*c+b^2)
^(1/2))^(1/2)/(c*x^4+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(4*a*c+b
^2)^(1/2))/a)^(1/2)*x,2/(-b+(4*a*c+b^2)^(1/2))*a*e/d,1/2*2^(1/2)*
((b+(4*a*c+b^2)^(1/2))/a)^(1/2)/(-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2-a}(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 - a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^4 + b*x^2 - a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(-a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2-a}(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^4 + b*x^2 - a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

3.396 $\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=293

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2c^{3/4}\sqrt{-a+bx^2-cx^4}} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{c^{3/4}\sqrt{-a+bx^2-cx^4}} - \frac{ex\sqrt{-a+bx^2-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[Out] -((e*x*Sqrt[-a + b*x^2 - c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)
)) - (a^(1/4)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3/4)*Sqrt[-a + b*x^2 - c*x^4
]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/
4)*Sqrt[-a + b*x^2 - c*x^4])
```

Rubi [A] time = 0.235354, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2c^{3/4}\sqrt{-a+bx^2-cx^4}} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{c^{3/4}\sqrt{-a+bx^2-cx^4}} - \frac{ex\sqrt{-a+bx^2-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/Sqrt[-a + b*x^2 - c*x^4],x]
```

```
[Out] -((e*x*Sqrt[-a + b*x^2 - c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)
)) - (a^(1/4)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3/4)*Sqrt[-a + b*x^2 - c*x^4
]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/
4)*Sqrt[-a + b*x^2 - c*x^4])
```

Rubi in Sympy [A] time = 33.8636, size = 260, normalized size = 0.89

$$-\frac{\sqrt[4]{a}e\sqrt{-\frac{-a+bx^2-cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}+\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{c^{\frac{3}{4}}\sqrt{-a+bx^2-cx^4}} - \frac{ex\sqrt{-a+bx^2-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{\sqrt{-\frac{-a+bx^2-cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}e+\sqrt{c}d\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}+\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2\sqrt[4]{a}c^{\frac{3}{4}}\sqrt{-a+bx^2-cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(-c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] -a**(1/4)*e*sqrt(-(-a + b*x**2 - c*x**4)/(sqrt(a) + sqrt(c)*x**2)
**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/
4)), 1/2 + b/(4*sqrt(a)*sqrt(c)))/(c**(3/4)*sqrt(-a + b*x**2 - c*
x**4)) - e*x*sqrt(-a + b*x**2 - c*x**4)/(sqrt(c)*(sqrt(a) + sqrt(
c)*x**2)) + sqrt(-(-a + b*x**2 - c*x**4)/(sqrt(a) + sqrt(c)*x**2)
**2)*(sqrt(a) + sqrt(c)*x**2)*(sqrt(a)*e + sqrt(c)*d)*elliptic_f(
2*atan(c**(1/4)*x/a**(1/4)), 1/2 + b/(4*sqrt(a)*sqrt(c)))/(2*a**(
1/4)*c**(3/4)*sqrt(-a + b*x**2 - c*x**4))
```

Mathematica [C] time = 0.448899, size = 295, normalized size = 1.01

$$-\frac{i\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}-b}+1}\sqrt{1-\frac{2cx^2}{\sqrt{b^2-4ac}+b}}\left(\left(e\left(b-\sqrt{b^2-4ac}\right)+2cd\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+e\left(\sqrt{b^2-4a\right.\right.}{2\sqrt{2}c\sqrt{-\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{-a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/Sqrt[-a + b*x^2 - c*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*((-b + Sqrt[b^2 - 4*a*c])*e*Ellip
ticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], (b
+ Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (2*c*d + (b - Sqr
t[b^2 - 4*a*c])*e)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt
[b^2 - 4*a*c]))]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])]))/(Sqrt[2]*c*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[-a + b*
x^2 - c*x^4])
```

Maple [A] time = 0.037, size = 357, normalized size = 1.2

$$\frac{d}{2}\sqrt{4+2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4-2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\mathit{EllipticF}\left(\frac{x}{2}\sqrt{-2\,\frac{-b+\sqrt{-4\,ac+b^2}}{a}},\frac{1}{2}\sqrt{-4+2\,\frac{b\left(b+\sqrt{-4\,a\right.}{ac}}\right.$$

$$+ae\sqrt{4+2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4-2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(\mathit{EllipticF}\left(\frac{x}{2}\sqrt{-2\,\frac{-b+\sqrt{-4\,ac+b^2}}{a}},\frac{1}{2}\sqrt{-4+2\,\frac{b\left(b+\sqrt{-\right.}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/2*d/(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(-4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*
x^4+b*x^2-a)^(1/2)*EllipticF(1/2*x*(-2*(-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+e*a/(-2*(-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2-a)^
(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*(-2*(-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-
EllipticE(1/2*x*(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{-cx^4+bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 - a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2+d}{\sqrt{-cx^4+bx^2-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 - a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(-c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(-a + b*x**2 - c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ex^2+d}{\sqrt{-cx^4+bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 - a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 - a), x)
```

3.397 $\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=399

$$\frac{\tan^{-1}\left(\frac{x\sqrt{-\frac{ae}{d}-b-\frac{cd}{e}}}{\sqrt{-a+bx^2-cx^4}}\right)}{2d\sqrt{-\frac{ae}{d}-b-\frac{cd}{e}}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2\sqrt[4]{a}\sqrt{-a+bx^2-cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\Pi\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\sqrt{-a+bx^2-cx^4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)}$$

```
[Out] ArcTan[(Sqrt[-b - (c*d)/e - (a*e)/d]*x)/Sqrt[-a + b*x^2 - c*x^4]]
/(2*d*Sqrt[-b - (c*d)/e - (a*e)/d]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(
2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a + b*x^2 - c*x^4]) - (((
Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[a]*((Sqrt[c]
*d)/Sqrt[a] - e)^2)/(4*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*((Sqrt[c]*d
)/Sqrt[a] - e)*Sqrt[-a + b*x^2 - c*x^4])
```

Rubi [A] time = 0.485705, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{\tan^{-1}\left(\frac{x\sqrt{-\frac{ae}{d}-b-\frac{cd}{e}}}{\sqrt{-a+bx^2-cx^4}}\right)}{2d\sqrt{-\frac{ae}{d}-b-\frac{cd}{e}}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2\sqrt[4]{a}\sqrt{-a+bx^2-cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{4\sqrt[4]{c}d\sqrt{-a+bx^2-cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)*Sqrt[-a + b*x^2 - c*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[-b - (c*d)/e - (a*e)/d]*x)/Sqrt[-a + b*x^2 - c*x^4]]
/(2*d*Sqrt[-b - (c*d)/e - (a*e)/d]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(
2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a + b*x^2 - c*x^4]) - (a^
(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a -
 b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d
 - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(
1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(4*c^(1/4)*d*(Sqrt[c]*d - Sq
rt[a]*e)*Sqrt[-a + b*x^2 - c*x^4])
```

Rubi in Sympy [A] time = 31.7169, size = 338, normalized size = 0.85

$$\frac{\operatorname{atan}\left(\frac{x\sqrt{-\frac{ae}{d}-b-\frac{cd}{e}}}{\sqrt{-a+bx^2-cx^4}}\right)}{2d\sqrt{-\frac{ae}{d}-b-\frac{cd}{e}}}-\frac{\sqrt[4]{c}\sqrt{-\frac{-a+bx^2-cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}+\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}e-\sqrt{c}d\right)\sqrt{-a+bx^2-cx^4}}+\frac{\sqrt{-\frac{-a+bx^2-cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}e+\sqrt{c}d\right)\Pi\left(-\frac{\sqrt{a}\left(e-\frac{\sqrt{c}d}{\sqrt{a}}\right)^2}{4\sqrt{c}de};2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}+\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{a}e-\sqrt{c}d\right)\sqrt{-a+bx^2-cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)/(-c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] atan(x*sqrt(-a*e/d - b - c*d/e)/sqrt(-a + b*x**2 - c*x**4))/(2*d*
sqrt(-a*e/d - b - c*d/e)) - c**(1/4)*sqrt(-(-a + b*x**2 - c*x**4)
/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_f
(2*atan(c**(1/4)*x/a**(1/4)), 1/2 + b/(4*sqrt(a)*sqrt(c)))/(2*a**
(1/4)*(sqrt(a)*e - sqrt(c)*d)*sqrt(-a + b*x**2 - c*x**4)) + sqrt(
-(-a + b*x**2 - c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + s
qrt(c)*x**2)*(sqrt(a)*e + sqrt(c)*d)*elliptic_pi(-sqrt(a)*(e - sq
rt(c)*d/sqrt(a))**2/(4*sqrt(c)*d*e), 2*atan(c**(1/4)*x/a**(1/4)),
 1/2 + b/(4*sqrt(a)*sqrt(c)))/(4*a**(1/4)*c**(1/4)*d*(sqrt(a)*e -
 sqrt(c)*d)*sqrt(-a + b*x**2 - c*x**4))
```

Mathematica [C] time = 0.21763, size = 207, normalized size = 0.52

$$-\frac{i\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}-b}+1}\sqrt{1-\frac{2cx^2}{\sqrt{b^2-4ac}+b}}\Pi\left(-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|-\frac{b+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}-b}\right)}{\sqrt{2}d\sqrt{-\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{-a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)*Sqrt[-a + b*x^2 - c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticPi[-((b + Sqrt[b^2 - 4*a*c]
)*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))
]*x], -((b + Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c]))])/(Sqrt
[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*d*Sqrt[-a + b*x^2 - c*x^4]
)
```

Maple [A] time = 0.035, size = 199, normalized size = 0.5

$$\frac{1}{d}\sqrt{1-\frac{bx^2}{2\,a}+\frac{x^2}{2\,a}\sqrt{-4\,ac+b^2}}\sqrt{1-\frac{bx^2}{2\,a}-\frac{x^2}{2\,a}\sqrt{-4\,ac+b^2}}EllipticPi\left(\sqrt{-\frac{1}{2\,a}\left(-b+\sqrt{-4\,ac+b^2}\right)}x,2\frac{ae}{\left(-b+\sqrt{-4\,ac+b^2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/d/(1/2*b/a-1/2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1-1/2*b*x^2/a+1/2/a
*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(1-1/2*b*x^2/a-1/2/a*x^2*(-4*a*c+b
```

```
^2)^(1/2))^(1/2)/(-c*x^4+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2)*x,2/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,1/2*2^
(1/2)*((b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/(-1/2*(-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 - a}(ex^2 + d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-c*x^4 + b*x^2 - a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-c*x^4 + b*x^2 - a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^2)\sqrt{-a + bx^2 - cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)/(-c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(-a + b*x**2 - c*x**4)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-c*x^4 + b*x^2 - a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.398 $\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}} dx$

Optimal. Leaf size=229

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(5d^3-10de^2+8e^3)F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{5\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{3e(x^2+2)x(5d^2-10de+6e^2)}{5\sqrt{x^4+3x^2+2}}-\frac{3\sqrt{2}e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(5d^2-10de+6e^2)E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}}+\frac{1}{5}e^2\sqrt{x^4+3x^2+2}x(5d-4e)+\frac{1}{5}e^3\sqrt{x^4+3x^2+2}x^3$$

```
[Out] (3*e*(5*d^2 - 10*d*e + 6*e^2)*x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^
4]) + ((5*d - 4*e)*e^2*x*Sqrt[2 + 3*x^2 + x^4])/5 + (e^3*x^3*Sqrt
[2 + 3*x^2 + x^4])/5 - (3*Sqrt[2]*e*(5*d^2 - 10*d*e + 6*e^2)*(1 +
 x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqr
t[2 + 3*x^2 + x^4]) + ((5*d^3 - 10*d*e^2 + 8*e^3)*(1 + x^2)*Sqrt[
(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(5*Sqrt[2]*Sqrt[2
 + 3*x^2 + x^4])
```

Rubi [A] time = 0.294469, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(5d^3-10de^2+8e^3)F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{5\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{3e(x^2+2)x(5d^2-10de+6e^2)}{5\sqrt{x^4+3x^2+2}}-\frac{3\sqrt{2}e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(5d^2-10de+6e^2)E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}}+\frac{1}{5}e^2\sqrt{x^4+3x^2+2}x(5d-4e)+\frac{1}{5}e^3\sqrt{x^4+3x^2+2}x^3$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```


```
[Out] (3*e*(5*d^2 - 10*d*e + 6*e^2)*x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^
4]) + ((5*d - 4*e)*e^2*x*Sqrt[2 + 3*x^2 + x^4])/5 + (e^3*x^3*Sqrt
[2 + 3*x^2 + x^4])/5 - (3*Sqrt[2]*e*(5*d^2 - 10*d*e + 6*e^2)*(1 +
 x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqr
t[2 + 3*x^2 + x^4]) + ((5*d^3 - 10*d*e^2 + 8*e^3)*(1 + x^2)*Sqrt[
(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(5*Sqrt[2]*Sqrt[2
 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 54.6225, size = 211, normalized size = 0.92

$$\frac{e^3x^3\sqrt{x^4+3x^2+2}}{5}+e^2x\left(d-\frac{4e}{5}\right)\sqrt{x^4+3x^2+2}+\frac{3ex(2x^2+4)(5d^2-10de+6e^2)}{10\sqrt{x^4+3x^2+2}}-\frac{3e\sqrt{\frac{2x^2+4}{x^2+1}}(4x^2+4)(5d^2-10de+6e^2)E\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{20\sqrt{x^4+3x^2+2}}+\frac{\sqrt{\frac{2x^2+4}{x^2+1}}(4x^2+4)\left(\frac{d^3}{8}-\frac{de^2}{4}+\frac{e^3}{5}\right)F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**3/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] e**3*x**3*sqrt(x**4 + 3*x**2 + 2)/5 + e**2*x*(d - 4*e/5)*sqrt(x**
4 + 3*x**2 + 2) + 3*e*x*(2*x**2 + 4)*(5*d**2 - 10*d*e + 6*e**2)/(
10*sqrt(x**4 + 3*x**2 + 2)) - 3*e*sqrt((2*x**2 + 4)/(x**2 + 1))*(
4*x**2 + 4)*(5*d**2 - 10*d*e + 6*e**2)*elliptic_e(atan(x), 1/2)/(
20*sqrt(x**4 + 3*x**2 + 2)) + sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x*
*2 + 4)*(d**3/8 - d*e**2/4 + e**3/5)*elliptic_f(atan(x), 1/2)/sqr
t(x**4 + 3*x**2 + 2)
```

Mathematica [C] time = 0.304077, size = 154, normalized size = 0.67

$$\frac{-3ie\sqrt{x^2+1}\sqrt{x^2+2}\left(5d^2-10de+6e^2\right)E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-5i\sqrt{x^2+1}\sqrt{x^2+2}\left(d^3-3d^2e+4de^2-2e^3\right)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right.\right.}{5\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (e^2*x*(2 + 3*x^2 + x^4)*(5*d + e*(-4 + x^2)) - (3*I)*e*(5*d^2 -
10*d*e + 6*e^2)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x
/Sqrt[2]], 2] - (5*I)*(d^3 - 3*d^2*e + 4*d*e^2 - 2*e^3)*Sqrt[1 +
x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(5*Sqrt[2
+ 3*x^2 + x^4])
```

Maple [C] time = 0.014, size = 380, normalized size = 1.7

$$\begin{aligned}
&-\frac{i}{2}d^3\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}+e^3\left(\frac{x^3}{5}\sqrt{x^4+3\,x^2+2}\right.\\
&-\frac{4\,x}{5}\sqrt{x^4+3\,x^2+2}-\frac{4\,i}{5}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}\\
&\left.+\frac{9\,i}{5}\sqrt{2}\left(EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)-EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}\right)\\
&+\frac{3\,i}{2}d^2e\sqrt{2}\left(EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)-EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}\\
&+3\,e^2d\left(1/3\,x\sqrt{x^4+3\,x^2+2}+\frac{i/3\sqrt{2}\sqrt{2\,x^2+4}\sqrt{x^2+1}EllipticF\left(i/2\sqrt{2}x,\sqrt{2}\right)}{\sqrt{x^4+3\,x^2+2}}\right.\\
&\left.-\frac{i\sqrt{2}\sqrt{2\,x^2+4}\sqrt{x^2+1}\left(EllipticF\left(i/2\sqrt{2}x,\sqrt{2}\right)-EllipticE\left(i/2\sqrt{2}x,\sqrt{2}\right)\right)}{\sqrt{x^4+3\,x^2+2}}\right)
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*d^3*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1
/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+e^3*(1/5*x^3*(x^4+3*x^2+2)
^(1/2)-4/5*x*(x^4+3*x^2+2)^(1/2)-4/5*I*2^(1/2)*(2*x^2+4)^(1/2)*(x
^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2)
)+9/5*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)
*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^
(1/2))))+3/2*I*d^2*e*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3
*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I
*2^(1/2)*x,2^(1/2)))+3*e^2*d*(1/3*x*(x^4+3*x^2+2)^(1/2)+1/3*I*2^(
1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(
1/2*I*2^(1/2)*x,2^(1/2))-I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/
(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE
(1/2*I*2^(1/2)*x,2^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^3}{\sqrt{x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/sqrt(x^4 + 3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}{\sqrt{x^4+3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/sqrt(x^4 + 3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)/sqrt(x^4 + 3
*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^3}{\sqrt{(x^2+1)(x^2+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**3/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**3/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^3}{\sqrt{x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^3/sqrt(x^4 + 3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

3.399 $\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=168

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,(3d^2-2e^2)\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{2ex\,(x^2+2)\,(d-e)}{\sqrt{x^4+3x^2+2}}-\frac{2\sqrt{2}e\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(d-e)E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{1}{3}e^2x\sqrt{x^4+3x^2+2}$$

```
[Out] (2*(d - e)*e*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (e^2*x*Sqrt[2 +
 3*x^2 + x^4])/3 - (2*Sqrt[2]*(d - e)*e*(1 + x^2)*Sqrt[(2 + x^2)/
(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + ((3
*d^2 - 2*e^2)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTa
n[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.174077, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,(3d^2-2e^2)\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{2ex\,(x^2+2)\,(d-e)}{\sqrt{x^4+3x^2+2}}-\frac{2\sqrt{2}e\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(d-e)E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{1}{3}e^2x\sqrt{x^4+3x^2+2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (2*(d - e)*e*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (e^2*x*Sqrt[2 +
 3*x^2 + x^4])/3 - (2*Sqrt[2]*(d - e)*e*(1 + x^2)*Sqrt[(2 + x^2)/
(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + ((3
*d^2 - 2*e^2)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTa
n[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 27.162, size = 148, normalized size = 0.88

$$\frac{e^2x\sqrt{x^4+3x^2+2}}{3}+\frac{ex\,(d-e)\,(2x^2+4)}{\sqrt{x^4+3x^2+2}}-\frac{e\sqrt{\frac{2x^2+4}{x^2+1}}\,(d-e)\,(4x^2+4)\,E\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{2\sqrt{x^4+3x^2+2}}+\frac{\sqrt{\frac{2x^2+4}{x^2+1}}\left(\frac{d^2}{8}-\frac{e^2}{12}\right)(4x^2+4)\,F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] e**2*x*sqrt(x**4 + 3*x**2 + 2)/3 + e*x*(d - e)*(2*x**2 + 4)/sqrt(
x**4 + 3*x**2 + 2) - e*sqrt((2*x**2 + 4)/(x**2 + 1))*(d - e)*(4*x
**2 + 4)*elliptic_e(atan(x), 1/2)/(2*sqrt(x**4 + 3*x**2 + 2)) + s
qrt((2*x**2 + 4)/(x**2 + 1))*(d**2/8 - e**2/12)*(4*x**2 + 4)*elli
ptic_f(atan(x), 1/2)/sqrt(x**4 + 3*x**2 + 2)
```

Mathematica [C] time = 0.201345, size = 127, normalized size = 0.76

$$\frac{-i\sqrt{x^2+1}\sqrt{x^2+2}\left(3d^2-6de+4e^2\right)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-6ie\sqrt{x^2+1}\sqrt{x^2+2}(d-e)E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+e^2x\left(x^4+3x^2\right.}{3\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (e^2*x*(2 + 3*x^2 + x^4) - (6*I)*(d - e)*e*Sqrt[1 + x^2]*Sqrt[2 +
 x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - I*(3*d^2 - 6*d*e + 4*e
^2)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2
])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.014, size = 235, normalized size = 1.4

$$-\frac{i}{2}d^2\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}$$
$$+e^2\left(\frac{x}{3}\sqrt{x^4+3x^2+2}+\frac{i}{3}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}\right.$$
$$\left.-i\sqrt{2}\left(EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)-EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}\right)$$
$$+ide\sqrt{2}\left(EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)-EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*d^2*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1
/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+e^2*(1/3*x*(x^4+3*x^2+2)^(
1/2)+1/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1
/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-I*2^(1/2)*(2*x^2+4)^(1/2)*
(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1
/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2))))+I*d*e*2^(1/2)*(2*x^2+4)
^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)
*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^2+d\right)^2}{\sqrt{x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/sqrt(x^4 + 3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^2x^4+2\,dex^2+d^2}{\sqrt{x^4+3x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/sqrt(x^4 + 3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex^2\right)^2}{\sqrt{\left(x^2 + 1\right)\left(x^2 + 2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**2/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2}{\sqrt{x^4 + 3\,x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^2/sqrt(x^4 + 3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

3.400 $\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}} dx$

Optimal. Leaf size=122

$$\frac{d(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{ex(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

```
[Out] (e*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*e*(1 + x^2)*Sqrt
[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 +
 x^4] + (d*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x
], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0831953, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{d(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{ex(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (e*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*e*(1 + x^2)*Sqrt
[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 +
 x^4] + (d*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x
], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 13.8699, size = 116, normalized size = 0.95

$$\frac{d\sqrt{\frac{2x^2+4}{x^2+1}}(4x^2+4)F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{8\sqrt{x^4+3x^2+2}}+\frac{ex(2x^2+4)}{2\sqrt{x^4+3x^2+2}}-\frac{e\sqrt{\frac{2x^2+4}{x^2+1}}(4x^2+4)E\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{4\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] d*sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_f(atan(x),
1/2)/(8*sqrt(x**4 + 3*x**2 + 2)) + e*x*(2*x**2 + 4)/(2*sqrt(x**4
+ 3*x**2 + 2)) - e*sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*ell
iptic_e(atan(x), 1/2)/(4*sqrt(x**4 + 3*x**2 + 2))
```

Mathematica [C] time = 0.081516, size = 73, normalized size = 0.6

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\left((d-e)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+eE\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(e*EllipticE[I*ArcSinh[x/Sqrt[2
]], 2] + (d - e)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]))/Sqrt[2 + 3*
```

```
x^2 + x^4]
```

Maple [C] time = 0.008, size = 108, normalized size = 0.9

$$-\frac{i}{2}d\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}+\frac{i}{2}e\sqrt{2}\left(EllipticF\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)-EllipticE\left(\frac{i}{2}\sqrt{2}x,\sqrt{2}\right)\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*d*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2
)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/2*I*e*2^(1/2)*(2*x^2+4)^(1
/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,
2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2+d}{\sqrt{x^4+3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex^2}{\sqrt{(x^2+1)\,(x^2+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{x^4 + 3x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2), x)
```

3.401 $\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=124

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}(d-e)}-\frac{e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\left(1-\frac{e}{d};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}d\sqrt{x^4+3x^2+2}(d-e)}$$

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(
Sqrt[2]*(d - e)*Sqrt[2 + 3*x^2 + x^4]) - (e*(1 + x^2)*Sqrt[(2 + x
^2)/(1 + x^2)]*EllipticPi[1 - e/d, ArcTan[x], 1/2])/(Sqrt[2]*d*(d
 - e)*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.253755, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}(d-e)}-\frac{e(x^2+2)\left(1-\frac{e}{d};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}d\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}(d-e)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(
Sqrt[2]*(d - e)*Sqrt[2 + 3*x^2 + x^4]) - (e*(2 + x^2)*EllipticPi[
1 - e/d, ArcTan[x], 1/2])/(Sqrt[2]*d*(d - e)*Sqrt[(2 + x^2)/(1 +
x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 35.01, size = 102, normalized size = 0.82

$$\frac{\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{8(d-e)\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}e\sqrt{x^4+3x^2+2}\left(1-\frac{e}{d};\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{2d\sqrt{\frac{x^2+2}{x^2+1}}(d-e)(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_f(atan(x), 1/
2)/(8*(d - e)*sqrt(x**4 + 3*x**2 + 2)) - sqrt(2)*e*sqrt(x**4 + 3*
x**2 + 2)*elliptic_pi(1 - e/d, atan(x), 1/2)/(2*d*sqrt((x**2 + 2)
/(x**2 + 1))*(d - e)*(x**2 + 1))
```

Mathematica [C] time = 0.0587892, size = 59, normalized size = 0.48

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\left(\frac{2e}{d};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{d\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[(2*e)/d, I*ArcSinh[x
/Sqrt[2]], 2])/(d*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.024, size = 55, normalized size = 0.4

$$\frac{-i\sqrt{2}}{d}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}EllipticPi\left(\frac{i}{2}\sqrt{2}x, 2\frac{e}{d}, \sqrt{2}\right)\frac{1}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -I/d*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*
EllipticPi(1/2*I*2^(1/2)*x,2/d*e,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{x^4+3x^2+2}(ex^2+d)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2+1)(x^2+2)}(d+ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(d + e*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3x^2+2}(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)), x)
```

3.402 $\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=316

$$\frac{e^2x\sqrt{x^4+3x^2+2}}{2d(d^2-3de+2e^2)(d+ex^2)}-\frac{ex(x^2+2)}{2d\sqrt{x^4+3x^2+2}(d^2-3de+2e^2)}-\frac{e(x^2+2)(3d^2-6de+2e^2)\,\Pi\left(1-\frac{e}{d};\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{2}d^2\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}(d-2e)(d-e)^2}+\frac{(x^2+1)\sqrt{\frac{x^2+2}{2x^2+2}}(2d-e)F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{2d\sqrt{x^4+3x^2+2}(d-e)^2}+\frac{e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}d\sqrt{x^4+3x^2+2}(d-2e)(d-e)}$$

```
[Out] -(e*x*(2 + x^2))/(2*d*(d^2 - 3*d*e + 2*e^2)*Sqrt[2 + 3*x^2 + x^4]
) + (e^2*x*Sqrt[2 + 3*x^2 + x^4])/(2*d*(d^2 - 3*d*e + 2*e^2)*(d +
 e*x^2)) + (e*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTa
n[x], 1/2])/(Sqrt[2]*d*(d - 2*e)*(d - e)*Sqrt[2 + 3*x^2 + x^4]) +
 ((2*d - e)*(1 + x^2)*Sqrt[(2 + x^2)/(2 + 2*x^2)]*EllipticF[ArcTa
n[x], 1/2])/(2*d*(d - e)^2*Sqrt[2 + 3*x^2 + x^4]) - (e*(3*d^2 - 6
*d*e + 2*e^2)*(2 + x^2)*EllipticPi[1 - e/d, ArcTan[x], 1/2])/(2*S
qrt[2]*d^2*(d - 2*e)*(d - e)^2*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 +
 3*x^2 + x^4])
```

Rubi [A] time = 0.491622, antiderivative size = 399, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$\frac{e^2x\sqrt{x^4+3x^2+2}}{2d(d^2-3de+2e^2)(d+ex^2)}-\frac{ex(x^2+2)}{2d\sqrt{x^4+3x^2+2}(d^2-3de+2e^2)}+\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(3d^2-6de+2e^2)F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{2}d\sqrt{x^4+3x^2+2}(d-2e)(d-e)^2}-\frac{e(x^2+2)(3d^2-6de+2e^2)\,\Pi\left(1-\frac{e}{d};\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{2}d^2\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}(d-2e)(d-e)^2}-\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}(d-2e)(d-e)}+\frac{e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}d\sqrt{x^4+3x^2+2}(d-2e)(d-e)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] -(e*x*(2 + x^2))/(2*d*(d^2 - 3*d*e + 2*e^2)*Sqrt[2 + 3*x^2 + x^4]
) + (e^2*x*Sqrt[2 + 3*x^2 + x^4])/(2*d*(d^2 - 3*d*e + 2*e^2)*(d +
 e*x^2)) + (e*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTa
n[x], 1/2])/(Sqrt[2]*d*(d - 2*e)*(d - e)*Sqrt[2 + 3*x^2 + x^4]) -
 ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/
(2*Sqrt[2]*(d - 2*e)*(d - e)*Sqrt[2 + 3*x^2 + x^4]) + ((3*d^2 - 6
*d*e + 2*e^2)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTa
n[x], 1/2])/(2*Sqrt[2]*d*(d - 2*e)*(d - e)^2*Sqrt[2 + 3*x^2 + x^4
]) - (e*(3*d^2 - 6*d*e + 2*e^2)*(2 + x^2)*EllipticPi[1 - e/d, Arc
Tan[x], 1/2])/(2*Sqrt[2]*d^2*(d - 2*e)*(d - e)^2*Sqrt[(2 + x^2)/(
1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi in Sympy [A] time = 77.5317, size = 332, normalized size = 1.05

$$-\frac{\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)F\left(\operatorname{atan}\left(x\right)\middle|\frac{1}{2}\right)}{16\left(d-2e\right)\left(d-e\right)\sqrt{x^4+3x^2+2}}+\frac{e^2x\sqrt{x^4+3x^2+2}}{2d\left(d-2e\right)\left(d-e\right)\left(d+ex^2\right)}-\frac{ex\left(2x^2+4\right)}{4d\left(d-2e\right)\left(d-e\right)\sqrt{x^4+3x^2+2}}+\frac{e\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)E\left(\operatorname{atan}\left(x\right)\middle|\frac{1}{2}\right)}{8d\left(d-2e\right)\left(d-e\right)\sqrt{x^4+3x^2+2}}+\frac{\sqrt{\frac{2x^2+4}{x^2+1}}\left(4x^2+4\right)\left(3d^2-6de+2e^2\right)F\left(\operatorname{atan}\left(x\right)\middle|\frac{1}{2}\right)}{16d\left(d-2e\right)\left(d-e\right)^2\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}e\left(3d^2-6de+2e^2\right)\sqrt{x^4+3x^2+2}\,\Pi\left(1-\frac{e}{d};\operatorname{atan}\left(x\right)\middle|\frac{1}{2}\right)}{4d^2\sqrt{\frac{x^2+2}{x^2+1}}\left(d-2e\right)\left(d-e\right)^2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] -sqrt((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_f(atan(x), 1
/2)/(16*(d - 2*e)*(d - e)*sqrt(x**4 + 3*x**2 + 2)) + e**2*x*sqrt(
x**4 + 3*x**2 + 2)/(2*d*(d - 2*e)*(d - e)*(d + e*x**2)) - e*x*(2*
x**2 + 4)/(4*d*(d - 2*e)*(d - e)*sqrt(x**4 + 3*x**2 + 2)) + e*sqr
t((2*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_e(atan(x), 1/2)/
(8*d*(d - 2*e)*(d - e)*sqrt(x**4 + 3*x**2 + 2)) + sqrt((2*x**2 +
4)/(x**2 + 1))*(4*x**2 + 4)*(3*d**2 - 6*d*e + 2*e**2)*elliptic_f(
atan(x), 1/2)/(16*d*(d - 2*e)*(d - e)**2*sqrt(x**4 + 3*x**2 + 2))
 - sqrt(2)*e*(3*d**2 - 6*d*e + 2*e**2)*sqrt(x**4 + 3*x**2 + 2)*el
liptic_pi(1 - e/d, atan(x), 1/2)/(4*d**2*sqrt((x**2 + 2)/(x**2 +
1))*(d - 2*e)*(d - e)**2*(x**2 + 1))
```

Mathematica [C] time = 0.759612, size = 175, normalized size = 0.55

$$\frac{\frac{e^2x\left(x^4+3x^2+2\right)}{\left(d^2-3de+2e^2\right)\left(d+ex^2\right)}+\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\left(\left(-3d^2+6de-2e^2\right)\Pi\left(\frac{2e}{d};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+d(d-e)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+deE\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{d(d-2e)(d-e)}}{2d\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((e^2*x*(2 + 3*x^2 + x^4))/((d^2 - 3*d*e + 2*e^2)*(d + e*x^2)) +
(I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(d*e*EllipticE[I*ArcSinh[x/Sqrt[2]
], 2] + d*(d - e)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (-3*d^2 +
6*d*e - 2*e^2)*EllipticPi[(2*e)/d, I*ArcSinh[x/Sqrt[2]], 2]))/(d*
(d - 2*e)*(d - e)))/(2*d*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.037, size = 443, normalized size = 1.4

$$\frac{e^{2} x}{2 \left(d^{2}-3 d e+2 e^{2}\right) d \left(e x^{2}+d\right)} \sqrt{x^{4}+3 x^{2}+2}
+\frac{\frac{i}{4} \sqrt{2}\, EllipticF\left(\frac{i}{2} \sqrt{2}\, x ,\sqrt{2}\right)}{d^{2}-3 d e+2 e^{2}} \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \frac{1}{\sqrt{x^{4}+3 x^{2}+2}}
-\frac{\frac{i}{4} e \sqrt{2}\, EllipticF\left(\frac{i}{2} \sqrt{2}\, x ,\sqrt{2}\right)}{\left(d^{2}-3 d e+2 e^{2}\right) d} \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \frac{1}{\sqrt{x^{4}+3 x^{2}+2}}
+\frac{\frac{i}{4} e \sqrt{2}\, EllipticE\left(\frac{i}{2} \sqrt{2}\, x ,\sqrt{2}\right)}{\left(d^{2}-3 d e+2 e^{2}\right) d} \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \frac{1}{\sqrt{x^{4}+3 x^{2}+2}}
-\frac{\frac{3 i}{2} \sqrt{2}}{d^{2}-3 d e+2 e^{2}} \sqrt{1+\frac{x^{2}}{2}}\, \sqrt{x^{2}+1}\, EllipticPi\left(\frac{i}{2} \sqrt{2}\, x ,2 \frac{e}{d},\sqrt{2}\right) \frac{1}{\sqrt{x^{4}+3 x^{2}+2}}
+\frac{3 i e \sqrt{2}}{\left(d^{2}-3 d e+2 e^{2}\right) d} \sqrt{1+\frac{x^{2}}{2}}\, \sqrt{x^{2}+1}\, EllipticPi\left(\frac{i}{2} \sqrt{2}\, x ,2 \frac{e}{d},\sqrt{2}\right) \frac{1}{\sqrt{x^{4}+3 x^{2}+2}}
-\frac{i e^{2} \sqrt{2}}{\left(d^{2}-3 d e+2 e^{2}\right) d^{2}} \sqrt{1+\frac{x^{2}}{2}}\, \sqrt{x^{2}+1}\, EllipticPi\left(\frac{i}{2} \sqrt{2}\, x ,2 \frac{e}{d},\sqrt{2}\right) \frac{1}{\sqrt{x^{4}+3 x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 1/2*e^2*x*(x^4+3*x^2+2)^(1/2)/d/(d^2-3*d*e+2*e^2)/(e*x^2+d)+1/4*I
/(d^2-3*d*e+2*e^2)*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x
^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-1/4*I*e/(d^2-3*d*e
+2*e^2)/d*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/
2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/4*I*e/(d^2-3*d*e+2*e^2)/d
*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Ellipt
icE(1/2*I*2^(1/2)*x,2^(1/2))-3/2*I/(d^2-3*d*e+2*e^2)*2^(1/2)*(1+1
/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*
2^(1/2)*x,2/d*e,2^(1/2))+3*I/(d^2-3*d*e+2*e^2)/d*e*2^(1/2)*(1+1/2
*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^
(1/2)*x,2/d*e,2^(1/2))-I/(d^2-3*d*e+2*e^2)/d^2*e^2*2^(1/2)*(1+1/2
*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^
(1/2)*x,2/d*e,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3 x^2+2}(e x^2+d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(e^{2} x^{4}+2\, d e x^{2}+d^{2}\right) \sqrt{x^{4}+3 x^{2}+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)^2),x, algorithm="fricas")
```

```
[Out] integral(1/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(x^4 + 3*x^2 + 2)), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2+1)(x^2+2)}\,(d+ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(d + e*x**2)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3\,x^2+2}(ex^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)^2), x)
```

3.403 $\int (c+ex^2)^q (a+cx^2+bx^4)^p \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\left(c+ex^2\right)^q\left(a+bx^4+cx^2\right)^p,x\right)$$

```
[Out] Unintegrable[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p, x]
```

Rubi [A] time = 0.0285223, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\left(c+ex^2\right)^q\left(a+cx^2+bx^4\right)^p,x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p,x]

[Out] Defer[Int][(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p, x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(c+ex^2\right)^q\left(a+bx^4+cx^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+c)**q*(b*x**4+c*x**2+a)**p,x)

[Out] Integral((c + e*x**2)**q*(a + b*x**4 + c*x**2)**p, x)
```

Mathematica [A] time = 0.113788, size = 0, normalized size = 0.

$$\int \left(c+ex^2\right)^q\left(a+cx^2+bx^4\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p,x]

[Out] Integrate[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p, x]
```

Maple [A] time = 0.109, size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)^q\left(bx^4+cx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x)
```

```
[Out] int((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4 + cx^2 + a\right)^p \left(ex^2 + c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^4 + cx^2 + a\right)^p \left(ex^2 + c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**q*(b*x**4+c*x**2+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4 + cx^2 + a\right)^p \left(ex^2 + c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q, x)
```

3.404 $\int \left(c+ex^2\right)^3 \left(a+cx^2+bx^4\right)^p \, dx$

Optimal. Leaf size=498

$$\frac{ex^3\left(-3be\left(ae(4p+5)+c^2\left(8p^2+26p+21\right)\right)+3b^2c^2\left(16p^2+48p+35\right)+c^2e^2\left(4p^2+16p+15\right)\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx\right.}{3b^2(4p+5)(4p+7)}$$
$$+\frac{cx\left(-3abe^2(4p+7)+ae^3(2p+5)+b^2c^2\left(16p^2+48p+35\right)\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{1}{2};\right.}{b^2(4p+5)(4p+7)}$$
$$+\frac{ce^2x(12bp+21b-2ep-5e)\left(a+bx^4+cx^2\right)^{p+1}}{b^2(4p+5)(4p+7)}+\frac{e^3x^3\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+7)}$$

```
[Out] (c*e^2*(21*b - 5*e + 12*b*p - 2*e*p)*x*(a + c*x^2 + b*x^4)^(1 + p
))/(b^2*(5 + 4*p)*(7 + 4*p)) + (e^3*x^3*(a + c*x^2 + b*x^4)^(1 +
p))/(b*(7 + 4*p)) + (c*(a*e^3*(5 + 2*p) - 3*a*b*e^2*(7 + 4*p) + b
^2*c^2*(35 + 48*p + 16*p^2))*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2
, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c
 + Sqrt[-4*a*b + c^2])])/(b^2*(5 + 4*p)*(7 + 4*p)*(1 + (2*b*x^2)/
(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2
]))^p) + (e*(c^2*e^2*(15 + 16*p + 4*p^2) + 3*b^2*c^2*(35 + 48*p +
 16*p^2) - 3*b*e*(a*e*(5 + 4*p) + c^2*(21 + 26*p + 8*p^2)))*x^3*(
a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - S
qrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*b^2*
(5 + 4*p)*(7 + 4*p)*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1
 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rubi [A] time = 1.59861, antiderivative size = 498, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{ex^3\left(-3be\left(ae(4p+5)+c^2\left(8p^2+26p+21\right)\right)+3b^2c^2\left(16p^2+48p+35\right)+c^2e^2\left(4p^2+16p+15\right)\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx\right.}{3b^2(4p+5)(4p+7)}$$
$$+\frac{cx\left(-3abe^2(4p+7)+ae^3(2p+5)+b^2c^2\left(16p^2+48p+35\right)\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{1}{2};\right.}{b^2(4p+5)(4p+7)}$$
$$+\frac{ce^2x(12bp+21b-2ep-5e)\left(a+bx^4+cx^2\right)^{p+1}}{b^2(4p+5)(4p+7)}+\frac{e^3x^3\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+7)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + e*x^2)^3*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (c*e^2*(21*b - 5*e + 12*b*p - 2*e*p)*x*(a + c*x^2 + b*x^4)^(1 + p
))/(b^2*(5 + 4*p)*(7 + 4*p)) + (e^3*x^3*(a + c*x^2 + b*x^4)^(1 +
p))/(b*(7 + 4*p)) + (c*(a*e^3*(5 + 2*p) - 3*a*b*e^2*(7 + 4*p) + b
^2*c^2*(35 + 48*p + 16*p^2))*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2
, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c
 + Sqrt[-4*a*b + c^2])])/(b^2*(5 + 4*p)*(7 + 4*p)*(1 + (2*b*x^2)/
(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2
]))^p) + (e*(c^2*e^2*(15 + 16*p + 4*p^2) + 3*b^2*c^2*(35 + 48*p +
 16*p^2) - 3*b*e*(a*e*(5 + 4*p) + c^2*(21 + 26*p + 8*p^2)))*x^3*(
a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - S
qrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*b^2*
(5 + 4*p)*(7 + 4*p)*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1
 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c+ex^2\right)^3 \left(a+bx^4+cx^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+c)**3*(b*x**4+c*x**2+a)**p,x)
```

```
[Out] Integral((c + e*x**2)**3*(a + b*x**4 + c*x**2)**p, x)
```

Mathematica [B] time = 18.3258, size = 1871, normalized size = 3.76

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + e*x^2)^3*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (3*4^(-1 - p)*c^3*(c + Sqrt[-4*a*b + c^2])*x*((c - Sqrt[-4*a*b +
c^2] + 2*b*x^2)/b)^(1 + p)*((c + Sqrt[-4*a*b + c^2] + 2*b*x^2)/b)
^(-1 + p)*(-2*a + (-c + Sqrt[-4*a*b + c^2])*x^2)^2*(a + c*x^2 + b
*x^4)^(-1 + p)*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + Sqrt[-4
*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])/((-c + Sqrt[-
4*a*b + c^2])*((c - Sqrt[-4*a*b + c^2])/(2*b) + x^2)^p*((c + Sqrt
[-4*a*b + c^2])/(2*b) + x^2)^p*(-3*a*AppellF1[1/2, -p, -p, 3/2, (
-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b +
 c^2])] + p*x^2*((-c + Sqrt[-4*a*b + c^2])*AppellF1[3/2, 1 - p, -
p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt
[-4*a*b + c^2])] - (c + Sqrt[-4*a*b + c^2])*AppellF1[3/2, -p, 1 -
 p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqr
t[-4*a*b + c^2])]))) + (5*2^(-2 - p)*b*c^2*(c + Sqrt[-4*a*b + c^2
])*e*x^3*((c - Sqrt[-4*a*b + c^2] + 2*b*x^2)/b)^(1 + p)*(-2*a + (
-c + Sqrt[-4*a*b + c^2])*x^2)^2*(a + c*x^2 + b*x^4)^(-1 + p)*Appe
llF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*
x^2)/(-c + Sqrt[-4*a*b + c^2])])/((-c + Sqrt[-4*a*b + c^2])*((c -
 Sqrt[-4*a*b + c^2])/(2*b) + x^2)^p*(c + Sqrt[-4*a*b + c^2] + 2*b
*x^2)*(-5*a*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*
b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + p*x^2*((-c + Sq
rt[-4*a*b + c^2])*AppellF1[5/2, 1 - p, -p, 7/2, (-2*b*x^2)/(c + S
qrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] - (c + S
qrt[-4*a*b + c^2])*AppellF1[5/2, -p, 1 - p, 7/2, (-2*b*x^2)/(c +
Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])]))) + (2
1*2^(-2 - p)*b*c*(c + Sqrt[-4*a*b + c^2])*e^2*x^5*((c - Sqrt[-4*a
*b + c^2] + 2*b*x^2)/b)^(1 + p)*(-2*a + (-c + Sqrt[-4*a*b + c^2])
*x^2)^2*(a + c*x^2 + b*x^4)^(-1 + p)*AppellF1[5/2, -p, -p, 7/2, (
-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b +
 c^2])])/(5*(-c + Sqrt[-4*a*b + c^2])*((c - Sqrt[-4*a*b + c^2])/(
2*b) + x^2)^p*(c + Sqrt[-4*a*b + c^2] + 2*b*x^2)*(-7*a*AppellF1[5
/2, -p, -p, 7/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(
-c + Sqrt[-4*a*b + c^2])] + p*x^2*((-c + Sqrt[-4*a*b + c^2])*Appe
llF1[7/2, 1 - p, -p, 9/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2
*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] - (c + Sqrt[-4*a*b + c^2])*App
ellF1[7/2, -p, 1 - p, 9/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (
2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])]))) + (9*2^(-2 - p)*b*(c + Sqr
t[-4*a*b + c^2])*e^3*x^7*((c - Sqrt[-4*a*b + c^2] + 2*b*x^2)/b)^(
1 + p)*(-2*a + (-c + Sqrt[-4*a*b + c^2])*x^2)^2*(a + c*x^2 + b*x^
4)^(-1 + p)*AppellF1[7/2, -p, -p, 9/2, (-2*b*x^2)/(c + Sqrt[-4*a*
b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])/(7*(-c + Sqrt[-4
*a*b + c^2])*((c - Sqrt[-4*a*b + c^2])/(2*b) + x^2)^p*(c + Sqrt[-
4*a*b + c^2] + 2*b*x^2)*(-9*a*AppellF1[7/2, -p, -p, 9/2, (-2*b*x^
2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])]
 + p*x^2*((-c + Sqrt[-4*a*b + c^2])*AppellF1[9/2, 1 - p, -p, 11/2
, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*
b + c^2])] - (c + Sqrt[-4*a*b + c^2])*AppellF1[9/2, -p, 1 - p, 11
/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*
a*b + c^2])])))
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int \left(e x^{2}+c\right)^{3} \left(b x^{4}+c x^{2}+a\right)^{p}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x)
```

```
[Out] int((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + c)^3 (bx^4 + cx^2 + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + c)^3*(b*x^4 + c*x^2 + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + c)^3*(b*x^4 + c*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^6 + 3\,ce^2x^4 + 3\,c^2ex^2 + c^3\right)\left(bx^4 + cx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + c)^3*(b*x^4 + c*x^2 + a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*c*e^2*x^4 + 3*c^2*e*x^2 + c^3)*(b*x^4 + c*x
^2 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+c)**3*(b*x**4+c*x**2+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + c)^3 (bx^4 + cx^2 + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + c)^3*(b*x^4 + c*x^2 + a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + c)^3*(b*x^4 + c*x^2 + a)^p, x)
```

3.405 $\int \left(c+ex^2\right)^2 \left(a+cx^2+bx^4\right)^p \, dx$

Optimal. Leaf size=358

$$-\frac{x\left(ae^2-bc^2(4p+5)\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)}{b(4p+5)}$$
$$+\frac{cex^3(8bp+10b-2ep-3e)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)}{3b(4p+5)}$$
$$+\frac{e^2x\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+5)}$$

```
[Out] (e^2*x*(a + c*x^2 + b*x^4)^(1 + p))/(b*(5 + 4*p)) - ((a*e^2 - b*c
^2*(5 + 4*p))*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2,
(-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b
+ c^2])])/(b*(5 + 4*p)*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p
*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (c*e*(10*b - 3*e +
 8*b*p - 2*e*p)*x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p, 5
/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*
a*b + c^2])])/(3*b*(5 + 4*p)*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^
2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rubi [A] time = 0.770099, antiderivative size = 345, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$x\left(c^2-\frac{ae^2}{4bp+5b}\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)$$
$$+\frac{1}{3}cex^3\left(2-\frac{e(2p+3)}{b(4p+5)}\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)+\frac{e^2x\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+5)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + e*x^2)^2*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (e^2*x*(a + c*x^2 + b*x^4)^(1 + p))/(b*(5 + 4*p)) + ((c^2 - (a*e^
2)/(5*b + 4*b*p))*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3
/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*
a*b + c^2])])/((1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2
*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (c*e*(2 - (e*(3 + 2*p))/(b
*(5 + 4*p)))*x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p, 5/2,
 (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b
 + c^2])])/(3*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*
b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c+ex^2\right)^2\left(a+bx^4+cx^2\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+c)**2*(b*x**4+c*x**2+a)**p,x)
```

```
[Out] Integral((c + e*x**2)**2*(a + b*x**4 + c*x**2)**p, x)
```

Mathematica [B] time = 6.05803, size = 1410, normalized size = 3.94

$$\frac{3\,4^{-p-1}c^2\left(c+\sqrt{c^2-4ab}\right)x\left(x^2+\frac{c-\sqrt{c^2-4ab}}{2b}\right)^{-p}\left(x^2+\frac{c+\sqrt{c^2-4ab}}{2b}\right)^{-p}\left(\frac{2bx^2+c-\sqrt{c^2-4ab}}{b}\right)^{p+1}\left(\frac{2bx^2+c+\sqrt{c^2-4ab}}{b}\right)^{p-1}\left(\left(\sqrt{c^2-4a\cdots\right.\right.}{\left(\sqrt{c^2-4ab}-c\right)\left(px^2\left(\left(\sqrt{c^2-4ab}-c\right)F_1\left(\frac{3}{2};1-p,-p;\frac{5}{2};-\frac{2bx^2}{c+\sqrt{c^2-4ab}},\frac{2bx^2}{\sqrt{c^2-4ab}-c}\right)-\left(c+\sqrt{c^2-4ab}\right)F_1\left(\frac{3}{2};-p,1-p;\cdots\right.\right.\right.}+\frac{5\,2^{-p-1}bc\left(c+\sqrt{c^2-4ab}\right)ex^3\left(x^2+\frac{c-\sqrt{c^2-4ab}}{2b}\right)^{-p}\left(\frac{2bx^2+c-\sqrt{c^2-4ab}}{b}\right)^{p+1}\left(\left(\sqrt{c^2-4ab}-c\right)x\cdots\right.}{3\left(\sqrt{c^2-4ab}-c\right)\left(2bx^2+c+\sqrt{c^2-4ab}\right)\left(px^2\left(\left(\sqrt{c^2-4ab}-c\right)F_1\left(\frac{5}{2};1-p,-p;\frac{7}{2};-\frac{2bx^2}{c+\sqrt{c^2-4ab}},\frac{2bx^2}{\sqrt{c^2-4ab}-c}\right)-\left(c+\sqrt{c^2\cdots}\right.\right.\right.}+\frac{7\,2^{-p-2}b\left(c+\sqrt{c^2-4ab}\right)e^2x^5\left(x^2+\frac{c-\sqrt{c^2-4ab}}{2b}\right)^{-p}\left(\frac{2bx^2+c-\sqrt{c^2-4ab}}{b}\right)^{p+1}\left(\left(\sqrt{c^2-4ab}-c\right)x\cdots\right.}{5\left(\sqrt{c^2-4ab}-c\right)\left(2bx^2+c+\sqrt{c^2-4ab}\right)\left(px^2\left(\left(\sqrt{c^2-4ab}-c\right)F_1\left(\frac{7}{2};1-p,-p;\frac{9}{2};-\frac{2bx^2}{c+\sqrt{c^2-4ab}},\frac{2bx^2}{\sqrt{c^2-4ab}-c}\right)-\left(c+\sqrt{c^2\cdots}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + e*x^2)^2*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (3*4^(-1 - p)*c^2*(c + Sqrt[-4*a*b + c^2])*x*((c - Sqrt[-4*a*b +
c^2] + 2*b*x^2)/b)^(1 + p)*((c + Sqrt[-4*a*b + c^2] + 2*b*x^2)/b)
^(-1 + p)*(-2*a + (-c + Sqrt[-4*a*b + c^2])*x^2)^2*(a + c*x^2 + b
*x^4)^(-1 + p)*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + Sqrt[-4
*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])/((-c + Sqrt[-
4*a*b + c^2])*((c - Sqrt[-4*a*b + c^2])/(2*b) + x^2)^p*((c + Sqrt
[-4*a*b + c^2])/(2*b) + x^2)^p*(-3*a*AppellF1[1/2, -p, -p, 3/2, (
-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b +
 c^2])] + p*x^2*((-c + Sqrt[-4*a*b + c^2])*AppellF1[3/2, 1 - p, -
p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt
[-4*a*b + c^2])] - (c + Sqrt[-4*a*b + c^2])*AppellF1[3/2, -p, 1 -
 p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqr
t[-4*a*b + c^2])]))) + (5*2^(-1 - p)*b*c*(c + Sqrt[-4*a*b + c^2])
*e*x^3*((c - Sqrt[-4*a*b + c^2] + 2*b*x^2)/b)^(1 + p)*(-2*a + (-c
 + Sqrt[-4*a*b + c^2])*x^2)^2*(a + c*x^2 + b*x^4)^(-1 + p)*Appell
F1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^
2)/(-c + Sqrt[-4*a*b + c^2])])/(3*(-c + Sqrt[-4*a*b + c^2])*((c -
 Sqrt[-4*a*b + c^2])/(2*b) + x^2)^p*(c + Sqrt[-4*a*b + c^2] + 2*b
*x^2)*(-5*a*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*
b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + p*x^2*((-c + Sq
rt[-4*a*b + c^2])*AppellF1[5/2, 1 - p, -p, 7/2, (-2*b*x^2)/(c + S
qrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] - (c + S
qrt[-4*a*b + c^2])*AppellF1[5/2, -p, 1 - p, 7/2, (-2*b*x^2)/(c +
Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])]))) + (7
*2^(-2 - p)*b*(c + Sqrt[-4*a*b + c^2])*e^2*x^5*((c - Sqrt[-4*a*b
+ c^2] + 2*b*x^2)/b)^(1 + p)*(-2*a + (-c + Sqrt[-4*a*b + c^2])*x^
2)^2*(a + c*x^2 + b*x^4)^(-1 + p)*AppellF1[5/2, -p, -p, 7/2, (-2*
b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^
2])])/(5*(-c + Sqrt[-4*a*b + c^2])*((c - Sqrt[-4*a*b + c^2])/(2*b
) + x^2)^p*(c + Sqrt[-4*a*b + c^2] + 2*b*x^2)*(-7*a*AppellF1[5/2,
 -p, -p, 7/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c
+ Sqrt[-4*a*b + c^2])] + p*x^2*((-c + Sqrt[-4*a*b + c^2])*AppellF
1[7/2, 1 - p, -p, 9/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*
x^2)/(-c + Sqrt[-4*a*b + c^2])] - (c + Sqrt[-4*a*b + c^2])*Appell
F1[7/2, -p, 1 - p, 9/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b
*x^2)/(-c + Sqrt[-4*a*b + c^2])])))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)^2\left(bx^4+cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x)

[Out] int((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + c\right)^2 \left(bx^4 + cx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + c)^2*(b*x^4 + c*x^2 + a)^p,x, algorithm="maxima")

[Out] integrate((e*x^2 + c)^2*(b*x^4 + c*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^4 + 2\,cex^2 + c^2\right)\left(bx^4 + cx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + c)^2*(b*x^4 + c*x^2 + a)^p,x, algorithm="fricas")

[Out] integral((e^2*x^4 + 2*c*e*x^2 + c^2)*(b*x^4 + c*x^2 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+c)**2*(b*x**4+c*x**2+a)**p,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + c\right)^2 \left(bx^4 + cx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + c)^2*(b*x^4 + c*x^2 + a)^p,x, algorithm="giac")

[Out] integrate((e*x^2 + c)^2*(b*x^4 + c*x^2 + a)^p, x)
```

3.406 $\int \left(c+ex^2\right) \left(a+cx^2+bx^4\right)^p \, dx$

Optimal. Leaf size=274

$$\frac{1}{3}ex^3\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)+cx\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)$$

```
[Out] (c*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/
(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(
(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sq
rt[-4*a*b + c^2]))^p) + (e*x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/2
, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c
 + Sqrt[-4*a*b + c^2])])/(3*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2
]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rubi [A] time = 0.511617, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{1}{3}ex^3\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)+cx\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(c + e*x^2)*(a + c*x^2 + b*x^4)^p,x]
```


```
[Out] (c*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/
(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(
(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sq
rt[-4*a*b + c^2]))^p) + (e*x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/2
, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c
 + Sqrt[-4*a*b + c^2])])/(3*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2
]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rubi in Sympy [A] time = 67.9417, size = 233, normalized size = 0.85

$$cx\left(\frac{2bx^2}{c-\sqrt{-4ab+c^2}}+1\right)^{-p}\left(\frac{2bx^2}{c+\sqrt{-4ab+c^2}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\operatorname{appellf_1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)+\frac{ex^3\left(\frac{2bx^2}{c-\sqrt{-4ab+c^2}}+1\right)^{-p}\left(\frac{2bx^2}{c+\sqrt{-4ab+c^2}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\operatorname{appellf_1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+c)*(b*x**4+c*x**2+a)**p,x)
```

```
[Out] c*x*(2*b*x**2/(c - sqrt(-4*a*b + c**2)) + 1)**(-p)*(2*b*x**2/(c +
 sqrt(-4*a*b + c**2)) + 1)**(-p)*(a + b*x**4 + c*x**2)**p*appellf
```

```
1(1/2, -p, -p, 3/2, -2*b*x**2/(c - sqrt(-4*a*b + c**2)), -2*b*x**
2/(c + sqrt(-4*a*b + c**2))) + e*x**3*(2*b*x**2/(c - sqrt(-4*a*b
+ c**2)) + 1)**(-p)*(2*b*x**2/(c + sqrt(-4*a*b + c**2)) + 1)**(-p
)*(a + b*x**4 + c*x**2)**p*appellf1(3/2, -p, -p, 5/2, -2*b*x**2/(
c - sqrt(-4*a*b + c**2)), -2*b*x**2/(c + sqrt(-4*a*b + c**2)))/3
```

Mathematica [B] time = 1.25916, size = 706, normalized size = 2.58

$$\frac{1}{3}2^{-p-3}x\left(\sqrt{c^2-4ab}+c\right)\left(x^2\left(\sqrt{c^2-4ab}-c\right)-2a\right)\left(\frac{c-\sqrt{c^2-4ab}}{2b}+x^2\right)^{-p}\left(\frac{-\sqrt{c^2-4ab}+2bx^2+c}{b}\right)^{p+1}\left(a+bx^4+cx^2\right)^{p-1}\left(\frac{5ex^2F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c+\sqrt{c^2-4ab}},\frac{2bx^2}{\sqrt{c^2-4ab}-c}\right)}{px^2\left(\left(\sqrt{c^2-4ab}-c\right)F_1\left(\frac{5}{2};1-p,-p;\frac{7}{2};-\frac{2bx^2}{c+\sqrt{c^2-4ab}},\frac{2bx^2}{\sqrt{c^2-4ab}-c}\right)-\left(\sqrt{c^2-4ab}+c\right)F_1\left(\frac{5}{2};-p,1-p;\frac{7}{2};-\frac{2bx^2}{c+\sqrt{c^2-4}}\right.\right.}-\frac{9cF_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c+\sqrt{c^2-4ab}},\frac{2bx^2}{\sqrt{c^2-4ab}-c}\right)}{px^2\left(\left(c-\sqrt{c^2-4ab}\right)F_1\left(\frac{3}{2};1-p,-p;\frac{5}{2};-\frac{2bx^2}{c+\sqrt{c^2-4ab}},\frac{2bx^2}{\sqrt{c^2-4ab}-c}\right)+\left(\sqrt{c^2-4ab}+c\right)F_1\left(\frac{3}{2};-p,1-p;\frac{5}{2};-\frac{2bx^2}{c+\sqrt{c^2-4ab}},\frac{2b}{\sqrt{c^2-4}}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + e*x^2)*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (2^(-3 - p)*(c + Sqrt[-4*a*b + c^2])*x*((c - Sqrt[-4*a*b + c^2] +
 2*b*x^2)/b)^(1 + p)*(-2*a + (-c + Sqrt[-4*a*b + c^2])*x^2)*(a +
c*x^2 + b*x^4)^(-1 + p)*((-9*c*AppellF1[1/2, -p, -p, 3/2, (-2*b*x
^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])
])/(3*a*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + Sqrt[-4*a*b +
c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + p*x^2*((c - Sqrt[-4
*a*b + c^2])*AppellF1[3/2, 1 - p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-
4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + (c + Sqrt[-
4*a*b + c^2])*AppellF1[3/2, -p, 1 - p, 5/2, (-2*b*x^2)/(c + Sqrt[
-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])) + (5*e*x^2
*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]),
(2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])/(-5*a*AppellF1[3/2, -p, -p,
 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-
4*a*b + c^2])] + p*x^2*((-c + Sqrt[-4*a*b + c^2])*AppellF1[5/2, 1
 - p, -p, 7/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c
 + Sqrt[-4*a*b + c^2])] - (c + Sqrt[-4*a*b + c^2])*AppellF1[5/2,
-p, 1 - p, 7/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-
c + Sqrt[-4*a*b + c^2])]))))/(3*((c - Sqrt[-4*a*b + c^2])/(2*b) +
 x^2)^p)
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)\left(bx^4+cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+c)*(b*x^4+c*x^2+a)^p,x)
```

```
[Out] int((e*x^2+c)*(b*x^4+c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)\left(bx^4+cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex^2+c\right)\left(bx^4+cx^2+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+c)*(b*x**4+c*x**2+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)\left(bx^4+cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p, x)
```

3.407 $\int (a + cx^2 + bx^4)^p \, dx$

Optimal. Leaf size=133

$$x\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)$$

```
[Out] (x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c
 - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1
 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt
[-4*a*b + c^2]))^p)
```

Rubi [A] time = 0.138497, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$x\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c
 - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1
 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt
[-4*a*b + c^2]))^p)
```

Rubi in Sympy [A] time = 30.3956, size = 112, normalized size = 0.84

$$x\left(\frac{2bx^2}{c-\sqrt{-4ab+c^2}}+1\right)^{-p}\left(\frac{2bx^2}{c+\sqrt{-4ab+c^2}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p \operatorname{appellf_1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+c*x**2+a)**p,x)
```

```
[Out] x*(2*b*x**2/(c - sqrt(-4*a*b + c**2)) + 1)**(-p)*(2*b*x**2/(c + s
qrt(-4*a*b + c**2)) + 1)**(-p)*(a + b*x**4 + c*x**2)**p*appellf1(
1/2, -p, -p, 3/2, -2*b*x**2/(c - sqrt(-4*a*b + c**2)), -2*b*x**2/
(c + sqrt(-4*a*b + c**2)))
```

Mathematica [B] time = 0.781785, size = 487, normalized size = 3.66

$$\frac{3\, 4^{-p-1}x\left(\sqrt{c^2-4ab}+c\right)\left(x^2\left(\sqrt{c^2-4ab}-c\right)-2a\right)^2\left(\frac{c-\sqrt{c^2-4ab}}{2b}+x^2\right)^{-p}\left(\frac{\sqrt{c^2-4ab}+c}{2b}+x^2\right)^{-p}\left(\frac{-\sqrt{c^2-4ab}+2bx^2+c}{b}\right)^{p+1}\left(\sqrt{c^2-\right.}{\left(\sqrt{c^2-4ab}-c\right)\left(px^2\left(\left(\sqrt{c^2-4ab}-c\right)F_1\left(\frac{3}{2};1-p,-p;\frac{5}{2};-\frac{2bx^2}{c+\sqrt{c^2-4ab}},\frac{2bx^2}{\sqrt{c^2-4ab}-c}\right)-\left(\sqrt{c^2-4ab}+c\right)F_1\left(\frac{3}{2};-p,1-p;\frac{5}{2}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (3*4^(-1 - p)*(c + Sqrt[-4*a*b + c^2])*x*((c - Sqrt[-4*a*b + c^2]
 + 2*b*x^2)/b)^(1 + p)*((c + Sqrt[-4*a*b + c^2] + 2*b*x^2)/b)^(-1
 + p)*(-2*a + (-c + Sqrt[-4*a*b + c^2])*x^2)^2*(a + c*x^2 + b*x^4
)^(-1 + p)*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + Sqrt[-4*a*b
 + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])/((-c + Sqrt[-4*a*
b + c^2])*((c - Sqrt[-4*a*b + c^2])/(2*b) + x^2)^p*((c + Sqrt[-4*
a*b + c^2])/(2*b) + x^2)^p*(-3*a*AppellF1[1/2, -p, -p, 3/2, (-2*b
*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2
])] + p*x^2*((-c + Sqrt[-4*a*b + c^2])*AppellF1[3/2, 1 - p, -p, 5
/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*
a*b + c^2])] - (c + Sqrt[-4*a*b + c^2])*AppellF1[3/2, -p, 1 - p,
5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4
*a*b + c^2])])))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \left(b x^{4}+c x^{2}+a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+c*x^2+a)^p,x)
```

```
[Out] int((b*x^4+c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^{4}+c x^{2}+a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + c*x^2 + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b x^{4}+c x^{2}+a\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + c*x^2 + a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + c*x^2 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+c*x**2+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+cx^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + c*x^2 + a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p, x)
```

3.408 $\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(\frac{\left(a+bx^4+cx^2\right)^p}{c+ex^2},x\right)$$

```
[Out] Unintegrable[(a + c*x^2 + b*x^4)^p/(c + e*x^2), x]
```

Rubi [A] time = 0.0305571, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\operatorname{Int}\left(\frac{\left(a+cx^2+bx^4\right)^p}{c+ex^2},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[(a + c*x^2 + b*x^4)^p/(c + e*x^2),x]

[Out] Defer[Int][(a + c*x^2 + b*x^4)^p/(c + e*x^2), x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+c*x**2+a)**p/(e*x**2+c),x)

[Out] Timed out
```

Mathematica [A] time = 0.0593028, size = 0, normalized size = 0.

$$\int \frac{\left(a+cx^2+bx^4\right)^p}{c+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2),x]

[Out] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2), x]
```

Maple [A] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+cx^2+a\right)^p}{ex^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+c*x^2+a)^p/(e*x^2+c),x)
```

```
[Out] int((b*x^4+c*x^2+a)^p/(e*x^2+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + cx^2 + a)^p}{ex^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^4 + cx^2 + a)^p}{ex^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + c*x^2 + a)^p/(e*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+c*x**2+a)**p/(e*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + cx^2 + a)^p}{ex^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c), x)
```

3.409 $\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(a+bx^4+cx^2)^p}{(c+ex^2)^2}, x\right)$$

```
[Out] Unintegrable[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2, x]
```

Rubi [A] time = 0.0283879, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\text{Int}\left(\frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}, x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2,x]

[Out] Defer[Int][(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2, x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+c*x**2+a)**p/(e*x**2+c)**2,x)

[Out] Timed out
```

Mathematica [A] time = 0.0892746, size = 0, normalized size = 0.

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2,x]

[Out] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2, x]
```

Maple [A] time = 0.077, size = 0, normalized size = 0.

$$\int \frac{(bx^4+cx^2+a)^p}{(ex^2+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x)
```

```
[Out] int((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+cx^2+a)^p}{(ex^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^4+cx^2+a)^p}{e^2x^4+2\,cex^2+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + c*x^2 + a)^p/(e^2*x^4 + 2*c*e*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+c*x**2+a)**p/(e*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+cx^2+a)^p}{(ex^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c)^2, x)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```